Entailment for Structured Specifications

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Entailment for Structured Specifications

\[
\begin{array}{c}
SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \\
\vdash \{\varphi_1, \ldots, \varphi_n\} \vdash_{\text{Sig}(SP)} \varphi \\
SP \vdash \varphi \\
\end{array}
\]

\[
\begin{array}{c}
\langle \Sigma, \Phi \rangle \vdash \varphi \quad \varphi \in \Phi \\
SP_1 \vdash \varphi \\
SP_1 \cup SP_2 \vdash \varphi \\
SP_2 \vdash \varphi \\
SP \vdash \sigma(\varphi) \\
SP \text{ hide via } \sigma(\varphi) \\
\end{array}
\]

**Clarifications:** INS = \langle Sign, Sen : Sign \to Set, Mod : Sign^{op} \to \text{Cat} \rangle \vdash \exists \Sigma \subseteq [\text{Mod}(\Sigma) \times \text{Sen}(\Sigma)]_{\Sigma \in \text{Sign}} is an institution that defines the logical system used for specifications, SP, SP_1 and SP_2 are structured \Sigma-specified over INS, where \Sigma is a signature in the category Sign. \varphi, \varphi_1, \ldots, \varphi_n are \Sigma-sentences, i.e. elements in Sen(\Sigma), \Phi is a set of \Sigma-sentences, and \sigma(\varphi) denotes \text{Sen}(\sigma)(\varphi), the translation of the sentence \varphi along \sigma : \Sigma \to \Sigma'. Structured specifications in INS are built from basic specifications \langle \Sigma, \Phi \rangle \vdash \varphi \subseteq |\Sigma| \vdash \text{Sen}(\sigma)(\varphi). SP functions \text{SP hide via } \sigma(\varphi)\) for hiding the symbols in SP not occurring in the image of \sigma : \Sigma \to \Sigma'. Sig(SP) is the signature of SP. Translations of \Sigma-sentences and \Sigma'-models along \sigma : \Sigma \to \Sigma' are required to preserve satisfaction: for any \varphi \in \text{Sen}(\Sigma) and M' \in [\text{Mod}(\Sigma')], M' \models_{\Sigma'} \text{Sen}(\sigma)(\varphi) \equiv \text{Mod}(\sigma)(M') \models_{\Sigma} \varphi. Finally, \langle \models_{\Sigma} \subseteq \text{Pow}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} is a sound entailment relation for the satisfaction relation \langle \models_{\Sigma} \subseteq \text{Sign} \rangle. The judgement SP \vdash \varphi is meant to capture the property that \varphi is satisfied in all models of SP.

**History:** The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

**Remarks:** The system is sound; completeness is shown in [3] for the first-order logic instance and in [5][6] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation \langle \models_{\Sigma} \subseteq \text{Sign} \rangle. [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [4] provides additional rules for observability operators.

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