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Optimization methods based on input impedance target functions have been proposed for the design of brass musical instruments. Criteria for target functions in trombone bore optimization are discussed, drawing on experimental input impedance data from a variety of high-quality trombones of differing sizes. An "inharmonicity plot" is introduced and used to aid the interpretation of impedance curves. An efficient optimization technique is described and is shown to be capable of predicting bore changes which achieve specified modifications to the input impedance curve while maintaining a smoothly-flaring bell contour. Further work is required to clarify the relationship between input impedance targets and the preferences of professional players.
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I. INTRODUCTION

A player produces a note on a brass musical instrument by expelling a stream of air through the lips, which are pressed against the mouthpiece of the instrument. For an appropriate mechanical configuration, and a suitable range of fluid parameters, the flow of air over the lip surfaces causes them to destabilize and to vibrate. The resulting periodic modulation of the volume flow through the lips acts as an acoustical source, and is the origin of the musical sound radiated by the instrument. When the lip vibration frequency is close to the frequency of one of the acoustical modes of the downstream resonator, a significant pressure variation develops in the mouthpiece, leading to an acoustical feedback from the resonator to the lips. This feedback loop encourages the development of a powerful self-sustained oscillation of the coupled system at a frequency close to that of the acoustical mode in question.

At least for moderate playing levels,[1] the brass instrument may be modeled as a non-linear flow control valve coupling a linear mechanical oscillator (the lips) to a linear acoustical resonator (the air column).[2] The playing behavior of the instrument depends on the properties of both the lips and the resonating tube. In learning to play the instrument, the player develops expertise in lip control; the role of the instrument designer is to optimize the acoustical behavior of the resonating tube when coupled to the lips of a player. The linear acoustical properties of the resonator are described by the input impedance $Z_{\text{in}}=p_{\text{in}}/U_{\text{in}}$, where p_{in} is the acoustic pressure in the mouthpiece and U_{in} the volume flow into the mouthpiece. For a conventional brass instrument such as a trumpet, trombone, or horn, Z_{in} is almost completely determined by the bore-profile $r(x)$, where r is the tube radius and x the axial distance from the mouthpiece entrance plane.

In the 1970s, Smith and Daniell developed a method for calculating the bore perturbations necessary to achieve specified changes in the input impedance curves of brass instruments.[3] More recently, Kausel[4] and Noreland[5] described approaches to the optimization of bore-profiles of brass instruments using targets based on input impedance curves. However, this approach relies on the ability to define the target input impedance curve; it is not yet clear which features of the input impedance curve correspond to optimum behavior as judged by the player.

Poirson *et al.*[6] conducted research into the effect of mouthpiece depth on trumpet timbre, using the spectral centroid to measure the brightness. The results show a correlation (but not necessarily a causal relationship) between the brightness and the inharmonicity, magnitude, and Q-factor of the input impedance peak corresponding to the second harmonic of the played frequency.

Bertsch and co-workers[7–9] made extensive studies into the correlation between input impedance and playing characteristics as determined through blind testing. Chick *et al.*[10] examined the inharmonicity of various French Horns in all valve positions. An extensive comparison of objective and subjective tests of trumpet leadpipes by Poirson *et al.* suggested that the optimum impedance curve as determined by player choices did not correspond to perfect harmonic alignment of acoustic resonances.[6]

Several previous studies[11–17] have identified minor discrepancies between measured and calculated input impedance curves for wind instruments. These discrepancies can arise from uncertainties and calibration problems in the experimental techniques, or from approximations made in the calculations. Real instruments have been successfully optimized in spite of these discrepancies,[18] although they may limit the application of this approach to very fine work. The development of improved optimization techniques is not constrained by this problem, since the efficacy of such techniques as applied to theoretical instruments is independent of the accuracy of the underlying impedance calculations.

In Sec. II of this paper, the use of the input impedance curve as a target for trombone optimization is further explored. Measurements of the input impedance of several trombones which are very highly rated by professional players are presented and analyzed, using a plotting method which allows simultaneous visualization of the resonance

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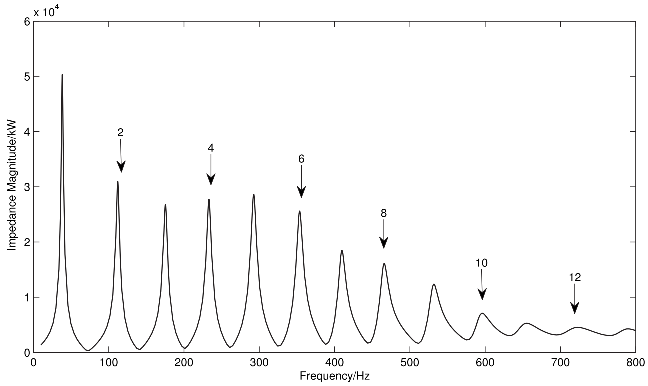

FIG. 1. Theoretical impedance curve of a tenor trombone with slide closed. Peaks comprising the regime of B♭$_2$ highlighted.

