MANAGEMENT AND AGGREGATE PRODUCTIVITY

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ABSTRACT. This paper proposes a novel mechanism to answer why firms in low income countries are badly managed, and quantifies the resulting productivity loss. First, I present empirical evidence on a significant positive correlation between the share of managerial workers and contract enforcement across countries. Second, I construct a tractable model that captures benefits to managerial delegation in large organizations. The model also features an agency problem between the owner of a firm and its middle management. Ineffective contract enforcement, allowing middle managers to steal from the firm, constrains firm size by limiting the efficient delegation of managerial authority. Third, I use a calibrated version of the model to measure the effect of lowering contract enforcement. Compared to the benchmark of US contract enforcement, no enforcement decreases the aggregate share of managerial workers by about 10 percentage points, typical of countries with income levels of about one-tenth of the US. The associated loss in aggregate labor productivity is roughly 18 percentage points. Auxiliary statistics on the mean firm size, self-employment and productivity dispersion offer additional empirical validation of these results.

1. Introduction

Why is income per capita so different across countries? Much of the evidence shows that factor endowments such as physical and human capital fall short of quantitatively matching the bulk of the difference. To explain the remaining total factor productivity (TFP) gap, the literature has increasingly focused on the institutional environment of economies. In this context, recently collected data on managerial performance across countries provide valuable insights. In a series of papers Bloom and his co-authors demonstrate that firms in poorer countries are badly managed. Their finding is that, by performing relatively minor and cheap changes in the daily management (e.g. improving monitoring, target setting and incentive schemes to modern management standards) firms could potentially boost output per worker significantly. Importantly, these studies suggest that one major source of managerial inefficiency in less developed countries is insufficient delegation of decision-making. Hence, many efficiency-enhancing measures are left on the
table; workers who are best informed about particular problems are not endowed with sufficient authority to solve them. This paper addresses the phenomenon of poor management in three ways. First, I present evidence that, in the aggregate, relatively few workers in less developed countries are employed in problem-solving positions, i.e. there are relatively few “managerial workers” as I refer to them henceforth. I then construct a theoretical model where the scarcity of managerial workers is a result of sub-optimal decentralization within firms. In the model, this results from insufficient property protection and hence a higher risk of expropriation by middle management. Third, I use the model as a measuring device to gauge to what extent the underlying institutional weakness impacts GDP per capita.

The theoretical model features firms that are heterogeneous in their efficiency and that hire production and managerial workers to produce output. The technology is such that more efficient firms, wishing to employ more workers, have an incentive to disperse managerial tasks across a larger number of managerial layers. I call this “delegation.” Delegation also implies that the relative share of problem solvers in the firm increases. At the same time, each additional layer of middle managers may divert revenue from the firm. In equilibrium, owners in the model compensate managerial workers for not stealing, which makes delegation more costly. Countries in the model differ institutionally by the share of revenue that managers can expropriate.

Theoretically, the model delivers the following findings. The share of managerial workers for similarly efficient firms is lower in countries with poor property protection. This is a direct consequence of a shorter managerial hierarchy. The resulting drop in the firm’s managerial quality - a manifestation of misallocation within the firm - translates into bad management practice. In addition, more efficient and therefore larger firms suffer relatively more from poor property protection than their smaller peers, especially since own-account workers and firms with single management layers face no incentive incompatibility. This implies that ineffective property protection inefficiently channels resources into relatively unproductive firms, creating misallocation across firms. Therefore, such an economy features smaller production units and depressed output.

The empirical application of the model rests on the observation depicted in Figure (1), namely that the overall share of managerial workers is significantly lower in poor relative to rich countries. Estimation results presented in the subsequent section show that differences in the level of education and - crucially for this paper - differences in an indicator of contract enforcement tend to explain the share of managerial workers across countries, not GDP per capita itself. As contract enforcement is difficult to quantify directly, I use the model’s prediction on the share of managerial workers to infer it indirectly. I calibrate the model to match the ratio of managerial workers in the US along with data on the US firm size distribution. Subsequently I vary the institutional parameter to match the managerial share in poorer countries.

I find that in the extreme case of no property protection, the simulated managerial share drops by 10 percentage points. Such a drop in the managerial share in the data is associated with countries having income levels of about one-tenth of the US. At the
same time the model predicts labor productivity in such an environment to decline by about 18 percent compared to the US. While the mechanism in the present paper does not per se generate the order of magnitude in productivity differences across countries that are observed in the data, it goes some way in addressing them by offering an additional source of inefficiency. Also, as the model abstracts from any kind of accumulated capital, productivity losses are equivalent to pure TFP losses.\footnote{Moreover, the model is only concerned with the non-agricultural business sector where productivity differences between the poorest and richest countries are far less pronounced than in the aggregate economy. The difference is a factor of 5 in the non-agricultural sector and about 32 overall according to Restuccia, Yang and Zhu (2008).}

The identification procedure appears robust in the sense that the model performs well on a variety of other features that characterize differences between rich and poor countries. It is shown that the lack of property protection induces a significant drop in the average firm size, a widely observed property in poor countries (see for instance Tybout (2000) for a review). Also, as individuals in the model know their productivity in running their own business, similarly to Lucas (1978), many choose to become self-employed rather than work for a relatively low wage. This generates a substantial amount of self-employment, another feature typical in poor countries (see for instance Gollin (2008)). Furthermore, I show that poor property protection causes a rise in misallocation, measured as productivity dispersion along the lines of Restuccia and Rogerson (2008), Hsieh and Klenow (2009) and Bartelsman, Haltiwanger and Scarpetta (2009).

This paper is closely related to the literature analyzing the link between credit frictions and TFP across countries, such as Greenwood, Sanchez and Wang (2009), Amaral and Quintin (2010), Buera, Kaboski and Shin (2011), Moll (2010) and Midrigan and Xu (2010) and Caselli and Gennaioli (2011). These papers have in common a game between the capital-provider and the entrepreneur where poor institutions decrease the flow of credit. Here, the friction is similar, only that the game is played out inside the firm rather than between the owner and his middle management. Another difference is that credit frictions can be partially circumvented through retained earnings, while the problem of trust within firms is a permanent state.

