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Mass Variance from Archival X-ray Properties of Dark Energy Survey Year-1 Galaxy Clusters


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ABSTRACT

Using archival X-ray observations and a log-normal population model, we estimate constraints on the intrinsic scatter in halo mass at fixed optical richness for a galaxy cluster sample identified in Dark Energy Survey Year-One (DES-Y1) data with the redMaPPer algorithm. We examine the scaling behavior of X-ray temperatures, $T_X$, with optical richness, $\lambda_{RM}$, for clusters in the redshift range $0.2 < z < 0.7$. X-ray temperatures are obtained from Chandra and XMM observations for 58 and 110 redMaPPer systems, respectively. Despite non-uniform sky coverage, the $T_X$ measurements are > 50% complete for clusters with $\lambda_{RM} > 130$. Regression analysis on the two samples produces consistent posterior scaling parameters, from which we derive a combined constraint on the residual scatter, $\sigma_{\lnT_X|\lambda} = 0.275 \pm 0.019$. Jointed with constraints for $T_X$ scaling with halo mass from the Weighing the Giants program and richness–temperature covariance estimates from the LoCuSS sample, we derive the richness-conditioned scatter in mass, $\sigma_{\lnM|\lambda} = 0.30 \pm 0.04_{\text{(stat)}} \pm 0.09_{\text{(sys)}}$, at an optical richness of approximately 100. Uncertainties in external parameters, particularly the slope and variance of the $T_X$–mass relation and the covariance of $T_X$ and $\lambda_{RM}$ at fixed mass, dominate the systematic error. The 95% confidence region from joint sample analysis is relatively broad, $\sigma_{\lnM|\lambda} \in [0.14, 0.55]$, or a factor ten in variance.

Key words: galaxies: clusters: general, X-rays: galaxies: clusters, galaxies: clusters: statistics

1 INTRODUCTION

Population statistics of galaxy clusters are acknowledged as a valuable probe of cosmological parameters (Allen et al. 2011; Weinberg et al. 2013; Huterer & Shafer 2018), as illustrated by analysis of modern cluster samples (e.g., Vikhlinin et al. 2009; Rozo et al. 2010; Benson et al. 2013; Mantz et al. 2014; de Haan et al. 2016; Bocquet et al. 2018), and anticipated from larger and deeper cluster samples being assembled. The Dark Energy Survey (DES, Dark Energy Survey Collaboration et al. 2016) is identifying clusters using color-based searches in five-band optical photometry. A small initial sample from the Science Verification survey phase, with 786
clusters, (Rykoff et al. 2016) is supplemented by a Year-1 (Y1) data sample containing ~ 7,000 clusters with 20 or more statistically galaxy members (McCintock et al. 2019).

The population statistics approach relies on comparing the number and spatial clustering of galaxy clusters, as a function of their observable properties and redshift, to theoretical expectations derived from simulations of dark matter halos, particularly the Halo Mass Function (HMF) (e.g., Jenkins et al. 2001; Evrard et al. 2002; Tinker et al. 2008; Murray et al. 2013). To connect halo and cluster properties, a probabilistic model commonly referred to as the mass–observable relation is employed to map host halo mass to multiple cluster observables. This paper focuses on the statistical relationships between optical and X-ray properties of a cluster and the underlying total mass of the halo hosting it.

Ensemble-averaging, or stacking, to estimate mean mass as a function of galaxy richness has been applied to the DES-Y1 cluster sample by McCintock et al. (2019). The process of stacking has the drawback that it integrates out the variance in halo mass, $M$, conditioned on galaxy richness, $\lambda$. A complementary inference technique is needed to determine the width and shape of the conditional probability distribution, $P(M | \lambda)$.

Both observations (Pratt et al. 2009; Reichert et al. 2011; Mardelli et al. 2013; Liu et al. 2016; Mantz et al. 2016a,b; Giles et al. 2017) and simulations (Evrard et al. 2008; Stanek et al. 2010; Farahi et al. 2018a) support a log-normal form for observable-mass conditional distributions. This form, coupled with a low-order polynomial approximation for the HMF, yields analytic expressions for the space density as a function of multiple observable properties as well as property-conditioned statistics of the massive halos hosting groups and clusters (Evrard et al. 2014, hereafter, E14). We employ this model in our analysis, with particular emphasis on conditional property covariance.

The red-sequence Matched-filter Probabilistic Percolation (redMaPPer) identifies clusters using an empirically-calibrated, matched-filter model for old, red galaxies (Rykoff et al. 2014). The algorithm outputs a probabilistic estimate of optical richness – the count of red galaxies inside a cluster – along with a mean cluster redshift and a set of up to five likely central galaxies. Previous studies found this photometric cluster-finder algorithm produces a highly complete and pure cluster sample with accurate redshift estimates (Rykoff et al. 2012; Rozo & Rykoff 2014; Rozo et al. 2015a,b).

The Sloan Digital Sky Survey (SDSS) DR-8 redMaPPer cluster sample (Rykoff et al. 2014) has recently been combined with ensemble-average weak lensing masses (Simet et al. 2017) to produce cosmological constraints (Costanzi et al. 2018). A similar analysis is underway for DES-Y1 (McCintock et al. 2019). In both of these works, marginalization over the weakly constrained scatter between mass and richness weakens posterior likelihoods of cosmological parameters. The aim of our work is to provide an empirical constraint on the mass-richness variance, a result that will be combined with other systematics calibration effort to refine and improve likelihood analysis of cluster counts for cosmology.

Integrated measures of clusters such as redMaPPer richness, $\lambda_{RM}$, X-ray temperature, $T_X$, and luminosity, $L_X$, are proxies for host halo mass in that each scales as a (typically) positive power of $M$. In general, each proxy has intrinsic variance generated by internal dynamics within halos, as well as extrinsic scatter caused by projection, measurement uncertainties and other effects. For the intrinsic component, the log-normal property covariance model of E14 provides expressions that link proxy properties to each other and to unobservable host halo mass. The expressions involve the local slope and curvature of the HMF because of the convolution required to map mass to the observed measures.

Here we study the scaling behavior of $T_X$ as a function of $\lambda_{RM}$ for a redMaPPer sample of clusters identified in DES-Y1 imaging data within the redshift range, $z \in [0.2, 0.7]$ (McCintock et al. 2019). X-ray properties of clusters contained in archival Chandra or XMM pointings are measured via the Mass Analysis Tool for Chandra (MATCha, Hollowood et al. 2018) or XCS data analysis pipelines (Giles et al. in preparation), respectively. We employ the Bayesian regression model of Kelly (2007) to estimate parameters of the conditional scaling, $Pr(T_X | \lambda_{RM})$.

The inference of mass scatter requires additional information, namely the $T_X-M_{500c}$ scaling relation and the $\lambda_{RM}-T_X$ covariance at fixed halo mass. These additional quantities are taken from previous studies (Mantz et al. 2016b; Farahi et al. 2019). Uncertainties on the inferred scatter are determined by marginalizing over uncertainties in the model priors.

The structure of this paper is as follows. In Section 2, we introduce the cluster sample and X-ray follow-up programs of the optically-selected clusters. In Section 3, we describe the regression algorithm and the population model employed to obtain an estimate of the mass–richness scatter, with results presented in Section 4. In Section 5, we discuss our treatment of systematic uncertainties. Finally, we conclude in Section 6. Appendix A provides the tables of cluster properties employed in this work. Appendix B provides corrections for a small number of richness measurements using the X-ray emission peak locations of Chandra and XMM observations. Finally in Appendices C and D, we present the richness–temperature correlation at fixed halo mass and upper limits on the running of temperature variance at fixed optical–richness, respectively.

