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# Center Vortices and the Asymptotic String Tension

L. Del Debbio<sup>a</sup>, M. Faber<sup>b</sup>, J. Greensite<sup>c</sup> \*, Š. Olejník<sup>d</sup>

<sup>a</sup>CPT, CNRS Luminy, Case 907, F-13288 Marseille Cedex 9, France

<sup>b</sup>Inst. für Kernphysik, Tech. Univ. Wien, A-1040 Vienna, Austria

<sup>c</sup>The Niels Bohr Institute, DK-2100 Copenhagen Ø, Denmark

<sup>d</sup>Inst. of Phys., Slovak Acad. of Sci., SK-842 28 Bratislava, Slovakia

We present a method for locating center vortices (“fluxons”) in thermalized lattice gauge field configurations. We find evidence, in lattice Monte Carlo simulations, that the asymptotic string tension of fundamental-representation Wilson loops is due to fluctuations in the number of center vortices linking those loops.

## 1. Introduction

In this contribution I would like to discuss a variety of numerical data, obtained recently by our group, which supports the Center Vortex Theory of confinement. This theory was proposed, in various forms, by ’t Hooft [1], Mack [2] and by Nielsen and Olesen [3] (the “Copenhagen Vacuum”), in the late 1970’s. Space limitations do not allow me to actually display very much of the relevant data here; for this I must refer the interested reader to some other recent conference proceedings [4].

The most popular theory of quark confinement is the abelian projection theory proposed by ’t Hooft [5]. In past years our group has been highly critical of this theory (as well as the center vortex theory), on the grounds that it fails to explain the existence of a linear potential between higher representation quarks in the Casimir scaling regime [6,7]. This failure is very significant, because it is in the Casimir regime that the confining force replaces Coulombic behavior, and in fact it is only in this regime that the QCD string has been well studied numerically. If we don’t understand Casimir scaling, then we don’t really understand how flux tubes form.

A possible response to this criticism is simply

to admit that the formation of flux tubes, at intermediate distances, remains to be understood, but that the abelian projection theory is nonetheless valid at very large distance scales, where Casimir scaling breaks down and color screening sets in. I will argue that there may be some truth to this response, but that the confining configurations relevant to this asymptotic regime seem to be  $Z_N$  vortices, rather than abelian monopoles.

## 2. Center Dominance

I will begin with the phenomenon of “center dominance,” which we reported, at the Lattice 96 meeting last year [6], as part of a critique of the abelian projection theory. The idea is as follows: In an  $SU(2)$  lattice gauge theory, begin by fixing to maximal abelian gauge [8]. Then go one step further, using the remnant  $U(1)$  symmetry to bring the abelian link variables  $A = \text{diag}[e^{i\theta}, e^{-i\theta}]$  as close as possible to the  $SU(2)$  center elements  $\pm I$ , by maximizing  $\langle \cos^2 \theta \rangle$ , leaving a remnant  $Z_2$  symmetry. This is the (**indirect**) **Maximal Center Gauge** (the center is maximized in  $A$ , rather than directly in the full link variables  $U$ ). We then define, at each link,  $Z \equiv \text{sign}(\cos \theta) = \pm 1$  which is easily seen to transform like a  $Z_2$  gauge field under the remnant  $Z_2$  symmetry. “Center Projection” is the replacement  $U \rightarrow Z$  of the full link variables by the center variables; we can then calculate Wilson loops,

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Creutz ratios, etc. with the center-projected  $Z$ -link variables. What we found was the following [6]:

1. Center-projected Creutz ratios  $\chi(R, R)$  scale very nicely with  $\beta$ , following the usual prediction of asymptotic freedom. Moreover, at fixed  $\beta$ , they are nearly  $R$ -independent, indicating that the Coulombic contribution is suppressed, and only the constant confining force remains.
2. If the  $Z$  variable is factored out of the abelian links, and loops are computed from the  $A/Z$  variables, the string tension disappears.

The fact that the  $Z$  variables seem to carry most of the information about the confining force is what we mean by “center dominance.”

The only excitations of  $Z_2$  lattice gauge theory with non-zero action are “thin”  $Z_2$  vortices, which have the topology of a surface (one lattice spacing thick) in  $D=4$  dimensions. We will call the  $Z_2$  vortices, of the center projected  $Z$ -link configurations, “Projection-vortices” or just **P-vortices**. These are to be distinguished from the hypothetical “thick” center vortices, which might exist in the full, unprojected  $U$  configurations. The first question to ask is whether the presence or absence of P-vortices, in a given center-projected lattice, is correlated with the confining properties of the corresponding unprojected lattice.

