Trading constraints and illiquidity discounts

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Trading Constraint and Illiquidity Discount: An Empirical Investigation and Theoretical Extension

Wenxuan Hou and Sydney Howell[1]

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[1] Wenxuan and Sydney are from Manchester Business School, University of Manchester. This paper is based on one part of Hou's PhD dissertation. The authors are grateful to MBS faculty for helpful discussions and comments.
Contact Information: whou@dom01.mbs.ac.uk; Address correspondence to: Office 5.19, Manchester Business School, Booth Street West, Manchester, M15 6PB, United Kingdom.

Abstract

This paper empirically investigates the effects of a particular type of illiquidity on asset finding the average illiquidity discount is around 77%, and the transaction of restricted shares slightly decreases the price of otherwise identical freely-traded shares, by 0.61%. We build an empirical model to study how the discounts vary with firm-specific and transaction-specific variables finding that illiquidity discounts are positively related to the length of trading constraints, and volatility. In the end, we make a comparison of discounts from empirical and theoretical models (Longstaff 1995 and 2001) and propose some explanations for the difference between them.

1. Illiquidity and Trading Constraints

The concepts liquidity and illiquidity refer to the degree of ease and certainty with which assets can be converted into cash. There are extensive bodies of literature studying the illiquidity issue. Early studies studies defined illiquidity in terms of bid-ask spread and transaction costs. Examples include Glosten and Milgrom (1985), Amihud and Mendelson (1986) Constantinides (1986), Easley and O' Hara (1987), Glosten (1987), Glosten and Harris (1998), Stoll (1989), Davis and Norman (1990), Grossman and Laroque (1990), Dumas and Luciano (1991), Jouini and Kallal (1998), etc. Under this definition of illiquidity, an investor can still trade unlimited amounts whenever desired, albeit at some cost.

There is a more extreme case of illiquidity which we focus on in this paper. Due to some imposed contractual or legal constraints, certain shares are forbidden from trading for an extended period which may last for years, and may be of indefinite duration. The period of constraint duration is known as the constraint horizon. There are many important classes of assets restricted by such trading constraints. For example, in an IPO (Initial Public Offering), the purchaser gets the shares at a discount, but is not allowed to resell them immediately. Initial dominant shareholder sometime also promises not selling any shares in a certain period right after IPO.A CEO often receives stock as part of a compensation package, but he/she is usually not allowed to trade it very soon. In America, traded shares that are not registered in SEC (Securities and Exchange Commission) are not allowed to be resold within a two-year constraint horizon. “Letter stock” is a typical example used in past study of illiquid assets. The biggest scale of restricted shares in the world may be the restricted shares in China Stock Market, where more than two thirds of all shares are restricted from free trading.

Empirical evidence suggests that restricted assets, or assets showing illiquidity characteristics, are often valued at a large discount to comparable freely-traded assets. Discounts discussed in those studies are defined on the price differences between the price of freely-traded shares and price of restricted shares, expressed as percentages of the former. By summarising the evidence from eight different studies of restricted stock, spanning the 1966 to 1984 period, Pratt (1989) found the mean and median percentage discount range from 25.8% to 45.0%. This is consistent with a later study by Silber (1991), whose empirical results show that restricted stocks are sold at an average price discount of 33.75 percent, in a range from 12.7 percent to 84 percent and the amount of the discount varies with the firm and the issue characteristics.

Brenner, Eldor and Hauser (2001) are the first group of researchers to focus on the restricted options. They empirically examined the effect of trading constraints on option values rather than on its bid-ask spread by comparing the prices of restricted bank-issued option and freely-traded exchange-traded option in Israeli currency market. They document a 21% gap between the price of freely-traded options and restricted options on the exchange rate, and find that the difference cannot be arbitraged away, due to transactions costs and the risk that the exchange rate will change during the bidding process.

The results from theoretical studies are consistent with those findings. In early theoretical research, Mayers (1972,1973) and Brito (1977) shows that illiquidity discounts can occur in equilibrium models, and the size of the discount depends (inversely) on how closely the optimal portfolio strategy for an investor approximates the buy-and-hold. Longstaff (1995) derived analytical expressions for the upper bound of by using option-pricing theory. Longstaff (2001) investigated the influence of trading constraints on optimal portfolio choice and illiquidity discount of a portfolio with restricted shares and riskless asset relative to freely-traded shares with riskless asset.

Most recently Longstaff (2006) found in one equilibrium model that the existence of trading constraints affects not only the price of restricted assets, but also the price of freely-trade assets. Under liquidity constraints, the freely-traded share is priced at a premium, and restricted share is priced at a discount, relative to the intrinsic value which is identical to both of them. The approach of the end of the constraint horizon causes these two prices to converge. In empirical work, intrinsic value is not discussed in this way because it is not directly observable. In the present study we do not directly consider theoretical models of the liquidity discount relative to intrinsic value, but we borrow from these models the concept that price differentials may fall as a known or unknown constraint horizon emerges. We also borrow the convention of expressing discount as a percentage of the (observable) freely traded price, rather than of the unobservable intrinsic value, which is actually fundamental to the theory.

The objective of our study is to investigate empirically the effects of trading constraints along with other empirical correlates on the relative prices of fully traded and restricted shares in the China Stock Market. This market has the largest pool of restricted assets in the world, and it offers the special feature that the restricted shares and fully traded shares in the same company always have identical dividend and voting rights, and differ only in liquidity.

This paper is organised as following: data and methodology are introduced in the next section. We also introduce some background and features about the trading constraints in China Stock Market. Section three presents the statistical summary empirical investigation and analysis. Afterwards, we use a new benchmark NAVPS (Net Asset Value per Share) to study the price of restricted shares. In Section five, existing theoretical models are extended and applied to compare with empirical findings. Section six offers main finding and conclusion.

2. Data and Methodology

2.1 Background and Data

The China Stock Market provides a unique (and perhaps historically brief) opportunity for empirically investigating the effects of trading constraints on share prices. There are three categories of shares for every listed company in China: 1) State Shares, which are directly owned by Chinese government and effectively not freely traded; 2) Legal Person Shares, which are shares owned by corporations, including other shareholding companies, non-bank financial institutions (NBFIs) and State-owned-enterprises (SOEs) and are very infrequently bought and sold between these, by special permission; 3) Ordinary Shares, which can legally be held by private individuals, and are effectively continuously traded in the Shenzhen and Shanghai Exchanges.

