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Colour confinement as dual Meissner effect: $SU(2)$ gauge theory.

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We demonstrate that confinement in $SU(2)$ gauge theory is produced by dual superconductivity of the vacuum. We show that for $T < T_c$ (temperature of deconfining phase transition) the $U(1)$ symmetry related to monopole charge conservation is spontaneously broken; for $T > T_c$ the symmetry is restored.

I. INTRODUCTION

Dual superconductivity of the vacuum has been advocated as the mechanism for confinement of colour [1–3]: the chromoelectric field is channeled into Abrikosov [4] flux tubes, producing a static potential proportional to the distance between $q\bar{q}$ pairs.

Magnetic charges, defined as Dirac monopoles of a residual $U(1)$ symmetry selected by a suitable gauge fixing (abelian projection), should accordingly condense in the vacuum, in the same way as Cooper pairs do in an ordinary superconductor.

Evidence has been collected by numerical simulations of the theory on the lattice, that such monopoles do exist, and that their number density is correlated with the deconfining phase transition: we refer to [5] for a review of these results. For a recent updating we refer to [**?**] However a direct demonstration that confinement is produced by monopole condensation is still lacking.

In fact monopole condensation means that the ground state of the system is a superposition of states with different magnetic charges, or that the dual (magnetic) $U(1)$ symmetry is spontaneously broken, in the same way as the electric $U(1)$ symmetry is spontaneously broken in a ordinary superconductor [6]. Such a breaking is monitored by the non vanishing of the vacuum expectation value (*vev*) of any operator μ with nontrivial magnetic charge. $\langle\mu\rangle$ is called a disorder parameter: it is non zero in the broken phase, and vanishes in the ordered phase, at least in the thermodynamical limit $V \to \infty$.

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The construction of a disorder parameter for dual superconductivity has been presented in [7] where it has been successfully tested on compact $U(1)$ gauge theory.

In this paper we use the same construction to probe the vacuum condensation of the monopoles defined by abelian projection in $SU(2)$ gauge theory. We find that the abelian projection which diagonalizes the Polyakov line [8], identifies monopoles which condense in the confined phase and do not in the deconfined one.

Monopoles defined by the abelian projection which diagonalizes a component of the field strength [8] do not show any signal of condensation correlated with confinement.

We conclude that:

1) Confinement of colour is related to dual superconductivity of gauge theory vacuum.

2) Not all the abelian projections are equivalent, or define monopoles wich condense in the vacuum to confine colour.

In sect.2 we recall the basic ideas of the abelian projection, the definition of the corresponding monopoles, and their role in confinement. In sect.3 we present our results, in sect.4 the conclusions.

II. MONOPOLES IN Q.C.D.

Stable monopoles configurations in gauge theories are related to the first homotopy group Π_1 of the gauge group [9]. Since $\Pi_1[SU(N)]$ is trivial, the symmetry has to break down to some non simply connected subgroup, in order to define magnetic charges.

In the Georgi-Glashow [10] model with gauge group $SU(2)$ coupled to a scalar field Φ^a in the adjoint representation a spontaneous breaking $SU(2) \to U(1)$ allows to define an abelian field strength [11]

$$f_{\mu\nu} = \hat{\Phi}^a G^a_{\mu\nu} - \frac{1}{g}\varepsilon_{abc}\hat{\Phi}^a (D_\mu \hat{\Phi})^b (D_\nu \hat{\Phi})^c \tag{1}$$

which admits stable monopole configuration [11,12], behaving as Dirac monopoles at large distances. In Eq.(1) $\hat{\Phi}^a = \Phi^a/|\Phi|$. Putting

$$a_\mu = \hat{\Phi}^a A^a_\mu \tag{2}$$

one has [13]

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu - \frac{1}{g}\varepsilon_{abc}\hat{\Phi}^a (\partial_\mu \hat{\Phi})^b (\partial_\nu \hat{\Phi})^c \tag{3}$$

$f_{\mu\nu}$ in Eq.(1) is gauge invariant, since both $\hat{\Phi}$ and $G_{\mu\nu}$ are gauge covariant. a_μ in Eq.(2) is not. In the gauge in which

$$\hat{\Phi}^a = \delta^a_3 \tag{4}$$

Eq.(3) becomes

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu \tag{5}$$

Eq.(5) is the usual expression of the electromagnetic field strength in terms of the (abelian) potential a_μ. The choice of the gauge Eq.(4), which is defined up to an arbitrary gauge rotation around the third axis, is called an abelian projection.