We assume a flat ΛCDM cosmology with $\Omega_m = 0.3$ and $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Distances and masses, unless otherwise noted, are defined as physical quantities with this choice of cosmology, rather than in comoving coordinates. We denote the mass inside spheres around the cluster center as $M = M_{500c}$, corresponding to an overdensity of 500 times the critical matter density at the cluster redshift.

2 DES-Y1 DATA

This work is based on data obtained during the DES-Y1 observational season, between 31st August 2013 and 9th February 2014 (Drlica-Wagner et al. 2018). During this period 1,839 deg$^2$ was mapped out in three to four tilings using $g$, $r$, $i$, $z$ filters. This strategy produces a shallower survey depth compared to the full-depth Science Verification data, but it covers a significantly larger area. We use approximately 1,500 deg$^2$ of the main survey split into two contiguous areas, one overlapping the South Pole Telescope (SPT) Sunyaev-Zel’dovich Survey area, and the other overlapping the Stripe-82 (S82) deep field of SDSS. The sky footprint is illustrated in Fig. 1 of McCintock et al. (2019).

We first describe the main data products used in our analysis and refer the reader to corresponding papers for a more detailed overview. Imaging and galaxy catalogs associated with the redMaPPer catalog used here are publicly available$^1$ in the first DES data release (DES DR-1, Abbott et al. 2018) and the Y1A1 GOLD wide-area object catalog (Drlica-Wagner et al. 2018).

$^1$ https://des.ncsa.illinois.edu/releases/dr1
2.1 Optical cluster catalog

We employ a volume-limited sample of galaxy clusters detected in the DES-Y1 photometric data using version 6.4.17 of the redMaPPer cluster-finding algorithm (Rykoff et al. 2016). The redMaPPer algorithm identifies clusters of red-sequence galaxies in the multi-dimensional space of four-band magnitudes and sky position. Starting from an initial spectroscopic seed sample of galaxies, the algorithm iteratively fits a model for the local red-sequence. It then performs a matched filter step to find cluster candidates and assigns membership probabilities to potential members. Starting with an initial spectroscopic seed sample of galaxies, the algorithm iteratively fits a model for the local red-sequence. It then performs a matched filter step to find cluster candidates and assigns membership probabilities to potential members. Starting from an initial spectroscopic seed sample of galaxies, the algorithm iteratively fits a model for the local red-sequence. It then performs a matched filter step to find cluster candidates and assigns membership probabilities to potential members.

2.2 Supplemental X-ray catalogs

Table 1 summarizes contents of the XMM and Chandra samples employed in this work, and Fig. 1 shows the distribution of cluster samples as a function of their observables. The X-ray catalogs are provided in Appendix A and will be available from the online journal in machine-readable format. In the following, we detail how the redMaPPer clusters are matched to the X-ray sources identified in Chandra and XMM archival data, and how the X-ray properties of the matched sources are measured.

The two methods produce independent luminosity and temperature estimates, and we adjust the latter to remove the known spectral bias between the two X-ray telescopes (Schellenberger et al. 2015). The two catalogs have similar depth (median redshifts of 0.41) while differing in their coverage of halo mass scale, reflected in Table 1 by offsets in the median values of mass proxies. The median richness is 76 for Chandra, 38 for XMM, and the respective median X-ray temperatures are 7.45 and 4.41 keV. In terms of natural logarithms, these offsets are 0.48 and 0.52, respectively. Both samples have range of a factor of ten in both $\lambda_{RM}$ and $T_X$ dimensions.

We are concerned about the relation between the properties of the redMaPPer-selected cluster observables and its host halo. Therefore, we need to correct for the fraction of the mis-centered population. Instead of modeling the mis-centered population, we correct our cluster observables with an associated X-ray center and re-estimate the optical–richness at X-ray peak (see Appendix B for more detail). In the following, richness, $\lambda_{RM}$, implies the optical–richness assigned by the redMaPPer algorithm at the X-ray peak, unless otherwise mentioned.

<table>
<thead>
<tr>
<th>Source</th>
<th>$N_{sam}$</th>
<th>$z_{med}$</th>
<th>$\lambda_{med}$</th>
<th>$kT_{X,med}$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandra</td>
<td>58</td>
<td>0.41</td>
<td>76</td>
<td>7.45</td>
</tr>
<tr>
<td>XMM</td>
<td>110</td>
<td>0.41</td>
<td>38</td>
<td>4.41</td>
</tr>
</tbody>
</table>

2.2.1 Chandra-redMaPPer Catalogs

The analysis of Chandra observations was conducted with the MATCHa pipeline described in Hollowood et al. (2018). We briefly outline the steps here. Starting from the volume-limited, $\lambda \geq 20$ redMaPPer catalog, we analyze all archival Chandra data, public at the time of the analysis, which overlapped redMaPPer cluster positions. In brief, after standard data reduction and cleaning, we search for a significant X-ray cluster detection starting from the redMaPPer position and iteratively re-centering toward the X-ray peak using an initial 500 kpc aperture. If the cluster is X-ray detected (SNR > 5), a spectrum is extracted, and we attempt to fit for the X-ray temperature, $T_X$. An iterative process is employed to center, determine cluster temperature and luminosity in the same X-ray band, and estimate cluster radius based on the $T_X$ fit. To evaluate $T_X$, the metal abundance is fixed at 0.3 $Z_\odot$, using the model from Anders & Grevesse (1989). For clusters with sufficiently well-sampled data, the output of the MATCHa algorithm includes the centroid location, $L_X$, and $T_X$ within a series of apertures, 500 kpc, $r_{500}$, and core-cropped $r_{500}$. In this work, we only use core included $r_{500}$ $T_X$ values.

In addition, we estimate the X-ray emission peak position of each detected cluster for use in studying the redMaPPer centering distribution (Hollowood et al. 2018; Zhang et al. 2019). The peak is determined, after smoothing the point-source subtracted cluster image with a Gaussian of 50 kpc width, as the brightest pixel within 500 kpc of redMaPPer position. We perform a visual check and then remove clusters for which the position, source spectrum, or background spectrum were significantly affected by instrumental chip edges or where the identified X-ray cluster was a foreground or background cluster not matched to the redMaPPer cluster.

2.2.2 XCS-redMaPPer Catalogs

For the the XMM-redMaPPer analysis (Giles et al. in preparation), the redMaPPer sample is matched to all XMM ObsIDs (with usable EPIC science data) under the requirement that the redMaPPer position be within 13′ of the aim point of the ObsID. Next, the XMM observations were filtered based upon exposure time. The exposure time is determined within a radius of 5 pixels centred on the redMaPPer position, with the mean and median required to be $>3$ks and $>1.5$ks respectively. Here the mean is taken to be the exposure time averaged over the sum of each pixel, while the median refers to 50 percent of the pixels in the enclosed region. These cuts are applied to ensure the redMaPPer cluster of interest is within the XMM FOV and has a sufficiently long exposure time for reliable SNR and $T_X$ measurements.

X-ray sources for each ObsID were then detected using the XCS Automated Pipeline Algorithm (XAPA, Lloyd-Davies et al. 2011). At the position of the most likely central galaxy of each redMaPPer cluster, we match to all XAPA-defined extended sources within a comoving distance of 2 Mpc. Cutout DES and XMM images are then produced and visually examined to assign a XAPA source to the optical cluster. Through this process, the final XMM-redMaPPer sample contains 110 clusters.