The three categories of shares in each company enjoy identical voting rights and dividends per share, and the only difference is that the trading constraints are imposed on first two categories of shares in order to prevent losing state direct and indirect control of those companies. Although the trading constraints affect a majority of all shares, they are not expected to remain permanently in place. The trading constraint has uncertainty in the proportion of restricted shares that can be transferred or auctioned occasionally, and in the length of the constraint horizon, in the end of which all shares becomes freely-traded. The trading constraint has been progressively abolished by the process of so-called Division Reform started in August 2005 and expected to mainly finish in 2010. Trading constraints are not imposed on companies newly listed after the launch of Division Reform. The effects of releasing existing trading constraints will be discussed in our future paper in detail.

As figure 1 shows, restricted shares account for around two third of total number shares in the markets. Though the restricted shares are forbidden from free trading, they can change hands (only between SOEs and agencies of government), after approval from administrative commissions. A transaction in State Shares has normally taken the form of a private placement, whereas a transaction in Legal Person Shares is normally in the form of auction. Those occasional transactions provide us a chance to observe the price of restricted shares. In this paper, we focus only on restricted shares traded in auction, since these transactions are more numerous, more transparent, and hypothetically more free of unobservable private intra- and inter-corporate agendas.

The data covering 4084 valid auctions have been collected from three sources: the Centre of China Economic Research Services (CCER), Chinese Security Regulation Committee (CSRC) and DataStream. They coverall auctions of restricted shares in China Stock Market history (with varying accuracy and completeness) and the potential sample is finite, because auction, as a way of transferring restricted shares, was forbidden as soon as Division Reform was launched in 2005.

In the earliest version of this paper, we used DataStream as the source of the prices of freely-traded shares, for comparison with the auction process of restricted shares. However the resulting estimates of price discounts had very erratic distributions. On checking carefully, we found many errors in the price of freely-traded shares given by DataStream; hence we decided to use the prices of freely-traded shares from CCER, although the CCER sample has 1000 missed values compared with DataStream.

2.2. Methodology

The main hypothesis of our study is that restricted shares are priced at a discount relative to their freely-traded counterparty, but the goal was to establish the size of the discount and to look for correlates with the size of the discount, using candidate variables drawn from theoretical or previous empirical work.

In order to be consistent with the theoretical model by Longstaff (1995, 2001, 2006), and us, we use the discount D_t, rather than relative price as the dependent variable.

$$D_t = \frac{(FP_t - RP_t)}{FP_t} \quad (1)$$

where FP_t the price of is freely-traded shares and RP_t is their restricted counterparty.

In selecting the potential empirical descriptive variables, we incorporate those in the empirical model by Silber (1991). There are two categories of variables; firm specific and transaction specific variables.

Firm-specific Variables

P/B (Price of freely-traded share to Book Value) gives idea about whether investors pay too much for what would be left if the company went bankrupt immediately. A lower P/B ratio could mean that the stock is undervalued or is fundamentally wrong.

P/E (Price of freely-traded share to Earning) is popular measure of share’s valuation. The higher the ratio, the more favourable the share is. A high P/E Ratio reflects investors' expectation of a higher earning growth in the future compared to companies with a lower P/E.

FR: The number of freely-traded shares divided by the total number of shares of the listed firm. It is a proxy of the marketability of firm's shares.

lnMC: The natural logarithm of the market capitalisation of the freely-traded shares listed in exchange. It measures a company's total value of all A-shares[2].

Volatility: It refers to the amount of risk about the change of a share's value.

NAVPS: Net Asset Value per Shares is an indicator giving an estimate of the value of a share after all assets are sold and all liabilities are paid off. It is usually below the market price in that the current market value of the shares is higher than the value appearing on the historical financial statements used in calculation of NAVPS.

Dividend (Dividend Yield): Dividend yield shows how much a company pays out in dividends each year relative to its share price. It is a way to measure how much cash flow investors get from dividends.

Age: The number of years since the company gets listed.

Exchange (Dummy Variable): 1 if the firm listed in Shanghai Stock Exchange; 0 if the firm listed in Shenzhen Stock Exchange.

SEO (Dummy Variable): 1 if the firm is a state-owned enterprise; 0 otherwise

Profit-Status (Dummy Variable): 1 if the firm have normal profit status; 0 is the firm bear loss in past 3 years and has been put ST (special treated) in front of the firm by Exchange.

Transaction-specific Variables

RTT: the ratio of the number of restricted shares in the transaction relative to the total number of shares of the listed company.

SqRTT: the square of ARRT.

RTR: the ratio of the number of restricted shares in the transaction relative to the total number of restricted shares of the listed company.

T: The number of years till trading constraints in China Stock Market is released. *T* is an ex post value in that it was unknown to all investors even to the government when

[2] Freely-traded shares priced in Renminbi (Yuan) are A-shares; Freely-traded shares priced in Dollars (in Shanghai Exchange) and Hong Kong Dollars (in Shenzhen Exchange) are B-shares. B-shares used to be only open for foreigner before Sep. 2001.

those transactions occurred (because those transactions only happened before Division Reform, until which the *T* was specified by government). We assume investor has a more or less accurate expectation of it i.e. it equals the length (in years) between the transaction day and 2010 when Division Reform finishes.

3. Empirical Findings

3.1 Descriptive Statistics

Table 1 shows the mean transaction size of restricted shares in auction is 1,165,013 shares per transaction. *RTT*, the number of restricted shares auctioned relative to the total number of shares in the listed firm is 0.61% indicating that only a small proportion of shares involved and auctions are unlikely to lead to changes in the control of a listed firm. *RTR*, the number of restricted shares auctioned, relative to the number of restricted shares in the listed firm is also a small fraction, 1.00% only showing the permission of occasional auction is only for a small proportion of restricted shares. The medians of *AT* and *AR* are even smaller - both below 0.05%. The trading constraint is not absolute but still very strict.

Table 2 displays the overall descriptive statistics of the illiquidity discount observed in the auction. The AD' column is the illiquidity discount in auction is calculated based on the price of freely-traded shares the day before the auction.

$$AD' = \frac{FP_{t-1} - RP_t}{FP_{t-1}} \qquad (2)$$

The AD column is calculated based on the closing price on the announcement day.

$$AD = \frac{FP_t - RP_t}{FP_t} \qquad (3)$$

The *Difference* column shows the difference between those two discounts and calculated by:

$$AD - AD' = \frac{RP_t}{FP_{t-1}} - \frac{RP_t}{FP_t} \qquad (4)$$

If $AD - AD' > 0$, then $FP_{t-1} < FP_t$ implying the price of the freely-traded share increases on the auction of its restricted counterpart.
If $AD - AD' < 0$, then $FP_{t-1} > FP_t$ implying the price of the freely-traded share decrease on the auction of its restricted counterpart.

