Lattice-Constrained Parametrizations of Form Factors for Semileptonic and Rare Radiative $B$ Decays

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We describe the form factors for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ and $B \to K^* \gamma$ decays with just two parameters and the two form factors for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ with three parameters. The parametrizations are constrained by lattice results and are consistent with heavy quark symmetry, kinematic constraints and light cone sum rule scaling relations.

We obtain a simple yet phenomenologically useful description of the form factors for semileptonic and rare radiative heavy-to-light meson decays for all $q^2$, the squared momentum transfer to the leptons or photon. Lattice calculations determine the form factors over a limited region at high $q^2$. We use model input to extend the results to $q^2=0$, seeking consistency with:

- kinematic constraints: $F_1(0) = F_0(0)$ and $T_1(0) = iT_2(0)$
- heavy quark symmetry (HQS)
- light cone sum rule (LCSR) scaling relations: all form factors scale like $M^{-3/2}$ as $M \to \infty$ at $q^2=0$, where $M$ is the heavy meson mass
- dispersive constraints

The normalisation is fixed using lattice results. The outcome is a two parameter fit for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ or $B \to K^* \gamma$ and a three parameter fit for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$. More details can be found in $\footnote{1}$. The leading order HQS analysis shows that heavy-to-light $P \to P$ decay form factors are determined by two universal (“Isgur-Wise”) functions, while $P \to V$ decays are governed by four more such functions ($P$ and $V$ denote pseudoscalar and vector mesons respectively). We adopt a model of Stech $\footnote{2}$ which keeps just one universal function for $P \to P$ and one more for $P \to V$.

Lattice simulation details can be found in $\footnote{2}$ with details of the chiral extrapolation for $\bar{B}^0 \to \pi^+ l^- \bar{\nu}_l$ in $\footnote{2}$. All form factors are calculated for four values of the heavy quark mass around the charm mass and for a variety of $q^2$. In our previous work $\footnote{3}$, the form factors were extrapolated at fixed four-velocity recoil, $\omega = v/(p_{P,V}/m_{P,V})$, near the zero recoil point $\omega = 1$, using the heavy-quark scaling relations:

$$f \Theta M^{\epsilon_f/2} = \gamma_f \left( 1 + \frac{\delta_f}{M} + \frac{\epsilon_f}{M^2} + \cdots \right)$$

where $n_f = -1,1,-1,1,-1,1,-1,1$ for $f = F_1, F_0, A_0, V, A_1, A_2, T_1, T_2$ and $\gamma_f, \delta_f$ and $\epsilon_f$ are fit parameters. $\Theta$ comes from leading logarithmic matching and is chosen to be 1 at the $B$ mass. This procedure neglects the fact that for $M \to \infty$, HQS predicts $A_1 = 2iT_2$ and $V = 2T_1$ at fixed $\omega$ not too far from $q^2_{\max}$. We enforce this condition by performing a combined fit, at fixed $\omega$, of the pairs $(A_1, T_2)$ and $(V, T_1)$ imposing the constraints: $\gamma_{A_1} = 2i\gamma_{T_2}$ and $\gamma_V = 2\gamma_{T_1}$. This guarantees that the extrapolated form factors are consistent with HQS in the infinite mass limit and reduces statistical errors by decreasing the number of parameters.

1. $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ AND $B \to K^* \gamma$ DECAYS

We use the freedom to adjust quark masses in lattice calculations and consider two situations for the light quark $q$ into which the $b$ decays:

1. Decays
Table 1
Form factor results for $B^0 \to \rho^+ l^- \bar{\nu}_l$ and $B \to K^{*}\gamma$. For $B^0 \to \rho^+ l^- \bar{\nu}_l$ the fit parameters are: $A_1(0) = 0.27(5)$, $M_1 = 7.0(40)$ GeV, $\chi^2$/dof = 24/20. For $B \to K^{*}\gamma$: $A_1(0) = 0.29(4)$, $M_1 = 6.8(7)$ GeV, $\chi^2$/dof = 27/20.

<table>
<thead>
<tr>
<th>$q^2$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_0$</th>
<th>$V$</th>
<th>$T_1$</th>
<th>$T_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.27(5)</td>
<td>0.26(3)</td>
<td>0.30(6)</td>
<td>0.35(5)</td>
<td>0.16(7)</td>
<td></td>
</tr>
<tr>
<td>$q^2_{\text{max}}$</td>
<td>0.46(7)</td>
<td>0.88(3)</td>
<td>1.80(3)</td>
<td>2.07(11)</td>
<td>0.90(5)</td>
<td>0.25(1)</td>
</tr>
</tbody>
</table>

A $q=u$: matrix elements of $\bar{u}\sigma^{\mu\nu}(1+\gamma^5)b$ are unphysical but constrain $B^0 \to \rho^+ l^- \bar{\nu}_l$.

B $q=s$: $\bar{s}\gamma^{\mu}(1-\gamma^5)b$ is unphysical but constrains $B \to K^{*}\gamma$.