In a different vein, this paper also builds on theoretical studies of the problem of delegation of authority within firms. The classical trade-off is that of the costly state-verification process. The principal would like to delegate tasks but needs to employ resources to control for the outcome, as for instance in Townsend (1979). Aghion and Tirole (1997) show that principals (firm owners) may have an interest in exercising less control over agents when the latter are better informed because this boosts their incentive for initiatives. Another delegation problem is considered in Rajan and Zingales (2001) where the principal faces a trade-off between enhancing productivity by delegating knowledge and the risk of encouraging the creation of spin-off from the firm. Dessein (2002) combines an environment in which communication is costly with an agency problem to analyze under what circumstances delegation from uninformed principals to informed agents is optimal. Finally, the present paper is most closely related to Garicano (2000) where delegation is represented by a knowledge hierarchy in which the most important tasks are optimally delegated to the bottom of the hierarchy to save on communication costs, while upper echelons of the hierarchy specialize in solving less common tasks. The model here captures the rationale for a hierarchy more tractably by leaving out a more structural interpretation. It also add a commitment problem between middle managers and the entrepreneur on top of the technological trade-off.

The next section describes the empirical motivation. Section 3 then presents the model environment and Section 4 the theoretical implications of the stationary equilibrium. In
Section 5 I describe the calibration of the model and the model’s predictions for different institutional environments. I end with concluding remarks.

2. EMPIRICAL MOTIVATION

Figure (1) presents a striking negative cross-country correlation between the share of what I label “managerial workers” and GDP per capita. The employment data stems from the ILO and builds mainly on labor force surveys. For each country I compute the number of employees categorized as managers, professionals and administrative workers (categories 0, 1 and 2, 3 in the ILO classification) and divide it by the total working population excluding agricultural and non-classifiable workers (categories 6 and X). Non-managerial workers, labeled “production workers” henceforth, are clerks, service and sales workers, craft and related trade workers, plant and machine operators and workers in elementary occupations (4, 5, 7, 8 and 9). I then compute averages over the sampling years 1999 to 2008. GDP per capita is taken from the Penn World Tables and represents averages over the same time period.

\[
\log \left( \frac{\text{share}_i}{\text{share}_{US}} \right) = \alpha + \sum_j \beta_j \log \left( \frac{x_{j,i}}{x_{j,US}} \right) + \epsilon_i.
\]

Table (1) presents the resulting OLS regressions. The first column reports the regression with GDP per capita (the US income per capita normalized to 1) being the only explanatory variable. This reflects the observation discussed in Figure (1), i.e. for a decrease in GDP per capita of 1 percent (relative to the US), the proportion of managerial workers is expected to drop by 0.23 percent vis-a-vis the US (which has a managerial share of 0.35). The correlation is strongly significant in the statistical sense.

In the second and third column I control for the sectoral composition of the workforce by adding the share of workers employed in services and agriculture as well as the share of government spending, which arguably acts as a proxy for the share of public employees.
The data are derived from the World Bank and the Penn World Tables, and represent averages over the period 1999 through 2008. The share of service workers as well as government spending come out statistically significant, which indicates that managerial workers may be more prevalent in the service industry and the public sector. Notice that GDP per capita, however, keeps its strong explanatory power. In the next column I add another obvious candidate, which is the share of the population with completed or attempted tertiary education, taken from the dataset in Barro and Lee (2005). This statistic is again constructed as country averages over the period of interest. It seems intuitive that countries with higher educational attainments have more managerial workers assuming that the latter are characterized by higher skills. Education turns out to be an important explanatory variable yet GDP per capita still appears highly significant (both statistically and economically) in explaining occupations across countries.

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coeff (SE)</th>
<th>Coeff (SE)</th>
<th>Coeff (SE)</th>
<th>Coeff (SE)</th>
<th>Coeff (SE)</th>
<th>Coeff (SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.025 0.013 0.038 0.054 0.134** 0.166**</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP/capita (%)</td>
<td>0.230*** 0.160** 0.164*** 0.151** 0.049 0.001</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Services employment (%)</td>
<td>0.364** 0.348** 0.043 0.113 0.197</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agriculture employment (%)</td>
<td>0.037 0.035 0.007 −0.032 −0.027</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government spending (%)</td>
<td>0.101 0.145* 0.131* 0.163*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tertiary education (%)</td>
<td>0.150*** 0.172*** 0.158***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contract enforcement (%)</td>
<td>0.707*** 0.676***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Self-employment (%)</td>
<td>−0.079</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>125 122 122 105 99 93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.349 0.380 0.393 0.436 0.472 0.495</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Subsequently, I add to the regression an indicator for contract enforcement, as in the remainder I will argue that contract enforcement (or rather the lack thereof) is the crucial driving factor of the share of managerial workers across countries. For this I use data from the World Bank’s Doing Business database on the cost of suing for a claim as a percentage of the value of the claim and subtract it from 1 (i.e. the percent of the claim that is expected to be recovered in a lawsuit). Again, I use averages over the years of interest. Interestingly GDP per capita now loses statistical significance while contract enforcement turns out very significant, along with education and, to a lesser extent, the size of the public sector. Since the model will also create cross-country differences in firm size and the share of self-employment, I finally also add the proportion of employers and self-employed in the population that is available from the ILO and similarly covers the period 1999-2008. One could argue that very small firms (captured by a high number of employers and self-employed) demand relatively fewer managerial workers, but the variable turns out not to matter at all in the regression.

I take from these regression results two things. First, contract enforcement appears to matter in determining the managerial share. Second, in the remainder I will not use the above indicator for contract enforcement, but rather calibrate it indirectly and discipline
the calibration by targeting the number of managerial workers. Variations in the model-based indicator of contract enforcement will vary the share of managerial workers. To get a sense of what the model-based outcome in the share of managerial workers corresponds to in terms of a country’s GDP per capita, I will use the regression results in column 4, which does not use any explanatory variables assumed or generated in the model. In this sense, a country with GDP per capita equal to 0.1 of the US is expected to have a managerial share implicit from log \( \frac{\text{share}}{0.35} \) = 0.054 + 0.151\log(0.1), i.e. 0.26, as opposed to the US managerial share of 0.35.

