Critical single domain grain sizes in chains of interacting greigite particles: Implications for magnetosome crystals

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Magnetotactic bacteria contain chains of magnetically interacting crystals (magnetosomes), which aid navigation (magnetotaxis). To improve the efficiency of magnetotaxis, magnetosome crystals (which can consist of magnetite or greigite) should be magnetically stable single domain (SD) particles. Larger particles subdivide into nonuniform multidomain (MD) magnetic structures that produce weaker magnetic signals, while small SD particles become magnetically unstable due to thermal fluctuations and exhibit superparamagnetic (SP) behavior. In this study, we determined the stable SD range as a function of grain elongation and interparticle separation for chains of identical greigite grains using fundamental parameters recently determined for greigite. Interactions significantly increase the stable SD range. For example, for cube-shaped greigite grains the upper stable SD threshold size is increased from 107 nm for isolated grains to 204 nm for touching grains arranged in chains. The larger critical SD grain size for greigite means that, compared to magnetite magnetosomes, greigite magnetosomes can produce larger magnetic signals without the need for intergrain interactions.

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1. Introduction

Magnetotactic bacteria produce chains of magnetic crystals (magnetosomes) that usually consist of magnetite (Fe₃O₄) or greigite (Fe₃S₄) (Figure 1). These magnetosome chains are found in both unicellular bacteria and in larger multicellular magnetotactic prokaryotes (MMP) [Silveira et al., 2007; Faivre and Schuler, 2008; Perantoni et al., 2009]. The primary purpose of magnetosomes is thought to be navigation (magnetotaxis), therefore natural selection should ensure that magnetosomes provide a strong magnetic signal to maximize their efficiency [Kopp and Kirschvink, 2008]. The magnetic state that best exhibits this property is the stable single domain (SD) state. The magnetic domain state of a crystal is strongly dependent on both size and shape [Butler and Banerjee, 1975]. When SD particles are smaller than a critical threshold size, they are no longer magnetically stable because thermal energy can easily overcome the energy barrier that otherwise prevents domain switching. Such particles then have superparamagnetic (SP) behavior. Larger grains above the SD threshold size form complex nonuniform or multidomain (MD) structures, which leads to inefficient magnetotaxis [Frankel et al., 1998] because the magnetic remanence per unit volume is far less than that in SD grains. Determining the critical stable SD size range as a function of morphology is important for determining magnetosome function and possible magnetotaxis efficiency. Critical sizes for stable SD behavior are also of interest to Earth and materials scientists because stable SD grains have the most ideal recording fidelity, and because the easily identifiable magnetic characteristics of stable SD grains are useful indicators of grain size.

It is common to assess the domain state of magnetosome crystals by plotting their length versus grain elongation axial ratio (AR; short-axis/long-axis or width/length) on domain-state phase diagrams [Thomas-Keperta et al., 2000], which were initially determined analytically by Evans and McElhinny [1969] and Butler and Banerjee [1975]. Butler and Banerjee [1975] calculated the SP to stable SD critical size and the SD to MD critical size as functions of AR for individual particles of magnetite and titanomagnetite. Subsequently, both the SP to stable SD transition size [Winklhofer et al., 1997; Muxworthy et al., 2003a] and the SD to MD transition size [Fabian et al., 1996; Newell and Merrill, 1999; Witt et al., 2005; Muxworthy and Williams, 2006] have been re-examined and revised for individual magnetite particles through application of the numerical micromagnetic equations of Brown [1963].

Muxworthy and Williams [2006] demonstrated that it is deceptive to compare magnetosome sizes with critical grain size boundaries derived for individual crystals because magnetosomes nearly always occur in magnetostatically interacting chains [Dunin-Borkowski et al., 1998; Faivre and Schuler, 2008]. Muxworthy and Williams [2006, 2009] performed calculations to include chains of...
magnetically interacting rectangular cuboids of magnetite, and demonstrated that for cube-shaped grains, interactions within chains increases the stable SD to MD threshold size for magnetite from ~70 nm to a maximum of ~200 nm. *Muxworthy and Williams* [2006, 2009] showed that magnetostatic interaction fields are sufficient to cause the largest observed magnetite magnetosome crystals found in living bacteria [length = 250 nm (AR = 0.84); *Lins et al.*, 2005] to be in a stable SD state; without magnetostatic interactions they would be in a MD state and would have a far lower magnetotaxis efficiency. Interactions also decrease the SP/SD size for cube-shaped magnetite grains from ~26 nm to a minimum of ~12 nm for a thermal relaxation time of 60 s. That interactions decrease the SP/SD threshold size supports earlier observational data for magnetosomes [*Dumin-Borkowski et al.*, 1998; *McCartney et al.*, 2001].

