Entailment for Structured Specifications

Citation for published version:
Entailment for Structured Specifications (1988)

\[
\begin{array}{c}
SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \quad (\varphi_1, \ldots, \varphi_n) \vdash_{\Sigma, \Phi} SP \vdash \varphi \\
\Sigma, \Phi \vdash \varphi \in \Phi \\
SP_1 \vdash \varphi \quad SP_2 \vdash \varphi \\
SP_1 \cup SP_2 \vdash \varphi \\
SP \vdash \varphi \quad SP \vdash \sigma(\varphi) \\
SP \text{ with } \sigma \vdash \sigma(\varphi) \\
\end{array}
\]

Clarifications: INS = \langle \text{Sign}, \text{Sen} : \text{Sign} \to \text{Set}, \text{Mod} : \text{Sign}^{op} \to \text{Cat}, (\models_{\Sigma} \subseteq [\text{Mod}(\Sigma) \times \text{Sen}(\Sigma)]_{\Sigma \in \text{Sign}}) \rangle is an institution that defines the logical system used for specifications, SP, SP_1 and SP_2 are structured \Sigma-specifications over INS, where \Sigma is a signature in the category Sign, \varphi, \varphi_1, \ldots, \varphi_n are \Sigma-sentences, i.e., elements in Sen(\Sigma), \Phi is a set of \Sigma-sentences, and \sigma(\varphi) denotes Sen(\sigma)(\varphi), the translation of the sentence \varphi along \sigma : \Sigma \to \Sigma'. Structured specifications in INS are built from basic specifications \langle \Sigma, \Phi \rangle which are finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation ⟨|\models \subseteq |\text{Mod}(\Sigma)| \times |\text{Sen}(\Sigma)| \rangle_{\Sigma \in \text{Sign}} is a sound entailment relation for the satisfaction relation \langle |\models \rangle_{\Sigma \in \text{Sign}}.

The judgement \text{SP } \vdash \varphi \text{ is meant to capture the property that } \varphi \text{ is satisfied in all models of } \text{SP}.

History: The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

Remarks: The system is sound; completeness is shown in [3] for the first-order logic instance and in [5, 6] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation ⟨|\models \rangle_{\Sigma \in \text{Sign}}. [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [4] provides additional rules for observability operators.