3. Economic environment

The economy is populated by atomless infinitely-lived agents of measure 1. Each agent is endowed with one unit of time per period and a fixed and known level of project quality (or talent to run a firm) \( z \in \mathbb{Z} \) drawn from the cumulative distribution function \( G(z) \). An agent’s discounted utility reads \( U = \sum_{t=0}^{\infty} \beta^t c_t \) where \( \beta \in (0, 1) \) is the discount factor of time \( t \). I assume linear utility to switch off the risk component, which can be interpreted as a short-cut for having completeness of financial markets.

3.1. Occupational choice

At the beginning of each time period an agent with project quality \( z \) can run his own business and earn \( V^e(z) \) or enter the labor market to become a worker and earn an expected value \( V^{lm} \). Agents optimally choose \( V(z) = \max \{ V^e(z), V^{lm} \} \), with \( z \) denoting the threshold value such that \( V^e(z) = V^{lm} \). The value of being an entrepreneur is \( V^e(z) = \frac{\pi(z)}{1-\beta} \) where \( \pi(z) \) represents period profits, i.e. the agents’ choice of becoming entrepreneur is permanent.\(^7\)

Workers are ex-ante identical on the labor market. They have the same (endogenous) probability \( q_l(s) \in [0,1] \) of becoming managerial workers of firm \( s \in \mathbb{Z} \) in managerial position \( l \in \{1, 2, ..., L(s) - 1\} \). Ex-post, workers are only different to the extent that nature randomly assigns them according to the above probability distributions to be the first to negotiate a particular occupation and firm-specific contract. Workers are able to accept or decline the contract that they are offered, and in equilibrium they will always accept.

Signing a contract as a production worker hence procures \( V^n = w + \beta V^{lm} \), i.e. a worker earns a competitive wage \( w \) and subsequently receives the expected value of searching in the labor market. Signing a contract as a managerial worker in firm \( s \) and position \( l \), on the other hand, is valued at \( V^{xi}(s) \). The expected value of entering the labor market is hence \( V^{lm} = \int_{s \geq z} \sum_{l=1}^{L(s)-1} q_l(s) V^{xi}(s) + \left( 1 - \int_{s \geq z} \sum_{l=1}^{L(s)-1} q_l(s) \right) V^n \).

3.2. Production

An active entrepreneur with quality \( z \) maximizes profits by deciding whether to be an own-account worker, a firm with one single management layer (i.e. himself) or whether to run a business with multiple managerial layers, so \( \pi(z) = \max \{ \pi^{ml}(z), \pi^{ml}(z), \pi^{ml}(z) \} \).

\(^7\)The fact that this choice is permanent rests on the assumption that in the initial period there are no incumbent workers in any firm. However, if there were incumbent workers, the permanent occupational choice would just as well come about in the stationary equilibrium because there is exogenous separation, as will become clear below.
3.2.1. Self-employment and single-layer firms

Upon paying a period fixed cost $\kappa^{se}$, period profits of the self-employed entrepreneur are given by

$$\pi^{se}(z) = \max_{n,x} y - \kappa^{se} = \max_{n,x} \gamma n^{1 - \frac{\kappa}{\gamma}} - \kappa^{se},$$

subject to $n + x = 1$ where $n$ is the amount of production hours worked and $x$ is the amount of hours spent managing the business and solving problems. I assume that $\gamma, \theta \in [0,1)$ and - for the problem to be well-defined - that $\gamma + \theta \leq 1$. The parameter $\gamma$ is the technical intensity of managerial work, while the parameter $\theta$ is interpreted as the communication cost between managerial work and the hierarchy beneath, in this case the layer of production work. When $\theta = 0$, there is no communication cost and no loss of control, and the production function has constant returns to scale in problem-solving and production work. When $\theta = 1 - \gamma$, on the other hand, production work produces no value because it is not channeled at all into solving problems.

Alternatively, the entrepreneur pays a higher fixed cost $\kappa^{sf} > \kappa^{se}$ and runs an employer firm. This allows him to go beyond his size-constraint by employing workers. With a single management layer the firm’s profits then equal

$$\pi^{sl}(z) = \max_{n,x} [y - w(n + x - 1)] - \kappa^{sf} = \max_{n,x} \left[ zx^{\gamma} n^{1 - \frac{\kappa}{\gamma}} - w(n + x - 1) \right] - \kappa^{sf},$$

subject to $x \leq 1$.

3.2.2. Multi-layer firms

The entrepreneur’s third choice consists of setting up a firm with multiple management layers that take discrete values $L = \{2, 3, \ldots\}$. The profit function is

$$\pi^{ml}(z) = \max_{L=\{2,3,\ldots\},n,(x_{l})_{l=1}^{L-1},m \geq 1} \left[ y - wn - w \sum_{l=1}^{L-1} m_{l} x_{l} \right] - \kappa^{sf}$$

$$= \max_{L=\{2,3,\ldots\},n,(x_{l})_{l=1}^{L-1},m \geq 1} \left[ z \prod_{l=1}^{L-1} x_{l}^{\theta^{l-1}} n^{1 - \frac{\kappa}{\gamma}} - wn - w \sum_{l=1}^{L-1} m_{l} x_{l} \right] - \kappa^{sf},$$

assuming that the entrepreneur occupies the top echelon of the managerial hierarchy ($x_{L} = 1$), and subject to the participation constraint(s) of each middle manager $x_{l} \in \{1, 2, \ldots, L-1\}$,

$$V^{x_{l}}(z) \geq V^{x_{l},out}(z).$$

To gain understanding, first abstract from the participation constraints and the managerial markups $m$, and consider the trade-off faced by the entrepreneur who adds a second managerial layer ($L = 2$). The benefit consists of the option to increase the first line of problems solvers $x_{1}$ at will rather than facing the constraint $x_{1} \leq 1$. The cost, as compared to running a single-layer firm, is the entrepreneur’s opportunity cost of not employing his time at solving problems in this particular layer as he now oversees the production process from the highest hierarchical position, i.e. in this case $x_{2} = 1$. It is obvious that for a high enough $z$ he has an interest in doing so. Now consider the choice of employing additional layers of middle managers such that $L = 3, 4, 5, \ldots$ With decreasing returns to scale, it is evident that initial units of an additional managerial layer have a relatively stronger impact on productivity than marginal units of preceding layer. Note, however, that an additional layer $l$ only increases production for $x_{l} > 1$, which is clearly desirable only for a sufficiently high efficiency $z$. Also, average units in the additional layer of management are relatively less effective as the exponent $\theta^{l}$ decreases.
One possible economic interpretation for the concavity of each single managerial layer is that an additional managerial layer allows for more specialization in tasks. For the additional layer to be effective, however, it needs to be of a minimum size, which captures fixed costs in setting up a longer chain-of-command. Also, the decreasing effectiveness of each managerial layer arises naturally from a hierarchy in which communication is costly and higher layers are further away from the production problems. This mechanism can hence be interpreted as a reduced-form variant of the knowledge-hierarchy modeled in Garicano (2000). The notion of delegation or decentralization in the present model is hence simply the length of the chain-of-command.