The average discounts one day before and after auction are 78.13% and 77.18%. Those three medians are all greater than corresponding means. The *Difference* is 0.62% indicating an auction of restricted shares slightly decreases the price of their freely-traded counterparts. Although after one occasional transaction, trading constraints still exist, investor may regard those transactions as an implicit signal of releasing trading constraint. Such decrease reflects their fear of releasing trading constraint dilutes the share price of freely-traded share.

3.2 Regression Model

Table 3 shows that the percentage discounts from one day before the auction are strongly correlated with all chosen variables; expect the volatility which is a proxy of risk. Discounts from the auction day are strongly correlated with all variables and have exactly the same signs with figures on their left column.

Negative correlations are found between discounts and *Exchange* identity of Shanghai, *SOE*, *Profit*, *Ln-MC*, *NAVPS*, *Dividend*, and *Age* showing that SOEs, large firms, firms listed in Shanghai, firms with normal profit status are associated with lower discount. Companies with higher NAVPS and dividend yield offer smaller discount. On the contrary, the discounts vary positively with *FR*, *RTT*, *RTR*, *P/B*, *P/E* and *T* showing restricted shares from companies with bigger proportion of freely-traded shares is associated with bigger discount. Growth companies offer smaller discount. In addition, with the approach to the Division Reform, the illiquidity discounts become small.

The third column shows the difference between discounts from one day before the auction and from the auction day. This negatively varies with *Exchange*, *Profit*, *lnMC*, *NAVPS*, *Age* and *T* indicating that transaction of restricted shares from companies listed in Shanghai, with normal profit, with bigger market capitalisation, with higher net asset value per share, or with bigger age tend to decrease the price of their liquid counterparts.

The table 4 shows for both discounts from the day before announcement and those from the auction day, nine explanatory variables left after the filtering steps of stepwise method used to build our regression model. The intercepts and all significant variables from 1st and 2nd column are with same signs. This fact is not surprising in that those two discounts are close.

Discounts from company listed in Shanghai Exchange are 3% smaller than discounts from those listed in Shenzhen Exchange. The reason might be caused by the characteristics of those two exchanges: Companies listed in Shanghai normally with greater market capitalisation. Natural logarithm of market capitalisation is an explanatory variable in our model. The sign of its coefficient is consistent with the sign of *Exchange*. It is possible that both variable measures same characteristics of companies. Moreover, transactions volume in Shanghai Exchange is greater. Volume is a proxy of liquidity although different from the type of illiquidity we address in this paper. High trading volume is associated with price premium and consequently larger discount. If this is the case, the result contradicts to the finding. A possible explanation could be investors believe those two types of illiquidity are linked in a positive way, so they offer a higher price on those restricted shares whose liquid counterparts enjoy higher degree of liquidity in terms of high trading volume.

The sign of *FR* is not as we expect: If the company has a higher proportion of freely-traded shares, when its restricted shares become freely-traded, the freely counterparts are under less pressure of dilution. A possible explanation is that, normally companies with high *FR* are small in market capitalisation. *FR* and *RTR* does not significant showing that the proportion of shares involved in the transaction does not matter. *Sq-RTT* is an explanatory variable to capture the effects that the transaction price positively varies with the proportion until when the proportion reach a level that makes the purchaser obtain the control of this company. The effect does not appear significantly in the sample. The possible reason is that as showed in Table 1 proportion involved in auction is generally too small to lead change in control.

B/P has a small negative influence on the discount showing that discounts from growth companies are smaller. *NAVPS* and *Dividend* both have negative coefficients indicating that company with one more unit of *NAVPS* lead 1.66% smaller discount and company with one unit increase of dividend yield lead 1.58% decrease in discount. With every one year aged, the discount decreased by 1.80%.

Volatility is an important variable in theoretical model. For a single asset, the level of volatility determines the opportunity cost for being restricted; for portfolio, the level of volatility determines the optimal portfolio strategy and consequently the discount. As we expected, higher volatility lead to larger discount. *T* is another variable in all theoretical models to capture the main characteristics of the trading constraints. Consistent with theoretical model, it has a positive influence on the discount. With a bigger *T*, the investors lose more in that they cannot rebalance the portfolio to take advantage of the price increase and prevent deep loss in decrease.

The third column shows that auctions of restricted shares from older companies and those with bigger proportion of freely-traded shares raise the price of freely-traded shares, whereas auction of restricted shares from companies with high *NAVPS* decrease the price. Moreover, with approach of Division Reform, the influence of auction on freely-traded shares decreases. In general however, beyond these small effects, the market reacted little to the announcements of auctions. This may reflect the small fraction of shares sold in the auctions, which might have had little economic effect at all, but it may also reflect efficient pricing of the shares.

4. An Alternative Measurement

In the China Stock Market, a benchmark often used to estimate the lower bound on the value of restricted shares is Net Asset Value per Share. This measure (although lacking any obvious economic foundation) is frequently discussed in the press and by regulatory commission who approve occasional transaction of restricted shares. We therefore tested NAVPS as a possible metric for comparing the relative prices of restricted shares and freely-traded shares. Table 5 shows that freely-traded shares are priced at a premium of 1000.39% relative to NAVPS. The premiums from restricted shares are much smaller: only 126.73% indicating that restricted shares are priced one fourth greater than their NAVPS. The difference between them is huge, namely 877.46% showing that due to imposed trading constraints, the premium from restricted shares is around one ninth of that from freely-traded shares. This in effect confirms the finding from other sources that the discount (unweighted by size of transaction) is a large fraction of the price of the fully traded shares.

Table 6 shows the regression model. Difference from the model based on discount, all dummy variables here are significant, two key variables (*T* and *volatility*) in theoretical models are not significant, and RTT becomes significant. R^2, smaller than R^2 from the model from the previous measurement 0.359, is only 0.225. We may conclude that, although NAVPS is often referred as the benchmark to evaluate restricted shares, it is unlikely investors and regulatory use discount as a more directly benchmark as a way of measurement.

5. Comparison with Theoretical Results.

5.1 Upper Bound of Illiquidity Discount

Longstaff (1995ab) presents a simple analytical upper bound for the illiquidity discount. Consider there is only one risky asset with price of S_t traded in the economy with dynamics:

$$DS = \mu Sdt + \sigma dZ \quad (5)$$

where μ and σ are constants and Z is a standard Brownian motion.

Assume an investor endowed with one unit of asset who is planning to maximise wealth at some time horizon *T*. A hypothetical investor with perfect market timing ability can sell the asset and reinvest in the riskless asset so as to maximise the wealth at time *T*, calling this U_T where

$$U_T = \max_{0 \le t \le T} (e^{r(T-t)} S_t) \quad (6)$$

where *t* could be any time between 0 and *T*, *r* is the interest rate. As showed in the function (6), the investor is only allowed to change his position once.