The fossil record contains a limited number of exceptions [*Schumann et al.*, 2008; *Chang et al.*, 2012]; these studies reported several morphologies, including a spearhead-like magnetite magnetofossil that occurs with lengths up to 4000 nm (AR ~0.4). The function of this spearhead-like magnetofossil is unlikely to be magnetotaxis [*Chang et al.*, 2012], and there are indeed contemporary examples of metazoaos that use magnetic minerals for their hardness, rather than for their magnetic properties, e.g., chiton (marine molluscs that cling to rocks) have magnetite-coated “teeth” [*Lowenstein*, 1962], and scaly foot gastropods found at deep ocean vents have greigite-bearing sclerites on their foot [*Warén et al.*, 2003; *Suzuki et al.*, 2006].

In order to determine the ideal particle size range for magnetotaxis, it is important to make micromagnetic calculations for all relevant magnetic biominerals. This size range is now well known for magnetite [*Muxworthy and Williams*, 2006, 2009], but the range for greigite has not yet been established. *Diaz-Ricci and Kirschvink* [1992] made analytical estimates similar to those of *Butler and Banerjee* [1975] for individual greigite particles; however, their calculations were based on crude estimations of the fundamental parameters of greigite, which were not well constrained at the time due to the lack of chemically pure samples and instead were deduced from comparison with magnetite [for more detailed discussion, see *Chang et al.*, 2008; *Roberts et al.*, 2011]. *Diaz-Ricci and Kirschvink* [1992] determined the SD/MD threshold to be ~250 nm for cube-shaped grains. More robust analytical and micromagnetic calculations of the SD state phase diagram for greigite are now possible because all of the required fundamental magnetic parameters have recently been determined on chemically pure samples [*Chang et al.*, 2008, 2009; *Roberts et al.*, 2011; M. Winklhofer et al., On the magnetocrystalline anisotropy constant of greigite (cubic Fe3S4), submitted to *Journal of Geophysical Research*, 2013].

As is the case for magnetite magnetosomes, greigite magnetosomes often occur as rectangular cuboids [*Bazylnski et al.*, 1995] or in cubo-octahedral shapes [*Pósfai et al.*, 1998] in interacting chains [*Kasama et al.*, 2006a]. Furthermore, inorganic greigite often occurs in strongly interacting clusters of cubo-octahedral grains [e.g., *Rowan and Roberts*, 2006; *Roberts et al.*, 2011]. Both of these common scenarios make it important to understand how the SD size range is affected by interactions. In this paper, we present results for chains of greigite magnetosomes. Using the physical parameters measured by *Chang et al.*, 2008, 2009] and *Winklhofer et al.* (Winklhofer et al., submitted manuscript, 2013), we have determined the domain-state phase diagram for chains of rectangular greigite cuboids with varying interaction spacings and AR values. We model interacting elongated rectangular cuboids as an approximation to magnetosome crystals. For individual magnetite grains, rectangular cuboids have been shown to yield slightly lower estimates for the SD/MD threshold size than numerical estimates for more magnetosome-like morphologies [*Witt et al.*, 2005], yet the differences for magnetite are small compared to the effect of magnetic interactions [*Muxworthy and Williams*, 2006]. The same is likely to be true for greigite.

While greigite magnetosomes are commonly found in double or multiple chains [*Kasama et al.*, 2006a, 2006b], direct imaging of their magnetic structures using off-axis electron-holography [*Kasama et al.*, 2006a, 2006b] indicates that the magnetic interaction fields along the individual chains are much stronger than between the strands in a chain, i.e., as a first-approximation the inter-chain chain interactions can be ignored allowing us to use the same micromagnetic approaches detailed by *Muxworthy and Williams* [2006, 2009] for chains of magnetite crystals.

Absolute values for magnetite’s SD/MD threshold size for individual rectangular cuboids determined by *Witt et al.* [2005] are less than those of *Muxworthy and Williams* [2006]. The
rectangular cuboid threshold predictions of Muxworthy and Williams [2006] are closer to the predictions of Witt et al. [2005] for magnetosome-like morphologies rather than to their predictions for rectangular cuboids. This difference between the two model predictions for rectangular cuboids is most likely because Witt et al. [2005] used a conjugate-gradient rather than the dynamic numerical solver used by Muxworthy and Williams [2006]; rapid conjugate-gradient solvers are now generally considered to produce less robust solutions than dynamic solvers [Suess et al., 2002].