Employing outside managerial workers is associated with a cost over and above the regular wage due to institutional reasons. I assume that the revenue of production flows upward in the hierarchy, i.e. it is first collected by the lowest managerial rank and then successively handed up through the layers until reaching the entrepreneur. This assumption rests on the premise that the managers closest to the production process have immediate control over the value of and only lose this authority when they transfer revenue up to the next hierarchy rank. Given these circumstances I assume that in each managerial layer, middle managers can coordinate and expropriate up to a fraction 1 − λ of the firm’s revenue where λ ∈ [0, 1] governs the quality of institutions. In particular λ reflects the degree of property protection and can be interpreted as the fraction of stolen output that the entrepreneur can formally or informally recover. I will further assume that it takes one unit of time for the entrepreneur to discover the expropriation.

The entrepreneur plays a game of Stackelberg leader with each of his middle managers. More to the point, the entrepreneur has the possibility to offer each middle manager a contract that specifies a time-invariant wage markup such that the middle manager does not steal from him. At the same time he threatens to fire the middle manager upon discovering any revenue loss. This threat is perfectly credible as it is costless for the owner to exchange middle managers. Let β ∈ (0, 1) be the time discount factor and δ ∈ (0, 1) an exogenous separation rate. We have that the staying value for a middle manager is then

\[ V^{xz_l}(z) = m_l w + \beta \left[ \delta V^{lm} + (1 - \delta) V^{xz_l}(z) \right] = \frac{m_l w + \beta \delta V^{lm}}{1 - \beta (1 - \delta)}. \]

where \( V^{lm} \) is the value of re-entering the labor market. Alternatively, if a middle manager decides to divert resources, his value is

\[ V^{xz_l, out}(z) = (1 - \lambda) \frac{y}{x_l} + w + \beta V^{lm}. \]

where \( \frac{y}{x_l} \) implies that at each layer managers divide the stolen revenue proportionally amongst themselves. Let \( 1 + a \equiv \frac{1 - \beta}{\beta} V^{lm} \). It can be checked that the participation constraint (3) holds with equality for

\[ m_l w = [1 + \beta(1 - \delta)a]w + (1 - \lambda)[1 - \beta(1 - \delta)] \frac{y}{x_l}. \]

so that

\[ V^{xz_l}(z) = \frac{[1 + \beta(1 - \delta)a]w + (1 - \lambda)[1 - \beta(1 - \delta)] \frac{y}{x_l} + \beta \delta V^{lm}}{1 - \beta (1 - \delta)}. \]  

The markup hence consists of two parts. The second part is the discounted private benefit of expropriation, \( (1 - \lambda)[1 - \beta(1 - \delta)] \frac{y}{x_l} \), which depends positively on the amount that can potentially be stolen and negatively on the effective discount factor \( \beta(1 - \delta) \), which expresses the opportunity cost of not being able to steal from that particular firm in the future. The first part \( w + \beta(1 - \delta)aw \) is the regular wage plus the outside opportunity cost of re-entering the labor market. To the extent that there exists a positive labor
market premium in the economy, agents employed in the labor market earn in expectation strictly more than the discounted sum of wages, i.e. $V^{lm} > \frac{w}{1-\beta}$ and $a$ is positive as will be shown below. The outside option depends positively on the discount factor because the possibility of a relatively high remuneration in the future strengthens the middle manager’s bargaining position.

The profit function of firms with multiple managerial layers can hence be rewritten more concisely as

$$\pi^{ml} (z) = \max_{L=\{2,3,...,n\}, \{x_i\}_{i=1}^{L-1}} \left\{ [1 - (1 - \lambda) (L - 1) (1 - [\beta(1 - \delta)])] z \prod_{l=1}^{L-1} x_l^{q_l^{e_l}} \right.$$  
$$- wn - w (1 + a [\beta(1 - \delta)]) \sum_{l=1}^{L-1} x_l - \kappa e f \right\}$$  

(5)

4. EQUILIBRIUM

4.1. Characterization

4.1.1. Labor market value

The characteristic of this labor market is that incumbent managerial workers have an advantage. First, note that each agent offered a managerial contract with firm $s$ as described above accepts the contract. To see this, consider the agents who are randomly chosen to be the first to negotiate a managerial contract with the highest paying job $(s,l)$. They have an interest in signing the contract because trying to find any other match in the labor market would result in lower managerial rents. As to the firm, it also has an interest in signing such a contract because paying less (or equally seeking workers willing to undercut the agreed wage) would result in managers stealing, which would decrease profits. Similarly, the agents matched with the second-highest paying job will sign their contract because the only alternative contract offering more is provided by a position that is already filled. This logic continues for all initial negotiations, implying that agents matched for negotiation as production workers have no other period option but to take a production worker contract, which is competitive.