### 3. Vortex-Limited Wilson Loops

We will say that a plaquette is “pierced” by a P-vortex if, upon going to maximal center gauge and center-projecting, the projected plaquette has the value  $-1$ . Likewise, a given lattice surface is pierced by  $n$  P-vortices if  $n$  plaquettes of the surface are pierced by P-vortices.

In a Monte Carlo simulation, the number of P-vortices piercing the minimal area of a given loop  $C$  will, of course, fluctuate. Let us define  $W_n(C)$  to be the Wilson loop evaluated on a sub-ensemble of configurations, selected such that precisely  $n$  P-vortices, in the corresponding center-projected configurations, pierce the minimal area of the loop. It should be emphasized

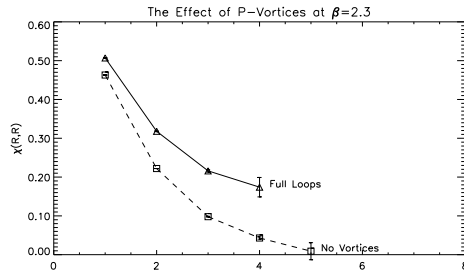


Figure 1. Creutz ratios  $\chi_0(R, R)$  extracted from loops with no P-vortices, as compared to the usual Creutz ratios  $\chi(R, R)$ , at  $\beta = 2.3$ .

here that the center projection is used only to select the data set. The Wilson loops themselves are evaluated using the full, *unprojected* link variables. Then, if the presence or absence of P-vortices in the projected configuration is unrelated to the confining properties of the corresponding unprojected configuration, we would expect

$$\chi_0(I, J) \approx \chi(I, J) \quad (1)$$

at least for large loops.

The result of this test is shown in Fig. 1. Quite contrary to our original expectations, the confining force vanishes if P-vortices are excluded. This does not necessarily mean that the confining configurations of  $SU(2)$  lattice gauge theory are thick center vortices. It does imply, however, that the presence or absence of P-vortices in the projected gauge field is strongly correlated with the presence or absence of confining configurations (whatever they may be) in the unprojected gauge field.

The next question is whether these confining configurations are, in fact, thick center vortices. If they are, then a short argument (see ref. [4,9]) leads to the prediction that

$$W_n[C]/W_0[C] \rightarrow (-1)^n \quad (2)$$

as the loop size increases. Figure 2 shows the ratio  $W_1/W_0$ , which seems to confirm this prediction;

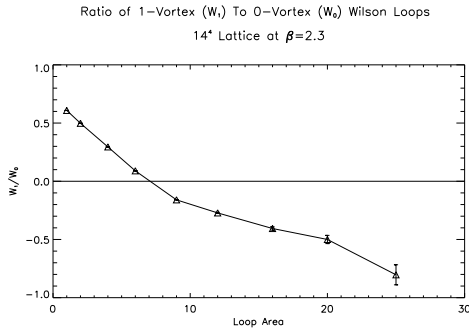


Figure 2. Ratio of the 1-Vortex to the 0-Vortex Wilson loops,  $W_1(C)/W_0(C)$ , vs. loop area at  $\beta = 2.3$ .

we have other data showing that  $W_2/W_0 \rightarrow +1$ . We have also considered loops pierced by only even, or only odd, numbers of P-vortices, and found that the string tension in those cases vanishes, and that  $W_{odd}[C]/W_{even}[C] \rightarrow -1$ . This data, in conjunction with center dominance, is a strong indication that P-vortices are correlated with thick center vortices, and that these thick vortices are responsible for the confining force. It is also consistent, we believe, with related results reported by Kovács and Tomboulis at this meeting [10].

#### 4. Latest Results

I would like to briefly mention some further developments [4]:

1. We have introduced a **(direct) Maximal Center Gauge**, which brings the entire link variable (not just the abelian part) as close as possible to  $\pm I$ . In this gauge there is not only scaling, but also very close numerical agreement of the center-projected string tension with currently accepted values for the asymptotic string tension.
2. We have found, in the (indirect) maximal center gauge, that almost all monopoles found in the abelian projection lie on P-

vortices, and that virtually all of the excess field strength of (unprojected) monopole cubes, above the lattice average, is directed along the P-vortices. Monopoles would appear to be an artifact of the abelian projection; they are condensed because the underlying vortices from which they emerge are condensed.

Finally, there is the issue of the Casimir scaling of higher-representation string tensions, in the intermediate distance regime. We have not forgotten the point that Casimir scaling, whose importance we have often emphasized [6,7], does not seem to be explained by the center vortex theory [9]. Nor has this point gone unnoticed by other people at this meeting [11]. Very recently we have found a possible explanation for Casimir scaling within the framework of the vortex theory. This explanation will be reported elsewhere.

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