The luminosities and temperatures for the XMM-redMaPPer sample are derived using the XCS Post Processing Pipeline (Lloyd-Davies et al. 2011), with updates presented in Giles et al. (in preparation). Cluster spectra are extracted and fit in the 0.3 – 7.9 keV band with an absorbed MeKαL model (Liedahl et al. 1995). The fits are performed using the xspec package (Arnaud 1996), with the metallicity fixed at 0.3 $Z_\odot$. Using an iterative procedure, spectra...
are extracted within $r_{2500}$. We estimate an initial temperature within
the XAPA source detection region (Lloyd-Davies et al. 2011), and
an initial $r_{2500}$ estimated from the $r_{2500}$–$kT$ relation of Arnaud et al.
(2005). A temperature is estimated within this $r_{2500}$, and hence an
updated $r_{2500}$ estimated as above. This process is iterated until
$r_{2500}$ converges to 10%. To assess the reliability of temperature esti-
mates, variance of the temperature is calculated for each iteration.
This involves generating a grid of 5-by-5 pixels and estimating the
temperature for each region. We assign a mean temperature to each
cluster which satisfies $\sigma(T_x) \langle T_x \rangle \leq 0.25$, where $\sigma$ is the standard
deviation and $\langle T_x \rangle$ is the mean of estimated temperatures. Similar
to the Chandra analysis, the peak is determined, after smoothing
the point-source subtracted cluster image with a Gaussian of 50 kpc
width, as the brightest pixel within 500 kpc of redMaPPer position.

The different X-ray detection method, combined with the
larger collecting area of XMM compared to the Chandra observa-
tory, produces an X-ray sample for XMM that both is larger and
extends to lower richness than the Chandra detections. We defer a
detailed analysis of the X-ray selection processes used here to fu-
ture work. Here, we first analyze each sample independently, com-
bining them after demonstrating consistency of posterior scaling
parameters.

2.2.3 X-ray temperature as primary mass proxy

While X-ray luminosities, $L_X$, are measured for a larger number
of clusters than are temperatures, the larger variance in non-core
excised $L_X$ (Fabian et al. 1994; Mantz et al. 2016b) and the com-
plexities of modeling the supplemental survey masks motivate the
choice of $T_X$ as the primary link to halo mass. If we assume each
archival data can be treated as an $L_X$ limited sample, then there
could be a secondary selection function. This secondary selection
is mainly a function of $L_X$. Due to the complexity of modeling this
secondary $L_X$-selection, we do not perform and report $L_X$–$M_{\text{gas}}$ re-
ation. As we will see, systematic uncertainties limit the precision
with which we can recover the scatter in underlying halo mass.

An important systematic effect that we address is the misalign-
ment of X-ray cluster temperatures derived from the instruments on
the Chandra and XMM observatories (Schellenberger et al. 2015).
Since we are particularly interested in population variance, it is im-
portant to align the $T_X$ measurements before performing a joint
sample regression. We use the calibration of Rykoff et al. (2016)
based on 41 SDSS redMaPPer-selected clusters,

$$\log_{10}(T_X^{\text{Chandra}}) = 1.0133 \log_{10}(T_X^{\text{XMM}}) + 0.1008, \quad (1)$$

with temperatures in units of keV. Rykoff et al. (2016) note that
the above relation is consistent with that of Schellenberger et al.
(2015). We employ the Chandra temperature scale in analysis be-
low.

Within our sample, there are < 20 clusters with both Chandra
and XMM temperatures. The calibration relation from these clusters
alone is consistent with that of Rykoff et al. (2016), but with larger
uncertainties.

$M_{\text{gas}}$ is another low-scatter mass proxy (Mulroy et al. 2019).
Currently, $M_{\text{gas}}$ measurement for these sets of clusters is unavail-
able. We are planning on employing $M_{\text{gas}}$ measurement as another
cluster mass proxy in a future work.

2.2.4 X-ray Completeness

The supplemental samples, with fewer than 200 clusters, are far
from complete relative to the full DES-Y1 redMaPPer population
of $\sim$ 7,000 clusters. The incompleteness is primarily due to the
limited sky coverage of the two observational archives to the depths
required to detect distant clusters.

If the X-ray signal-to-noise-ratio (SNR) is $> 5$, typically a
few hundreds of photons, there is enough signal to measure the
X-ray luminosity; but at least 1,000 photons are needed to get a re-
liable estimation of the $T_X$. Fewer counts leads to larger errors, but
not excluded from the sample. The variable depths of the archival
pointings produce a complex pattern of flux limits across the optical
sample. The observations also have different levels of background
noise, adding to the complexity of X-ray selection modeling. Thor-
ough synthetic observations (Bahé et al. 2012; ZuHone et al. 2014)
are needed to accurately model this selection function. We defer
such modeling to future work.

The archival nature of the follow-up also produces a mix of
previously known and newly detected X-ray systems. Fig. 2 shows the
fraction of redMaPPer clusters with $T_X$ measurements as a function

![Figure 1. Supplementary sample properties. Left and right panels show the richness and hot gas temperatures as a function of cluster redshift obtained from XMM and Chandra archival data. The optical-richness is re-measured at the location of the X-ray emission peak (see Section 2.2 for more detail). XMM temperatures are scaled to the Chandra according to equation 1. The error bars are 68% measurement errors.](https://example.com/figure1.png)
of redMaPPer richness and redshift. As may be expected, completeness is high for the largest clusters. The sample is more than half complete in X-ray temperature at high optical richness values, $\lambda_{\text{RM}} \gtrsim 130$. At lower richness, the completeness falls off, with more systems being found in the XMM archival analysis. We note in Appendix D that the posterior scaling parameters found in §4 are relatively insensitive to an imposed minimum richness threshold (see Fig. D), but the effects of X-ray selection may affect our estimates of variance at low richness, particularly at high redshift.

3 POPULATION STATISTICS

The observed richness and X-ray temperature reflect properties of its host halo, subject to additional contributions from projected line-of-sight structure and other source of noise. Costanzi et al. (2019) develop a probabilistic model that maps intrinsic richness, $\lambda_{\text{true}}$, to measured richness, $\lambda_{\text{RM}}$. Generically, projection both widens the variance in $\lambda_{\text{RM}}(M)$ and adds a moderate degree of skewness. We do not apply corrections for projection effects, taking instead an approach that assumes $\lambda_{\text{RM}}$, $T_{\text{X}}(M, z)$ is a bivariate log-normal.

The integrated stellar and gas mass fractions in halos extracted from recent hydrodynamic simulations follow a log-normal form, as validated at percent-level accuracy by Farahi et al. (2018a). This form is also supported by previous cosmological simulations (Evrard et al. 2008; Stanek et al. 2010; Truong et al. 2018). Below, we show that normalized residuals in the measured scaling relation are consistent with a log-normal form, supporting this choice for recent hydrodynamic simulations.

$\lambda_{\text{true}} = \lambda_{\text{RM}} \pm \sigma_{\lambda_{\text{RM}}}$

where $\sigma_{\lambda_{\text{RM}}}$ is the slope, $\lambda_{\text{med}} = 70$ is the median richness of the joint sample, $\sqrt{\frac{\lambda_{\text{med}}}{\pi}}$ is the logarithmic intercept at $z = 0$, and $E(z) \equiv H(z)/H_0$ is the evolution of the Hubble parameter.

We regress $T_{\text{X}}$ on $\lambda_{\text{RM}}$, rather than the other way around, because optical richness is the primary selection variable. Under the assumption that the X-ray temperature at fixed optical richness is either complete or randomly selected, explicit modeling of X-ray selection process is not required (see Kelly 2007, Section 5.1.1). We use the regression method of Kelly (2007), which returns posterior estimates of the slope and normalization along with the residual variance, $\sigma_{\ln T_{\text{X}}}^2$.