[9] Subsequent to publication of Muxworthy and Williams [2009], who used a micromagnetic approach to calculate the SP/SD threshold size for interacting chains of magnetite, Newell [2009] proposed an entirely different algorithm for making such calculations using polynomial homotopy continuation. While Muxworthy and Williams [2009] calculated the switching field of the chain to estimate the thermal fluctuation field needed to overcome this field, Newell [2009] directly calculated the energy barriers between possible magnetic states in the chain. For a chain of touching cubic crystals of magnetite, he determined an SP/SD transition size of ≈10 nm for a measurement time of 10^5 s, compared to ≈18 nm in Muxworthy and Williams [2009]. The approaches of Muxworthy and Williams [2009] and Newell [2009] both make assumptions, e.g., the paths calculated by Newell [2009] are only guides and are not the actual path followed by the magnetization during thermally assisted transitions. These approximations may be the reason why Newell [2009] determined a higher energy barrier than Muxworthy and Williams [2009]. For consistency and direct comparison with the results of Muxworthy and Williams [2009], we use their approach here to determine the SP/SD threshold size for interacting grains.

2. The Micromagnetic Model

[10] The numerical algorithms used in this paper are identical to those described by Muxworthy and Williams [2006, 2009], except that we use the physical parameters for greigite instead of magnetite. In the model, a grain is subdivided into a number of subcubes. Each subcube represents the averaged magnetization direction of many hundreds of atomic magnetic dipole moments. All of the subcubes have magnetic moments of equal magnitude, but the magnetization of the different subcubes can vary in direction. To determine the magnetic structures using this finite difference model, two approaches were considered; a combination of both a conjugate-gradient (CG) algorithm [Williams and Dunlop, 1989] and a dynamic algorithm [Suess et al., 2002], and the CG algorithm alone. The reason for the combined approach is that the dynamic algorithm gives a more rigorous solution; however, it is computationally slow compared to the CG method. In the combined approach, we use the CG algorithm to rapidly generate a magnetic structure, which is then put into the dynamic solver as an initial estimate. This increases the efficiency of the algorithm by roughly an order of magnitude compared to the dynamic solver alone.

[11] In the CG algorithm, the domain structure is calculated by minimizing the total magnetic energy, \( E_{tot} \), which is the sum of the exchange energy (\( \propto A \)), the magnetostatic energy (\( \propto M_S^2 \), where \( M_S \) is the spontaneous magnetization), and the anisotropy energy (\( \propto K \)). \( E_{tot} \) is calculated using a fast-Fourier transform (FFT) to give a local energy minimum (LEM) for the assemblage. The increased efficiency with which the demagnetizing energy can be calculated in Fourier space allows the high resolution needed to examine large arrays of interacting grains. The dynamic algorithm solves the dynamic Landau-Lifshitz-Gilbert equation. We used a finitely damped solver detailed by Brown et al. [1989]. In effect, instead of minimizing the energy, the solver minimizes the torque on each magnetic moment by solving for the effective field.

[12] Values for \( A (2 \times 10^{-12} \text{ J m}^{-1}) \) and \( M_S (59 \text{ Am}^2 \text{ kg}^{-1} = 241 \times 10^3 \text{ Am}^{-1}) \) are from Chang et al. [2008, 2009]. The value for \( K_1 \) ((cubic) \( \propto -1.7 \times 10^9 \text{ J m}^{-3} \)), which was determined from ferromagnetic resonance (FMR) powder spectra reported by Winklhofer et al. (submitted manuscript, 2013), is different to the value reported by Roberts et al. [2011] \( (+3 \times 10^4 \text{ J m}^{-3}) \). There is still uncertainty about the sign of \( K_1 \), because the measured FMR spectra of the greigite powder sample are consistent with two kinds of cubic anisotropy models, one with the \( K_1 < 0 \) solution used here, the other with \( K_1 > 0 \) and a large contribution from \( K_2 (K_2/K_1 = 3) \) (Winklhofer et al., submitted manuscript, 2013). For the lack of an independent physical justification of a large \( K_2/K_1 \) ratio in greigite, we here used the \( K_1 < 0 \) solution, but point out that both anisotropy models are equivalent in terms of the magnetocrystalline anisotropy field (\( \sim 100 \text{ mT} \)).
In determining the SD/MD boundary threshold size (sections 3 and 4 below), the initial CG estimate for the nonuniform magnetic structures in the most elongated grains was found to be effectively the same as the solution produced by the dynamic solver. This convergence between the two algorithms is due to smoothing of the energy surfaces as the grains become more elongated. For more symmetrical grains with more uneven energy surfaces, the CG algorithm can become hooked on small saddle points and trapped in shallow minimum energy states. Therefore, as grains become more elongated, the CG algorithm is less likely to stall and the CG and dynamic solutions converge. Because the CG solver is approximately an order of magnitude faster than the combined algorithm, and it is more memory efficient than the dynamic algorithm for some of the larger arrays, i.e., for the largest chains of elongated grains, calculations were made using only the CG algorithm.