Managerial workers therefore remain in their position as long as they are not exogenously separated. New managerial job openings become vacant only when incumbents are exogenously separated, which occurs at the rate $\delta$. Let $N(s)$ denote the total number of workers of firm $s$, of which $N(s) - 1$ are employed workers. The endogenous probability of becoming a managerial worker in firm $s$, position $l$, is therefore $q_l(s) = \delta \frac{x_l(s) dG(s)}{\int_{s \geq \gamma} [N(s) - 1 - (1 - \delta) \sum_{l=1}^{L(s)-1} x_l(s)] dG(s)}$. Combining this with $V^{x_l}(s)$ from (4) gives

$$(1 - \beta) V^{lm} = w + \delta \int_{s \geq \gamma} \sum_{l=1}^{L(s)-1} \left[ \beta(1 - \delta) \right] aw + (1 - \lambda) \frac{y_l(s)}{x_l(s)} \left( 1 - [\beta(1 - \delta)] \right) x_l(s) dG(s)$$

Finally, substituting in $a \equiv \frac{1-\beta}{w} V^{lm} - 1$, the expected period value of entering the labor market is $V^{lm} = \frac{w}{1-\beta} (1 + a)$ where

$$a = \frac{(1 - \lambda) \delta}{w} \int_{s \geq \gamma} \left[ L(s) - 1 \right] y(z) dG(s)$$

$$\int_{s \geq \gamma} [N(s) - 1 - (1 - \delta) \sum_{l=1}^{L(s)-1} x_l(s)] dG(s) \geq 0.$$  

(6)

The labor market premium $a$ is equal to the total economy-wide revenue that can potentially be expropriated by middle managers, weighted by the probability of a managerial re-shuffle, and divided by the number of employees in the economy. Note that the premium $a$ is equal to 0 for the extreme cases where stealing is not possible ($\lambda = 1$) and/or where workers do not switch firms and where managerial workers hence face no
opportunity cost in remaining with a given employer \((\delta = 0)\) and/or there are no firms with positive middle management, \(L(s) = 1, \forall z\). Otherwise, the value of \(a\) is strictly positive because managerial workers earn rents over and above the market wage. Notice also that in the partial equilibrium (i.e. for a given value of the wage \(w\)) an economy with a low value of \(\lambda\) is one where entering the labor market is relatively more interesting as opposed to running a firm. Finally, while property protection \((\lambda)\) has a direct negative impact on \(a\), in the general equilibrium that relation may be overturned by an indirect effect. As will be shown, \(\lambda\) is also likely to positively impact the optimal number of layers \(L(s)\), which implies that there is more scope for stealing in the labor market, pushing up the labor market premium.

4.1.2. Organizational choice

Each firm’s optimal organizational pattern depends on its relative productivity \(z/w\), the relative premium of entering the labor market \(a\), and the institutional parameter \(\lambda\). Here I summarize the most relevant choice variables. All computations and proofs are relegated to the Appendix.

Proposition 1. The optimal level of managerial layers \(L(z/w, a; \lambda)\) is weakly increasing in \(z/w\), weakly increasing in \(\lambda\) and weakly decreasing in \(a\).

Proof. See the Appendix.

From the profit function it is immediate that adding a managerial layer boosts production if and only if the additional layer of middle management employs a minimum amount of workers (i.e. \(x_1 > 1\)). Firms that are productive enough have an interest in doing so. Also, it is obvious that an increase in the parameter \(\lambda\) as well as a decrease in the wage premium \(a\) renders managerial workers less costly and hence facilitates the introduction of additional middle layers. As argued above, the relation between \(a\) and \(\lambda\) is not clear. If we suppose that it is of second-order importance (as suggested by all simulations with reasonable parameter values), then the Proposition above states that all else equal, firms in countries with poor property protection are likely to have relatively few managerial layers. In this sense, there is misallocation in the degree of delegation.

Proposition 2. The ratio of employed managerial workers to employed production workers is weakly increasing in \(L(z/w, a; \lambda)\) and weakly decreasing in \(a\).

Proof. See the Appendix.

Notice that this Proposition only refers to employed workers. The overall ratio of managerial to production workers depends also on the entrepreneur’s activity, but this naturally wanes in importance for firms that have a large number of workers. Ignoring the weight of the entrepreneurs and assuming again that the movement in \(a\) is of second-order importance, the Proposition suggests that countries with a low degree of property protection have relatively few managerial workers. This is the key relationship in the model that I exploit to infer the value of \(\lambda\) across countries.

4.2. Stationary equilibrium definition

The stationary equilibrium is a list of firm managerial layers \(L(z)\), output \(y(z)\), production workers \(n(z)\), total employees \(N(z)\), profits \(\pi(z)\), managerial workers \(x_l(z)\), managerial markups \(m_l(z)\) and consumption \(c(z)\), \(\forall z \in Z\), \(\forall l \in \{1, 2, ..., L(z)\}\); the ex ante individual value functions \(V(z)\), \(V^x(z)\), \(V^{x, out}(z)\) and the expected values \(V^n, V^x\) and \(V^{lim}\), a wage \(w\), the probability of becoming a manager in firm \(z\) in position \(l\), \(q_l(z), \forall z \in Z\), \(\forall l \in \{1, 2, ..., L(z) - 1\}\), a labor market premium \(a\), a cutoff productivity \(\Sigma\) and a lump-sum transfer \(T\) such that:
i) all firms solve their profit maximization problem;
ii) all agents solve their occupational problem;
iii) \( V^{zi}(z) = \frac{(1+\beta(1-\delta)[n_a+n_l+(1-\lambda)]\frac{\beta(1-\delta)}{\beta+1]}{1-\beta(1-\delta)} , \forall z \in Z, \forall l \in \{1, 2, ..., L(z)\} \);
iv) \( V(z) = \max \left\{ \frac{\pi(z)}{1-\beta}, V^{lm} \right\} , \forall z \in Z; \)
v) \( q(z) = \delta \frac{\int_{z \geq \pi} [N(z)-1-(1-\delta)\sum_{l=1}^{\pi(z)} x_l(z)]dG(z)}{\int_{z \geq \pi} [N(z)-1-(1-\delta)\sum_{l=1}^{\pi(z)} x_l(z)]dG(z)}; \)
vii) feasibility reads \( \int_{z \geq \pi} y(z)^{G(z)} = \int_{z \geq \pi} c(z)dG(z); \)
viii) the labor market clears, i.e.
\[
\int_{z \geq \pi} \left( n(z) + \sum_{l=1}^{L} x_l(z) \right) \frac{dG(z)}{1-G(z)} = 1.
\]
ix) all fixed costs are rebated lump-sum to the agents, i.e.
\[
\int_{z \geq \pi} z^{se} \frac{dG(z)}{1-G(z)} + \int_{z \geq \pi} z^{se} \frac{dG(z)}{1-G(z)} = T,
\]
with \( z^{se} \) being the productivity cutoff for the self-employed.