The strategy for relating confinement of colour to dual superconductivity in Q.C.D. is to make a guess of a possible effective Higgs field $\hat{\Phi}$, belonging to the adjoint representation, and then perform the abelian projection Eq.(4) and look for condensation of the corresponding Dirac monopoles.

In most of the lattice investigations on the problem, the density of monopoles, or quantities related to it, have been studied: of course the density of magnetic charges is not a disorder parameter for dual superconductivity, in the same way as the number density of electrons is not for ordinary superconductivity: a non vanishing *vev* of an operator with non trivial charge is needed, while the density of charge commutes with total charge operator (which is in fact neutral). We will instead make use of a genuine disorder parameter that we have constructed and checked on the $U(1)$ gauge theory [7].

Another question is if all abelian projections are physically equivalent, a possibility suggested in [8]. We will study the projection in which $\hat{\Phi}$ is the Polyakov line, and the one in which $\hat{\Phi}$ is any component of the field strength $G_{\mu\nu}$ [8].

For reasons which will be explained in sect.3, we have technical difficulties (computing power) to explore the so called maximal abelian gauge [5].

III. THE DISORDER PARAMETER: NUMERICAL RESULTS.

As for the $U(1)$ case [7] we define the operator which creates a monopole at the point $\vec{z}$ and time z_0

$$\mu(\vec{z}, z_0) = \exp\left[\frac{\mathrm{i}}{g}\int \mathrm{d}^3y\, f_{0i}(\vec{y}, z_0) b_i(\vec{y} - \vec{z}\,)\right] \tag{6}$$

where f_{0i} is the electric field strength Eq.(1) and b_i/g is the vector potential produced by a Dirac monopole, with the Dirac string subtracted. $\vec{b}(\vec{r}\,)$ is given by

$$\vec{b}(\vec{r}\,) = \frac{\vec{r} \wedge \vec{n}}{r(r - \vec{r} \cdot \vec{n})} \tag{7}$$

if the gauge is chosen in such a way that the string singularity is in the direction $\vec{n}$. The equal time commutator between the vector potential a_i (Eq.(2) and f_{0i} is

$$\left[a_k(\vec{x}, x^0), f_{0j}(\vec{y}, x^0)\right] = \mathrm{i}\delta_{kj}\, \delta^3(\vec{x} - \vec{y}) \tag{8}$$

as in the $U(1)$ gauge theory f_{0i} is the conjugate momentum to a_i, and as a consequence μ (Eq.(6)) is an operator which translates the field $a_i(x)$ by $b_i(\vec{x} - \vec{z})/g$.

A proper definition of the *v.e.v.* of μ is [7]

$$\langle\mu\rangle = \frac{\int \mathcal{D}A_\mu \exp\left[-\beta S\right]\, \mu(\vec{z}, z^0)}{\int \mathcal{D}A_\mu \exp\left[-\beta S\right]\, \gamma(z^0)} \tag{9}$$

where $\gamma(z^0)$ is a traslation of the field a_i by a time independent $\vec{g}(\vec{x})$ with $\vec{\nabla}\wedge\vec{g} = 0$

$$\gamma(z^0) = \exp\left[\frac{\mathrm{i}}{g}\int \mathrm{d}^3x\, f_{0i}(\vec{y}, z^0) g_i(\vec{y})\right] \tag{10}$$

subjected to the constraint

$$\int \mathrm{d}^3x\, \vec{b}^2(x) = \int \mathrm{d}^3x\, \vec{g}^2(x) \tag{11}$$

After Wick rotation Eq.(9) can be written

$$\langle\mu\rangle = \frac{\int \mathcal{D}A_\mu \exp\left[-\beta(S + S_b)\right]}{\int \mathcal{D}A_\mu \exp\left[-\beta(S + S_g)\right]} \tag{12}$$

with

$$S_b(\vec{x}, \vec{x}^0) = \int \mathrm{d}^3y b_i(\vec{y} - \vec{x}) f_{0i}(\vec{y}, x^0) \tag{13a}$$

$$S_g(\vec{x}, \vec{x}^0) = \int \mathrm{d}^3y g_i(\vec{y}) f_{0i}(\vec{y}, x^0) \tag{13b}$$