However, if during the period from time 0 to *T*, a trading constraint is imposed on this investor, he cannot change his position at all and must receive cash flow worth S_T at the end of constraint horizon *T*. The difference between U_T and S_T is the incremental cash flow the investor would receive if trading constraint is released i.e. the upper bound on the value of being unrestricted. Its present value at time 0 is

$$F(S,T) = e^{-rT} E[U_T] - e^{-rT} E[S_T] \qquad (7)$$

where S is the value of the asset at t_0 and r is interest rate.

By using the density function for the maximum of a Brownian motion over an interval of length *T* (Harrision 1985), the close-form solution for the upper bound is obtained:

$$F(S,T) = S\left(2 + \frac{\sigma(s)^2 T}{2}\right) N\left(\frac{\sqrt{\sigma^2 T}}{2}\right) + S\sqrt{\frac{\sigma(s)^2 T}{2\pi}} \exp\left(-\frac{\sigma(s)^2 T}{T}\right) - S \qquad (8)$$

where $N(\cdot)$ is the cumulative normal distribution function. $\sigma(s)$ is the volatility of share price.

Table (7) and figure (2) show the above upper bound *F(S,T)* of the illiquidity discount for different values of volatility and *T* They show that the illiquidity discount is positively related to the length of the trading constraint and the volatility of *S*. The directions of these relationships as agree with our empirical models of illiquidity discounts.

The size of discounts from empirical and theoretical results could not be directly compared directly in that the input parameter *T* was not explicit for the restricted shares in China until Oct 2005 when the government announces the Division Reform a process to release all trading constraints[3], a process lasts around 5 years to 2010. We did not conduct comprehensive survey about investors' expected *T* in China Stock Markets before 2005. However, if we assume that investors did correctly forecast *T* where *T=2010-t* i.e. $E[T] \approx T$, what we can conclude is that illiquidity discount observed from the data of restricted shares in China stock market is within the range of upper bound suggested by the above theoretical model (Longstaff 1995).

The gap between empirical discount and upper bound is still greater than the gap between US empirical discount for letter stocks and the Longstaff (1995) upper bound. The US gap is so small that Longstaff (1995) argues that his upper bound is tight, and actually provides useful an approximation to the empirical value of marketability for US letter stocks. It is not surprising because the hypothetical investor in the model does not have such perfect timing ability which can make him sell the share at the highest price over *T*, but rationality to trade. Moreover, the investor is only allow to trade freely-traded shares once, particularly from the position of risky asset to riskless asset making the investor cannot make as much money as he could make if he can change position freely between those two types of assets and can borrow money. Those two assumptions limited the potential of wealth growth during from 0 to *T* and make the obtained upper bound close to the empirical discount.

For our sample, however, the *T* for restricted shares in China is much longer than *T* for letter stocks. With a 10-year constraint horizon *T*, shares with volatility greater than 0.3 have a Longstaff (1995) upper bound on their illiquidity discount close to 100% i.e. the price of a restricted share is close to 0. The empirical discount is far smaller. For a large *T*, the assumption of the model that assuming volatility is constant is less likely to be fulfilled; hence, the obtained upper bound is questionable. This may also because the holders of restricted shares are mostly agencies of the Chinese government whose investment horizon is presumably long, and they were obliged to continue holding the shares in state owned enterprises, in order to allow the Chinese government to continue to control these SOEs. Transfers of restricted shares between Chinese agencies at prices close to zero were rare, and may represent bureaucratic transfers of power between agencies, rather than either rational trading as the

[3] Our next paper addresses this issue in detail.

Longstaff upper bound might suggest or the abuse of economic power. Moreover, different from the scale of letter stocks, the scale of restricted shares in China is in big scale accounting for around one third of total number shares in the market making the alternatives of investment are limited, hence investors have to keep holding restricted shares.

As an alternative to assume $E[T] \approx T$ *=2010-t*, we can assume that the empirically observed discount was roughly correct as the case of letter stocks, but investors held different expectations about *E*[*T*]. In this case, we can calculate the implied *E*[*T*] from the observed value of *F(S,T)* in equation (8). The implied *E*[*T*] is the length of trading constraint required to equate the Longstaff (1995) upper bound to the illiquidity discount seen in empirical data. In this case, because our observed discount is around 77%, the implied *T* is 6 to 7 year, with variance 0.3. This implied *T* is shorter than real *T*, hence it is possible that when Chinese agencies were trading with each other before 2005, they may have been acting as rational profit maximisers, but were too optimistic about how soon trading constraints would be released.

5.2 Optimal Portfolio Choice and Illiquidity Discount

Different from the Longstaff (1995) framework in which investor can only change position once on freely-traded asset and in which vitality is constant, a continuous-time trading framework involving two assets with stochastic vitality in the economy is proposed by Longstaff (2001). The first asset is a riskless money market account with price $B(t)$ and interest rate $r(t)=0$. The second asset is risky with instantaneous stochastic volatility of returns follows the dynamic process

$$dV(t) = \sigma V(t) dZ_1(t) \qquad (9)$$

where σ is a constant and $Z_1(t)$ is a standard Brownian motion.

The second asset has price dynamics given by:

$$dV(t) = (\mu + \lambda V^2(t)) S(t) dt + V(t) S(t) dZ_2(t) \qquad (10)$$

Where μ and λ are constants, $V(t)$ is the instantaneous volatility of returns, and $Z_2(t)$ is a standard Brownian motion independent of $Z_1(t)$.

The investor's wealth at time t is given by

$$W(t) = N(t)S(t) + M(t) \qquad (11)$$

Where $N(t)$ and $M(t)$ are the number of risky and riskless asset hold by the investor. The wealth dynamics are expressed as:

$$dW(t) = (\mu + \lambda V^2(t))w(t)W(t)dt + V(t)w(t)W(t)dZ(t) \qquad (12)$$

Where the portfolio weight $w(t) = \dfrac{N(t)S(t)}{W(t)}$

The derived utility of wealth $J(W,V,t)$ is expressed as:

$$J(W,V,t) = \max_{w(t)} E[\ln W(T)] \qquad (13)$$

Optimal portfolio weight is given by

$$w^*(t) = \frac{\mu + \lambda V^2(t)}{V^2(t)} \qquad (14)$$

The utility of terminal wealth is expressed as:

$$\begin{aligned} J(W,V,t) &= \max_{w(t)} E[\ln W(T)] \\ &= \ln W(t) + \lambda\mu(T-t) + \frac{\mu^2}{6\sigma^2}\frac{1}{V^2(t)}(e^{3\sigma^2(T-t)} - 1) + \frac{\lambda^2}{2\sigma^2}V^2(t)(e^{\sigma^2(T-t)} - 1) \end{aligned} \qquad (15)$$

Equation (14) and (15) provide a complete solution to the investor's portfolio choice problem in this stochastic volatility framework without trading constraints. When trading constraints is imposed, we assumed that the investor cannot trade any shares within the constraint horizon.