The redshift dependence of the normalization in equation (3) reflects a self-similar expectation, based on virial equilibrium, that $T \propto [E(z)M]^{2/3}$ (Kaiser 1991; Bryan & Norman 1998). For observations spanning a range of redshift, the quantity $E^{-2/3}(z)kT$ should be a closer reflection of halo mass, $M$, than temperature alone. Over the redshift interval of our analysis, the $E^{-2/3}(z)$ factor decreases modestly, from 0.94 at $z = 0.2$ to 0.77 at $z = 0.7$.

While the slope and scatter of scaling relations may be scale-dependent and/or evolving with redshift (e.g., Farahi et al. 2018a; Ebrahimipour et al. 2018), our data are not yet rich enough to model these effects. We crudely test richness dependence by splitting both samples into two non-overlapping samples, with different characteristic scales, at their pivot richness, and find no evidence of scale dependence in the posterior scaling relation parameters.

![Figure 2. X-ray completeness of the supplementary samples. Lines show the fractions of DES-Y1 redMaPPer clusters with both $L_x$ and $T_x$ measurements from Chandra (left) and XMM (right) archival data. Solid, dashed, and dash-dot lines denoting increasing redshift bins given in the legend. Here, we employ the redMaPPer original richness.](https://academic.oup.com/mnras/article-abstract/doi/10.1093/mnras/stz2689/5574403/10.1038/msb.2018.657)
The regression method of Kelly (2007) includes uncertainties associated with the independent variable, here $\ln \lambda_{\text{RM}}$, by assuming a mixture model in that variable. The number of mixture elements is a free parameter in the method. We use two components in our analysis, and have performed tests to demonstrate that our results are insensitive to this hyperparameter.

### 3.2 Mass scatter inference

In the E14 population model, the variance in temperature of a sample conditioned on the selection variable, $\lambda$, is set by the joint (X-ray-optical) selection mass variance scaled by the slope, $\alpha_{T1_M}$, of the temperature–mass relation,

$$\sigma_{\ln T}\sigma_{\ln M} = \alpha_{T1_M} \left[ \frac{\sigma_{\ln M}\sigma_{\ln T} - 2\alpha_{T1_M}\sigma_{\ln M\ln T}}{\sigma_{\ln M\ln T}} \right],$$  

(4).

where $\sigma_{\ln M}\sigma_{\ln T}$ is the variance in halo mass at fixed optical richness (and similarly for temperature) and $r_{\ln T}$ is the correlation coefficient between log-richness and log-temperature at fixed halo mass. We use the variance relationship, $\sigma_{\ln M\ln T} = \sigma_{\ln M}^2/\sigma_{\ln T}^2$, and assume that all parameters are constant with mass and redshift.

Rearranging the expression isolates what this study is after, the mass variance conditioned on optical richness,

$$\sigma_{\ln M}\sigma_{\ln T} = \left[ \frac{\sigma_{\ln T}^2 - (1 - \alpha_{T1_M})^2}{\sigma_{\ln M}} \right]^{1/2} + \alpha_{T1_M} r_{\ln T}. \quad (5)$$

We ignore local curvature in the mass function, but note that its effect is to reduce the mass variance amplitude in equation (5). This suggests that the upper limits we derive below are somewhat conservative.

If there is no property correlation, $r_{\ln T} = 0$, the above expression simplifies to

$$\sigma_{\ln M}\sigma_{\ln T} = \sigma_{\ln M}^2 \left( \frac{\sigma_{\ln T}^2}{\sigma_{\ln M}^2} - 1 \right). \quad (6)$$

Note that the first term inside the parentheses is guaranteed to be greater than one because, when $r_{ab} = 0$, the simple Euclidean condition

$$\sigma_{\sum a}^2 = \sigma_{\sum b}^2 + \frac{\sigma_{\sum b}^2 \sigma_{\sum c}^2}{\sigma_{\sum c}^2} \sigma_{\sum b}^2,$$

holds for any pair of properties $\{a, b\}$.

### 4 RESULTS

In this section, we present temperature-richness scaling parameters derived from *Chandra* and *XMM* data. Consistent posterior constraints are found, motivating a joint analysis. We then introduce additional priors on the missing elements of the residual mass variance conditioned on observed richness in § 4.2, and present the resulting constraints.

#### 4.1 redMaPPer richness – hot gas temperature relation

Figure 3 shows the $T_{\lambda_{\text{RM}}}$ relation for the *Chandra* and *XMM* data, and joint samples respectively, and best-fit parameters are listed in Table 2. The blue lines and shaded regions present best-fit and 68% confidence intervals for the mean log scaling, equation (3). In this regression, $\lambda_{\text{RM}}$ is remeasured at the location of the X-ray peak, for each cluster resulting in mostly small corrections and a small number of significant adjustments, as detailed in Appendix B.

We find consistent slopes of $0.56 \pm 0.09$ (*Chandra*) and $0.61 \pm 0.05$ (*XMM*). The *XMM* temperature normalization, expressed in the *Chandra* system via the adjustment of equation (1), is 4.88 $\pm$ 0.15 keV, roughly 1σ lower than the *Chandra* value of 5.23 $\pm$ 0.26. The *XMM* temperature normalization, before the adjustment of equation (1), is 3.82 $\pm$ 0.12.

Gray shaded regions show the residual scatter about the mean relation. There are a small number of outliers, particularly toward low values of $T_{\lambda_{\text{RM}}}$, or equivalently, a larger $\lambda_{\text{RM}}$ than expected given their $T_{\lambda_{\text{RM}}}$. Such systems are likely to have a boosted richness due to lower-mass halos along the line of sight that boost $\lambda_{\text{RM}}$ more than $T_{\lambda_{\text{RM}}}$.

We test the shape of Pr$(kT_{\lambda_{\text{RM}}}, z)$ by examining the normalized residuals of the data about the best-fit mean scaling,

$$\delta_{\lambda_{\text{RM}}} = \frac{\ln \left( E^{-2\ln(i_{\text{err}})kT_{\lambda_{\text{RM}}}} - \sigma_{\lambda_{\text{RM}}} \ln(i_{\text{err}}) - \pi_{\lambda_{\text{RM}}} \right)}{\sigma_{\lambda_{\text{RM}}} \sigma_{\ln(i_{\text{err}})}}^{1/2}. \quad (8)$$

using posterior maximum likelihood estimates of the parameters $\pi_{\lambda_{\text{RM}}}, \alpha_{\lambda_{\text{RM}}}$ and $\sigma_{\ln(i_{\text{err}})}$ and the index $i_{\text{err}}$ corresponds to the $i_{\text{th}}$ cluster. The quadratic inclusion of $\sigma_{\ln(i_{\text{err}})}$, the square of the measurement uncertainty in $\ln(i_{\text{err}})$, is appropriate if measurement errors are both accurately estimated and also uncorrelated with the underlying astrophysical processes responsible for the residual scatter.

Figure 4 shows quantile-quantile (Q-Q) plots of the residuals in both samples. The Q-Q plot compares the quantiles of the rank-ordered residuals, expressed in units of the measured standard deviation, equation (8), to those expected under the assumed Gaussian model. The Q-Q form of both samples support the log-normal likelihood, equation (2), as shown by the proximity of the measured quantiles to the dashed line of unity. There is a very slight skew in the distribution, with more weight to the low-temperature side as expected from projection effects (Cohn et al. 2007). In a previous work, Mantz et al. (2008) employed the Q-Q plot and illustrated the log-normality of their cluster sample (see their Fig. 4).