To accurately model nonuniform structures, it is necessary to have a minimum model resolution of two cells per exchange length (exchange length = \(4(A/K_d)\), where \(K_d = \mu_0 M_S^2/2\) and \(\mu_0\) is the permeability of free space [Rave et al., 1998]). For greigite at room temperature the exchange length is 7.5 nm. The minimum resolution was maintained at all times in our calculations, i.e., each cell in the micromagnetic model was \(\sim 3.5\) nm.

3. SD/MD Critical Sizes for Individual Elongated Grains

There are several methods for determining the SD/MD critical size \(d_0\). Here the unconstrained method is employed [Fabian et al., 1996; Witt et al., 2005; Muxworthy and Williams, 2006]. In this approach, a small grain, say \(\sim 20\) nm in length, with an initial SD structure (Figure 2a) is gradually increased in size until the domain structure collapses to a vortex (i.e., MD) state at \(d_{\text{max}}\) (Figures 2b and 3). The grain size is then decreased until the vortex structure becomes SD at \(d_{\text{min}}\) (Figure 3). The \(d_{\text{min}}\) and \(d_{\text{max}}\) values are interpreted to represent the lower and upper bounds, respectively, of a range where both SD and vortex structures can coexist. For the most elongated grains, i.e., AR < 0.4, \(d_{\text{min}}\) and \(d_{\text{max}}\) are poorly defined because the collapse is gradual and less abrupt. In such cases, the \(d_{\text{min}}\) and \(d_{\text{max}}\) values are estimated at the point where the reduced magnetization passes through 0.8 on the increasing/decreasing curves, where the reduced magnetization is the magnetic moment divided by the magnetic moment of an ideal SD grain.

In addition to calculating \(d_{\text{min}}\) and \(d_{\text{max}}\) as a function of AR (Figure 4), we have considered the relationship between the relative orientation of the cubic magnetocrystalline anisotropy [Roberts, 1995; Roberts et al., 2011] and the particle elongation for greigite. We model two extreme cases:

![Figure 2. Domain states in cube-shaped grains of greigite at room temperature for a grain with an edge length of 83 nm: (a) single domain (flower state), and (b) single vortex state. In this paper, the term “SD state” refers not just to homogeneous magnetization structures, but also to nonuniform domain structures as shown in (a), which are essentially SD-like with a degree of flowering toward the edges of the grain. In (a) and (b), the crystallographic <100> direction is aligned with the x axis.](image-url)
first, where the elongation is in the easy direction (i.e., \(\text{h}_{111}\) axes for \(K_{1} < 0\)) (yielding \(d_{\text{min}}^{\text{easy}}\) and \(d_{\text{max}}^{\text{easy}}\)) and second where it is in the hard direction (\(d_{\text{min}}^{\text{hard}}\) and \(d_{\text{max}}^{\text{hard}}\)). In the scenario with elongation along the easy direction, the magneto-crystalline anisotropy enhances the shape effect, i.e., it encourages the magnetization to align along the elongation axis. In contrast, when the elongation is along the hard direction, the magneto-crystalline anisotropy competes with the shape effect.

[17] In Figure 4, the y axis is the particle length, as used by Butler and Banerjee [1975] and Diaz-Ricci and Kirschvink [1992], rather than the mean diameter as used by Witt et al. [2005]. Use of the particle length enables easier comparison with the results of Diaz-Ricci and Kirschvink [1992], but the figure is more complicated to understand because the volume of the grains changes with movement along the x axis, i.e., there is a change both in shape that contributes to \(d_{0}\) and in volume that contributes to \(d_{0}\).

