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Discount rates for long-term projects:
the cost of capital and social discount rate compared

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Research on the cost of capital and on the social discount rate (SDR) has developed largely along separate paths. This paper offers an overview and comparison of both concepts. The consumption-based theory of discount rates is common to both, but there are striking differences in how the cost of capital and SDR are estimated. A project’s cost of capital is inferred in practice from market data, by a well-established package of techniques, and project risk makes a large difference. In contrast, the SDR is estimated by applying judgement about the welfare of future generations, in the setting of consumption-based theory. Project risk has tended to be ignored under the SDR approach.

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1. **Introduction**

The discount rate has a massive impact on the present value (PV) of long-lived projects, and especially on projects where most of the benefits arise in the fairly distant future, say after 100 years. Many projects have lifetimes of decades or centuries, including investment in infrastructure, housing, energy, research and development, and investment to protect the environment and reduce global warming. Such projects are found extensively in the private as well as the public sector. They are an important category of investment.

The discount rate for projects in the private sector is known as the cost of capital, and is discussed in the finance literature. The discount rate for projects in the public sector is known as the social discount rate (SDR), and is discussed in the literature on public sector economics. Research on the two rates has developed quite separately, especially on the finance side where the SDR is almost never mentioned. Are the two really the same concept, or do they differ in some ways? Is a project’s value affected by whether it is undertaken in the private or the public sector? The paper compares the two concepts, and the ways of thinking associated with each. No systematic comparison has previously been made, though the applicability or otherwise of the cost of capital to public projects is a major theme in research on the SDR. Long-lived projects are the focus, because much of the literature on the SDR, and especially on climate change, is concerned with the impact of investment decisions on future generations. At times the *Stern Review* (2006) on climate change is referred to, as a high-profile application of the SDR that has stimulated renewed interest.

There are several points to highlight from the discussion. The first concerns the role of theory. Consumption-based theory provides the analytical framework for thinking about discount rates in both the finance and SDR approaches. But the two use the theory differently. The finance approach uses the theory to try to understand what determines discount rates, and to predict what discount rates should be observed, given plausible assumptions about risk aversion and the other variables in a consumption-based model. If the predictions from such a model differ from expected rates of return estimated from market data – and they tend to differ – the estimates from market data, for all their problems, are the ones which are used for the purpose of estimating the cost of capital in practice. In contrast, the SDR approach uses consumption-based theory to help guide the application of judgement by a public sector executive about the welfare of future people. Judgement involves appeal to evidence, including market data, but the latter do not take priority. SDRs tend to be lower, sometimes much lower, than estimates of the cost of capital for private-sector projects.
Second, risk and the premium for bearing risk are of central importance in the practical application of the cost of capital. Differences in estimated risk result in large differences in the cost of capital. Risk is much less prominent in the SDR approach. Much of the SDR literature uses the ‘Ramsey model’ of the discount rate, which assumes that the project is risk free. In principle this is a mistake, since public projects do not in general provide risk-free benefits to society. The risk or consumption beta of a public project should be estimated from the covariance of the project’s benefits with consumption per head. Such a method is analogous to estimating the conventional beta of a private project from the covariance of its payoffs with the value of the stock market. But even if risk is allowed for in public projects, the risk premiums that arise in standard consumption-based models are small, less than 0.5% per year, compared with the risk premiums estimated in practice for private sector projects. This is a corollary of the famous equity premium puzzle: consumption-based theory predicts a much smaller premium than is estimated from market data.

Third, discounting using the cost of capital in practice always involves a constant risk premium over multiple future periods. The equity risk premium is usually estimated to be at least three per cent per year. This is the premium that is used in valuing project equity with average risk, ie a beta of around one. The impact of a premium of a few percentage points on the PV of a long-term project is enormous. Compounding a constant premium implies that the valuer’s uncertainty about the future cash flows increases steadily with the distance of the cash flows into the future. If the valuer’s uncertainty does not, in fact, increase in this way with the time horizon, it is not correct to compound a fixed premium. So the relationship between the valuer’s uncertainty and the time horizon is critical, and should be a first-order consideration in the valuation of long-term projects. This is not a new point, but we suggest that it is one that deserves more prominence.