Within the statistical uncertainties, the slope of temperature-richness scaling is consistent with a simple self-similar expectation of $2/3$, the result obtained if the star formation efficiency is constant, so that $\lambda \propto M_{\text{star}} \propto M$, and the temperature scales as $T \propto [E(i_{\text{err}})M]^{2/3}$ from virial equilibrium (Kaiser 1991; Bryan & Norman 1998). However, both dynamical and weak lensing analysis of the same SDSS sample produce mean behavior $M \propto \lambda_{\text{RM}}^{1.3 \pm 0.1}$, which shows deviation from the self-similar expectation (Simet et al. 2017; Farahi et al. 2016), and the Weighing the Giants (WtG) analysis yielding $T \propto M^{0.62 \pm 0.04}$.

If $\lambda_{\text{RM}}$ is close to zero and there is no scatter about the mean relation, then it is expected that temperature scales with mass with slope $\alpha_{T1_M} = 0.81 \pm 0.10$. There is moderate tension with our result of $0.62 \pm 0.04$ for the joint sample, which we suspect reflects the

<table>
<thead>
<tr>
<th>Sample</th>
<th>Normalization $e^{\pi T_{\lambda_{\text{RM}}} R_{T}}$ [keV]</th>
<th>Slope $\alpha_{T1_{\lambda}}$</th>
<th>Residual scatter $\sigma_{\lambda_{\text{RM}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandra</td>
<td>$5.23 \pm 0.26$</td>
<td>$0.56 \pm 0.09$</td>
<td>$0.260 \pm 0.032$</td>
</tr>
<tr>
<td>XMM</td>
<td>$4.88 \pm 0.15$</td>
<td>$0.61 \pm 0.05$</td>
<td>$0.289 \pm 0.025$</td>
</tr>
<tr>
<td>Joint</td>
<td>$4.97 \pm 0.12$</td>
<td>$0.62 \pm 0.04$</td>
<td>$0.275 \pm 0.019$</td>
</tr>
</tbody>
</table>

Table 2. Best-fit parameters for the $T_{\lambda_{\text{RM}}}$ relation, equation (3).
Joint 0.15 < z < 0.3, with 33 overlapping the SDSS sample region. The redMaPPer richness estimates for those systems range from 27 to 181, with the median value near 100. Using the same model framework as this paper, modified to include the original X-ray selection criteria, Farahi et al. (2019) derive the first empirical constraint on the correlation coefficient, \( r_{\chi \lambda} = -0.25^{+0.24}_{-0.22} \).

With these additional elements, we can now derive an estimate of the richness-conditioned mass scatter, equation (5), resulting in \( \sigma_{\ln M|\chi} = 0.30 \pm 0.10 \). To get this result, we employ the values derived from the joint sample temperature variance conditioned on richness. Since the correlation coefficient is broad and slightly asymmetric (see Fig. C1), we use Monte Carlo sampling of the terms on the right-hand side of equation (5), discarding any combinations that produce unphysical results (negative values inside the square root). The resultant posterior distributions are shown in Fig. 5 for the Chandra, XMM, and joint analysis with values of \( \sigma_{\ln T|\chi} \) from Table 2.

For the joint sample analysis, the median value of the posterior mass scatter is

\[
\sigma_{\ln M|\chi} = 0.30 \pm 0.04 \text{ (stat)} \pm 0.09 \text{ (sys)},
\]

where the quoted uncertainties are 68% confidence level. The statistical error derives from the \( T_{\chi} - \lambda_{\text{RM}} \) residual variance uncertainty.

Figure 3. X-ray temperature–redMaPPer richness scaling behavior from the Chandra, XMM, and joint archival data samples (left to right) using a pivot richness of 70. XMM temperatures have been modified using equation (1) to align with Chandra estimates. In each panel, the blue line and blue shaded region are the best-fit and 68% confidence interval of the mean logarithmic relation, equation (3). Gray shaded regions show 1σ, 2σ, and 3σ residual scatter about the scaling relation. Fit parameters are given in Table 2. Richness errors are provided directly by the redMaPPer algorithm.

Figure 4. Normalized, ranked residuals, equation (8), of the Chandra (top, left), XMM (top, right), and joint (bottom) samples follow closely a log-normal form, as indicated by the close proximity of the points to the dashed line of equality.

Table 3. External constraints required for the richness-conditioned mass variance, equation (5). Uncertainties are 68% confidence intervals.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{\ln T/M} )</td>
<td>0.16 ± 0.02</td>
<td>Mantz et al. (2016b)</td>
</tr>
<tr>
<td>( \sigma_{T/M} )</td>
<td>0.62 ± 0.04</td>
<td>Mantz et al. (2016b)</td>
</tr>
<tr>
<td>( \sigma_{\ln M/T} )</td>
<td>0.26 ± 0.04</td>
<td>inferred from above</td>
</tr>
<tr>
<td>( r_{\chi T} )</td>
<td>-0.25^{+0.24}_{-0.22}</td>
<td>Farahi et al. (2019)</td>
</tr>
</tbody>
</table>

lack of low luminosity and temperature systems having low optical richness in the X-ray archives.
while the systematic uncertainty is derived from the quoted errors of the required external parameters. The overall 95% confidence region is broad, spanning 0.14 to 0.55.

5 DISCUSSION

Here we review our treatment of systematic uncertainties, including priors, before turning to a comparison with existing estimates of the mass scatter using optical proxies in both observed cluster samples and simulated halo ensembles.

5.1 Systematic uncertainties

The richness-conditioned mass variance is inferred from the observed temperature variance via equation (5). Uncertainties in the additional elements (see Table 3) propagate to broaden the uncertainty in $\sigma_{\ln M|\lambda}$.

Figure 6 explores the contribution of each element’s uncertainty by systematically setting the error in specific terms to zero — i.e., fixing the value of each element. The yellow curve fixes the $T_X-M_X$ relation parameters, both slope and scatter, while the red curve fixes the correlation coefficient, $r_{XT}$. Finally, the blue curve shows the impact of having perfect knowledge of all above parameters.

The temperature–mass relation uncertainties make the largest contribution to the uncertainty in mass variance. We note that the contribution of the temperature–mass uncertainty in the slope is negligible and it is dominated by the uncertainty in the scatter. The width of the blue curve, the statistical uncertainty, is 0.04.

5.2 Comparing to previous studies

Using a smaller set of redMaPPer clusters identified in science verification (DES-SV) data, Rykoff et al. (2016) estimated the slope and the scatter of $T_X-\lambda_{\text{RM}}$ scaling relation to be 0.60 ± 0.09 and 0.28 ± 0.03, respectively. Employing a set of rich redMaPPer clusters identified from SDSS DR8 data, Ge et al. (2019) also estimated the slope and the scatter of $T_X-\lambda_{\text{RM}}$ scaling relation to be 0.79 ± 0.06 and 0.24 ± 0.03, respectively. With a larger sample size, we find values in agreement with those of Rykoff et al. (2016) and Ge et al. (2019), but with smaller parameter uncertainties.

Our measurements can also be compared to those of Rozo & Rykoff (2014). That work set is similar in spirit to this study but differs in some key details. The correlation coefficient, $r_{XT}$, was set to zero, and the analysis did not propagate uncertainties in the $T_X-M$ relation. Employing sub-samples of X-ray selected clusters from the literature, including the XCS (Mehrtens et al. 2012), MCXC (Piffaretti et al. 2011), ACCEPT (Cavagnolo et al. 2009), and Mantz et al. (2010) cluster samples, they estimate $\sigma_{\ln M|\lambda} = 0.26$ ± 0.03, with the quoted uncertainty being entirely statistical. We central value is consistent with theirs, but a key step of our analysis is to more carefully revise uncertainties by incorporating a coherent multi-property model.

In a separate work, Rozo et al. (2015a) directly estimated the scatter in richness at fixed SZ-mass by comparing the redMaPPer catalog to the Planck SZ-selected cluster catalog (Planck Collaboration et al. 2014). They estimate $\sigma_{\ln M|\lambda} = 0.277$ ± 0.026, with the reported uncertainties again being purely statistical. We note that the SZ-masses are inferred from $Y_{SZ}$–$M$ relation, so covariance between $\lambda_{\text{RM}}$ and $Y_{SZ}$ needs to be taken into account in this analysis.

