When Salop meets E-commerce: Consumer Heterogeneity and Multi-Equilibria

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Abstract

We introduce consumer heterogeneity and E-commerce into classical Salop model. Due to free entry, market typically generates multi-equilibria. Although E-commerce delivery cost determines equilibria pattern, equilibrium price set by traditional firms does not depend on it. Compared to classical Salop model, E-commerce competition may either decrease or increase market price.

1 Introduction

E-commerce becomes increasingly important. We try to analyze how E-commerce competition affects industrial structure and pricing strategy of conventional mer-
chants when consumers are heterogeneous in terms of their ability to use E-commerce. For expositional convenience, we call the consumers with high using cost as $H$ consumers, and those with low using cost as $L$ consumers. When E-commerce is available, consumers must decide whether to buy from E-commerce or from conventional stores. Correspondingly, their behavior will affect pricing and entry/exit strategies of conventional stores.

In this paper, we intend to address the following questions: (1) What is the market equilibrium in the presence of E-commerce? Will E-commerce always decrease the price level set by conventional merchants? (3) Will E-commerce always increase social welfare?

To analyze these issues, we extend the classical Salop model by introducing consumer heterogeneity and E-commerce and characterize the corresponding equilibria.

Intuitively, when E-commerce as a potential competition is introduced, market competition will become more intense, and hence market price will be driven down. However, our analysis show that this argument might not be right. Indeed, when E-commerce has attracted $L$ consumers, conventional firms will be forced to only target at $H$ consumers, and correspondingly, the conventional stores will increase rather than decrease their prices.
2 The Model

Consumers of measure 1 are uniformly distributed on a Salop circle city. There are two types of consumers, with proportion of $\lambda$ being $L$ consumers who have low using cost for E-commerce and proportion of $1 - \lambda$ being $H$ consumers who have high cost. Every consumer has unity demand. The reservation utility of customers is normalized to zero. The utility derived from buying the product, $u$, is assumed to be large enough such that in equilibrium, every consumer always buys.

To highlight our points, we assume for simplicity that $L$ consumers’ using cost for E-commerce is zero while $H$ consumers’ using cost is prohibitively high; therefore, only $L$ consumers has the possibility to buy from E-commerce while $H$ consumers never do that. There are two kinds of firms, i.e., conventional firms and E-commerce firms. Both have zero production cost. E-commerce firms are perfectly competitive. In reality, they are like the online stores opened in Taobao and Ebay platform. Typically, they charge similar price, denoted by $\delta$ to deliver the goods to consumers regardless to their locations. Equivalently, we can imagine that all the online stores are located at the center of the circle, only $L$ consumers can go there to buy and the transportation cost is $\delta$. Conventional firms can freely enter the market and entry incurs a sunk cost $F$. As usual, we omit integer constraint of numbers of the conventional firms.
3 Market Outcome

We focus on symmetric Nash equilibria in which all the conventional firms are evenly located along the circular city and charge the same price. Suppose that in equilibrium there are $N_e$ conventional firms and they are charging $P_e$. To find the symmetric Nash equilibrium, we consider how a representative conventional firm $i$ chooses its price $P_i$ given that other $N_e - 1$ conventional firms have chosen equilibrium price $P_e$. We characterize the equilibria in the following cases.

- Inactive E-commerce

Omit for the moment E-commerce competition. Then our analysis repeats exactly the classical Salop model. Given $P_i$, $P_e$ and that all $N_e$ conventional firms are evenly located on the circle city, the consumer (either $H$ or $L$) who is indifferent between buying from firm $i$ and its immediate neighbours is

$$\hat{x} = \frac{1}{2N_e} + \frac{P_e - P_i}{2t},$$

(1)

and firm $i$’s demand is thus $D_i = 2\hat{x}$. Firm $i$ chooses its price $P_i$ to maximize its profit

$$\max_{P_i} 2P_i \left( \frac{1}{2N_e} + \frac{P_e - P_i}{2t} \right),$$

and the corresponding first order condition (FOC) is

$$\frac{1}{2N_e} + \frac{P_e - 2P_i}{2t} = 0.$$
Using symmetry condition, the candidate equilibrium price is \( P_e = \frac{t}{N_e} \), and the equilibrium profit is \( \pi_i = \frac{t}{N_e} \). Due to free entry, each conventional firm in the market can only get zero profit, that is, \( \pi_i = F \). The numbers of conventional firms and corresponding equilibrium price are respectively \( N_I = \sqrt{\frac{t}{F}} \) and \( P^I = \sqrt{tF} \).