Similarly the correlation function can be defined of any number of monopoles and anti-monopoles: for example for a pair $m\,\bar{m}$ at equal time and distance d

$$\langle\mu(\vec{d})\mu(0)\rangle = \frac{\int \mathcal{D}A_\mu \exp\left[-\beta(S + S_{b\bar{b}})\right]}{\int \mathcal{D}A_\mu \exp\left[-\beta(S + S_g)\right]} \tag{14}$$

$$S_{b\bar{b}} = \int \mathrm{d}^3y f_{0i}(\vec{y}, x^0)\left[b_i(\vec{y} - \vec{d}) - b_i(\vec{y})\right] \tag{15}$$

and g is now subjected to the constraint

$$\int \mathrm{d}^3y\, \vec{g}^2(y) = \int \mathrm{d}^3y\, \left|\vec{b}(\vec{y} - \vec{d}) - \vec{b}(\vec{y})\right|^2 \tag{16}$$

Instead of $\langle\mu\rangle$ we will measure

$$\rho = \frac{d}{d\beta}\ln\langle\mu\rangle = \langle S + S_g\rangle_{S+S_g} - \langle S + S_b\rangle_{S+S_b} \tag{17}$$

In terms of ρ

$$\langle\mu\rangle = \exp\left[\int_0^\beta \rho(\beta)\mathrm{d}\beta\right] \tag{18}$$

For $b\bar{b}$

$$\rho_{b\bar{b}} = \langle S + S_g \rangle_{S+S_g} - \langle S + S_{b\bar{b}} \rangle_{S+S_{b\bar{b}}} \tag{19}$$

If there is monopole condensation $\langle \mu \rangle \neq 0$ or by cluster property

$$\langle \mu(x)\mu(0) \rangle \underset{|x|\to\infty}{\longrightarrow} |\langle \mu \rangle|^2 \neq 0 \tag{20}$$

In terms of ρ the cluster property Eq.(20) reads

$$\rho_{b\bar{b}} \underset{|x|\to\infty}{\longrightarrow} 2\rho_b \tag{21}$$

We have measured ρ for a single monopole and for a $m\bar{m}$ pair at different distances, across the deconfining phase transition of an $SU(2)$ gauge theory. The abelian projection in the gauge which diagonalizes the Polyakov line gives a clear signal of condensation: Fig.1 shows ρ for a $12^3 \times 4$ lattice; Fig.2 for a $16^3 \times 6$ lattice.A clear signal is observed of transition from superconductivity to normal vacuum at $\beta = \beta_c$. The (known) deconfining β_c for the two lattices ($N_T = 4$, $N_T = 6$) is indicated by the vertical lines in figures 1 and 2. In Fig.3 $\rho_{b\bar{b}}$ of a $m\bar{m}$ pair at distance $d = 10$ lattice spacing is compared to $2 \cdot \rho_b$, corresponding to a single monopole, checking successfully Eq.(21).

No signal is observed in the abelian projection which diagonalizes a component (say F_{12}) of the field strength.

A few points about the lattice version of the approach. f_{0i} of Eq.(6) is defined by Eq.(1). For the abelian projection in which $\hat{\Phi}$ is the direction of log of the Polyakov loop,L, the second term in Eq.(1) is absent when μ or ν take the value 0, since $D_0 L = 0$. Then constructing f_{0i} is simply a projection of $\vec{G}_{0i}$ on the direction of L: of course on the lattice G_{0i} can be taken as the imaginary part of the plaquette Π_{0i}.

For the abelian projection in which $\Phi = \ln \Pi_{12}$ the second term of Eq.(1) is not zero, but is computable.

In the case of the so called "maximal abelian gauge" [5], in which the gauge is fixed by maximizing the quantity

$$M = \sum_{\mu,n} \mathrm{Tr}\left[U_\mu(n) \sigma_3 U_\mu^\dagger(n) \sigma_3 \right]$$

the effective Higgs $\hat{\Phi}$ to introduce in Eq.(1) is not known explicitely, but must be determined by the maximization on each configuration. This is a serious problem from the numerical point of view, since at each change of the configuration in the updating procedure to compute ρ by Eq.(19) the maximization must be repeated to determine f_{0i} and S_b. A detailed finite size scaling analysis to extract the thermodinamical limit from our data is under study.