[18] Generally \(d_{\text{min}}^{\text{easy}}, d_{\text{max}}^{\text{easy}}, d_{\text{min}}^{\text{hard}}, \) and \(d_{\text{max}}^{\text{hard}}\) increase as AR decreases (Figure 4). The \(d_{\text{max}}^{\text{easy}}\) value is the largest of the four for all values of AR. Orienting the magnetization along the easy axis enhances the effect of elongation, while orientation along the hard direction increases curling of the magnetization at the edges of the grains, which breaks symmetry, encourages nucleation of vortex states and decreases \(d_{0}\). As AR is reduced, the difference between \(d_{\text{min}}^{\text{easy}}\) and \(d_{\text{max}}^{\text{easy}}\) decreases. The same is true for \(d_{\text{min}}^{\text{hard}}\) and \(d_{\text{max}}^{\text{hard}}\) as the contribution of the magneto-crystalline anisotropy is reduced.

[19] For comparison, the calculated results of Diaz-Ricci and Kirschvink [1992] are depicted in Figure 4. Our micromagnetic estimates of the SD/MD threshold size are significantly lower than those obtained from the analytical results of Diaz-Ricci and Kirschvink [1992], who lacked experimentally well-constrained material parameters. For example, for a cubic grain, the micromagnetic
model gives an estimate of \( \sim 60 \) nm \( (d_{\text{min}}) \) compared to \( 250 \) nm from Diaz-Ricci and Kirschvink [1992]. This discrepancy can readily be explained by the different parameter combinations used. With \( M_s' = 1/2 \) \( M_s \) and \( A' = 5 \) \( A \), where the primed and unprimed symbols refer to parameters assumed in Diaz-Ricci and Kirschvink [1992] and here, respectively, we obtain for the exchange lengths \( L_{\text{ex}} = 4.4 L_{\text{ex}} \). According to the generic \( d_{\text{min}} \) calculations for equant particles by Rave et al. [1998], \( d_{\text{min}} = 7.6 L_{\text{ex}} = 250 \) nm for \( Q_1' = (K_1/4)/K_d' = 0.027 \) and \( d_{\text{min}} = 7.9 L_{\text{ex}} = 60 \) nm for \( Q_1 = (-K_1/12)/K_d = 0.12 \). Therefore, the threshold SD/MD threshold sizes of Diaz-Ricci and Kirschvink [1992] (250 nm) and ours \( (d_{\text{min}} \sim 60 \) nm) are in excellent agreement with the respective analytical values \( d_{\text{min}}' \) and \( d_{\text{min}} \).

4. SD/MD Critical Sizes for Magnetostatically Interacting Elongated Grains

To model the effect of magnetostatic interactions on \( d_0 \), we consider the behavior of the middle grain in a chain of three grains. Modeling of interactions in this paper was simply done by masking blank cells, setting cell magnetizations to zero, and thereby creating a “void” between neighboring magnetic regions of our finite-difference mesh [Muxworthy et al., 2003b]. We examine the behavior of chains, as opposed to a three-dimensional grid, because this is likely to produce the largest difference compared to noninteracting grains [Muxworthy and Williams, 2004], and should be seen as an upper limit: greigite magnetosomes often do not align in linear chains (Figure 1), unlike magnetite magnetosomes. A chain of three grains is short; however, it was chosen so that we could calculate \( d_0 \) for elongated grains with large intergrain spacings using a full resolution model (where both the magnetic grains and intergrain spaces contribute to the calculation time). Nevertheless, given the relatively large SD/MD threshold size compared to magnetite, we were unable to calculate the SD/MD threshold size domain phase diagram as completely as Muxworthy and Williams [2006] did for magnetite. For example, the largest solutions considered had \( > 10 \) million elements.

To estimate \( d_0 \), a slightly different procedure was used compared to that described above in section 3. Rather than growing the domain structure, an initial SD state was assumed at each grain size, and the model structure was minimized. This procedure produces only a single (upper bound) value for \( d_0 \). The only orientation of the magnetocrystalline anisotropy considered was with alignment along the elongation axis, i.e., \( d_{\text{max}} = d_{\text{easy}} \). All three grains in the chain had the same magnetocrystalline anisotropy orientation. We considered a number of minimum nontouching separations defined by the ratio \( s = \text{spacing/length} \), and calculated \( d_0 \) for touching grains, i.e., \( \text{spacing/length} = 0.0 \). It is readily seen that both elongation and magnetostatic interactions cause significantly increased \( d_0 \) (Figure 5). For cubic grains, the increase is from \( (d_{\text{max}} = 107) \) to \( (d_0 = 204 \) nm (Figure 5).