5. Empirical results

5.1. Calibration procedure

The calibration proceeds as follows. I first choose the time period to be year, under the assumption that this is the time that firms need to discover wrongdoings per managerial layer. This can be defended on the grounds that the diagnosis of the firm’s performance and the analysis of necessary adjustments are medium-term projects. I hence choose the discount factor to be \( \beta = 0.95 \). I set the exogenous separation rate \( \delta \) to 0.152 so that \( 1/\delta \) equals the average US job tenure, which was about 6.6 years in the nineties 1997 according to Auer, Berg and Coulibaly (2005). As the calibration is obviously quite sensitive to the notion of a time period I later analyze the sensitivity of the model to other values.

Second, I fix the distribution \( G(z) \) to be log-normal such that \( \log z \sim N(1, \sigma^2) \) on the support \([0, z_{max}] \). This leaves 7 parameters \( (\sigma, z_{max}, \lambda, \theta, \sigma, \sigma_{eff} \) and \( \lambda_{se} \), which I choose jointly to minimize the sum of the quadratic discrepancy of 7 model moments from their empirical counterparts for the U.S. around 2005. Notice that in the absence of the possibility to add managerial layers, the firm size distribution in terms of the number of workers would directly inherit the properties of \( z \) and therefore feature a thin tail, which is at odds with the evidence according to which the right tail of the US firm size distribution closely follows a Pareto distribution. The possibility of adding layers, however, implies that firms can dampen the decreasing returns to scale, which thickens the right tail of the distribution. According to Luttmer (2007) the proportion of firms with more than \( n \) employees approximately equals \( n^{-1.06} \). To match this I regress the log of the inverse distribution of firms from 100 workers onward on the log of firm size to back out the slope. If the distribution was perfectly Pareto, this statistic should ideally equal \( -1.06 \). Since the right side of the distribution has thicker tails the higher is \( \lambda \) and the higher is the dispersion of the distribution \( \sigma \), these two parameters are key in matching the right tail,
together with the highest efficiency level $z_{\text{max}}$. To discipline the resulting distribution I require that firms with more than 10,000 employees account for about 25.4 percent of employment (US Business Census (2011)) and that the largest firm (Walmart) employs about 2 mio. workers.

Next, the parameters $\gamma$ and $\theta$ are crucial determinants of the firms’ returns to scale. I require that the average profit share of business firms (all firms that are not own-account workers) to roughly match 0.15, which is the computed US residual share not accruing to labor and physical capital as summarized by Atkeson and Kehoe (2005). At the same time, the model also ought to match the overall share of managerial workers in the US economy, which is about 0.35 according to the ILO data presented in section 2. Finally, the average size of business firms and the share of own-account workers in the economy is principally determined by $\kappa_{ef}$ and $\kappa_{se}$. The corresponding moments to match from the US economy are 20.4 (US Business Census (2011)) and 0.07 (Bureau of Labor Statistics (2011)). The resulting parameters are summarized in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount factor ($\beta$)</td>
<td>0.950</td>
<td>Interest rate</td>
<td>0.050</td>
<td>-</td>
</tr>
<tr>
<td>Separation rate ($\delta$)</td>
<td>0.152</td>
<td>Avg job tenure</td>
<td>6.600</td>
<td>-</td>
</tr>
<tr>
<td>Std deviation of log $z$ ($\sigma$)</td>
<td>1.081</td>
<td>Pareto tail of firm CDF</td>
<td>-1.060</td>
<td>-1.103</td>
</tr>
<tr>
<td>Largest efficiency ($z_{\text{max}}$)</td>
<td>5.916</td>
<td>Largest firm</td>
<td>2 mil.</td>
<td>2 mil.</td>
</tr>
<tr>
<td>Expropriation ($\lambda$)</td>
<td>0.483</td>
<td>Emp share large firms</td>
<td>0.254</td>
<td>0.264</td>
</tr>
<tr>
<td>Managerial share ($\gamma$)</td>
<td>0.276</td>
<td>Share of managers</td>
<td>0.350</td>
<td>0.366</td>
</tr>
<tr>
<td>Communication cost ($\theta$)</td>
<td>0.465</td>
<td>Profit share firms</td>
<td>0.150</td>
<td>0.150</td>
</tr>
<tr>
<td>Fixed cost employer firms ($\kappa_{ef}$)</td>
<td>1.062</td>
<td>Average firm size</td>
<td>20.40</td>
<td>20.24</td>
</tr>
<tr>
<td>Fixed cost self-employed ($\kappa_{se}$)</td>
<td>0.290</td>
<td>Share of self-employed</td>
<td>0.070</td>
<td>0.071</td>
</tr>
</tbody>
</table>

Table 2: Benchmark calibration

5.2. Model outcome

Figure (2) summarizes the main firm characteristics in equilibrium as a function of firm size. First, notice how firm size is related to the firm-specific efficiency level. In a standard model of heterogenous firms with decreasing returns to scale in labor, the plot in the left upper panel would typically be a straight upward sloping line. Here, the possibility for the firm to mitigate decreasing returns to scale implies that the plot is a succession of upward sloping lines whose slope is decreasing in each managerial layer. It reflects that a unit increase in efficiency $z$ at high levels of $z$ is associated with a larger percentage increase in labor than a unit change at low levels of $z$ as labor can be leveraged more with a longer chain-of-command. The associated relation between the optimal number of layers and firm size is traced in the upper right panel. At the benchmark calibration, the longest hierarchy has 5 layers. The choice of layers seems roughly in accordance with a spatial interpretation of a hierachy. Small firms are constrained by management at about 10 workers (say, a team). The next jumps occur at roughly 50 (a department), 800 (a production unit) and 200,000 workers (a conglomerate).

The lower left panel depicts the span-of-control, which is defined as the sum of managerial workers in the first managerial layer per top manager. Finally, the last subplot traces the managerial ratio. For own-account workers it is just the time spent solving problems.\footnote{Another natural interpretation is that own-account workers specialize in either production work or managerial work (a typical example of the latter being independent professional workers). In this sense the above managerial ratio can simply by viewed as the proportion of aggregate workers specializing in managerial tasks.} For small firms the ratio is decreasing as the number of managers is constrained...
Figure 2. Equilibrium implications: model-specific statistics

(consisting only of the entrepreneur himself) while the number of production workers is unlimited. As firms increase layers, the ratio of employed managers to production workers increases. For a given layer, the ratio is slightly downward sloping as the entrepreneur himself represents one more manager.