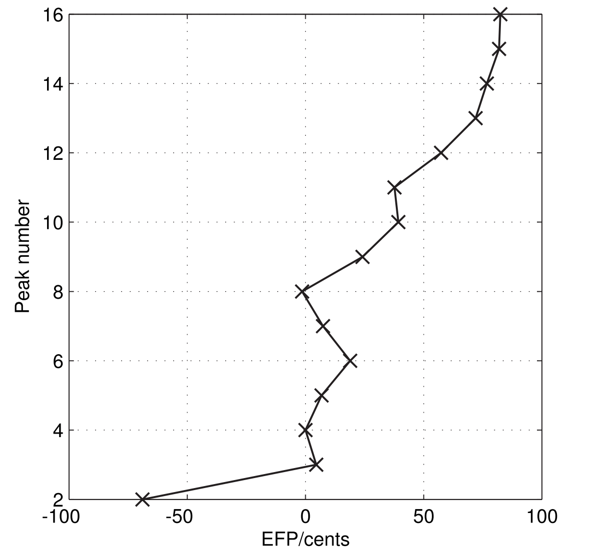

FIG. 2. EFP plot of a typical large-bore tenor trombone, showing resonances 2–16 (F=58.3 Hz).

frequencies, peak amplitudes, and peak widths. In Sec. III an improved computational method for bore optimization is described, and its use is illustrated in Sec. IV by predicting the bore changes required to achieve specified changes in the frequencies and amplitudes of input impedance peaks of a trombone. Finally, Sec. V discusses the usefulness and limitations of the currently available techniques, and reviews the work which needs to be done to provide a firm foundation for the choice of input impedance targets for brass instrument optimization.

II. INPUT IMPEDANCE TARGETS FOR OPTIMIZATION

A. Measurement and analysis of input impedance curves

A typical input impedance curve for a tenor trombone is illustrated in Fig. 1. Peaks in the curve occur at frequencies corresponding to air column standing waves with a pressure antinode at the mouthpiece. Since the lips behave as a pressure-controlled valve, each impedance peak is close in frequency to one of the pitches which can be strongly sounded; these pitches are known as the natural notes of the instrument.

While an input impedance plot unquestionably contains the essential information about the linear acoustical behavior of an instrument, a number of important features are not obvious when examining such a plot. The most important of these features is probably the mode inharmonicity, which describes the extent to which the peak frequencies deviate from a single harmonic series. Chick *et al.*[10] used the method of *Equivalent Fundamental Pitch* to plot the mode frequencies of horns in a manner which highlights the deviation from perfect harmonicity. For the ith resonance f_i, the fundamental frequency of which it is an exact ith harmonic is calculated, along with the intonation of this equivalent fundamental pitch relative to an arbitrary reference frequency F. This gives a measure, in cents, of the individual harmonic alignment of the peak frequencies, and therefore how closely the peaks collectively match a harmonic series. An example EFP plot is given in Fig. 2; the modes of an instrument with perfect harmonicity would lie on a vertical line on this plot. In this paper, F is chosen such that the fourth resonance lies on the vertical axis.

Although the inharmonicity of the resonances is easily read from an EFP plot, one major disadvantage is the lack of data describing the magnitude and Q-factor of the peaks. The influence of a small, narrow, harmonic peak may be somewhat less than that of a tall, broad inharmonic peak; the EFP does not represent this and can therefore be misleading. For this purpose, an *inharmonicity plot* is introduced; this is a development of the EFP plot including both the magnitude and the Q-factor. For each mode, the impedance magnitude at a series of frequencies corresponding to a pitch interval ± 100 cents around the reference frequency F is plotted on an EFP plot as a thin horizontal band, with color or grayscale denoting the impedance magnitude. Figure 3 shows the inharmonicity plot of a typical tenor trombone (this, along with the EFP plot in Fig. 2, is taken from the impedance curve in Fig. 1). The inharmonicity, relative magnitudes, and shapes of the peaks can be viewed simultaneously on this plot; an instrument with strong narrow and harmonic impedance peaks would appear as a narrow white vertical band.

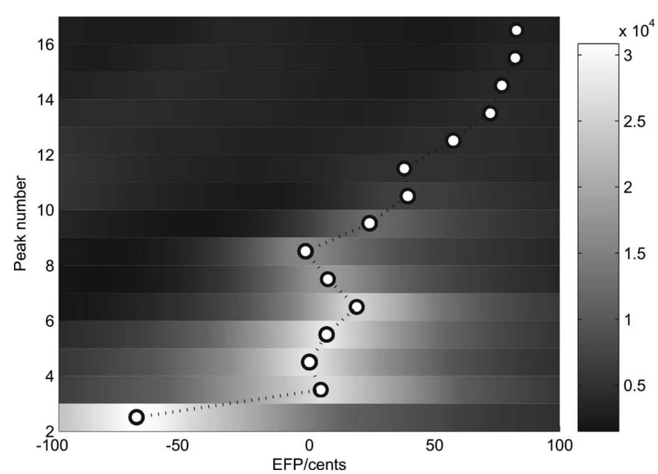

FIG. 3. Inharmonicity plot of a typical large-bore tenor trombone, showing resonances 2–16 (F=58.3 Hz).