To ensure that this candidate equilibrium is really an equilibrium, we should check whether to assume away the competition from the E-commerce side is reasonable. This boils down to verifying whether the \( L \) consumers located “right in the middle” of two conventional stores \(^1\) are indeed unwilling to buy from E-commerce; while this requires

\[
P^I + \frac{t}{2N_I} \leq \delta. \tag{2}
\]

Substituting \( P^I \) and \( N_I \) into equation (2), we obtain the condition that validate the equilibrium output:

\[
\delta \geq \frac{3}{2} \sqrt{tF}. \tag{3}
\]

Define \( \theta = \frac{\sqrt{tF}}{\delta} \), as an index of competitive advantage of E-commerce firms compared to conventional stores,\(^2\) Equation (3) can be simplified to \( \theta \leq \frac{2}{3} \). We sumnerize this finding in the following lemma:

\(^1\)that is, consumers locate at \( x = \frac{1}{2N_I} \).

\(^2t \) and \( F \) jointly determine the consumer’s cost to obtain products from conventional stores, while \( \delta \) is the delivery cost of E-commerce. Therefore, a small \( \theta \) means that it is easier for the conventional stores to attract consumers, while a large \( \theta \) implies a reverse tendancy.
Lemma 1 When $\theta \leq \frac{2}{3}$, E-commerce has no real effects on conventional firms, and the equilibrium is $\{P^I = \sqrt{tF}, N^I = \sqrt{t/F}\}$.

- Contestable E-commerce

Contestable equilibrium here means that although still no $L$ consumer buys from E-commerce, E-commerce exerts competitive pressure on conventional firms. More concretely, a contestable equilibrium is characterized by the following condition:

$$P_e + \frac{t}{2N_e} = \delta. \quad (4)$$

Namely, given that conventional firms all charging $P_e$, the consumers located at $x = \frac{1}{2N_e}$ is indifferent to buying from E-commerce or from conventional firms.

Combining the free entry condition $P_e = N_eF$, we have

$$2P_e^2 - 2\delta P_e + F = 0.$$  

When $\theta < \frac{\sqrt{2}}{2}$, the equation has two real roots $P_e = \frac{\delta}{2}(1 \pm \sqrt{1 - 2\theta^2})$, and correspondingly, $N_e = \frac{\delta}{2F}(1 \pm \sqrt{1 - 2\theta^2})$.

Now we check whether $\{P_e, N_e\}$ can constitute a contestable equilibrium. To be a valid contestable equilibrium, it must satisfy two conditions: (i) the conventional stores cannot profitably deviates by increasing their price, which leave a positive market share to E-commerce; (ii) the conventional stores cannot profitably deviate by decreasing their prices to steal customers from their neighbours.
Let \( \pi^+ (\pi^-) \) define the deviating profit of a conventional store charging a price \( P' \) higher (lower) than the candidate equilibrium price \( P_e \), we have

\[
\pi^+ = 2P' \left[ \frac{\delta - P'}{2} + (1 - \lambda) \left( \frac{1}{2N_e} + \frac{P_e - P'}{2t} \right) \right] \quad \text{if} \quad P' > P_e, \tag{5}
\]

\[
\pi^- = 2P' \left( \frac{1}{2N_e} + \frac{P_e - P'}{2t} \right) \quad \text{if} \quad P' < P_e. \tag{6}
\]

For \( \{P_e, N_e\} \) to be a contestable equilibrium, we should have

\[
\frac{\partial \pi^+}{\partial P'} = 2 \left[ \frac{\delta - 2P'}{2} + (1 - \lambda) \left( \frac{1}{2N_e} + \frac{P_e - 2P'}{2t} \right) \right] < 0 \quad \text{for} \quad P' \to P_e^+,
\]

\[
\frac{\partial \pi^-}{\partial P'} = 2 \left( \frac{1}{2N_e} + \frac{P_e - 2P'}{2t} \right) > 0 \quad \text{for} \quad P' \to P_e^-.
\]

Substituting equation (4) into the above equations yields \( \frac{2}{3+\lambda} \delta < P_e < \frac{2}{3} \delta \).

Since \( \frac{2}{3} (1 - \sqrt{1 - 2\theta^2}) < \frac{2}{3} < \frac{2}{3+\lambda} \delta \), \( P_e = \frac{2}{3} (1 - \sqrt{1 - 2\theta^2}) \) can not be supported as an equilibrium. So the only possible contestable equilibrium entails \( P_e = \frac{\delta}{2} (1 + \sqrt{1 - 2\theta^2}) \). It is easy to show that

\[
\frac{\delta}{2} (1 + \sqrt{1 - 2\theta^2}) < \frac{2}{3} \delta \Leftrightarrow \theta > \frac{2}{3}, \quad \frac{\delta}{2} (1 + \sqrt{1 - 2\theta^2}) > \frac{2}{3+\lambda} \delta \Leftrightarrow \theta < \frac{2\sqrt{1 + \lambda}}{3 + \lambda}.
\]

Therefore, when \( \frac{2}{3} < \theta < \frac{2\sqrt{1 + \lambda}}{3 + \lambda} \), we have the contestable equilibrium in which the price and number of conventional firms are respectively \( P^{II} = \frac{\delta}{2} (1 + \sqrt{1 - 2\theta^2}) \).