Figure 3. Equilibrium implications - observables

The next four graphs in Figure (3) plot equilibrium characteristics that be quite readily compared to their counterparts in the data. The first subplot depicts productivity, which takes a U-shaped form. The jumps from self-employment to a single-layer employment
firm and then on a multi-layer firm are associated with productivity losses. These major changes in the firm structure demand high fixed cost outlays in terms of additional workers, akin to an overhead labor cost. Multi-layer firms, on the other hand, see their productivity increase with each additional layer as the changes in the structure are used to increase the leverage of the entrepreneur’s project. The fact that over the most relevant part of the support there is a positive correlation between productivity and firm size is arguably a realistic feature of the present model, as opposed to standard models where productivity across firms is flat.\footnote{See Bartelsman, Haltiwanger and Scarpetta (2011) on the dispersion of labor productivity. Also, Syverson (2011) offers a comprehensive review of the literature on multi-factor productivity dispersion.} Furthermore, as depicted in upper right panel, the model predicts that the average wage increases with firm-size. This is consistent with a large body of evidence from the US according to which larger firms indeed pay higher wages than their less productive peers (see for instance Idson and Oi (1999)).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{Efficiency}
\end{figure}

Another property of the model is that the profit share is decreasing, as can be observed in the lower left panel. Hard evidence on this is hard to come by, given that profits here are rents on organizational capital rather than physical capital. Finally, the last panel plots the firms size distribution as well as a line with the approximate slope of the US size distribution of firms with more than 100 employees. Though the slope parameter is targeted in the calibration on \textit{average}, there could still be large deviations from the Pareto distribution around that average. But that departure does not seem particularly strong so that the model is rather consistent with a thick-tailed distribution, except at the very end of the distribution where firm growth seems size dependent.

5.3. Simulations

We are now ready to measure the impact of changes in the degree of property protection on several variables of interest.
5.3.1. Individual firms

To make the changes clear, I first analyze how firms vary their optimal choice in response to changes in $\lambda$ from its benchmark value of 0.483 to the halving of that value to 0.282 and to 0, respectively.

![Figure 5. Productivity](image)

From Figure (4) it is immediately evident that the worsening of property protection induces firms to be smaller. In particular, very efficient firms are reluctant to invest in additional managerial layers and hence cannot leverage their efficiency to the extent similar firms can do in an environment characterized by a high $\lambda$. It is also clear from the graph that firms of similar size in the three scenarios have shorter hierarchies the worse is property protection. This can be viewed as a direct model counterpart of the notion that the delegation of decision-making in poor countries is low.

Next, Figure (5) shows how productivity moves across the different environments. Interestingly, note that both the largest as well as small single-layer firms tend to be more productive in countries with more property protection. On the one hand this reflects that the most efficient firms can scale up in such an environment, on the other hand it means that the smallest firms tend to have higher intrinsic efficiency levels as there are fewer operating firms given that wages are higher.

Another key variable of interest is the composition of occupations across firms. From Figure (6) we have that for a given firm size, firms in the more adverse environment do not have a lower managerial share. In fact, the share is higher because the labor market premium $a$ drops with a decline in $\lambda$, making managerial workers relatively less costly given a particular hierarchy length. If the managerial share in the aggregate decreases, it must be because of a composition effect. This consists of the fact that firms in the low $\lambda$ economy have shorter hierarchies, and the fact that there is a large mass of firms willing to circumvent agency issues by operating with a single managerial layer. The fact that these firms are more prevalent in the adverse environment is visible from Figure (7). Note that lower levels of $\lambda$ translate into a significantly thinner (and shorter) tail of the firm size distribution.
5.3.2. Aggregate economy

The effect of the institutional parameter $\lambda$ on the aggregate economy is summarized in Figure (8), where $\lambda$ varies between 0 and its benchmark value.

Aggregate productivity, the actual variable of interest in the present paper, is depicted in the upper left panel. Passing from the US level of property protection to no protection is associated with a productivity loss of about 18 percent. It is also apparent that the loss is more sensitive to changes at higher levels of $\lambda$. Following the estimation results in the first Section, the lowest possible managerial share of 0.255 is associated with countries that have income levels at around 10 percent of the US.

The lower two panels of Figure (8) offer some evidence to believe that the simulated economy of $\lambda$ could well represent a country with income levels of about a tenth the
US. We have that the average employer firm size decreases in the most adverse economic environment by roughly one-half, to about 10 workers. This is very much in line with evidence on firm size in countries that have income levels of about a tenth of the US, as shown in Tybout (2000). Also, the level of self-employment increases strongly, for which there is also ample support, as in Gollin (2008).

Finally, Figure (9) offers additional qualitative support for the model. It shows the coefficient of variation of two variables. The first is firm labor productivity (output by the number of workers) and the second is a the theoretical underlying firm-specific efficiency, measured as a residual from 
\[
z = \frac{y(z)}{(n(z) + x(z))^{0.85}}
\]
where \(y\) and \(n + x\) are...
simulated firm-specific production and employment levels. Bartelsman, Haltwanger and Scaripta (2009) argue that microeconomic misallocation in poor countries is reflected in a higher productivity dispersion, while Restuccia and Rogerson (2008) and Hsieh and Klenow (2009) argue that generic tax wedges can be imputed from comparing predicted theoretical efficiency distributions to ones measured as residuals. As can be seen, the present model provides a fundamental for misallocation as manifested by the dispersion in these two statistics.

6. Concluding remarks

This paper proposes a tractable model to address the technological benefits of delegation in large organizations. Adding additional layers increases the span of control of the entrepreneur, but is associated with a fixed cost. Besides, adding an additional layer is associated with an extra cost that increases with the lack of property protection as middle managers can steal from the firm they work for. The threat of expropriation affects the firm’s organization choice, which consists of the number of managerial layers as well as the number of managerial and production workers. The calibrated version of the model predicts that countries with poor contract enforcement have smaller firms on average and more self-employed workers as firms are reluctant to add layers to their managerial structure. Importantly, an environment with no property protection generates a 10 percentage points fall in the aggregate share of managerial workers, which is a value associated with countries with one-tenth of US GDP per capita. The model predicts an associated drop in the aggregate labor productivity of 18 percent. This result is not trivial given that the productivity drop only concerns the non-agricultural sector and given that it represents a pure TFP loss.