B. Inharmonicity and mode-locking

Backus and Hundley[19] first suggested that an approximately harmonic alignment of the resonator peaks would help to maintain stable oscillations of the lips. For example, B♭$_2$ played on a trombone excites the resonances numbered 2, 4, 6, etc., represented by the corresponding input impedance peaks (Fig. 1). Fletcher[20] pointed out that, if the inharmonicity of the modes were sufficiently small, a self-sustained vibration regime could be established in which several modes maintained a fixed phase relationship. This mode-locked regime was described by Benade[21] as a "co-operative regime of oscillation."

Because of the non-linear character of the lip valve, the relative influence of each of the higher resonances on the regime is dependent on the dynamic level played; qualitatively, at pianissimo the excitation is nearly sinusoidal and only the sounded resonance has a significant influence, whereas at forte (excluding "brassy" cases[22]) the higher resonances have a much larger effect. It should be noted that some peaks (e.g., 4, 6, and 8) are part of the regimes of several sounded notes, whereas others (those of prime index) contribute only to one regime.

Worman[23] attempted to describe the overall intonation and resonance characteristics of a regime based on the sum-function

$$\mathrm{SF}(f) = \frac{1}{n}\sum_{n} \mathrm{Re}(Z_0(nf)), \tag{1}$$

where n is the harmonic number, Z_0 the input impedance, and f the frequency in question. Under this assumption, a perfectly-harmonic set of resonances would provide the tallest possible peaks in the sum-function, and therefore offer the strongest response to the player. The harmonic content in the radiated sound would be maximized (for the given peak magnitudes) and the intonation of the played regimes would have integer relationships. A modified version of the sum-function, altering the weighting such that higher peaks have a progressively greater influence at higher dynamics, was originally proposed by Worman and has recently been implemented in commercial software.[24]

The most strongly coupled oscillation regime would occur for a perfectly-harmonic series of acoustical resonances. However, it is not obvious that this should be the goal of the brass instrument designer, who must balance various issues including timbre, playability, stability, and intonation. A perfectly-harmonic series of bore resonances may produce a very stable oscillation, provided the player can match the embouchure to the note, but the resulting regime would be very difficult to "bend" or "lip" in musically expressive ways that alter the playing frequency of the note. Jazz players, and performers on natural instruments like the baroque trumpet, often require to bend a note away from its nominal pitch, a technique that is certainly easier when the instrument bore has some inharmonicity.[25]

The structure of musical scales also implies that a perfectly-harmonic series of resonances would not necessarily yield perfect musical intonation in all circumstances. Apart from the octave, none of the intervals of the equally-tempered scale corresponds exactly to an interval of the harmonic series; playing in tune requires frequent subtle modifications in playing pitch to fit the musical context. The balance to be drawn between stability and flexibility of intonation will depend on the playing style and repertoire of the individual musician.

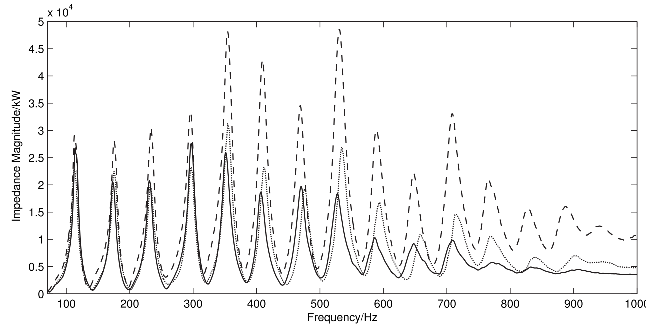

FIG. 4. Measured impedance plots for trombones A (dashed), B (dotted), and C (solid).

C. Input impedance measurements on high-quality trombones

In choosing an instrument, a professional player takes many different criteria into account.[6,8] To investigate how these choices are reflected in the input impedance curves, impedance measurements were made on three contrasting professional-standard models of trombone, all from the same highly-regarded British manufacturer. Trombone A was a medium-bore jazz trombone, B was a large-bore orchestral tenor, and C a bass trombone; each was fitted with an appropriate mouthpiece. The measurements were made using the BIAS input impedance measurement system.[24]

Figure 4 shows the input impedance magnitude plots. The most striking difference is in the peak heights for trombone A, which are consistently taller than for the larger bore instruments. The peak heights of the bass trombone C are generally, but not always, lower than those for B, particularly in the upper range of the instrument. The peaks of A in the upper range are particularly tall, which may be linked with the comparative ease of playing in this register with this instrument.

From these magnitude plots are produced inharmonicity plots (Fig. 5). It is clear that the peak frequencies do not represent an exact harmonic series. For each trombone, the first mode is several semitones below the nominal pitch of B$^\flat_1$ and is not shown on the EFP plot. The second peak is also rather flat, although markedly less so for the bass trombone C than for the tenor trombones. Given the similar peak heights and shapes in the lower ranges of these instruments, and the knowledge that the bass is generally preferred to the tenor for low pitch music, this preference can be tentatively linked with the large difference in the alignment of mode 2.

