Entailment for Structured Specifications

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Entailment for Structured Specifications

\[
\begin{array}{c}
SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \quad (\varphi_1, \ldots, \varphi_n) \vdash_{\text{Sig}[SP]} \varphi \\
\hline
SP \vdash \varphi \\
(\Sigma, \Phi) \vdash \varphi \quad \varphi \in \Phi \\
SP_1 \vdash \varphi \\
SP_1 \cup SP_2 \vdash \varphi \quad SP_2 \vdash \varphi \\
SP \vdash \varphi \\
SP \vdash \sigma(\varphi) \\
\hline
\text{SP with } \sigma \vdash \sigma(\varphi) \\
\text{SP hide via } \sigma \vdash \varphi
\end{array}
\]

Clarifications: INS = \langle \text{Sign}, \text{Sen}\rangle : \text{Sign} \to \text{Set}, \text{Mod} : \text{Sign}^{\text{op}} \to \text{Cat}, \langle \models \Sigma \subseteq \text{Mod}(\Sigma) \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} is an institution that defines the logical system used for specifications. SP, SP_1 and SP_2 are structured \Sigma-specifications over INS, where \Sigma is a signature in the category Sign, \varphi, \varphi_1, \ldots, \varphi_n are \Sigma-sentences, i.e. elements in Sen(\Sigma). \Phi is a set of \Sigma-sentences, and \sigma(\varphi) denotes Sen(\sigma(\varphi)), the translation of the sentence \varphi along \sigma : \Sigma \to \Sigma'. Structured specifications in INS are built from basic specifications (\Sigma, \Phi), the union of \Sigma-specifications SP_1 \cup SP_2, the translation “SP with \sigma” of SP along a signature morphism \sigma : \Sigma \to \Sigma', and hiding “SP hide via \sigma" for hiding the symbols in SP not occurring in the image of \sigma : \Sigma' \to \Sigma. Sig[SP] is the signature of SP. Translations of \Sigma-sentences and \Sigma'-models along \sigma : \Sigma \to \Sigma' are required to preserve satisfaction: for any \varphi \in Sen(\Sigma) and M' \in \text{Mod}(\Sigma'), M' \models_{\Sigma'} Sen(\sigma(\varphi)) \Leftrightarrow \text{Mod}(\sigma(M')) \models_{\Sigma} \varphi. Finally, \langle \models \Sigma \subseteq \text{Pow}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} is a sound entailment relation for the satisfaction relation \langle \models \Sigma \rangle_{\Sigma \in \text{Sign}}.

The judgement SP \vdash \varphi is meant to capture the property that \varphi is satisfied in all models of SP.

History: The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [5], Sect. 9.2.

Remarks: The system is sound; completeness is shown in [3] for the first-order logic instance and in [5][6] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation \langle \models \Sigma \rangle_{\Sigma \in \text{Sign}}. [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [4] provides additional rules for observability operators.