5. SP/SD Critical Sizes for Individual and Magnetostatically Interacting Elongated Grains

SD grains below a certain critical volume (the blocking volume, \( v_b \)) are magnetically unstable and have SP behavior. In systems of interacting SD grains, both SP and stable SD grains contribute to the interaction field. The magnetic interaction field generated by a stable SD grain is constant during rotation of a neighboring interacting SP or stable SD grain. This makes it possible to treat magnetic interaction fields due to stable SD grains as effectively static [Spinu and Stancu, 1998]. The effect of magnetostatic interactions due to stable...
SD grains is to increase/decrease the (micro)coercive ($H_K$) force of a crystal by the interaction field $\pm H_S$ [Dunlop and West, 1969]. For SP grains, the situation is more complicated. The behavior of a magnetic assemblage of SP particles falls into one of three regimes depending on the interparticle interactions [Dormann et al., 1997]: (i) a pure SP case (noninteracting), (ii) an SP state modified by interactions, and (iii) a collective state.

The properties of state (iii), which is called the glass collective state [Dormann et al., 1999], are close to those of spin glasses with a phase transition. This state is not fully understood and there is no analytical model for the collective state. However, Muxworthy and Williams [2009] showed that for chains of nearly identical, interacting grains, it is only necessary to consider state (ii), an SP state modified by magnetostatic interactions. The interaction field due to SP grains fluctuates at a high rate near the blocking volume or blocking temperature where relaxation is important. These dynamic interactions are qualitatively different from static ones. Dynamic systems are not in thermodynamic equilibrium and hence cannot be directly modeled using Boltzmann statistics. Several approaches have been developed to address this problem [Dormann et al., 1988]. These models demonstrate that the effect of interactions due to SP grains is to increase the relaxation time $t_m$, given by Néel [1949]:

$$ t_m = \tau_0 \exp \left( \frac{E_B}{kT} \right) $$

by increasing the energy barrier $E_B = E_A$ (where $E_A$ is the anisotropy energy barrier), by an amount termed the interaction energy $E_{\text{int}}$, i.e., $E_B = E_A + E_{\text{int}}$, where $k$ is Boltzmann’s constant, $T$ is the temperature and $\tau_0$ is the atomic reorganization time ($\sim 10^{-9}$) [Worm, 1998]. The relaxation time $t_m$ can be a few nanoseconds for SP particles that undergo thermal relaxation during laboratory experiments to billions of years for stable SD particles in geological samples.

For an SD assemblage with both dynamic SP and static stable SD interactions, using the experimentally verified theory of Dormann et al. [1988] and Muxworthy [2001] showed that the blocking volume, $v_b$, is given by:

$$ v_b = \frac{-E_{\text{int}} + 2kT(\ln(t_m/\tau_0))}{\mu_0 M_S (H_K \pm H_S)} $$

where $\mu_0$ is the permeability of free space. This equation is strictly for a system with only two possible states; however, in elongated and highly interacting chains this is likely to be the case. For symmetrical samples with higher order magnetocrystalline anisotropy, small errors will occur in blocking volume estimations.

### 5.1. Determining the Blocking Volume for a Chain of Identical Grains: A Model for Magnetosome Crystals

For SD assemblages with identically shaped particles and a distribution of grain volumes, it is necessary to calculate both $E_{\text{int}}$ and $H_S$ [Muxworthy, 2001]. However, Muxworthy and Williams [2009] argued that if it is assumed that every grain is either blocked or unblocked, i.e., all particles including the end grains have identical behavior, then the blocking volume is the volume where the magnetostatic interaction fields alone, i.e., $E_{\text{int}} \equiv 0$, are sufficient to overcome the thermal fluctuation fields. All that is required to find the blocking volume of a chain of identical grains is the value of $H_S$ such that $H_K \geq H_S$ ($\equiv$ the coercive force $H_C$) overcomes thermal fluctuations.

Muxworthy and Williams [2009] determined $H_C$ using a micromagnetic hysteresis simulation and demonstrated that a chain of only seven particles was sufficient. They argued that only hysteresis needs to be calculated for a simulated field applied along the chain length if the grain elongation and chain-length extension are aligned. Nevertheless, this approach provides only a lower limit for the SP/SD threshold for particles with nonuniaxial anisotropy: the application of an external field distorts the energy surface, therefore the in-field energy barrier determined using this approach will not have the same structure as the zero-field barrier. The relative degree of distortion will decrease as the interaction field increases, i.e., this assumption is more appropriate for strongly interacting magnetic particles.