Each instrument shows a general right-hand diagonal trend on the EFP plot, denoting that higher peaks are tuned to a progressively sharper equivalent fundamental; the effect is considerably lesser in the smaller trombone. Similar results were taken with the same BIAS equipment on French Horns,[10] and demonstrate that B♭ horns have peaks which

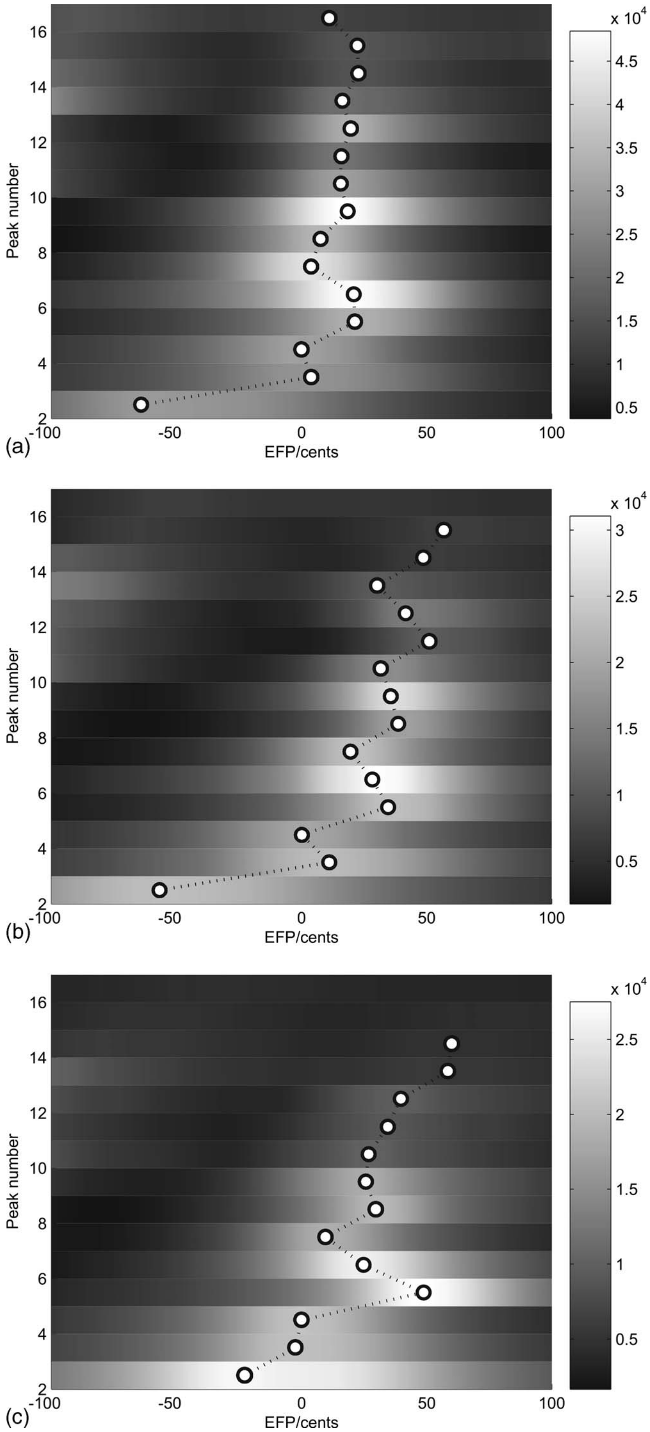

FIG. 5. Inharmonicity plots, showing resonances 2–16.

are considerably closer to a harmonic series than those of the trombones here. From these two studies, given that these instruments are considered of first-rate quality, it can be concluded that inharmonicity of this kind is a desirable feature contributing to the characteristic sound of the trombone, and particularly the large-bore trombone.

III. BORE OPTIMIZATION USING SPECIFIED INPUT IMPEDANCE TARGETS

For some years now, computer optimization techniques have been applied to brass instrument design problems. By taking an established model to calculate the impedance of an instrument with given bore-profile, and combining it with a search algorithm, a computer can be used to find the bore-profile whose impedance best matches specified performance characteristics (such as the location of resonances). Such optimization techniques allow many possible instrument designs to be tried quickly, without the material expenses of building many prototypes. Optimization can be used for *bore reconstruction* (deducing an unknown bore from a known impedance) and for *performance optimization* (modifying an existing design to alter certain characteristics while maintaining others).

The first attempt at such optimization was by Kausel,[4] representing the instrument using a piece-wise linear interpolation of points along the bore ("point-wise") and optimizing with the 0th-order Rosenbrock search algorithm. This method is commercially available and has been used by a manufacturer to design trumpet leadpipes.[18] Noreland[5,26] used a point-wise representation of the second derivative of the bore radii and the gradient-based Levenberg–Marquardt algorithm to give smoother bore-profiles. Both of these techniques assume plane-wave propagation. Similar techniques have been applied to the optimization of xylophones[27,28] and to leak detection.[29]

This section describes briefly the computer optimization technique developed by Braden, further details of which are given elsewhere.[15,30] This technique combines n-mode impedance modeling with the Rosenbrock optimization algorithm to search within a space of possible instrument designs to find that one whose input impedance best matches target criteria. The technique given is well-suited to optimizing complete instruments with realistic geometries, giving smooth bell flares. We show here results demonstrating the latter, using a design space constrained to include only "reasonable trombone shapes."

Any design optimization problem must combine a numerical representation of the object being designed, a quantitative method of evaluating possible designs, and an algorithm for the next "try" given previous tries.