\(^3\)It is easy to check that the second order derivative of equation (5) and (6) are strictly smaller than 0; thus, the concavity of profit functions guarantee that the global maximum conditions coincide with the local ones.
and \(N_{II} = \frac{\delta}{2F}(1 + \sqrt{1 - 2\theta^2})\).\(^4\)

**Lemma 2** When \(\frac{2}{3} < \theta < \frac{2\sqrt{1+\lambda}}{3+\lambda}\), the contestable equilibrium is \(\{P_{II} = \frac{1}{2}(\delta + \sqrt{\delta^2 - 2tF})\), \(N_{II} = \frac{1}{2F}(\delta + \sqrt{\delta^2 - 2tF})\), in which no \(L\) consumers buys from E-commerce, but E-commerce has competitive pressure on conventional firms.

- Direct Competition

Consider a representative firm \(i\) and assume that other \(N_e - 1\) conventional firms have chosen equilibrium price \(P_e\). Define \(\hat{y} = \frac{\delta - P_i}{t}\). Then, by assumption, all \(L\) consumers located at \(x \in (\hat{y}, \frac{1}{2N_e})\) and all \(H\) consumers located at \(x \in (0, \hat{x})\) will buy from firm \(i\). Firm \(i\) chooses \(P_i\) to maximize its profit

\[
\max_{P_i} 2P_i \left[ \lambda \frac{\delta - P_i}{t} + (1 - \lambda) \left( \frac{1}{2N_e} + \frac{P_e - P_i}{2t} \right) \right].
\]

(7)

The FOC is

\[
\lambda \frac{\delta - 2P_i}{t} + (1 - \lambda) \left( \frac{1}{2N_e} + \frac{P_e - 2P_i}{2t} \right) = 0.
\]

Using symmetry condition, the FOC becomes

\[
\lambda \frac{\delta - 2P_e}{t} + (1 - \lambda) \left( \frac{1}{2N_e} - \frac{P_e}{2t} \right) = 0.
\]

\(^4\)In this range, the pre-condition for real roots \(\theta < \sqrt{2}\) is also satisfied; that is, \(\frac{2\sqrt{1+\lambda}}{3+\lambda} < \sqrt{2}\).

see appendix for details.
Under this circumstance, a conventional firm’s equilibrium profit is

\[
2P_e \left[ \frac{\delta - P_e}{t} + (1 - \lambda) \frac{1}{2N_e} \right] = 2P_e \left[ \frac{\delta - 2P_e}{t} + (1 - \lambda) \left( \frac{1}{2N_e} - \frac{P_e}{2t} \right) + \lambda \frac{P_e}{t} + (1 - \lambda) \frac{P_e}{2t} \right] = P^2 \frac{1 + \lambda}{t},
\]

where the second equality uses the FOC.

The free entry condition implies \( \frac{1 + \lambda}{t} P_e^2 = F \), it is easy to get the equilibrium price and number of conventional firms: \( P^{III} = \frac{\delta \theta}{\sqrt{1 + \lambda}}, N^{III} = \frac{t(1 - \lambda)}{(1 + 3\lambda)P^{III} - 2\delta} \).

Also the candidate equilibrium must satisfy the followings:

- The \( L \) consumers located at a conventional firm’s place will not buy from E-commerce, i.e., \( 0 < \hat{y} = \frac{\delta - P^{III}}{t} \), which implies \( \theta < \sqrt{1 + \lambda} \).

- The \( L \) consumers located at \( x = \frac{1}{2N^{III}} \) should buy from E-commerce, i.e.,

\[ \hat{y} = \frac{\delta - P^{III}}{t} < \frac{1}{2N^{III}}, \text{ which implies } \theta > \frac{2\sqrt{1 + \lambda}}{3 + \lambda}. \]

To ensure that there is no profitable deviation, given \( N_e \) and that other conventional firms have set price \( P_e \), we should check whether a conventional firm \( i \) has incentives to undercut the prices and steal consumer demand directly from its neighbors.\(^5\) And we proved that it is not feasible for the conventional firms to

\(^5\)From the representative conventional firm’s maximization problem (7), we know that it is not optimal for the conventional firm to decrease the price to steal consumers from the E-commerce; however, it still remains unclear whether the conventional firms want to further decrease the price to exclude the E-commerce and directly steal customers from their neighbors.
To summarize, we have the following lemma:

**Lemma 3** When $\frac{2\sqrt{1+\lambda}}{3+\lambda} < \theta < \sqrt{1+\lambda}$, an equilibrium is $\{P^{III} = \sqrt{tF/(1+\lambda)}, N^{III} = t(1-\lambda)(1+3\lambda)\sqrt{tF/(1+\lambda)-2\lambda\delta} \}$, in which the $L$ consumers located at $(\frac{\delta-P^{III}}{t}, \frac{1}{2N^{III}})$ buy from E-commerce.