IV. CONCLUSIONS

We conclude that

1) gauge theory vacuum is a dual superconductor: the monopoles defined by the abelian projection diagonalizing the Polyakov line do condense in the confined phase, and the corresponding dual $U(1)$ symmetry is restored in the deconfined phase.

2) Not all abelian projections are physically equivalent: the monopoles in the gauge in which the field strength is diagonal are irrelevant to confinement.

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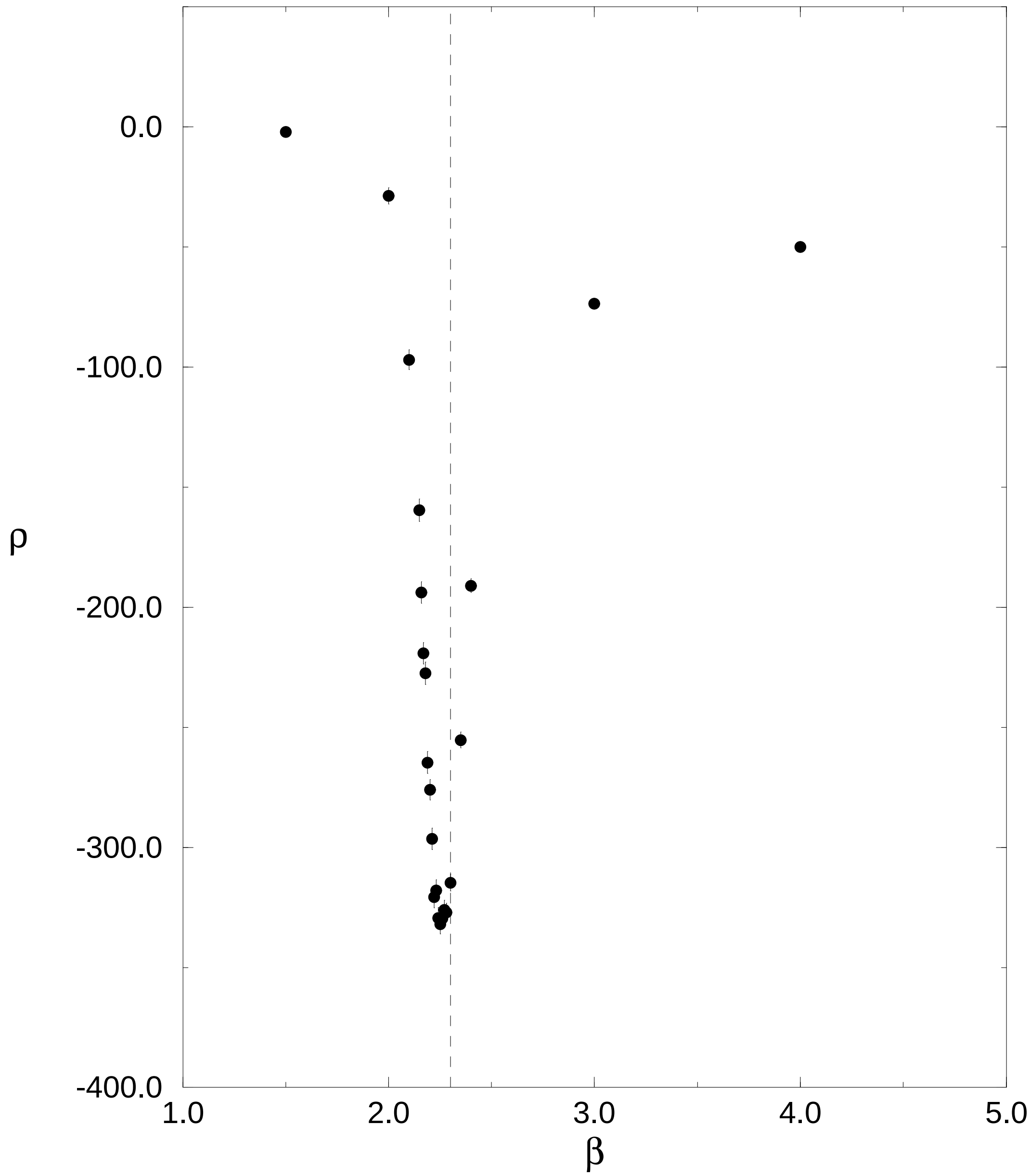