A. Objective function

Input impedance magnitude is an objective acoustic property, contains much useful information about the performance of the instrument, is experimentally measurable, and can be calculated numerically; this combination of features allows us to center our optimization around it. The peak locations (frequency and magnitude) have the greatest influence on the performance, though other properties, such as the width and shape of peaks, also have an effect. The aim is to find the instrument shape whose impedance properties best match certain targets; these targets will be defined in terms of another impedance curve and/or certain specific characteristics of the impedance peaks.

Given an instrument shape, which is denoted symbolically by α, a method for computing the input impedance must be chosen. Previously, this has been a plane- or spherical-wave model,[31] which both Kausel and Noreland employed; the results in this paper were also computed using this method. However, the optimization technique described here has also been successfully employed[15] with a model including higher modes.[12,14,32] Qualitatively similar optimization results were found with either model (with appropriate targets), so either model could have been chosen in principle. In practice, the computationally rather less expensive plane-wave model is more convenient for repeated experiments investigating the full capabilities of the optimizer, as is the primary goal of this paper, and was therefore chosen.

An objective function must be chosen to compare the impedance properties of each "tested" instrument to target impedance data. Three objective functions are used here in different combinations depending on the task, each giving scores between 0 (a perfect match) and 1 (no match). Typical tuning-parameter values are given. The objective functions are formulated as a minimization problem. The first function is a windowed linear least-squares comparison of the entire (discrete) impedance magnitude curve with N_{pt} points:

$$\mathcal{O}_1(\alpha) = \frac{1}{N_{\mathrm{pt}}} \sum_{i=1}^{N_{\mathrm{pt}}} \begin{cases} \dfrac{(\delta z_i)^2}{\nu^2} & \text{if } \delta z_i < \nu, \\ 1 & \text{if } \delta z_i \geqslant \nu, \end{cases} \tag{2}$$

where $\delta z_i = |z(f_i) - \bar{z}(f_i)|$, i.e., the difference between tested impedance magnitude z and target $\bar{z}$ measured at a series of frequencies f_i, and ν is the half-width of the window (usually 100–300 kΩ). Any valid method for computing $z(f_i)$ from α may be used. Both Kausel and Noreland used this least-squares approach but without the windowing, which eliminates cases where a very bad feature can overwhelm the more subtle difference in score between another feature being good and very good, thus improving overall convergence. In addition to the least-squares approach, a windowed-Gaussian comparison is used, scoring only the peak frequencies:

$$\mathcal{O}_2(\alpha) = \frac{1}{N_{\mathrm{pk}}} \sum_{i=1}^{N_{\mathrm{pk}}} \begin{cases} \dfrac{1 - \exp\left(\dfrac{-\mu_\phi (\delta\phi_i)^2}{\nu_\phi^2} \right)}{1 - \exp(-\mu_\phi)} & \text{if } \delta\phi_i < \nu_\phi, \\ 1 & \text{if } \delta\phi_i \geqslant \nu_\phi, \end{cases} \tag{3}$$

where $\delta\phi_i = |\phi_i - \bar{\phi}_i|$, i.e., the difference between tested peak-frequency ϕ_i and target peak-frequency $\bar{\phi}_i$ of peak i, the window half-width ν_ϕ (typically 10 Hz), μ_ϕ is a "strictness" parameter (typically 20), and N_{pk} the number of impedance peaks being tested. The function joins smoothly at the window bounds $\phi_i \pm \nu_\phi$ to take a value of 1. The gradient of the Gaussian function is much steeper at a moderate distance from the target than the equivalent least-squares function, and consequently offers improved convergence speed in this region.

A very similar function $\mathcal{O}_3$ is also used for the peak heights, using difference $|z(\phi_i) - \bar{z}(\bar{\phi}_i)|$ and magnitude-dependent window half-width $\nu_z z(\phi_i)$, with parameters ν_z and μ_z (typically 0.05 and 10, respectively). An overall objective function is then defined as

$$\mathcal{O}(\alpha) = \frac{\zeta_1 \mathcal{O}_1(\alpha) + \zeta_2 \mathcal{O}_2(\alpha) + \zeta_3 \mathcal{O}_3(\alpha)}{\zeta_1 + \zeta_2 + \zeta_3}, \tag{4}$$

where $\zeta_{1,\ldots,3}$ are weights which can take any real value, but are usually 0 or 1. The goal is to find the α which minimizes $\mathcal{O}(\alpha)$.

B. Instrument representation

The convergence rate of optimization algorithms is strongly dependent on the number of design variables N_v and the size of the design space; by reducing each of these, the performance of the optimization can be improved. Care must be taken to avoid pre-determining the eventual solution by constraining the space too restrictively; the optimizer must be given enough freedom to explore many reasonable solutions.

In Kausel's method, no attempt is made to constrain inappropriate designs, so many outlandish shapes are possible; for example, Fig. 6 in Ref. 33 is too "jagged" to be a realistic instrument. Noreland's smoothing method makes jagged results unlikely, but instead "wiggly" shapes are common (for example, Fig. 14 in Ref. 26). Designs such as these would be rejected out of hand by any manufacturer, regardless of their playing characteristics; the present method attempts to remove them from the design space altogether.