We complete the parametrization by specifying one of the form factors. To meet all our requirements, including the LCSR scaling condition at $q^2 = 0$, we choose

$$A_1(q^2) = \frac{A_1(0)}{1 - q^2/M_1^2}$$

with free parameters $A_1(0)$ and $M_1$. This allows $A_1$, $A_2$ and $T_2$, which receive contributions from $1^+$ resonances, to diverge at larger $q^2$ than the more singular $V$, $A_0$ and $T_1$. We also tried other parametrizations but all results below will use $A_1(q^2)$ in eq. (1). Figure 1 shows the fit for a final state with the mass of the $K^*$, and Table 1 gives results for the form factors.

2. $B^0 \to \pi^+ l^- \bar{\nu}_l$ DECAYS

Stech’s model makes $F_0(q^2_{\text{max}})$ vanish in the chiral limit, contradicting our results and made unlikely by unitarity bounds. Furthermore, the $B^*$ which contributes a pole very close to $q^2_{\text{max}}$ in $F_1$, induces the same singularity in $F_2$ in the model. This provokes a much stronger $q^2$ dependence for $F_0$ than seen in the lattice results or induced by the nearest $0^+$ resonance. Therefore we restrict to polar-type $q^2$-dependences, consistent with the kinematical constraint, $F_1(0) = F_0(0)$, HQS and unitarity bounds. Our preferred model, consistent with LCSR scaling relations at $q^2 = 0$, is

$$F_1(q^2) = \frac{F(0)}{(1-q^2/m_1^2)^2}, \quad F_0(q^2) = \frac{F(0)}{(1-q^2/m_0^2)}.$$  

The result of the fit is: $F(0) = 0.27(11)$, $m_1 = 5.79(58)$ GeV and $m_0 = 6.1(15)$ GeV with $\chi^2$/dof = 0.1/3. All results below will be quoted using this pole/dipole model.

3. PHENOMENOLOGY

Using our fits we can calculate total rates and differential decay spectra in $q^2$ and lepton energy $E$ for the decays $B^0 \to \rho^+ l^- \bar{\nu}_l$ (Figure 3) and $B^0 \to \pi^+ l^- \bar{\nu}_l$. In Table 2 we give our results, illustrating the good agreement of our form factor values at $q^2=0$ with LCSR calculations, and compare rates and ratios for semileptonic decays. For $B \to K^{*}\gamma$ we evaluate the ratio $R_K = \Gamma(B \to K^{*}\gamma)/\Gamma(b \to s\gamma) = 16(3)\%$, to be compared with $(18 \pm 7)\%$ from experiment.
Table 2
Form factor values at $q^2 = 0$ with $B \to \pi, \rho$ semileptonic decay rates and ratios from this calculation and from light cone sum rules (LCSR). Decay rates are given in units of $|V_{ub}|^2 \text{ps}^{-1}$. $\Gamma_{\rho/\pi} = \Gamma(B^0 \to \rho^+ l^- \bar{\nu}_l)/\Gamma(B^0 \to \pi^+ l^- \bar{\nu}_l)$ and $\Gamma_{L/T}$ denotes the ratio of rates to longitudinally and transversely polarised rho mesons in $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$. $l$ denotes a massless lepton.

<table>
<thead>
<tr>
<th>$F_1(0)$</th>
<th>$A_1(0)$</th>
<th>$A_2(0)$</th>
<th>$V(0)$</th>
<th>$T_1(0)$</th>
<th>$\Gamma_{\pi\ell\nu}$</th>
<th>$\Gamma_{\rho\ell\nu}$</th>
<th>$\Gamma_{\rho/\pi}$</th>
<th>$\Gamma_{L/T}$</th>
<th>$\Gamma_{\pi\tau\bar{\nu}_\tau}$</th>
<th>$\Gamma_{\rho\tau\bar{\nu}_\tau}$</th>
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</thead>
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<tr>
<td>0.27(11)</td>
<td>0.27(5)</td>
<td>0.26(5)</td>
<td>0.35(5)</td>
<td>0.16(7)</td>
<td>8.5(13)</td>
<td>16.5(35)</td>
<td>1.9(2)</td>
<td>0.80(4)</td>
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<td>8.8(14)</td>
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LCSR

<table>
<thead>
<tr>
<th>$\rho/\pi$</th>
<th>$\pi\tau\bar{\nu}_\tau$</th>
<th>$\rho\tau\bar{\nu}_\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.27(5)</td>
<td>0.28(5)</td>
<td>0.35(7)</td>
</tr>
<tr>
<td>0.24(4)</td>
<td>0.28(6)</td>
<td>0.16(3)</td>
</tr>
<tr>
<td>0.24–0.29</td>
<td></td>
<td>8.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.15(3)</td>
</tr>
</tbody>
</table>

Figure 2. Differential decay spectra for $\bar{B}^0 \to \rho^+ l^- \bar{\nu}_l$ for massless leptons: (a) $d\Gamma/dq^2$ in units of $10^{-12}|V_{ub}|^2 \text{GeV}^{-1}$, (b) $d\Gamma/dE$ in units of $10^{-12}|V_{ub}|^2$. The dashed lines show the envelope of the 68% bootstrap errors computed separately for each value of $q^2$ or $E$ respectively.

REFERENCES