- Separating Market

When all $L$ consumers buy from E-commerce, a representative conventional firm $i$ only target at $H$ consumers and all the analysis is the same as classical Salop model except that the market become “thinner.” Now the equilibrium price still has the form of $P_e = \frac{t}{N_e}$, but the free entry condition becomes $\frac{(1-\lambda)t}{N_e} = F$. Consequently, the candidate equilibrium number and price of conventional firms will be $\{P^{IV} = \frac{\delta\theta}{\sqrt{1-\lambda}}, N^{IV} = \sqrt{\frac{(1-\lambda)t}{F}} \}$.

The corresponding parameter range must satisfy that the $L$ consumers located at $x = 0$ buy from E-commerce, which requires $P^{IV} > \delta$, that is, $\theta > \sqrt{1-\lambda}$.

For $\{P^{IV}, N^{IV} \}$ to be indeed an equilibrium, we also characterise the condition when the representative conventional firm $i$ can not make profit by setting price $P_i < \delta$ to attract $L$ consumers from E-commerce firms. In the following Lemma, we give the explicit parameter range in which such deviations are not profitable.

**Lemma 4** When $\theta > \frac{\sqrt{1+\lambda}+\sqrt{1-\lambda}}{2}$, deviating by setting $P' < \delta$ is not profitable.

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6See appendix.
Proof. If firm $i$ sets price $P' < \delta$, the deviating profit is

$$\pi^d = 2P'[\lambda\hat{y} + (1 - \lambda)\hat{x}]$$

$$= 2P'[\lambda\frac{\delta - P'}{t} + (1 - \lambda)\left(\frac{1}{2NIV} + \frac{PIV - P'}{2t}\right)]$$

$$= \frac{2P'}{t}[\lambda(\delta - P') + (1 - \lambda)(PIV - \frac{P'}{2})]$$

$$= \frac{P'}{t}[2\lambda\delta + 2(1 - \lambda)PIV - (1 + \lambda)P']$$

where we use superscript $d$ to denote deviation profit.

Obviously, $\pi^d$ is concave, therefore the optimal deviating price $P'^* = \frac{\lambda\delta + (1 - \lambda)PIV}{1 + \lambda}$.

If $P'^* > \delta$, i.e., $\theta > \frac{1}{\sqrt{1 - \lambda}}$, then $\pi^d$ takes its maxima at $P_i = \delta$. So deviation by setting $P_i < \delta$ is unprofitable. However, by the construction of the candidate equilibrium, setting $P_i = \delta$ is strictly dominated by setting $P_i = P^{IV}$.

If $P'^* < \delta$, $\pi^d$ takes its maxima at $P'^*$. The profit of the deviating firm $i$ is

$$\pi^{d*} = (P'^*)^2(1 + \lambda)/t$$

So deviating by setting price $P_i < \delta$ is unprofitable when $\pi^{d*} < F$.

$$\pi^{d*} = \frac{\lambda^2\delta^2 + (1 - \lambda)tF + 2\lambda\delta\sqrt{(1 - \lambda)tF}}{(1 + \lambda)t}$$

$$= F - \frac{2\lambda\delta^2}{(1 + \lambda)t}\left[\theta^2 - \sqrt{(1 - \lambda)}\theta - \frac{1}{2}\lambda\right]$$

$$= F - \frac{2\lambda\delta^2}{(1 + \lambda)t}(\theta - \frac{\sqrt{1 - \lambda}}{2} - \frac{\sqrt{1 + \lambda}}{2})(\theta - \frac{\sqrt{1 - \lambda}}{2} + \frac{\sqrt{1 + \lambda}}{2}),$$

which is smaller than $F$ when the second term is negative. Aligned with the pre-condition $\theta > \sqrt{1 - \lambda}$, this requires $\theta > \frac{\sqrt{1 - \lambda}}{2} + \frac{\sqrt{1 + \lambda}}{2}$. ■

The results are presented in the following lemma.
Lemma 5 When $\theta > \frac{\sqrt{1+\lambda}+\sqrt{1-\lambda}}{2}$, the equilibrium involves $\{P^IV = \sqrt{tF/(1-\lambda)}, N^IV = \sqrt{(1-\lambda)t/F}\}$, in which all $L$ consumers buys from E-commerce.

In this case, compared to classical Salop model, introducing E-commerce increases rather than decreases the equilibrium market price. Since $\theta$ is large, the competitiveness of E-commerce is very strong and the conventional firms have to target only at $H$ consumers who has no access to E-commerce. As a consequence, the market becomes “thinner”. Given that the entry cost $F$ is fixed, the “distance” between any two neighbouring conventional firms becomes larger, which soften the competition between any pair of neighbouring conventional firms and induce higher price.

Notice that since the market is separated, a marginal change of $\delta$ has no effect on conventional firms in this case.

We summerize our findings in proposition 1.