To a first approximation, a trombone is a long cylindrical tube, of given length and radius, connected smoothly to a Bessel-horn, defined as

$$r(x) = b(-x)^{-\gamma}, \tag{5}$$

where r is the bore radius a distance x along the axis of instrument, γ is a flare constant, and b is a constant defined in terms of specified length and input/output radii. Given appropriate constraints, it is possible to define a five-dimensional space (i.e., $N_v = 5$) of possible designs, all of which are reasonable (albeit simple) trombone shapes. Bessel-horns, however, are not exact matches for real trombone bells, so the design space here is too limited. The bell is therefore described as a number of shorter Bessel-horns placed end-on-end,[30] to achieve much closer *piece-wise Bessel-horn interpolations* of real instruments. By confining the design space in this manner, many unacceptable possible solutions have been eliminated, significantly reducing the size of the design space. As with other piecewise interpolation techniques, a small discontinuity of gradient at the joints between pieces must be accepted; some unrealistic designs with large gradient discontinuities therefore exist in the space, but are proportionately far less frequent than for other representations.

In practice, the shape of a real trombone bell can be very closely described by five Bessel-horns of different flares, giving $N_v = 12$, where one variable is the length of the sections, six describe the input and output radii of the sections, and five give their flare coefficients (previous approaches typically require $N_v \sim 100$). Use of such a higher-order parametrization of the design of the instrument therefore confers a significant reduction in the number of design variables as well as a reduction in the size of the design space, giving an increase in optimization speed.

Other practical necessities in realistic instrument design, such as discontinuities caused by the joins between sections, tuning slides, etc., can be included in such parametrizations. In this manner, a description can be given of the complete general shape of the instrument to be designed;[30] this is termed a *design template*, and it is a representation of the detailed geometry of the instrument without the exact dimensions. A template can be constructed that can describe all trombones fairly closely; however, in practice it is more effective to have more detailed templates for more specific problems, such as tenor and bass trombones. Preset mouthpiece geometries can be specified and held fixed throughout the optimization.

Approaching optimization problems in this manner trades off generality against improved convergence. One of the advantages of the pointwise approach was the suitability to any given problem with little or no prior knowledge of the instrument shape—so-called "black-box" problems. The approach given here assumes that the rough shape of the instrument is already known, while this is less attractive in principle, the disadvantage is negligible in practice. Given that the primary objectives are the reconstruction and optimization of brass instruments, it is not unreasonable to assume that the class of the instrument (e.g., trumpet or trombone) is already known, and therefore that the rough shape is also known; a template can thus be chosen from prior knowledge of these instruments. The constraints are set to be large enough that the design space includes all known instruments of that class, so that it is still general enough to include all solutions to the specific problem in hand, but removes many of the unreasonable solutions that another, more general, approach would include. This approach favors pragmatism over rigorous generality, but it should be noted that templates can be constructed to consider general problems in the same (point-wise) way as Kausel,[4] so, rather than being lost, this generality is merely put aside unless needed.

C. Optimization algorithm

Noreland successfully applied a gradient-based method to instrument optimization. Use of such methods, however, requires a derivative of the objective function with respect to the design variables, and therefore a derivative of the input impedance. Noreland provided such a derivative for the plane-wave model. The technique described here was developed to be applicable to a broader range of impedance techniques, including methods for higher modes[14] and bent waveguides.[34] In principle, derivatives of these methods can be found and combined with a gradient-based method. Such derivation is outside the scope of the current work and has not yet been attempted; the present study is therefore restricted to the use of optimization algorithms which do not require derivatives. In the vein of Kausel, the Rosenbrock[35] method is chosen; this has been found to be superior to evolutionary approaches even in low-N_v parametrized problems.[30]

IV. OPTIMIZATION RESULTS

A. Tuning peak 2

It has been established through experimental comparison of a tenor and bass trombone that the tuning of peak 2 relative to the higher even-numbered peaks (particularly peak 4) has a significant effect on the lower playing register of the instrument. In this experiment, an attempt was made to improve a popular design of tenor trombone by shifting peak 2 into a closer alignment without compromising any other resonance properties.

The starting point was based on detailed geometrical measurements of a large-bore tenor trombone. The impedance of the instrument was then calculated with the spherical-wave model and the peak frequencies set as an optimization target, with the following modifications: the target frequency of peak 2 was shifted from 112.0 to 115.0 Hz (a shift of +45 cents), and the target frequency of peak 5 was shifted from 292.5 to 295.0 Hz (a shift of +15 cents). It was found that allowing freedom in the location of peak 5 granted greater flexibility in the location of peak 2.

Figure 6 shows that the optimizer produced a solution which closely matches the target set. Peaks 2 and 5 are now located at 114.7 and 295.1 Hz, respectively. Peaks 3 and 4 have not been significantly altered. It is evident from the EFP plot [Fig. 6(b)] that the desired re-alignment of peak 2 has occurred. As a compromise, the higher peaks have all been shifted by between +5 cents and +20 cents; these are all rather smaller than the +41 cents shift of peak 2. The impedance magnitudes of the peaks have been affected, notably peaks 3–6, which have been shortened, and peaks 8 and 9, which are taller.