Fourth, there are arguments, developed in research on the SDR, for using a declining discount rate for long-term projects. A declining discount rate means either that the rates applied to future periods decline with distance into the future, or that the (fixed) rate used should be negatively related to the lifetime of the project. The possibility of a declining rate for long-term projects is absent from the finance-driven package of techniques used to estimate the cost of capital. We suggest that the arguments for a declining discount rate be given more attention, in the context of conventional discounting using the cost of capital. However, since a project’s discount rate is uncertain, it is natural for the PV to be estimated as an average of the PVs using different possible rates. The PV using the lowest rate will increasingly dominate the average as project life increases. This procedure has the same
impact on the best-guess PV as does discounting by a single rate that is lower for longer-term projects.

Finally, we conclude that the cost of capital and the SDR are different concepts. The cost of capital is an opportunity cost which is given by the expected rate of return, estimated from market data, on a traded asset of the same risk as the project in question. The SDR as applied in recent years is in part a vehicle for reflecting judgements about the welfare of future generations. The SDR is, therefore, not an opportunity cost given by market data, and this is a fundamental difference. Under the SDR approach, the amounts of utility from future cash flows, including the cash flows that represent externalities, can be judged to differ from the amounts of utility that are implicit in a cost of capital estimated from market data. As a result, the SDR applied to a given project can differ from its estimated cost of capital, with a smaller SDR reflecting a greater weight for future utility. It is intrinsic to the concept of the SDR that it is a vehicle to capture the weight given to the welfare of future generations, as a result of judgement rather than inference from market data.

2. Consumption-based theory

2.1 Outline

The cost of capital and the SDR share a common theoretical background, which we introduce here.\textsuperscript{1} Consumption is fundamental in theory because utility is assumed to derive from consumption of goods and services, not from financial wealth. People are assumed to make decisions in order to maximise their ‘lifetime utility’, that is, the utility from their current (date 0) consumption together with the utility from their expected consumption during the rest of their life. Individuals differ in terms of preferences, beliefs, and wealth. To avoid the resulting complications, it is customary to assume that all individuals are identical, so that their collective behaviour can be modelled by the behaviour of a representative individual, or by what we will call society. The aim of society is to maximise lifetime utility \( U \). With one future date, \( U \) is given by

\[
U = U(C_0) + U(C_1)
\]

where \( C_t \) is consumption per head at date \( t \).

It is usual to assume that society has a utility function with constant relative risk aversion (CRRA), which is of the form

\[
U(C) = C^{1-\eta}/(1-\eta)
\]

\textsuperscript{1} See Armitage (2005), Danthine and Donaldson (2005), or Gollier (2013), for fuller expositions, all relatively accessible in terms of mathematics.
so

\[ MU(C) = C^\eta \]  

(3)

where \( MU = dU(C)/dC \), the marginal utility of consumption, and \( \eta \) is the elasticity of marginal utility, or the coefficient of relative risk aversion. If \( \eta > 0 \), the utility function is concave and society is risk-averse. Starting with a given \( C \), the utility gained from a marginal increase in \( C \) is less than the utility lost from a marginal decrease in \( C \). CRRA means that society’s risk aversion with respect to a given percentage change in consumption is constant as consumption changes. For a given loss of \( C \) measured in absolute terms, society’s aversion to (= loss of utility from) this absolute loss of \( C \) diminishes as consumption rises, which seems plausible.

Reasonable values for \( \eta \) are usually considered to be between one and four. They are based on empirical evidence about how risk-averse people are, and on judgement about what seems plausible. To gain a feel for the impact of \( \eta \), suppose that consumption at the present date will turn out to be either plus or minus 10% (50%) of its expected value, with equal probability. An individual with \( \eta = 2.0 \) in equation (2) would pay 1% (25%) of the expected value of consumption to avoid this uncertainty. CRRA means that these proportions are the same whatever the absolute amount of expected consumption.

Society maximises lifetime utility by saving to the point at which the marginal utility from saving, ie from the resulting higher consumption at date 1, is the same as the marginal utility from consumption at date 0. This applies whether the amount saved is invested in risk-free or risky assets. Assume for the moment that the future is known for certain. Saving one unit of consumption at date 0 provides \( e^{\tau F} \) units at date 1, where \( r_F \) is the continuously compounded risk-free interest rate. The condition for maximising lifetime utility can now be stated as

\[ MU_0 = e^{\tau F}MU_1e^{-\delta} \]  

(4)

where \( MU_t \) is marginal utility at date \( t \), and \( \delta \) is the discount rate for utility, or the rate of pure time preference. A positive \( \delta \) means that the utility to be experienced from a given amount of consumption at date 1 contributes less to lifetime utility than utility from the same amount of consumption today. The standard justification for a positive \( \delta \) is the assumption that people are impatient; they prefer to consume now rather than in the future.