In this case, the investor's derived utility of wealth is expressed as

$$J(W,N,S,V,t) = \max_{w(0),\gamma(t)} E[\ln W(T)] \qquad (16)$$

Where the number of assets the investor can trade per period $\gamma(t) = 0$.

$$J(W,N,S,V,t;w(0)) = \ln W(t) + E\left[\int_t^T (\mu + \lambda V^2(s)w(s) - \frac{V^2(s)}{2}w^2(s)ds\right] \quad (17)$$

We apply Longstaff and Schwartz (2001) approach termed as LSM algorithm to solve this problem. First, we discretize the investment horizon $[0,T]$ into 0.05 years equal intervals and investment horizon T is expressed in units of the discretization interval. We normalise the initial values of W_0 and S_0 to 1. Second, we simulate 100,000 paths of V and S using the standard Euler approximation to the dynamics of the volatility and share price in equation (9) and (10).

Third, we assign all possible initial portfolio weight $w(0) = \frac{N(0)S(0)}{W(0)} = N(0)$ from 0% to 100% in step of 1% to the 10,000 paths of S making $\ln W_{j,t} = \ln(N_{j,0} S_{j,t} + M_{j,0})$ where $M_{j,0} = 1 - N_{j,0}$. We then take the average value of $\ln W_T$ alone all paths:

$$\overline{\ln W_T}(S_T; w_0) = \frac{\sum_{j=1}^{10,000} \ln W_{j,T}(S_T; w_0)}{10,000} \qquad (18)$$

The largest average terminal value must be give by the optimal initial portfolio weight $w^*(0)$, which equal to N_0.

Existence of trading constraint reduces the derived utility of wealth: $J(W, N, S, V, t; w^*(0)) \leq J(W, V, t)$. In order to make an investor hold the restricted asset, compensation must be offered. The illiquidity is expressed as:

$$D_t = 1 - \frac{1}{\exp(J(W,V,t) - J(W,N,S,V,t; w^*(0)))} \qquad (19)$$

Table 8 reports the optimal initial portfolio choice with trading constraints for different V_0 and σ. The initial optimal portfolio weight is negatively related to the initial volatility of share price. For *T=1*, with the increase of initial volatility, investor put 100% to 19% initial wealth on risky asset. This decrease is not obvious when initial volatility smaller than 0.4. σ also negatively influences $w^*(0)$ in most cases. The size of decrease is bigger for V_0 larger than 0.4, whereas it for extreme small and large V_0 the corresponding change of $w^*(0)$ is small. T is also an proxy for the illiquidity risk: when T is bigger, there are of more uncertainty. As it shows, larger T is associated with smaller initial weight i.e. investor put less initial wealth in risky asset when the level of uncertainty is bigger.

Table 9 summarises the illiquidity discount for different V and σ under optimal portfolio strategy. Unlike the positive relationship between discount and V in Longstaff (1995) model, it is negative in Longataff (2001). The reasons that the framework of Longstaff (1995) does not allow borrow and continuous rebalance at every time step for freely-traded assets but the framework of Longstaff (2001) does. In the framework, when volatility changes, μ does not, implying that the investor can enjoy same return with smaller risk. In this case, investor borrows a lot to invest in the asset, the portfolio return is huge. Particularly, when the volatility is smaller than 0.3, investors borrow money to invest in risky asset, whereas when the volatility is greater than 0.5, only less than 20% initial wealth is put into risky asset. On the contrary, investor cannot take a leverage position when trading constraint is imposed, ($0\% \leq w^*(0) \leq 100\%$) in order to in order to guarantee positive wealth along the time paths (Longstaff 2001).

σ is positively related to discount in most cases. Bigger σ makes bear higher risk for being restricted. The effect of T on illiquidity discount is negative as well: with a longer constraint horizon, the illiquidity discount is deeper.

It is also important to note that, although $w^*(0)$ is numerically solved and discount from those strategies are obtained, this framework allow choosing the optimal portfolio weight for restricted risky asset at t_0 i.e. trading constraint is only imposed after time 0; we however, consider a situation in which investors cannot trade even at time 0, a situation as case of China Stock Market. For the case in China, some investors such as SOEs and agencies of government get an endowment at t_0 and their position is locked until the trading constraint is released. In this case, closer his/her position to $w^*(0)$, higher the value of their portfolio, consequently smaller the discount: $J(W,N,S,V,t;w^*(0))) \geq J(W,N,S,V,t;w(0)))$ as illustrate in Table 10.

For those investors whose portfolio weight after endowment is not optimal, the discount is bigger than what suggest in Table 9. However, this effect is offset by another fact that those such as agencies of government and SEOs are very patient: in order to keep control of SOEs, they do not sell shares or do rebalance even they are allowed to do so. Our next paper will take subjective discount rate into the model.

Furthermore, investor cannot borrow as much as he/she wants to invest in the stock market in China i.e. $w(t)=\min\left[\left(\frac{\mu+\lambda V^2(t)}{V^2(t)}\right),1\right]$. In this case, investor cannot borrow to invest in risky asset even when the volatility is low leading to a smaller utility of the liquid benchmark. Consequently, the discount is smaller that what suggests in Table 9. Unlike the previous problem of tradability at t_0, the problem cannot solved by simply input $w(t)=\min\left[\left(\frac{\mu+\lambda V^2(t)}{V^2(t)}\right),1\right]$ into the numerical process because $w(t)$ itself already allow for the possible borrowing in future. We need a new solution for the optimal portfolio weight. We will address this issue in our another paper.

6. Conclusion

By using the most complete set of available data on auctions of restricted shares in China, we examined the price discounts of the restricted shares relative to their otherwise identical freely-traded counterparts. We find that restricted shares were priced at a deep discount of around 77% relative to their freely traded counterparts and price freely-traded shares decrease slightly when their restricted counterparts are involved in an occasional transaction. Our empirical model is not totally consistent with previous empirical one by Silber (1991) reflecting some difference of characteristic of restricted shares in China and in the US. This paper is first piece of empirical work testing the effects of trading constraints on illiquidity discount and confirmed the theoretical findings of Longstaff (1995, 2001, 2005) that the length of the constraint horizon T and the volatility positively affects the illiquidity discount of restricted shares. In the end, we make some comparison between empirical and theoretical models and between different theoretical frameworks, and then give some possible explanations for the difference. Empirical and theoretical investigation the effects of releasing trading constraints will be addressed in our further study.