The paper offers several extensions worth pursuing. One of them is the introduction of the choice of human capital on the part of the workers in conjunction with the assumption that managerial workers are the only workers making use of human capital. Intuitively, in an environment in which the probability of becoming a managerial worker is low, workers ought to have a reduced incentive ex ante to invest in human capital. Such an outcome would be consistent with the data discussed in the regression results. It could possibly also increase the productivity loss associated with low property protection. Another potentially fruitful extension involves modeling the entrepreneur’s stock of trustworthy relations, i.e. the number of middle managers that he can hire without the need for extra compensation. If entrepreneurs were different along this additional characteristic, then misallocation could occur via an extra channel, namely the possibility that incompetent but well-connected entrepreneurs would suboptimally drain too much labor from the labor market. This would parallel the literature on credit constraints where the entrepreneur’s wealth is typically an important determinant on how much he can borrow. Finally, the addition of physical capital and credit constraints would allow to analyze the relative quantitative importance of firms’ external versus internal constraints.

7. Bibliography


8. Appendix

8.1. Computations

The optimizing behavior of firms with multiple layers ($L \geq 2$) involves the following first order conditions with respect to (5):

$$n = \left[ \frac{1 - \theta - \gamma}{1 - \theta} \left[ 1 - (1 - \lambda) (L - 1) (1 - [\beta(1 - \delta)]) \right] \right] \frac{z}{w} \prod_{l=1}^{L-1} x_l^{\theta^{l-1}} \left[ \frac{1 - \theta}{\gamma} \right]^{1 - \theta},$$

$$\frac{x_1}{n} = \frac{\gamma (1 - \theta)}{1 - \theta - \gamma (1 + a [\beta(1 - \delta)])},$$

and, for $L \geq 3$

$$\frac{x_l}{x_1} = \theta^{l-1}, \forall l \in \{1, ..., L - 1\}.$$

The sum of employed managerial workers (excluding the entrepreneur) $x \equiv \sum_{l=1}^{L-1} x_l$ as a function of production labor is

$$x = \frac{\gamma}{1 - \theta - \gamma (1 + a [\beta(1 - \delta)])} \left( \frac{1 - \theta^{L-1}}{n}. \right)$$
Production labor demand equals
\[
n = \left( \frac{\gamma (1 - \theta)}{1 - \theta - \gamma (1 + a [\beta(1 - \delta)])} \right)^{\frac{1 - \theta}{\gamma - \delta}} \left[ \frac{1 - \theta - \gamma}{1 - \theta} \right] \frac{1}{1 - \theta - \gamma} \left[ 1 - (1 - \lambda) \left( L - 1 \right) \left( 1 - \left[ \beta(1 - \delta) \right] \right) \right] \frac{z}{w} \frac{\theta^\gamma \sum_{i=1}^{L-1} \delta^{i-1}}{w} \frac{1 - \theta}{\theta^{\delta-1}}.
\]

Production equals
\[
y = \frac{1 - \theta}{1 - \theta - \gamma \left[ 1 - (1 - \lambda) \left( L - 1 \right) \left( 1 - \left[ \beta(1 - \delta) \right] \right) \right]} w \frac{n}{w} = \frac{\gamma \theta^{L-1}}{1 - \theta - \gamma} n
\]
and gross profits relative to the wage level are given by
\[
\frac{\pi^{ml} + \kappa^{bf}}{w} = \frac{\gamma \theta^{L-1}}{1 - \theta - \gamma} w \frac{n}{w} = \frac{\theta^{L-1} (1 + a [\beta(1 - \delta)])}{1 - \theta} \left[ \frac{\Xi}{\frac{\gamma}{w} \sum_{i=1}^{L-1} (\delta^{i-1})} \frac{1 - \theta}{\theta^{\delta-1}} \right]
\]
where
\[
\Xi \equiv \frac{\gamma}{1 - \gamma} \left( \frac{1 - \theta - \gamma}{1 - \theta} \right) \frac{1 - \theta - \gamma}{1 - \theta} \left( 1 + a [\beta(1 - \delta)] \right) \frac{1}{\frac{1 - \theta - \gamma}{1 - \theta} (1 - (1 - \lambda) \left( L - 1 \right) (1 - \left[ \beta(1 - \delta) \right])}
\in (0, 1)
\]

We also have that the gross profit ratio equals
\[
\frac{\pi^{ml} + \kappa^{bf}}{y} = \frac{\gamma \theta^{L-1}}{1 - \theta} \left[ 1 - (1 - \lambda) \left( L - 1 \right) \left( 1 - \left[ \beta(1 - \delta) \right] \right) \right]
\]

8.2. Proofs

8.2.1. Proposition 1:
Consider the profit function (9). Notice that since $\theta \in (0, 1)$, an increase in $L$ is profit-maximizing only if $\Xi \frac{z}{w} \theta^\gamma \sum_{i=1}^{L-1} (\delta^{i-1}) > 1$, which in turn is true only for a sufficiently high level of $\frac{z}{w}$. Notice also that $\forall \frac{z}{w}, \exists L$ such that $\Xi \frac{z}{w} \theta^\gamma \sum_{i=1}^{L-1} (\delta^{i-1}) < 1$. It follows that for each level of $\frac{z}{w}$ there exists an optimal finite level of $L$, with $L$ being weakly increasing in $\frac{z}{w}$. It is also immediate that $L$ is increasing in $\lambda$ and decreasing in $a$.

8.2.2. Proposition 2:
Single-layer firms do not have any employed managerial workers. Multi-layer firms have a ratio between employed managerial and production workers equal to $\frac{\gamma}{1 - \theta - \gamma} \frac{\left( 1 - \theta (\frac{z}{w}, a, \lambda) \right)^{L-1}}{1 + a [\beta(1 - \delta)]}$. 