Figure 6(c) shows the optimized bore-profile. The bore design is smooth, realistic, and could be built by a manufacturer with no additional difficulty; it can be described as a modified version of an existing design. The bore of the cylindrical section (i.e., the main slide) has been reduced from 6.95 mm radius (or, in the conventions of the instrument industry, 0.547 in. diameter) to 6.63 mm radius (0.522 in. diameter). The overall instrument length is some 26 mm shorter than before, and the bell contour has been subtly altered, as has the taper of the tuning slide. Given the performance target, these results are somewhat surprising; since the optimization is attempting to replicate a feature of a bass trombone, it might have been expected that the bore of the cylindrical section would have increased.

Clearly the trombone design space is complicated. Certain regions of high performance are well-established in the industry, and it may be that the optimizer has uncovered a region which might have otherwise been unexplored. Musical judgments of the success of the optimization will not be available until an example has been built and tested; however, the exercise has demonstrated the capability of the software to perform whole-instrument intonation optimization.

B. Modifying peak magnitudes

In a second test of the optimization software, an attempt was made to modify the magnitudes of specified impedance peaks. The target impedance was again a modification of the

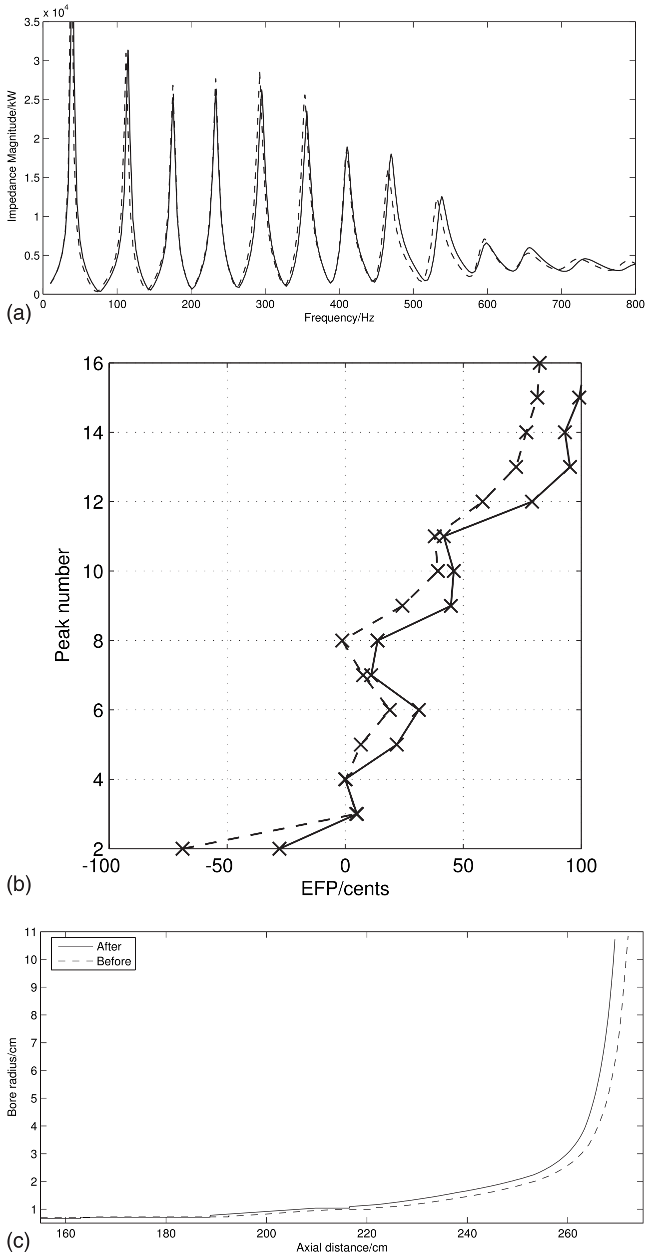

FIG. 6. Results of tuning peak 2 before (dashed) and after (solid) optimization.

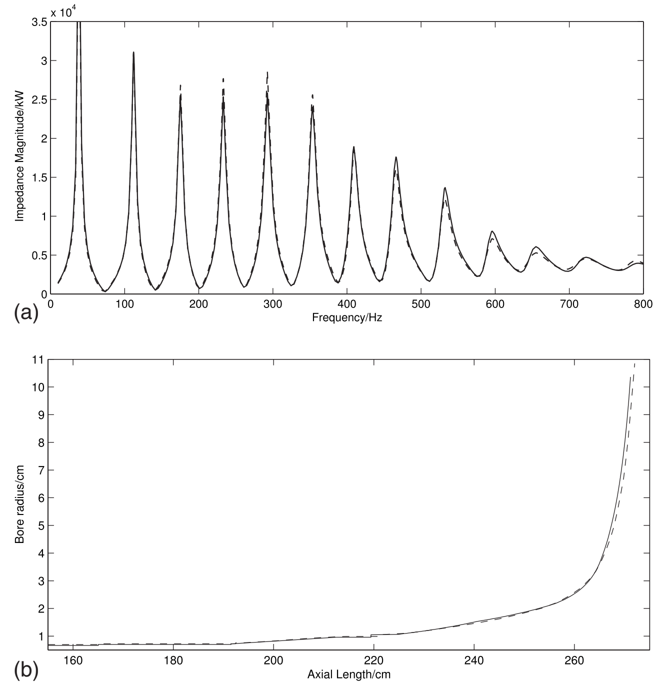

FIG. 7. Results of increasing the magnitudes of peaks 8–10 before (dashed) and after (solid) optimization.

measured impedance curve for the large-bore tenor trombone: this time peaks 8, 9, and 10 were targeted to increase in magnitude by 10%, and no target was set for the magnitude of peaks 1–7. The peak frequencies were left unchanged.