**Proposition 1** Depending on $\theta$, the possible equilibria are as follows:

1. $\theta < \frac{2}{3}$: E-commerce is inactive, $\{P = \sqrt{tF}, N = \sqrt{t/F}\}$;

2. $\frac{2}{3} < \theta < \frac{2\sqrt{1+\lambda}}{3+\lambda}$: E-commerce still owns no market share, while the $L$ market is contestable. $\{P^{II} = \frac{1}{2}(\delta + \sqrt{\delta^2 - 2tF}), N^{II} = \frac{1}{2F}(\delta + \sqrt{\delta^2 - 2tF})\}$;

3. $\frac{2\sqrt{1+\lambda}}{3+\lambda} < \theta < \sqrt{1+\lambda}$: E-commerce owns positive market share in the $L$ market. $\{P^{III} = \sqrt{tF/(1+\lambda)}, N^{III} = \frac{t(1-\lambda)}{(1+3\lambda)\sqrt{tF/(1+\lambda)-2\lambda\delta}}\}$;
4. \( \theta > \frac{\sqrt{1+\lambda} + \sqrt{1-\lambda}}{2} \): E-commerce dominates the L market. \( \{ P^{IV} = \sqrt{tF/(1-\lambda)}, N^{IV} = \sqrt{(1-\lambda)t/F}\} \).

[!!!Need to be re-organised!!!]

**Remark 1** Except in the contestable equilibrium, although the parameter \( \delta \) determines which equilibrium will be realized, it does not affect the equilibrium price and number of conventional firms.

**Remark 2** Multi-equilibria may occur. More concretely:

(1) When \( \frac{1}{\sqrt{1+\lambda}} \equiv \theta_{21} < \theta < \frac{2}{\sqrt{1+\lambda} + \sqrt{1-\lambda}} \equiv \theta_{3} \), both equilibria \( E^{II} \) and \( E^{III} \) are self-contained. That is, either the conventional firms will deter the E-commerce by limit pricing or they just target at \( H \) consumers.

**Remark 3** Compared to classical Salop model “without” outside goods, in the presence of consumer heterogeneity, introducing E-commerce may increase market price when market equilibrium is \( E^{III} \).

4 Policy Discussion

4.1 Welfare Analysis

Since every consumer has unity demand and will purchase one product in equilibrium, maximizing social welfare is equivalent to minimizing total social cost.
Here we identify two kinds of welfare optimalities: ex-ante and ex-post. From ex-ante point of view, the social costs incurred are composed of three parts:

- The transportation cost when consumers buy from conventional firms;
- The delivery cost when consumers buy from E-commerce;
- The fixed entry cost for conventional firms.

While from the ex-post point of view, the number of conventional firms are already determined and the entry costs are sunk, therefore, only the first two items remains. It is thus straightforward to characterize the ex-post social welfare optimality: Since the production cost is normalized to zero, the ex-post social optimal is equivalent to the situation that all the product prices match their marginal cost: that is, the E-commerce set price to $\delta$ and the conventional firms set prices to zero. If $\delta < \frac{1}{2N}$, then the $L$ consumers located between $(\frac{\delta}{1}, \frac{1}{2N})$ should buy from E-commerce; if $\delta > \frac{1}{2N}$, it is optimal to have E-commerce out of the market.

Taking into account the entry decision of conventional firms, we discuss the ex-ante welfare in two cases:

- E-commerce does not own any market share.

From the ex-post welfare analysis, we know that in this case, $\delta > \frac{1}{2N_0}$, where $N_0$ denotes the efficient number of conventional firms along the Salop circle. The
total social cost $T_0$ thus is

$$T_0^* = \min_{N_0} T_0$$

$$T_0 = 2N_0 \int_0^{\frac{1}{2N_0}} tx dx + NF = \frac{t}{4N_0} + N_0 F$$

and the optimal number of conventional firms is $N_0^* = \frac{1}{2} \sqrt{\frac{t}{F}}$, and the minimized social cost is $T_0^* = \sqrt{Ft}$.

substitute the optimal value of $N_0^*$ into the pre-condition $\delta > \frac{t}{2N}$, we have $\delta > \sqrt{Ft}$, that is, $\theta < 1$.

- E-commerce owns a positive market share.

Clearly, $\delta < \frac{t}{2N_1}$ is the necessary condition to guarantee that some $L$ consumers buy from E-commerce, where $N_1$ denotes the number of conventional firms in this case. Since $L$ consumers located between $[\frac{\delta}{t}, \frac{1}{2N_1}]$ buy from E-commerce, the corresponding total social cost $T_1$ is

$$T_1^* = \min_{N_1} T_1$$

$$T_1 = 2N_1 \lambda \int_0^{\frac{\delta}{t}} tx dx + 2N_1 \lambda \int_{\frac{\delta}{t}}^{\frac{1}{2N_1}} \delta dx + (1 - \lambda)2N_1 \int_0^{\frac{1}{2N_1}} tx dx + N_1 F$$

$$= \lambda \delta - \frac{\lambda \delta^2}{t} N_1 + \frac{(1 - \lambda)t}{4N_1} + N_1 F$$

Cost minimization implies that the optimal number of conventional firms is
\[ N_1^* = \frac{t}{2} \sqrt{\frac{1 - \lambda}{Ft - \lambda\delta^2}} \]

and the corresponding total social cost will be

\[ T_1^* = \lambda\delta + \sqrt{1 - \lambda} \sqrt{Ft - \lambda\delta^2} \]

Substituting \( N_1^* \) into the pre-condition \( \delta < \frac{t}{2N_1} \), we have \( \delta < \sqrt{Ft} \) or, \( \theta > 1 \).