In an independent analysis using abundance and stacked weak lensing profiles for roughly 8,000 SDSS redMaPPer clusters with richness, $20 < \lambda_{\text{RM}} < 100$, and redshift, $0.1 < z < 0.33$, Murata et al. (2018) derive $\sigma_{\ln M|\lambda} \sim 0.46 \pm 0.05$ at a pivot mass scale of $3 \times 10^{14} h^{-1} M_\odot$, equivalent to a richness of 24, from their mean scaling relation. In their analysis, the scatter is allowed to run with mass, and they find that $\sigma_{\ln M|\lambda} \propto M^{-0.17 \pm 0.03}$. Evaluating their result at a richness of 70, or a mass scale roughly a factor of 4 larger, leads to a mass scatter of 0.36, consistent with our findings.

This work is concerned about mass scatter conditioned on optical-richness. To estimate the richness scatter at fixed halo mass, Saro et al. (2015) modeled the total richness variance conditioned on halo mass with a Poisson term and a log-normal scatter term. If this additional Poisson contribution, at pivot richness of joint sample $\lambda_{\text{RM}} \sim 72$, is subtracted from the total variance, the richness variance conditioned on the halo mass yields

$$\text{Var}(\ln M) \equiv \frac{\text{Var}(\lambda|\ln M)}{\sigma_{\ln \lambda}^2} = \exp(-\langle \ln M \lambda \rangle) + \sigma_{\ln \lambda}^2,$$  (10)
where \( \text{Var}(\mu \ln \lambda) = 0.093^{+0.082}_{-0.047} \) is the halo mass variance conditioned on optical-richness, and \( \sigma^2_{\mu \ln \lambda} = 1.356 \pm 0.052 \) is the slope of \( M-\lambda_{\text{RM}} \) relation (McClintock et al. 2019). Plugging these numbers into Eq. 10, we infer
\[
\sigma_{\ln \lambda M} = 0.20^{+0.10}_{-0.08}.
\] (11)

This result is consistent with what is previously found employing redMaPPer clusters from SDSS survey (Saro et al. 2015; Simet et al. 2017; Costanzi et al. 2018).

### 5.3 Redshift dependence

We find no evidence of the redshift evolution for the slope and the scatter of the \( T_X-\lambda_{\text{RM}} \) relation. We split the Chandra, the XMM, and the joint samples in half at \( z = 0.4 \). The \( T_X-\lambda_{\text{RM}} \) relation results are presented in Table 4.

Farahi et al. (2018a) studied the redshift evolution of integrated stellar mass – halo mass scaling relation employing the hydrodynamical simulations. They find a mild redshift evolution for both slope and the scatter of this relation. The statistical uncertainties of our sample are larger than the magnitude of the redshift evolution they noticed. Therefore, we cannot rule out or confirm such a small evolution using the current sample.

### 5.4 Effect of potential selection bias on \( \sigma_{\ln \lambda M} \)

A potential source of systematic uncertainty in our analysis is the lack of a selection model for the archival X-ray analysis. It is well-known that ignoring the sample selection can lead to biased estimates (Giles et al. 2017; Mulroy et al. 2019; Mantz 2019). The most conservative approach is to interpret the mass scatter of 0.1 to lower richness systems could be biased if there is strong running of the mass-richness scatter with scale. If the selection effect is not negligible, it does have to be very large, \( \sim 50\% \), in order to have a significant impact on our key result, i.e., \( \sigma_{\ln \lambda M} \). If we take a scenario in which \( \sigma_{\ln T_X^\lambda} \) is underestimated by 50\%, then our constraint on the mass scatter would be \( \sigma_{\ln M|\lambda} = 0.55^{+0.13}_{-0.12} \), a shift of about two \( \sigma \) uncertainty.

The most conservative approach is to interpret the mass scatter constraints we report as appropriate to richness \( \lambda_{\text{RM}} \geq 100 \), since the supplementary samples are, cumulatively, 50\% complete above this richness. Under Poisson statistics, the expected fractional scatter in richness at fixed mass would be 0.1 or less, which in turn implies minimum mass scatter of 0.1 \( \sigma_{\ln M|\lambda} \approx 0.13 \) for a mass–richness slope of 1.3 (Simet et al. 2017; Farahi et al. 2016). This value lies just outside the 2\( \sigma \) low tail of our posterior joint-sample constraints.

### 5.5 Sensitivity to \( T_X-M \) relation

Considering the size of systematic and statistical uncertainties, incorporating different \( T_X-M \) relation does not have a significant effect on the inferred \( \sigma_{\ln M|\lambda} \). We rely on \( T_X-M \) relation from the WtG program (Mantz et al. 2015, 2016b) to infer \( \sigma_{\ln M|\lambda} \). There are other estimates of \( T_X-M \) relation in the literature (Lieu et al. 2016; Farahi et al. 2018b; Mulroy et al. 2019), but the way that \( T_X \) is measured in this work is better aligned with the \( T_X \) measure of Mantz et al. (2015). For example, Lieu et al. (2016) and Farahi et al. (2018b) employ \( T_X \) measured within 300 kpc physical aperture while we use core included \( \lambda_{\text{WM}} \) values. For completeness, we can ask what the impact of employing a different \( T_X-M \) relation would be. If we use the \( M_{\text{WL}}-T_X \) relation reported in (Mulroy et al. 2019), we get \( \sigma_{\ln M|\lambda} = 0.32^{+0.21}_{-0.15} \). The scaling relation values by (Lieu et al. 2016) give \( \sigma_{\ln M|\lambda} = 0.20^{+0.17}_{-0.12} \). Both results are statistically consistent with our key result based on the WtG sample.

Our results should be interpreted with caution when the low temperature or low optical richness clusters population is concerned. The WtG sample (Mantz et al. 2015) mainly comprises clusters with \( T_X \leq 5 \text{[keV]} \). As a result, the inferred \( \sigma_{\ln M|\lambda} \) is mainly valid for the most massive systems. Generalization of this finding to clusters with low temperature should be taken with caution. A future direction is to study the X-ray and mass properties of systems with \( \lambda_{\text{RM}} \leq 70 \).

### 5.6 Application to DES cluster cosmology

The mass variance constraints we derive can inform priors for cluster cosmology studies. For the DES survey, the model that links observed cluster richness with halo mass (Costanzi et al. 2018) is more complex than the log-normal population model we apply here. In particular, Costanzi et al. (2019) develop an explicit model of projection that is a component of a hierarchical Bayes framework for \( \text{Pr}(\lambda_{\text{RM}}|M,\tau) \). The base of that framework is an intrinsic halo population variance frames as a Poisson distribution convolved with a Gaussian of width, \( \sigma_{\text{intra}} \).

However, Costanzi et al. (2018) find that a log-normal model for the intrinsic halo population gives cosmological constraints consistent with the Poisson plus Gaussian model, and posterior estimates of \( \text{Pr}(\lambda_{\text{RM}}|M,\tau) \) are found to be nearly log-normal. More work is needed to fully incorporate constraints of the type derived in this study into cosmological analysis pipelines.