In this study, we determine the SP/SD threshold for single crystals and chains of interacting greigite grains, where the grain elongation and chain alignment is along the easy axis (here one of the $<111>$ axes of the cubic system). For single crystals, we directly determined the energy barrier (equation (1)) from the anisotropy energy surface, i.e., a combination of the uniaxial shape anisotropy and the cubic magnetocrystalline anisotropy. For the interacting chains, the blocking volume was determined using equation (2) by first numerically calculating $H_C$ for a range of axial ratios. We only consider the case where the grains are touching
because this is the most reliable solution and represents the extreme limiting case. The model includes both the cubic magnetocrystalline anisotropy of greigite plus a “shape anisotropy,” which is calculated in the magnetostatic energy term. Values of \( t_m = 1 \mu s \) and 4 Gyr are plotted against grain volume in Figure 6. For single crystals, \( v_b \) decreases with increasing elongation (decreasing AR), in agreement with calculations for magnetite [Butler and Banerjee, 1975; Winklhofer et al., 1997; Muxworthy and Williams, 2009; Newell, 2009] and greigite [Diaz-Ricci and Kirschvink, 1992].

[28] In the interacting models, \( H_S \) (equation (2)) was positive. Generally, for systems with two and three-dimensional magnetostatic interaction fields, \( H_S \) is negative [Muxworthy and Williams, 2004]. Here it is positive because the particles are in a chain, which produces a linear interaction field. Positive magnetostatic interactions effectively increase \( H_C \), which results in a reduction of \( v_b \) (Figure 6), i.e., interactions decrease the SP to stable SD transition boundary. As the degree of interaction increases, \( v_b \) decreases.

[29] The blocking volume was determined for interacting chains for \( t_m = 1 \mu s \) and 4 Gyr (Figure 6). As was shown by Néel [1949] for individual grains, increasing the relaxation time results in increased blocking volume. The longer timescale was chosen because of potential interests in magnetic stabilities over the age of the Earth, while the shortest timescale is an extreme case to demonstrate variability in \( t_m \).

6. Domain-State Phase Diagram

[30] We have constructed a domain-state phase diagram for interacting chains of elongated greigite particles (Figure 7). Interactions both decrease the SP to stable SD transition size for chains of SD grains and increase the SD to MD transition size, which significantly expands the SD size range. For the SP/SD transition, an intermediate timescale of \( t_m = 100,000 \) s represents the stability time required for magnetotaxis (same order of magnitude as the lifespan of a bacteria). For example, for AR = 1 and \( t_m = 100,000 \) s, the noninteracting SD range is 46 nm < length < 107 nm, which increases to 23 nm < length < 204 nm for chains of touching grains. Although the absolute values for the critical domain-state threshold sizes are different to those for magnetite, the trends are the same. For comparison, interactions increase the SD to MD threshold size in a chain of magnetite from 73 nm for noninteracting grains to 198 nm for touching grains [Muxworthy and Williams, 2006]. The reason for the difference in absolute

Figure 6. Blocking volume curves on a volume versus axial ratio for single crystals and touching chains of greigite. Grain elongation and chain alignment is in the \(<111\) direction. Two relaxation times \( t_m \) are shown: 1 \( \mu \)s and 4 Gyr. The blocking volumes were determined directly from equation (1) for the single crystals, and for the touching grains by numerically solving equation (2).

Figure 7. Stable SD grain size ranges for individual particles and for chains of interacting greigite magnetosomes. Grain elongation and chain alignment is in the \(<111\) direction. Published observational data for MMP greigite magnetosomes [Pósfai et al., 2001] are plotted. Using the format of Butler and Banerjee [1975], length (long-axis) rather than volume is plotted versus AR for various grain spacing/length ratios. Use of length enables easier comparison with Butler and Banerjee [1975], but the figure is more complicated to understand because the particle volume changes across the x axis, i.e., there is a change in both shape and volume that contributes to the critical boundaries. Hence, the blocking size appears to increase with particle elongation, unlike in Figure 6. For noninteracting particles, there is a range of grain sizes marked by \( d_{\text{min}} \) and \( d_{\text{max}} \), where both SD and MD states are possible. For the SP/SD transition an intermediate timescale of \( t_m = 100,000 \) s represents the stability time required for magnetotaxis.
size for the domain-state thresholds is due to relative differences in values for the exchange energy, spontaneous magnetization and magnetocrystalline anisotropy for the two minerals.