**Proposition 2** Social welfare optimality is as follows: (1) when \( \theta < 1 \), the number of conventional firms is \( N_0^* = \frac{1}{2} \sqrt{t} \) and no \( L \) consumers buy from E-commerce firms; (2) when \( \theta > 1 \), the number of conventional firms is \( N_1^* = \frac{t}{2} \sqrt{\frac{1 - \lambda}{Ft - \lambda\delta^2}} \) and the \( L \) consumers located between \([\frac{\delta}{t}, \frac{1}{2N_1}]\) buy from E-commerce and others still buy from the conventional firms.

### 4.2 Regulating \( \delta \) and \( F \)

In our framework, the government has two potential policy tools: delivery cost \( \delta \) can be adjusted by taxing a marginal rate \( d \) on the postal service and delivery agents, while the fixed entry cost \( F \) can be adjusted by altering business license fee \( l \). In this subsection, we will focus on how the government uses these tools to ameliorate the market outcome in the four regimes of market outcome discussed in section 2. For simplicity, we assume that the government can directly tax the consumers for an arbitrary amount, that is, the government does not have a
budget constraint.

• $\theta < 1$

In this regime, the social optimality requires E-commerce out of the market. Therefore, the government can charge a sufficiently high tax $d$ to prevent E-commerce from entry. This also lead $\theta_r = \frac{\sqrt{(F+l)t}}{\delta + d}$, the regulated $\theta$, smaller than $2/3$.

The ex-post efficiency is satisfied, since no L customer buys from E-commerce. However, the number of firms in market outcome should be $\frac{1}{2} \sqrt{\frac{t}{F}}$ to satisfy the ex-ante efficiency condition, while the market equilibrium yields $N^I = \sqrt{t/F}$. In other words, there is excessive entry of local stores.

To restore the efficiency, the government can charge a higher business license fee $l^I = 3F$ to balance the entry incentives of local stores, the new equilibrium number of conventional firms is $N^I_r = \sqrt{\frac{t}{l+F}} = 1/2 \sqrt{t/F} = N^*_0$.

• $\theta > 1$

This situation is more complicated, since to aligned with the social optimality, the regulation objectives are $N^* = \frac{t}{2} \sqrt{\frac{1-\lambda}{Ft-\lambda t^2}}$ and the L consumers located between $[\frac{\delta}{l}, \frac{1}{2N^*}]$ buy from E-commerce and others still buy from the conventional firms. We first prove the following lemma:
Lemma 6 Define $\hat{\lambda} = \sqrt{2} - 1$. If $\lambda \in [\hat{\lambda}, 1]$ the economy can be regulated to the efficient state; if $\lambda \in (0, \hat{\lambda})$, the economy with $\theta > \hat{\theta}(\lambda)$ can be regulated to the efficient state.

Proof. To achieve the efficient allocation, the optimal $l$ and $d$ must satisfy:

$$\frac{t(1-\lambda)}{(1+3\lambda)\sqrt{t(F+l)/(1+\lambda)} - 2\lambda(d + \delta)} = \frac{t}{2}\sqrt{\frac{1-\lambda}{Ft - \lambda\delta^2}},$$

$$\frac{\delta + d - \sqrt{t(F+l)/(1+\lambda)}}{t} = \frac{\delta}{t}.$$

Solving the equations above yields

$$d = \frac{\sqrt{(1-\lambda)(Ft - \lambda\delta^2) + \lambda\delta}}{1+\lambda},$$

$$l = -\frac{\lambda}{(1+\lambda)t}\left[\delta^2 - 2\delta\sqrt{1-\lambda)(Ft - \lambda\delta^2)} + 2(Ft - \lambda\delta^2)]\right].$$

We also need to check whether $\theta_r$ is still in the range of $[\frac{2\sqrt{1+\lambda}}{3+\lambda}, \sqrt{1+\lambda}]$.

$$\theta_r = \frac{\sqrt{(F+l)t}}{\delta + d} = \frac{d\sqrt{1+\lambda}}{\delta + d}$$

$$= \sqrt{1+\lambda}\frac{\sqrt{(1-\lambda)(\theta^2 - \lambda) + \lambda}}{1 + \sqrt{(1-\lambda)(\theta^2 - \lambda) + \lambda}},$$

which is by construction smaller than $\sqrt{1+\lambda}$. To ensure that $\theta_r$ larger than $