Microsoft Equation 3.0

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Figure 1: The Proportion of Restricted Shares in China Stock Market (in percentage)

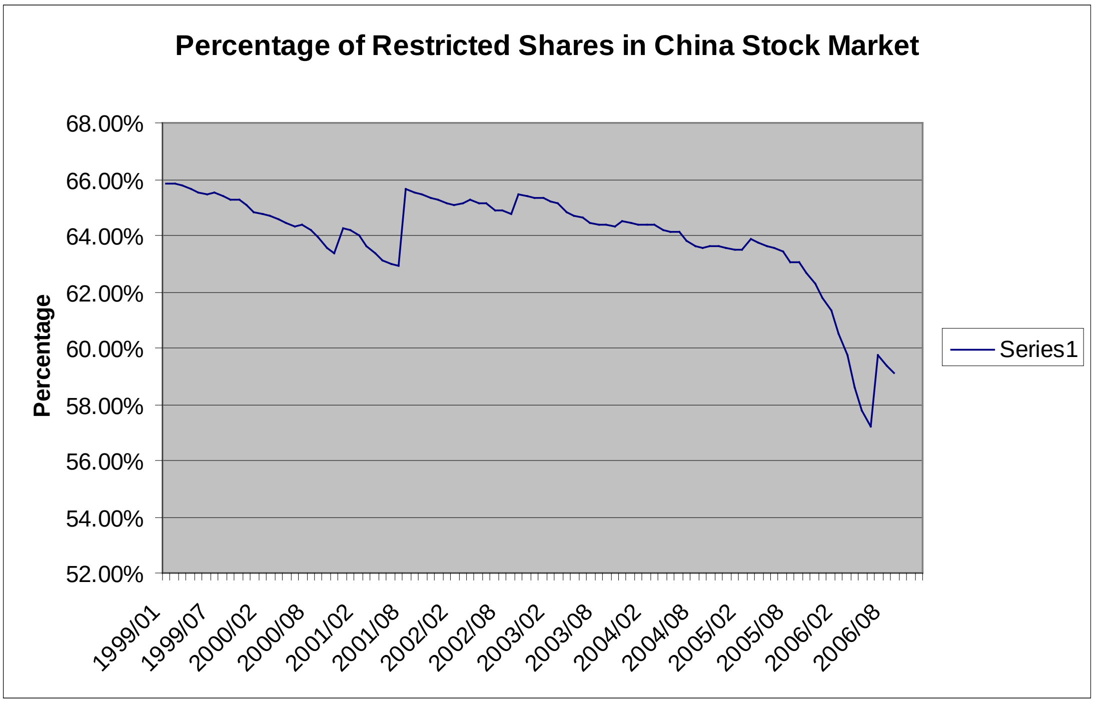

Table 1: Descriptive Statistics of Auction

Auction Size is the number of restricted shares auctioned in a transaction. *RTT* is the number of restricted shares auctioned relative to the total number of shares in the listed firm. *RTR* is the number of restricted shares auctioned relative to the number of restricted shares in the listed firm.

	Auction Size	***RTT***	***RTR***
Mean	1165013	0.61%	1.00%
Standard Error	123145.8113	0.000649	0.001121
Median	54700	0.03%	0.04%
Mode	50000	0.00%	0.00%
Standard Deviation	7868814.9894	0.0297	0.0513
Kurtosis	412.9206	71.9738	111.9466
Skewness	16.8347	7.8244	9.1609
Range	266520000	0.4286	1.0000
Minimum	0	0.0000	0.0000
Maximum	266520000	42.86%	100.00%

Table 2: Descriptive Statistics of Discounts and Difference.

A-1-D is the discount calculated based on the auction price of restricted shares and the closing price of their freely-traded counterparties one day before the auction. *A-D* is the discount calculated based on the auction price of restricted shares and the closing price of their freely-traded counterparties on the auction day. *A-Difference* is the difference between *A-1-D* and *A-D* in transaction.

	A-1-D	***A-D***	***A-Difference***
Mean	78.18%	77.18%	-0.62%
Standard Error	0.15%	0.18%	0.05%
Median	79.20%	78.60%	-0.14%
Mode	78.84%	75.00%	0.00%
Standard Deviation	7.39%	8.20%	2.11%
Sample Variance	0.55%	0.67%	0.04%
Kurtosis	226.15%	227.65%	5897.92%
Skewness	-82.60%	-84.07%	-647.78%
Range	73.26%	79.20%	34.33%
Minimum	26.74%	20.80%	-31.60%
Maximum	100.00%	100.00%	2.73%

Table 3: Correlation Matrix

Correlation Matrix of Discounts from Auction			
	A-1-Discount	A-Discount	Difference
Exchange	-0.2871**	-0.3223**	-0.0704**
	0.0000	0.0000	0.0031
SOE	-0.1044**	-0.0787**	-0.0330
	0.0000	0.0003	0.1665
Profit	-0.2102**	-0.2598**	-0.0564*
	0.0000	0.0000	0.0177
FR	0.0786**	0.0902**	0.0608*
	0.0009	0.0000	0.0107
RTT	0.1088**	0.1188**	0.0585*
	0.0000	0.0000	0.0139
sq-RTT	0.0704**	0.0791**	0.0399
	0.0031	0.0003	0.0942
RTR	0.1032**	0.1129**	0.0560*
	0.0000	0.0000	0.0186
lnMC	-0.3413**	-0.3862**	-0.0499*
	0.0000	0.0000	0.0362
Volatility	0.0332	0.0601*	0.0287
	0.1222	0.0109	0.2646
P/B	0.0919**	0.0854**	-0.0044
	0.0000	0.0001	0.8538
P/E	0.0611**	0.0476*	-0.0455
	0.0026	0.0295	0.0560
NAVPS	-0.3787**	-0.4129**	-0.0950*
	0.0000	0.0000	0.0001
Dividend	-0.3050**	-0.3096**	-0.0189
	0.0000	0.0000	0.4758
Age	-0.3183**	-0.3272**	-0.1116**
	0.0000	0.0000	0.0000
T	-0.2952**	-0.3074**	-0.1415**
	0.0000	0.0000	0.0000
**Correlation is significant at the 0.01 level (2-tailed). * Correlation is significant at the 0.05 level (2-tailed).			