It can be seen from Fig. 7(a) that the optimization was successful, increasing peak 8 from 16 110 to 17 602 kΩ, peak 9 from 12 358 to 13 668 kΩ, and peak 10 from 7094 to 8064 kΩ, respectively, increases of 9.2%, 10.6%, and 13.6%. Certain lower-index peaks were reduced in amplitude, most notably peak 5 from 28 671 to 25 820 kΩ, a decrease of 9.9%. The frequency (and therefore EFP) of peaks 2–10 was not significantly changed; higher peaks were each made roughly 5 cents sharper. The bore-profile [Fig. 7(b)] again shows a reduction in the radius of the main slide from 6.95 mm (0.547 in. diameter) to 6.65 mm (0.524 in. diameter), and some subtle changes (a maximum of 0.5 mm) to the bell contour. The experiment was repeated but with 20% increases specified; the optimizer was unable to converge to a geometrically-satisfying solution, suggesting that such a result was not possible within the constraints of the design space.

These results demonstrate that it is possible to modify the impedance envelope without substantially modifying the peak intonation. The changes made here to the envelope are of similar proportion to those found in the previous experiment, but had only a small effect on intonation. As a result of these changes, the optimized trombone would be expected to produce a somewhat brighter timbre than its predecessor and possess a higher register which is somewhat easier to play.

V. CONCLUSIONS

Modern brass instruments have sets of acoustic resonances which approximate closely to a harmonic series. When a normal note is sounded, the player's lips interact with a subset of these resonances to create a mode-locked regime of oscillation. The pitch of the radiated sound is that of the lip vibration, which will not necessarily correspond exactly to one of the acoustic resonance frequencies. The magnitude of the impedance at the sounding frequency and at its harmonics will affect the stability of the note and the timbre of the sound.

Is it a reasonable assumption that the peak frequencies of an "ideal" brass instrument should be harmonically related? Experimental measurements of several professional-standard models of trombone, intended for use in orchestral and in jazz idioms, suggest that this is not the case: having higher resonances tuned to a progressively sharper equivalent fundamental appears to be a desirable characteristic of trombones, particularly those of larger bore. From a purely musical standpoint, the designer's objective is not to maximize the stability or brightness of tones, but to produce an instrument which is easy to play, sufficiently in tune, and produces a timbre which is appropriate to the musical style and context of the performance.

Why might the inharmonicity present in a large-bore trombone be desirable? Consider a hypothetical trombone with many tall harmonic peaks (similar to trombone A in many respects). This instrument would be expected to have a large tonal range and produce a relatively bright timbre.[6,21] If this brightness were undesirable (as it generally is in orchestral settings), then a designer might combat this by reducing the heights of the middle and upper peaks, thus reducing the harmonic content in the sound. However, this would have the knock-on effect of reducing the stability of the higher tones. In order, therefore, to reduce the brightness without affecting the tonal range, some inharmonicity is necessary to reduce the influence of the higher peaks on the lower tones. In reality the orchestral tenor trombone may be thought of as a compromise between these two solutions, featuring shorter, less harmonic peaks than the jazz trombone, but having a similar tonal range. Trombonists are accustomed to making small corrections in the position of the hand-slide in any given musical situation for the purposes of fine intonation, and are therefore likely to be more tolerant of small alterations to the intonation of the instrument than would the player of a valved instrument.

The study described here has demonstrated the ability of an improved optimization procedure to propose technologically feasible and visually acceptable brass instrument bore shapes whose impedance curves correspond to a predetermined target. When the target requirements are compatible with the limitations imposed on the bore by non-acoustic design considerations, the optimization converges rapidly to a solution: roughly 1500 designs of the complete instrument are needed to converge to within a design tolerance of 0.01 mm. Lack of convergence indicates that the requirements of the target cannot be met without relaxing the limitations imposed on the design space.

Discrepancies between calculated and experimentally measured input impedance curves[12,15] currently limit the applicability of computer optimization to the solution of subtle problems in real instrument design. The optimization method described here is applicable to any impedance model, and any future improvements in numerical modeling could be immediately applied to optimization problems. This provides a strong motivation to develop further the underlying theoretical models and experimental impedance measurement techniques.

It should also be recognized that the input impedance describes only the linear behavior of the acoustic resonator. Since the interior acoustic pressure in a brass instrument can reach 180 dB in loud playing, non-linear sound propagation plays an important role in the development of timbre with increasing dynamic level. Once the bore-profile has been determined by the optimizer, non-linear effects can be calculated and their significance estimated.

The major limitation in the optimization of brass instrument design remains an inadequate understanding of the ideal target impedance function. The work of Poirson[6] on trumpets has shown the feasibility of a multidimensional scaling approach to finding correlations between player judgments of instruments and objective impedance criteria. Similar studies on a wide range of players and instrument types are required, in order to establish clearly which features of the impedance curves correlate closely with player ratings based on musical judgments. With this information available, the optimizer can become a valuable tool in the production of brass instruments of the highest quality.

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