\frac{2\sqrt{1+\lambda}}{3+\lambda}$, we need

$$\theta_r > \frac{2\sqrt{1+\lambda}}{3+\lambda} \Leftrightarrow \sqrt{1+\lambda}\frac{\sqrt{(1-\lambda)(\theta^2 - \lambda) + \lambda}}{1 + \sqrt{(1-\lambda)(\theta^2 - \lambda) + \lambda}} > \frac{2\sqrt{1+\lambda}}{3+\lambda},$$

$$\Leftrightarrow \frac{\sqrt{(1-\lambda)(\theta^2 - \lambda) + \lambda}}{1 + \sqrt{(1-\lambda)(\theta^2 - \lambda) + 2\lambda}} > \frac{2}{(3+\lambda)(1+\lambda)}, \quad (8)$$

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Notice that the LHS of the above equation increases along with the value of 
\[ \sqrt{(1 - \lambda)(\theta^2 - \lambda)}, \] which achieves its minimum value \( 1 - \lambda \) when \( \theta = 1 \), substituting the minimum value into equation (8) yields:

\[
\frac{\sqrt{(1 - \lambda)(\theta^2 - \lambda) + \lambda}}{1 + \sqrt{(1 - \lambda)(\theta^2 - \lambda) + 2\lambda}} \geq \frac{1}{2 + \lambda}.
\]

Therefore

\[
\frac{1}{2 + \lambda} > \frac{2}{(3 + \lambda)(1 + \lambda)} \iff \lambda^2 + 2\lambda - 1 > 0 \iff \lambda > \sqrt{2} - 1.
\]

Let \( \hat{\lambda} = \sqrt{2} - 1 \), any market with \( \lambda > \hat{\lambda} \) can be regulated to the social best state.

If \( \lambda < \hat{\lambda} \), it cannot be guaranteed that \( \theta_r > \frac{2\sqrt{1 + \lambda}}{3 + \lambda} \), equation (8) implicitly defines the scale of unregulated \( \theta \) in which the economy can be regulated to the first-best. ■

To better understand lemma 6, notice that

\[
\delta^2 - 2\delta \sqrt{(1 - \lambda)(Ft - \lambda\delta^2)} + 2(Ft - \lambda\delta^2) \geq \delta^2 - 2\delta \sqrt{Ft - \lambda\delta^2} + Ft - \lambda\delta^2 = (\delta - \sqrt{Ft - \lambda\delta^2})^2 \geq 0,
\]

therefore, when it is possible to achieve the first-best, \( d > 0 \) while \( l < 0 \). The government actually taxes the E-commerce and subsidies conventional firms to achieve the social optimum; Correspondingly, this result also implies that there is under-entry in the market of conventional firms.
The government want to solve two related problems: first the entry level of conventional firm should be promoted up to the social optimal level, while at the same time there must be some appropriate market share left for E-commerce companies. When $\lambda$ and $\theta$ are large enough, it is easy for the government to achieve this objectives through setting $d$ and $l$ properly.

**Proposition 3** The

5 Comparative Statics[!!!have not decided where to put it!!!]

<table>
<thead>
<tr>
<th>Equilibrium</th>
<th>$\lambda$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P^I = \sqrt{tF}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$N^I = \sqrt{\frac{t}{F}}$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$P^{II} = \sqrt{\frac{tF}{1+\lambda}}$</td>
<td>$-0$</td>
<td>0</td>
</tr>
<tr>
<td>$N^{II} = \frac{t(1-\lambda)}{(1+3\lambda)P^{II}-2\lambda\delta}$</td>
<td>$-0$</td>
<td>0</td>
</tr>
<tr>
<td>$P^{III} = \sqrt{\frac{tF}{1-\lambda}}$</td>
<td>+</td>
<td>0</td>
</tr>
<tr>
<td>$N^{III} = \sqrt{\frac{(1-\lambda)t}{F}}$</td>
<td>$-0$</td>
<td>0</td>
</tr>
<tr>
<td>$P^{IV} = \frac{\delta+\sqrt{\delta^2-2Ft}}{2}$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$N^{IV} = \frac{\delta+\sqrt{\delta^2-2Ft}}{2F}$</td>
<td>+</td>
<td>+</td>
</tr>
</tbody>
</table>
For reference: the sign of $\frac{\partial N}{\partial \lambda}$ is the same as

$$H = -\frac{4}{\sqrt{1 + \lambda}} + 2\theta + \frac{(1 - \lambda)(1 + 3\lambda)}{2(1 + \lambda)^{3/2}}.$$ 

Note that the equilibrium $\{P^II, N^II\}$ requires $\frac{1}{\sqrt{1 + \lambda}} < \theta < \frac{3 + \lambda}{2\sqrt{1 + \lambda}}$. So,

$$H < -\frac{4}{\sqrt{1 + \lambda}} + \frac{3 + \lambda}{\sqrt{1 + \lambda}} + \frac{(1 - \lambda)(1 + 3\lambda)}{2(1 + \lambda)^{3/2}} = -\frac{(1 - \lambda)^2}{(1 + \lambda)^{3/2}} < 0.$$