Table 4: Regression model for Discounts in Auction

Rregression Model	AD'		AD		Difference	
R Square		0.359		0.410	0.038	
Coefficents	c	t	c	t	c	t
(Constant)	1.2992	22.8436	1.4048	27.9156	0.0295	4.0491
Exchange	-0.0359	-9.0538	-0.0413	-12.2438		
SOE						
Profit						
FR	0.0376	2.3746	0.0310	2.2396	0.0101	2.1885
RTT						
Sq-RTT						
RTR						
LnMC	-0.0247	-8.4082	-0.0266	-10.3855		
Volatility	0.2168	3.6310	0.1133	2.3491		
P/B	-0.0003	-3.6067	-0.0003	-4.1831		
P/E						
NAVPS	-0.0166	-9.9451	-0.0184	-13.9945	-0.0017	-4.0288
Dividend	-0.0158	-5.8606	-0.0136	-5.8371		
Age	-0.0180	-6.7767	-0.0139	-6.5485	0.0018	2.5360
T	0.0098	3.6935	0.0047	2.1762	-0.0031	-4.2939

Table 5: Descriptive Statistics of Price Premium

Free Share Premium is the price premium of freely-traded shares relative to their NAVPS. Restricted Share Premium is the price premium of restricted shares relative to their NAVPS. Premium Difference is the difference between Free Share Premium and Restricted Share Premium.

	Free Share Premium	Restricted Share Premium	Premium Difference
Mean	1000.39%	126.73%	877.46%
Standard Error	51.67%	7.43%	42.04%
Median	550.85%	49.19%	499.41%
Mode	234.75%	1089.23%	285.51%
Standard Deviation	2331.56%	447.45%	1896.71%
Sample Variance	54361.71%	2002.10%	35975.26%
Kurtosis	14346.68%	13983.17%	14127.42%
Skewness	1082.87%	1095.93%	1063.52%
Range	40310.19%	7180.83%	34318.75%
Minimum	0.54%	-100.00%	25.42%
Maximum	40310.73%	7080.83%	34344.17%

Table 6: Regression model for Price Premium relative to NAVPS

Rregression Model	Free Share Premium		Restricted Share Premium		Premium Difference	
R Square	0.225		0.204		0.233	
Coefficents	c	t	c	t	c	t
(Constant)	132.818	7.977	22.168	7.120	110.275	8.194
Exchange	-5.701	-5.246	-0.684	-3.237	-5.074	-5.767
SOE	-9.163	-7.073	-1.812	-7.189	-7.231	-6.897
Profit	-36.607	-15.128	-7.106	-15.112	-30.047	-15.347
FR	-26.257	-5.863			-20.927	-5.769
RTT	-68.749	-3.744	-4.896	-5.673	-30.935	-3.418
Sq-RTT			-17.648	-4.944		
RTR						
LnMC	-3.350	-4.067	-0.541	-3.530	-2.791	-4.188
Volatility	49.281	3.077	9.347	3.008	40.750	3.144
P/E	0.003	2.529	0.001	4.286	0.002	2.429
Dividend	-2.557	-3.248			-2.222	-3.483
Age						
T						

Table 7: Upper Bound of Illiquidity Discount (Longstaff 1995)

	0.1	0.2	0.3	0.5	0.7
1	8.23%	16.98%	26.28%	46.56%	69.24%
2	11.79%	24.64%	38.60%	70.09%	100.00%
3	14.59%	30.78%	48.67%	89.99%	100.00%
4	16.98%	36.13%	57.59%	100.00%	100.00%
5	19.13%	40.98%	65.77%	100.00%	100.00%
6	21.09%	45.48%	73.44%	100.00%	100.00%
7	22.92%	49.71%	80.73%	100.00%	100.00%
8	24.64%	53.73%	87.72%	100.00%	100.00%
9	26.28%	57.59%	94.46%	100.00%	100.00%
10	27.84%	61.30%	100.00%	100.00%	100.00%
11	29.33%	64.89%	100.00%	100.00%	100.00%
12	30.78%	68.38%	100.00%	100.00%	100.00%
13	32.17%	71.77%	100.00%	100.00%	100.00%
14	33.53%	75.09%	100.00%	100.00%	100.00%
15	34.84%	78.34%	100.00%	100.00%	100.00%
16	36.13%	81.52%	100.00%	100.00%	100.00%
17	37.38%	84.64%	100.00%	100.00%	100.00%
18	38.60%	87.72%	100.00%	100.00%	100.00%
19	39.80%	90.74%	100.00%	100.00%	100.00%
20	40.98%	93.72%	100.00%	100.00%	100.00%

Figure 2 Upper Bound of Illiquidity Discount (Longstaff 1995)

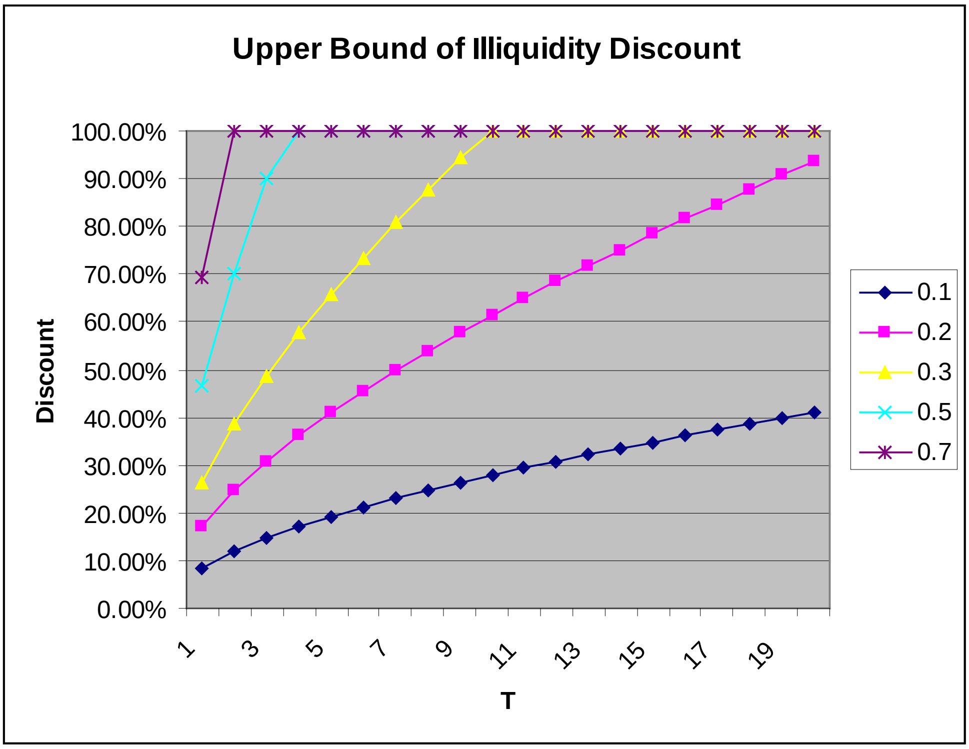

Table 8 Optimal Portfolio Choice

$w^*(0)$ is the initial optimal portfolio weight when trading constraint is imposed.