---

**Table 4.** Best-fit parameters for the \( T_X-\lambda_{\text{RM}} \) relation. Samples are split into two non-overlapping subsets with \( z > 4 \) and \( z \leq 0.4 \). The notation is similar to Table 2.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Slope ( \sigma_{T_X^\lambda \lambda} )</th>
<th>Residual scatter ( \sigma_{\ln T_X^\lambda \lambda} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chandra ( z \leq 0.4 )</td>
<td>0.60 ± 0.13</td>
<td>0.30 ± 0.06</td>
</tr>
<tr>
<td>XMM ( z \leq 0.4 )</td>
<td>0.56 ± 0.07</td>
<td>0.27 ± 0.04</td>
</tr>
<tr>
<td>Joint ( z \leq 0.4 )</td>
<td>0.59 ± 0.05</td>
<td>0.27 ± 0.03</td>
</tr>
<tr>
<td>Chandra ( z &gt; 0.4 )</td>
<td>0.51 ± 0.15</td>
<td>0.25 ± 0.05</td>
</tr>
<tr>
<td>XMM ( z &gt; 0.4 )</td>
<td>0.65 ± 0.08</td>
<td>0.32 ± 0.04</td>
</tr>
<tr>
<td>Joint ( z &gt; 0.4 )</td>
<td>0.65 ± 0.06</td>
<td>0.29 ± 0.03</td>
</tr>
</tbody>
</table>
6 CONCLUSION

We use archival X-ray observations of 168 redMaPPer clusters identified in DES-Y1 imaging to place limits on the mass variance at fixed galaxy richness, a critical component of cluster cosmology analysis. The X-ray observables, $T_X$ and $L_X$, of galaxy clusters at redshifts $0.2 < z < 0.7$ falling within archival Chandra or XMM archival data are extracted via MATCHa and XAPA processing pipelines, respectively. We determine parameters of a power-law $T_X - \lambda_{RM}$ relation, particularly the residual scatter in the log of temperature conditioned on richness, and infer the halo mass scatter at fixed optical richness using a log-normal multi-property population model.

Given the modest sample size and the lack of a detailed X-ray selection model, we do not attempt to add scaling of the mass variance with cluster richness or redshift. The median redshift of both samples is 0.41 while the median richness is 76 for Chandra and 47 for the larger XMM sample. We infer residual scatter in temperature at fixed richness, $\sigma_{\ln(T_X)} = 0.26 \pm 0.03$ (Chandra) and $0.29 \pm 0.03$ (XMM). The moderately larger variance in the lower-richness XMM sample may be providing a hint of mass dependence. Larger samples and a model for archival X-ray selection are required to address this issue.

Constraining the mass scatter requires additional information: the slope and variance of the $T_X - M$ relation as well as the correlation between $\lambda_{RM}$ and $T_X$ at fixed halo mass. Incorporating values from the Weighing the Giants and LoCuSS samples, respectively, and using the richness-conditioned temperature variance from the combined sample, we derive the mass scatter parameter, $\sigma_{\ln M/RM} = 0.30 \pm 0.04^{\text{stat}} \pm 0.09^{\text{sys}}$.

Our joint X-ray sample mainly consists of optically rich clusters, $\lambda_{RM} \geq 100$, with cumulative completeness of about 50% complete, and the prior on $T_X-M$ relation is taken from the WtG program, where the sample mainly consists of hot clusters with $T_X \leq 5$ keV. Therefore, our results should be interpreted as the mass scatter constraint on clusters of richness $\lambda_{RM} \geq 100$. A generalization to lower mass systems should be done with care as more work is needed to model selection in the sparsely sampled low richness population.

The contribution of the external parameter uncertainties in these systematics to the overall uncertainty budget is considerable. Therefore, as we make progress to better understand the scaling relations of multi-wavelength observables, it is necessary to pay attention to the off-diagonal elements of the mass-conditioned property covariance matrix. Mantz et al. (2016a) pioneered empirical estimates of the full covariance matrix for X-ray observables and Farahi et al. (2019) take the lead in combining optical, X-ray and SZ observables in the LoCuSS sample. Improved understanding of the broad property covariance matrix behavior will allow us to improve the mass variance constraints from studies such as this.

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APPENDIX A: CLUSTER CATALOGS

In Table A1 and A2, we provide the optical and the X-ray properties of Chandra and XMM clusters, i.e. the data vector, employed in this work. The MEM_MATCH_ID is the redMaPPer Cluster Identification Number that shall be used to match the X-ray clusters to the original redMaPPer clusters (McClintock et al. 2019). The full original redMaPPer DES Y1A1 catalogs will be available at http://risa.stanford.edu/redmapper/ in FITS format. LAMBDA_CHISQ and LAMBDA_CHISQ (X-ray peak) are the original redMaPPer optical richness and the new richness assigned to each cluster at the location of the X-ray emission peak, respectively. XCS_NAME in Table A2 is the unique source identifier which could be used to match with the XCS source catalog (Giles et al. in preparation). The full X-ray catalogs will be available from the online journal in machine-readable formats.
The mass scale of the redMaPPer host halos is studied in redMaPPer-selected cluster observables and its host halo. Specifically, we estimate the center of the host halo content of galaxy clusters traces the gravitational potential sourced by the host halo. To assign a center, we assume the hot gas associated X-ray center. To assign a center, we assume the hot gas associated X-ray center.

Table A1. Chandra Clusters.

<table>
<thead>
<tr>
<th>MEM_MATCH_ID</th>
<th>zA</th>
<th>LAMBDA_CHISQ</th>
<th>LAMBDA_CHISQ (X-ray peak)</th>
<th>kT_x [keV]</th>
<th>obsid(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.310</td>
<td>195.07 ± 6.78</td>
<td>200.65 ± 6.90</td>
<td>10.90^{+0.84}_{-0.81}</td>
<td>9331,15099</td>
</tr>
<tr>
<td>3</td>
<td>0.424</td>
<td>174.46 ± 5.07</td>
<td>171.91 ± 4.49</td>
<td>7.39^{+0.41}_{-0.32}</td>
<td>13396,16355,17536</td>
</tr>
<tr>
<td>4</td>
<td>0.307</td>
<td>146.24 ± 4.04</td>
<td>144.10 ± 4.00</td>
<td>10.24^{+0.26}_{-0.26}</td>
<td>12260,16127,16282,16524,16525,16526</td>
</tr>
<tr>
<td>5</td>
<td>0.355</td>
<td>178.84 ± 8.71</td>
<td>188.40 ± 10.06</td>
<td>14.89^{+0.59}_{-0.55}</td>
<td>4966</td>
</tr>
<tr>
<td>6</td>
<td>0.373</td>
<td>139.18 ± 4.67</td>
<td>138.85 ± 4.72</td>
<td>7.88^{+1.08}_{-0.80}</td>
<td>13395</td>
</tr>
<tr>
<td>7</td>
<td>0.243</td>
<td>135.48 ± 5.08</td>
<td>136.44 ± 4.69</td>
<td>12.16^{+0.36}_{-0.92}</td>
<td>15097</td>
</tr>
<tr>
<td>10</td>
<td>0.330</td>
<td>141.08 ± 5.96</td>
<td>142.26 ± 6.28</td>
<td>9.48^{+0.73}_{-0.53}</td>
<td>11710,16285</td>
</tr>
<tr>
<td>12</td>
<td>0.534</td>
<td>160.33 ± 6.45</td>
<td>159.39 ± 6.29</td>
<td>7.51^{+1.04}_{-1.41}</td>
<td>13466</td>
</tr>
<tr>
<td>14</td>
<td>0.282</td>
<td>129.00 ± 4.36</td>
<td>132.86 ± 4.36</td>
<td>9.46^{+0.66}_{-0.43}</td>
<td>3248,11728</td>
</tr>
<tr>
<td>15</td>
<td>0.610</td>
<td>169.08 ± 5.77</td>
<td>165.92 ± 5.63</td>
<td>7.71^{+0.84}_{-0.55}</td>
<td>12264,13116,13117</td>
</tr>
<tr>
<td>16</td>
<td>0.289</td>
<td>132.62 ± 4.75</td>
<td>130.37 ± 4.73</td>
<td>6.25^{+2.52}_{-1.30}</td>
<td>17162,16271,17162</td>
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<tr>
<td>17</td>
<td>0.597</td>
<td>144.88 ± 5.51</td>
<td>152.04 ± 5.00</td>
<td>14.32^{+0.52}_{-0.52}</td>
<td>13401,16135,16545</td>
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<tr>
<td>19</td>
<td>0.421</td>
<td>127.98 ± 4.61</td>
<td>124.99 ± 4.26</td>
<td>11.83^{+2.25}_{-0.90}</td>
<td>12259</td>
</tr>
<tr>
<td>20</td>
<td>0.231</td>
<td>136.78 ± 7.18</td>
<td>135.36 ± 6.69</td>
<td>9.88^{+0.79}_{-0.66}</td>
<td>15108</td>
</tr>
<tr>
<td>21</td>
<td>0.350</td>
<td>139.94 ± 7.49</td>
<td>125.67 ± 5.83</td>
<td>5.86^{+0.83}_{-0.37}</td>
<td>17185</td>
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Table A2. XMM Clusters.