Fig.1

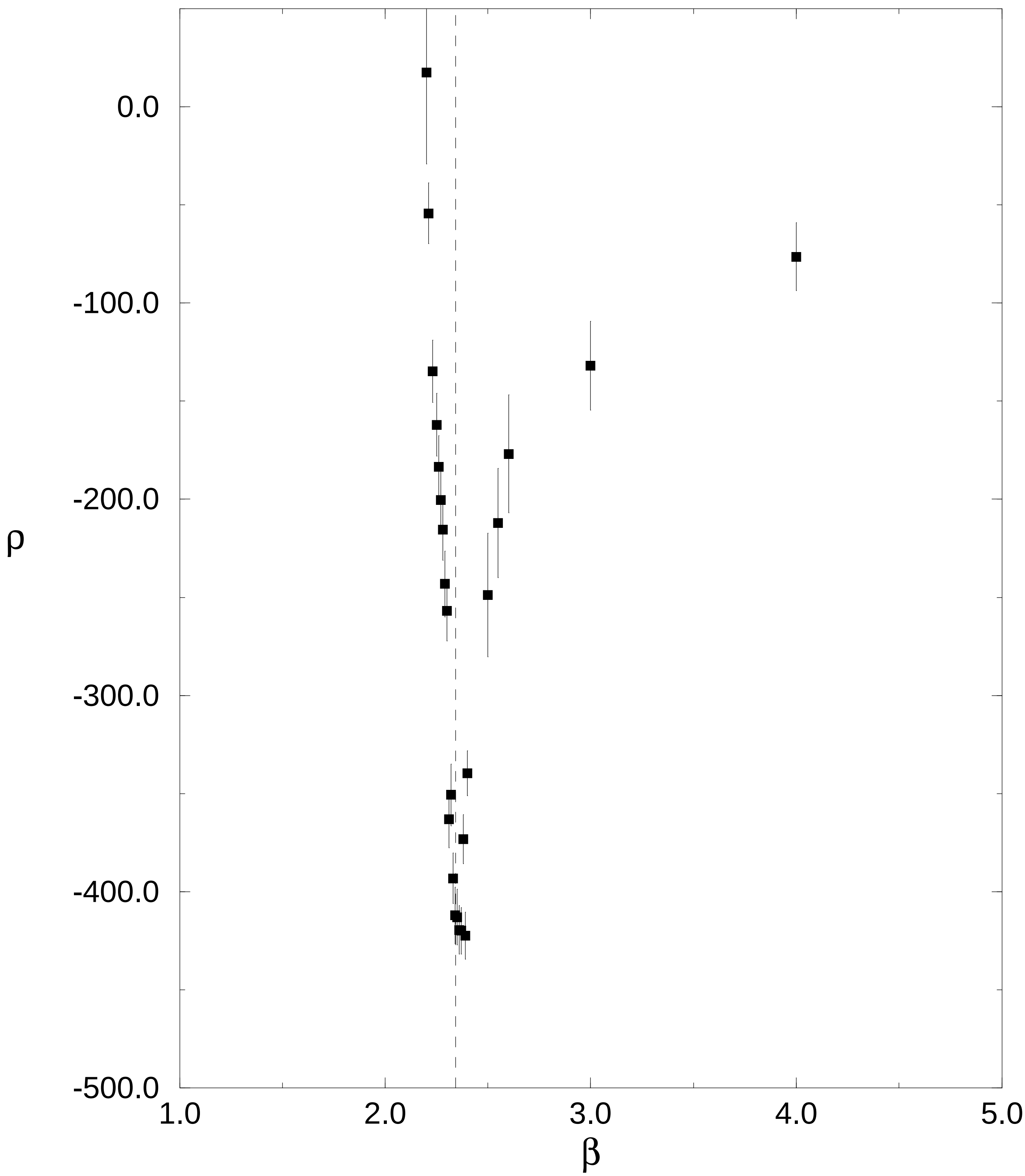

Fig.2

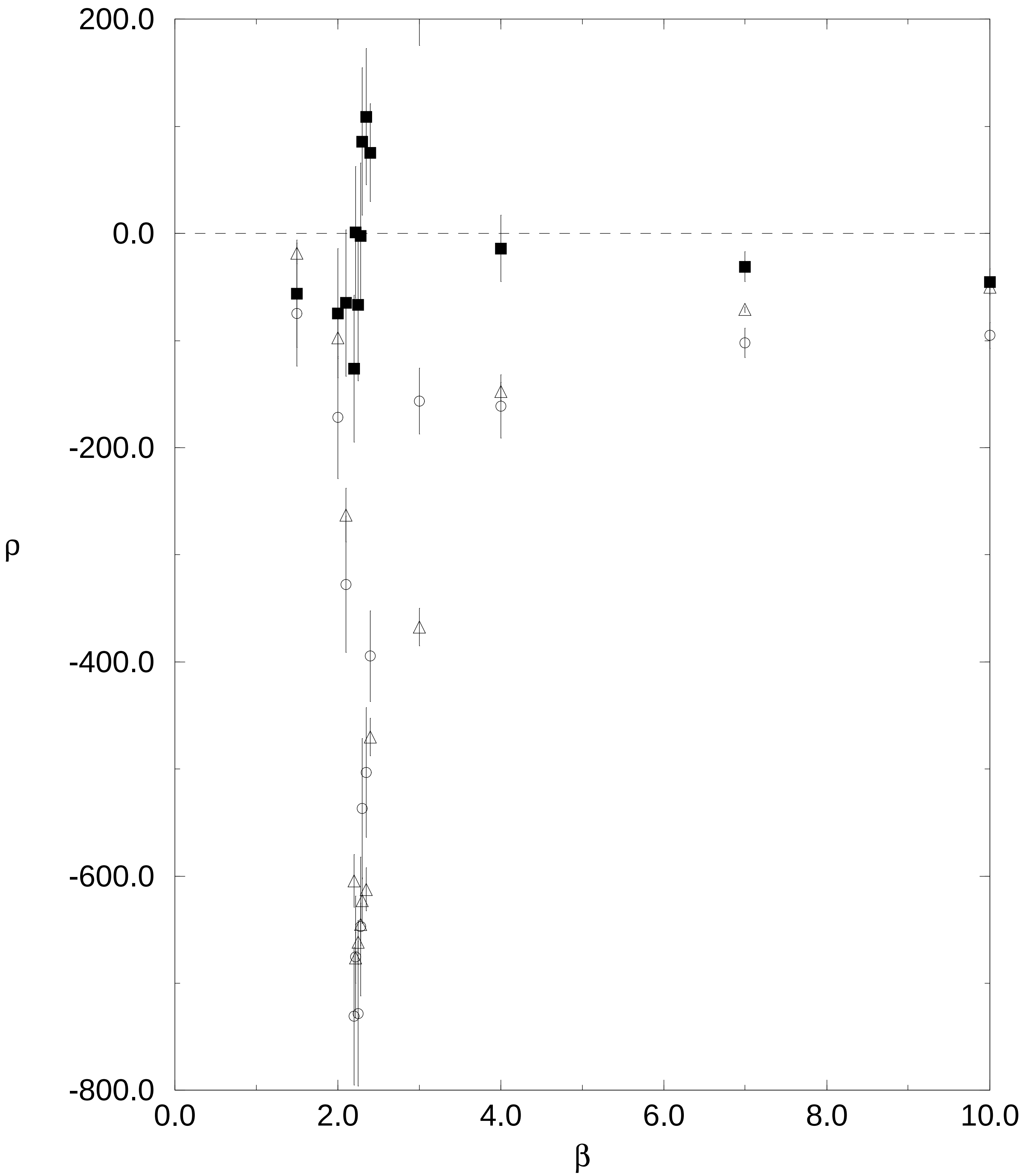

Fig.3

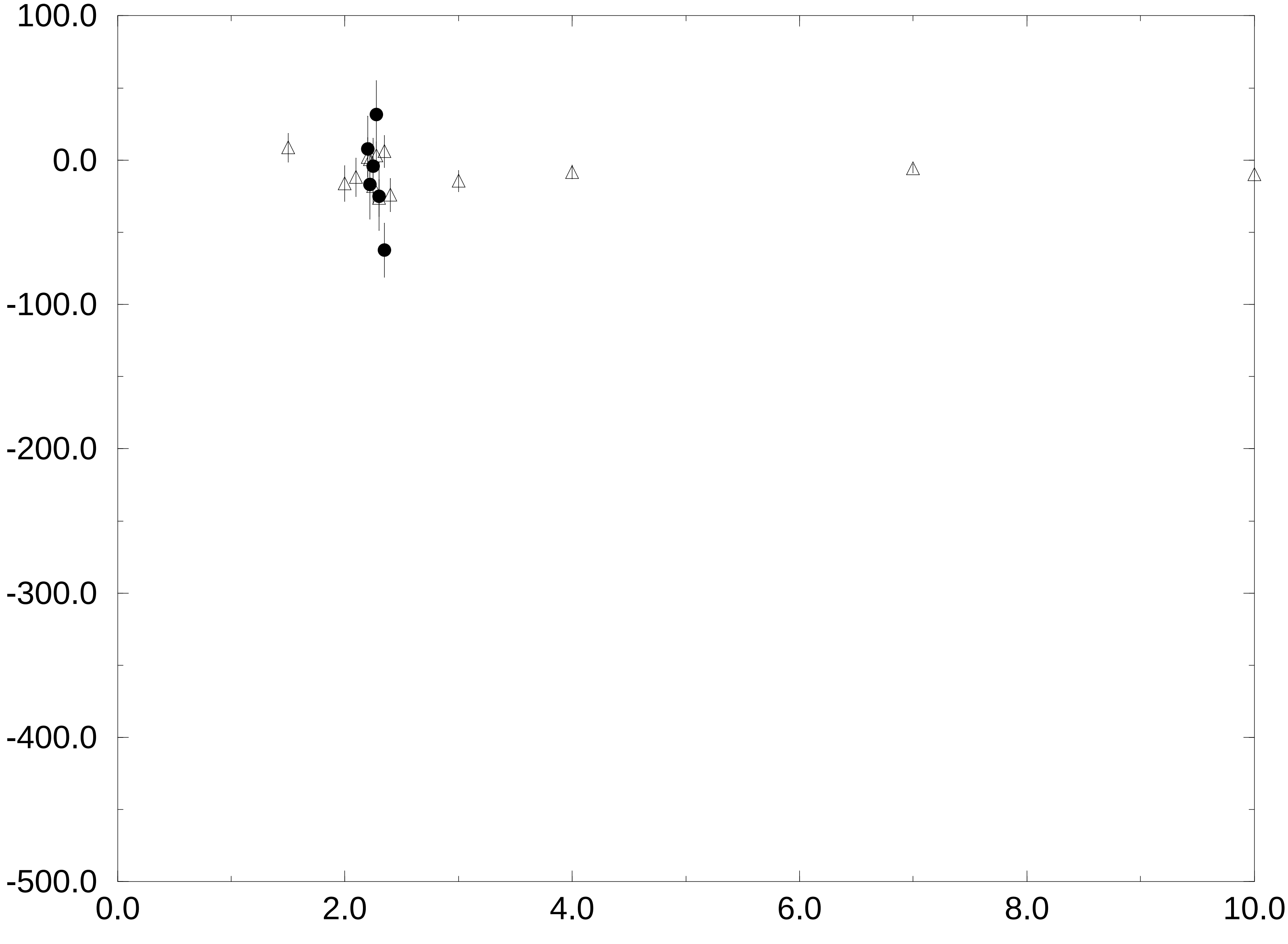

Fig.4

Figures Captions

Fig.1 ρ vs β on a $12^3 \times 4$ lattice. The vertical line denotes β_c corresponding to the deconfining transition.

Fig.2 ρ vs β on a $16^3 \times 6$ lattice. The vertical line denotes β_c corresponding to the deconfining transition.

Fig.3 $\rho_{b\bar{b}}(d)$ at $d = 10$ (circles) compared to 2ρ (triangles) and their difference (squares).

Fig.4 ρ for the abelian projection diagonalising F_{12} on $8^3 \times 4$ (dots) and $12^3 \times 4$ (triangles) lattices.