### 6.1. Model Uncertainties

[31] There are empirical errors associated with the three input constants used in the model: $M_S$, $A$, and $K_1$. The error in $M_S$ is thought to be relatively small and insignificant, but the uncertainty in the other two parameters is likely to be greater and potentially important.

[32] An accurate determination of $A$ requires inelastic neutron scattering experiments to be carried out on a large single crystal of greigite. Due to the difficulty in producing large stoichiometric samples of greigite, Chang et al. [2008] estimated $A$ by studying powdered samples and fitting a Bloch law to an $M_S$ versus temperature curve; hence, the estimate for $A$ should be considered a good estimate rather than a definitive answer. Rave et al. [1998] showed the critical SD/MD threshold size scales with the exchange length $\propto A$, i.e., a twofold increase in $A$ would increase $d_{\text{max}}$ by about 40%. The error in $A$ was not quantified by Chang et al. [2008], but could be as high as 50–80%. The $K_1$ used in this study has recently been determined using ferromagnetic resonance spectroscopy (FMR) on a greigite powder sample (Winklhofer et al., submitted manuscript, 2013). Ideally $K_1$ should be determined from single-crystal measurements (torque curves, FMR spectra) on a large oriented crystal; therefore; again, the estimate for $K_1$ is subject to an error. However, unlike the error in the exchange constant, the response of a magnetic domain structure to variation in $K_1$ is nonlinear making it difficult to quantify the effect this error will have on the SD/MD threshold size. Results from initial calculations based on an earlier estimate of $K_1$ ($+3 \times 10^4$ J m$^{-3}$, $Q_1 = 0.2$) are shown in the supporting information (Figures S1–S3). The change in sign of $K_1$ changes the easy direction to the $<100>$ direction. It can be seen that a larger $|K_1|$ significantly increases the upper SD/MD limit size $d_{\text{max}}$, i.e., it stabilizes the metastable SD state, particularly when the elongation is along the easy crystal axis. The effect on the lower SD/MD limit size $d_{\text{min}}$ is not so pronounced; e.g., for equant particles, $d_{\text{min}}$ changes from 60 to 70 nm when going from $Q_1 = 0.04$ to $Q_1 = 0.2$. Since the SP limit size decreases with increasing $|K_1|$, the combined effect of a larger $|K_1|$ is an enlargement of both the stable and metastable SD field in the phase diagram.

### 6.2. Comparison with Greigite Magnetosome Data

[33] Plotted in Figure 7 are observational data from Pósfai et al. [2001] for MMP greigite magnetosomes. Most of these magnetosome sizes plot well below the noninteracting SD/MD threshold size, which suggests that they have magnetically stable SD sizes regardless of the degree of magnetostatic interactions; MMP magnetosomes are usually found in poorly defined chains, i.e., they are weakly interacting [Pósfai et al., 1998, 2001; Kasama et al., 2006a, 2006b; Winklhofer et al., 2007]. Winklhofer et al. [2007] showed experimentally that interactions in MMP greigite stabilize the remanence state of clustered SP and SD particles.

[34] Pósfai et al. [2001] also reported that greigite particles from a Miocene marl from Poland (‘a’aka) have grain size and AR distribution shapes that are similar to those for MMP bacteria. Pósfai et al. [2001] hypothesized that these greigite particles are magnetofossils, with the largest crystals being 240 nm in size (AR ~0.7), i.e., above the noninteracting SD/MD threshold size, but within the size range for stable SD behavior for magnetically interacting particles (Figure 7). Other observations of suggested greigite magnetofossils [e.g., Vasiliev et al., 2008] also fall within the stable SD range.

### 7. Conclusions

[35] Using a three-dimensional micromagnetic algorithm, we have calculated and constructed the first domain-state phase diagram for interacting chains of elongated greigite particles (Figure 7). As was found for magnetite [Muxworthy and Williams, 2006, 2009], interactions both decrease the SP to stable SD transition size for chains of SD grains and they increase the SD to MD transition size, which significantly expands the SD size range. For cubic greigite grains, the SD range for noninteracting SD grains is increased from 46 nm < length < 107 nm ($t_m = 100,000$ s) to 23 nm < length < 204 nm for chains of touching grains.

[36] Most published greigite magnetosome sizes plot well below the noninteracting SD/MD threshold size (Figure 7). Although greigite has a smaller spontaneous magnetization than magnetite...
(~64%) [Chang et al., 2008], the larger SD/MD threshold size means that, regardless of interactions, it is possible for SD greigite particles to produce magnetic signals that are more than twice as large as those for magnetite particles.

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References