<table>
<thead>
<tr>
<th>MEM_MATCH_ID</th>
<th>zA</th>
<th>LAMBDA_CHISQ</th>
<th>LAMBDA_CHISQ (X-ray peak)</th>
<th>kT_x [keV]</th>
<th>XCS_NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.430</td>
<td>234.50 ± 7.52</td>
<td>238.88 ± 7.37</td>
<td>8.07^{+0.22}_{-0.21}</td>
<td>XMMXCSJ025417.8-585705.2</td>
</tr>
<tr>
<td>2</td>
<td>0.310</td>
<td>195.07 ± 6.78</td>
<td>198.50 ± 6.67</td>
<td>6.11^{+0.24}_{-0.14}</td>
<td>XMMXCSJ051636.6-543120.8</td>
</tr>
<tr>
<td>3</td>
<td>0.424</td>
<td>174.46 ± 5.07</td>
<td>171.91 ± 4.79</td>
<td>5.78^{+0.70}_{-0.59}</td>
<td>XMMXCSJ041114.1-481910.9</td>
</tr>
<tr>
<td>4</td>
<td>0.307</td>
<td>146.24 ± 4.04</td>
<td>149.23 ± 3.98</td>
<td>8.46^{+0.25}_{-0.24}</td>
<td>XMMXCSJ024529.3-530210.7</td>
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<tr>
<td>5</td>
<td>0.355</td>
<td>178.84 ± 8.71</td>
<td>190.51 ± 10.17</td>
<td>3.30^{+0.53}_{-0.56}</td>
<td>XMMXCSJ224857.4-443013.6</td>
</tr>
<tr>
<td>8</td>
<td>0.243</td>
<td>135.48 ± 5.08</td>
<td>135.74 ± 4.70</td>
<td>9.48^{+0.36}_{-0.34}</td>
<td>XMMXCSJ2133516.8-012600.0</td>
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<tr>
<td>10</td>
<td>0.330</td>
<td>141.08 ± 5.96</td>
<td>140.76 ± 5.94</td>
<td>7.06^{+0.78}_{-0.67}</td>
<td>XMMXCSJ213511.8-010258.0</td>
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<tr>
<td>14</td>
<td>0.282</td>
<td>129.00 ± 4.30</td>
<td>134.66 ± 4.43</td>
<td>7.45^{+0.24}_{-0.22}</td>
<td>XMMXCSJ233738.6-001614.5</td>
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<tr>
<td>15</td>
<td>0.610</td>
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<td>164.19 ± 5.68</td>
<td>7.29^{+0.76}_{-0.62}</td>
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<tr>
<td>17</td>
<td>0.597</td>
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<td>150.54 ± 4.93</td>
<td>12.11^{+0.07}_{-0.07}</td>
<td>XMMXCSJ234444.0-424314.2</td>
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<tr>
<td>19</td>
<td>0.421</td>
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<td>125.71 ± 4.58</td>
<td>8.31^{+0.24}_{-0.23}</td>
<td>XMMXCSJ043818.3-541916.5</td>
</tr>
<tr>
<td>20</td>
<td>0.231</td>
<td>136.78 ± 7.18</td>
<td>134.34 ± 6.71</td>
<td>8.05^{+0.48}_{-0.44}</td>
<td>XMMXCSJ023232.2-553504.7</td>
</tr>
<tr>
<td>24</td>
<td>0.494</td>
<td>126.99 ± 4.31</td>
<td>127.26 ± 4.33</td>
<td>5.70^{+1.17}_{-0.16}</td>
<td>XMMXCSJ024339.4-483338.3</td>
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<tr>
<td>25</td>
<td>0.427</td>
<td>130.39 ± 6.17</td>
<td>131.88 ± 6.31</td>
<td>5.26^{+0.71}_{-0.56}</td>
<td>XMMXCSJ213538.5-572616.6</td>
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<tr>
<td>26</td>
<td>0.450</td>
<td>138.53 ± 6.45</td>
<td>138.08 ± 6.31</td>
<td>6.41^{+0.26}_{-0.26}</td>
<td>XMMXCSJ304015.7-440153.0</td>
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</table>

APPENDIX B: X-RAY EMISSION PEAK CENTERING SENSITIVITY

We are concerned about the relation between the properties of the redMaPPer-selected cluster observables and its host halo. Therefore, we need to correct for the fraction of the mis-centered population. The mass scale of the redMaPPer host halos is studied in Mc- Clintock et al. (2019) by correcting for the mis-centered clusters. Instead of modeling, we correct our cluster observables with an associated X-ray center. To assign a center, we assume the hot gas content of galaxy clusters traces the gravitational potential sourced by the host halo. Specifically, we estimate the center of the host halo with the location of the X-ray emission peak. We run the redMaPPer algorithm and assign a new optical richness to each X-ray ex-
Figure C1. The marginalized posterior distribution for the $\ln(\lambda_{RM})$ and $\ln(T_X)$ correlation coefficient about fixed host halo mass employed in this work. This is taken from Farahi et al. (2019).

APPENDIX C: RICHNESS–TEMPERATURE COVARIANCE

Farahi et al. (2019) studied the full property covariance of ten observables, including redMaPPer richness and X-ray temperature, regressed on the weak-lensing mass. Their sample consist of a 41 X-ray luminosity selected, low-redshift clusters with weak-lensing mass measurement for each individual cluster. Figure C1 presents their marginalized posterior distribution for the $\ln(\lambda_{RM})$ and $\ln(T_X)$ correlation coefficient about weak-lensing mass, which is employed in this work. Clearly a strong positive and negative correlations are ruled out with a high statistical significance.

APPENDIX D: RUNNING OF VARIANCE WITH RICHNESS

We further study the change in the scatter parameter for a subset of clusters by progressively applying $\lambda_{RM} > \lambda_{cut}$ (Fig. D). We find that within the 68% statistical confidence intervals the estimated intrinsic scatter about the mean relation does not change. This implies that the bias caused by the X-ray analysis pipeline is negligible, or otherwise there is a miraculous running of the scatter that cancels the X-ray selection bias. Saying that, one should be cautious that a different subset of redMaPPer cluster sample, with a larger sample size or a different X-ray analysis pipeline, can have different characteristics.

Figure D1. Constraints on the scatter of the $T_X$–$\lambda_{RM}$ relation derived from subsamples of Chandra (blue, dashed line) and XMM (red solid line) data thresholded by redMaPPer richness, $\lambda_{RM} > \lambda_{cut}$. Shaded regions show 68% confidence intervals.