6 Appendix

6.1 Check the valid range of Lemma 2

Obviously, $\frac{2}{3} < \frac{\sqrt{2}}{2}$, the solution set is non-empty; therefore we only need to check whether $\frac{2\sqrt{1 + \lambda}}{3 + \lambda}$, the upper-limit of $\theta$ in lemma 2 is also smaller than $\frac{\sqrt{2}}{2}$.

$$\left(\frac{2\sqrt{1 + \lambda}}{3 + \lambda}\right)^2 = \frac{4(1 + \lambda)}{9 + 6\lambda + \lambda^2}$$

$$= \frac{4(1 + \lambda)}{(\lambda - 1)^2 + 8(1 + \lambda)}$$

$$< \frac{1}{2}.$$ 

Therefore, $\frac{2\sqrt{1 + \lambda}}{3 + \lambda} < \frac{\sqrt{2}}{2}$.

6.2 No Profitable Deviation

In such a circumstance, the deviating firm’s profit is:

$$\max_{P'} 2P'\left(\frac{1}{2N^III} + \frac{P'^{III} - P'}{2t}\right)$$
The FOC is
\[
\frac{t}{N^{III}} + P^{III} - 2P^* = 0
\]
which implies that the optimal deviating price \( P^* = \frac{t}{2N^{III}} + \frac{P^{III}}{2} \), and
\[
P^* - P^{III} = \frac{t}{2N^{III}} - \frac{P^{III}}{2}
\]
\[
= \frac{(1 + 3\lambda)P^{III} - 2\lambda\delta}{2(1 - \lambda)} - \frac{P^{III}(1 - \lambda)}{2(1 - \lambda)}
\]
\[
= \lambda \frac{2P^{III} - \delta}{1 - \lambda},
\]
which is smaller than 0 if and only if \( \theta < \sqrt{\frac{1+\lambda}{2}} \). However, since \( \forall \lambda < 1, 2\sqrt{\frac{1+\lambda}{3+\lambda}} > \sqrt{\frac{1+\lambda}{2}} \), within the parameter range \( \left[\frac{2\sqrt{1+\lambda}}{3+\lambda}, \sqrt{1+\lambda}\right] \), \( P^* > P^{III} \). Therefore, the FOC is always positive and undercutting other conventional firms price is not profitable.

6.3 Redundant Calculations [!!!will be deleted soon!!!]

However, this is not enough to replicate the efficient outcome, since the “regulated” \( \theta \), namely, \( \theta_r \), now becomes \( \theta_r = \sqrt{\frac{(I+F)\delta}{d}} = 2\theta \), which is not necessarily smaller than 2/3. Therefore, the government also need to tax \( d^I \geq \delta \) on top of the original delivery cost \( \delta \), so that \( \theta_r = \sqrt{\frac{(I+F)\delta}{d^I+\delta}} \leq \sqrt{\frac{(I+F)\delta}{2\delta}} = \theta < 2/3 \).

\[ \bullet \quad \frac{2}{3} < \theta < \frac{2\sqrt{1+\lambda}}{3+\lambda} \]

Since \( \frac{2\sqrt{1+\lambda}}{3+\lambda} < 1 \), contestable equilibrium output is still aligned with the ex-post optimality. However, The equilibrium number of conventional firms \( N^{II} > N^*_0 \).
so government intervention still target on discouraging the excessive entry by charging $l$ and $d$. An easy choice is to charge a sufficiently high $d$ in order to have the regulated $\theta_r < 2/3$, and then charge $l = 3F$ to restore the efficiency. Now we try to find whether it is possible to achieve efficiency and simultaneously keep the equilibrium contestable. In such a case, $l$ and $d$ are implicitly determined by the equations

\[
\frac{(\delta + d^{II}) + \sqrt{(\delta + d^{II})^2 - 2t(F + l^{II})}}{F + l^{II}} = \sqrt{\frac{t}{F}},
\]
\[
\frac{\sqrt{t(F + l^{II})}}{\delta + d^{II}} = \sqrt{\frac{t}{F}}.
\]

Solving the equations above yields $l^{II} = \left[\left(\frac{1+\sqrt{1-\theta^2}}{\theta}\right)^2 - 1\right]F$ and $d^{II} = \frac{1-\theta+\sqrt{1-\theta^2}}{\theta}\delta$.

- $\frac{2\sqrt{1+\lambda}}{3+\lambda} < \theta < \sqrt{1+\lambda}$

In this range of $\theta$, E-commerce and conventional firms compete head-to-head for the $L$-type consumers, while the social optimality requires the conventional firms occupy all the market when $\theta < 1$, while the E-commerce occupy the whole market share of $L$ consumers when $\theta > 1$. Therefore, we need to discuss two sub-cases:

1. $\frac{2\sqrt{1+\lambda}}{3+\lambda} < \theta < 1$: In this range, the government should help the conventional firms drive E-commerce out of the market by charging a sufficiently high $d$. Then the situation goes back to the two previous cases, in which the government can manipulate $l$ to eliminate excessive entry.