		σ						
		0.1	0.2	0.3	0.4	0.5	0.6	0.7
T	V_0	$w^*(0)$						
1	0.1	100%	100%	100%	100%	100%	100%	100%
1	0.2	100%	100%	100%	100%	100%	100%	100%
1	0.3	100%	100%	100%	99%	99%	85%	92%
1	0.4	59%	64%	58%	55%	54%	54%	49%
1	0.5	38%	37%	36%	35%	33%	34%	30%
1	0.6	23%	29%	27%	27%	22%	22%	24%
1	0.7	19%	19%	17%	15%	16%	14%	12%
2	0.1	100%	100%	100%	100%	100%	100%	100%
2	0.2	100%	100%	100%	100%	100%	97%	91%
2	0.3	100%	97%	92%	81%	78%	69%	67%
2	0.4	63%	63%	57%	51%	50%	45%	47%
2	0.5	36%	37%	34%	34%	32%	30%	28%
2	0.6	25%	25%	23%	21%	20%	23%	19%
2	0.7	17%	16%	16%	12%	12%	13%	13%
3	0.1	100%	100%	100%	100%	100%	100%	99%
3	0.2	100%	100%	100%	99%	96%	91%	87%
3	0.3	100%	92%	86%	76%	73%	72%	68%
3	0.4	64%	61%	55%	52%	51%	51%	53%
3	0.5	37%	36%	35%	34%	33%	37%	39%
3	0.6	22%	23%	22%	22%	21%	24%	27%
3	0.7	15%	14%	12%	13%	14%	16%	20%
4	0.1	100%	100%	100%	100%	99%	99%	51%
4	0.2	100%	100%	100%	96%	93%	89%	87%
4	0.3	100%	91%	83%	76%	74%	73%	73%
4	0.4	62%	60%	53%	53%	54%	55%	61%
4	0.5	38%	36%	36%	38%	41%	44%	51%
4	0.6	21%	22%	21%	23%	28%	33%	39%
4	0.7	14%	13%	14%	18%	20%	27%	31%
5	0.1	100%	100%	100%	100%	99%	51%	51%
5	0.2	100%	100%	99%	94%	90%	89%	51%
5	0.3	100%	90%	81%	77%	75%	76%	78%
5	0.4	67%	61%	56%	58%	59%	63%	65%
5	0.5	37%	37%	37%	40%	46%	53%	58%
5	0.6	23%	23%	26%	28%	35%	41%	52%
5	0.7	13%	14%	16%	19%	27%	34%	42%

Table 9 Illiquidity Discount

		σ						
		0.1	0.2	0.3	0.4	0.5	0.6	0.7
T	V_0							
1	0.1	36.38%	33.45%	34.61%	40.83%	54.92%	80.45%	99.80%
1	0.2	6.55%	5.67%	5.63%	7.34%	13.55%	29.86%	77.71%
1	0.3	2.82%	1.42%	1.43%	1.85%	4.77%	12.72%	47.75%
1	0.4	1.37%	1.13%	0.61%	1.05%	2.58%	7.53%	30.52%
1	0.5	0.82%	0.53%	0.50%	0.90%	1.75%	4.54%	20.80%
1	0.6	0.71%	0.38%	0.49%	0.74%	1.40%	3.35%	15.26%
1	0.7	0.55%	0.24%	0.48%	0.39%	1.06%	2.54%	11.01%
2	0.1	88.84%	72.61%	59.95%	55.79%	62.53%	82.78%	99.82%
2	0.2	38.19%	21.62%	12.96%	9.91%	13.07%	27.77%	76.94%
2	0.3	18.02%	9.22%	4.07%	2.73%	3.18%	11.11%	45.88%
2	0.4	10.20%	5.15%	2.33%	1.76%	1.94%	5.72%	29.11%
2	0.5	7.26%	3.53%	1.72%	1.08%	1.54%	4.34%	20.38%
2	0.6	5.28%	2.17%	1.61%	1.09%	1.16%	3.12%	14.61%
2	0.7	3.94%	2.13%	1.42%	1.24%	0.96%	2.52%	10.97%
3	0.1	100.00%	98.32%	87.20%	73.79%	71.40%	85.24%	99.84%
3	0.2	91.05%	59.72%	32.18%	17.98%	15.11%	27.34%	76.29%
3	0.3	64.86%	31.22%	13.44%	6.38%	2.97%	9.86%	43.64%
3	0.4	43.75%	18.55%	7.61%	3.08%	2.06%	4.83%	27.87%
3	0.5	30.78%	12.51%	5.46%	2.36%	1.53%	4.06%	19.51%
3	0.6	22.90%	9.37%	4.27%	2.15%	1.81%	3.13%	14.72%
3	0.7	17.62%	7.30%	3.50%	1.96%	1.98%	2.83%	11.09%
4	0.1	100.00%	100.00%	98.97%	89.47%	80.52%	87.72%	99.85%
4	0.2	100.00%	94.89%	62.44%	32.46%	19.19%	26.97%	75.64%
4	0.3	99.19%	71.98%	32.48%	12.22%	4.31%	7.76%	42.65%
4	0.4	93.19%	50.59%	19.16%	6.87%	3.10%	4.16%	27.04%
4	0.5	81.88%	35.87%	12.62%	4.26%	2.04%	3.62%	18.86%
4	0.6	69.44%	26.71%	9.47%	4.20%	2.40%	3.40%	14.60%
4	0.7	58.42%	20.38%	7.33%	2.89%	2.20%	3.16%	11.26%
5	0.1	100.00%	100.00%	100.00%	97.71%	88.54%	90.22%	99.87%
5	0.2	100.00%	99.99%	90.15%	52.18%	25.61%	27.82%	75.00%
5	0.3	100.00%	98.13%	61.64%	22.99%	7.48%	6.31%	40.46%
5	0.4	100.00%	89.03%	40.62%	12.58%	3.92%	3.38%	24.75%
5	0.5	99.95%	75.22%	27.66%	8.80%	3.22%	3.80%	18.50%
5	0.6	99.50%	62.25%	20.65%	6.84%	2.80%	3.53%	14.21%
5	0.7	97.97%	51.05%	15.65%	5.74%	2.85%	3.24%	11.30%

Table 10 Illiquidity Discount for all initial portfolio weight