The interest rate is determined as the rate which solves equation (4); it is the rate at which the marginal utility from saving is equal to the marginal utility from consumption today:

\[ r_F = \ln(e^{\delta MU_0}/MU_1) \]  

(5)
Let be $g$ be the continuously compounded growth rate of consumption, so $C_1 = C_0 e^g$. With the CRRA utility function, ie using equation (2), equation (5) can be written as

$$r_F = \ln[e^g C_0^{-\eta}(C_0 e^g)^{-\eta}]$$

$$= \delta + \eta g$$

(6)

The interest rate is positive if $C_1 > C_0$, and so $MU_1 < MU_0$; if society will be better off in the future, the interest rate needs to be positive for society to be indifferent between saving and consumption at date 0. If future utility is discounted ($\delta > 0$), this is a further reason for a positive interest rate. Equation (6) is the formula presented in Ramsey (1928), and it is the foundation of much of the literature on the SDR.\(^2\)

Now assume that future consumption is uncertain. Specifically, assume that the growth in consumption, $C_1/C_0$, is lognormally distributed, in which case $g$ is normally distributed. Let the expected value of $g$ be $\mu$ and the variance of $g$ be $\sigma^2$.

Consider first the risk-free asset. One unit of consumption invested in this asset provides $e^{r_F}$ units at date 1 for certain, whatever the growth of consumption turns out to be. The utility from the risk-free asset is therefore $e^{r_F}E(MU_1)e^{-\delta}$.\(^3\) It is the case that, for any variable $X$ which is lognormally distributed,

$$E(X) = \exp[E(\ln X) + 0.5\var(\ln X)]$$

(7)

where $\var(\ln X) = \text{variance of } \ln X$. Using this, we have

$$r_F = \ln[e^{\delta}MU_0/E(MU_1)]$$

$$= \ln\{e^{\delta}/\exp[(E(\ln e^g)^{-\eta}) + 0.5\var(\ln e^g)^{-\eta}]\}$$

$$= \delta + \ln\{\exp[-\eta \mu + 0.5\eta^2 \sigma^2]\}^{-1}$$

$$= \delta + \eta \mu - 0.5\eta^2 \sigma^2$$

(8)

Equation (8) reveals that the risk-free rate is positively related to the pure rate of time preference $\delta$, and expected growth rate of consumption $\mu$, and negatively related to the variance of the growth rate $\sigma^2$. Greater uncertainty about future consumption implies higher expected marginal utility from risk-free saving, and a lower risk-free rate. The reason is that, for a given expected value of future consumption, and with diminishing marginal utility, differences in the individual’s possible future consumption reduce expected utility. This means that expected marginal utility rises, and so $r_F$ falls, as the variance of consumption rises, for a given value of expected consumption. However, the third term in equation (8) is

\(^2\)See Ramsey’s equations (3) and (9); also Dasgupta (1982). Ramsey assumes non-increasing marginal utility, but does not specify a utility function. Equation (6) expressed in continuous time is

$$r_F = \delta - [U''(C_t)/U'(C_t)]d(C_t)/dt = \delta + \eta [d(C_t)/dt]/C_t = \delta + \eta g,$$

with $\eta$ defined as $-C_tU''(C_t)/U'(C_t)$.

\(^3\)The introduction of expected utility requires certain assumptions to be made about the rationality of society.
small for conventional values of $\eta$ and $\sigma^2$, which means that the relation between $r_F$ and aversion to risk is normally predicted to be positive. Equation (8) can be derived in the case of $t > 1$ time periods, if it is assumed that the growth rate per period $g_t = \ln(C_t/C_{t-1})$ follows a random walk (arithmetic Brownian motion). There will be a flat term structure of the discount rate; the same rate applies to each future period.

We now turn to risky assets. The uncertain payoff at date 1 from investing one consumption unit in risky asset $j$ at date 0 is $e^{r_j}$. The utility from $j$ is $E(e^{r_j}MU_1)e^{-\delta}$. For marginal consumption now and investment in the risky asset to provide the same utility requires that

$$MU_0 = E(e^{r_j}MU_1)e^{-\delta}$$

Assume that $e^{r_j}$, $(e^{r_j})^{-\eta}$, and $(e^{r_j})(e^{g})^{-\eta}$ are lognormally distributed. It is the case that

$$\ln E(XY) = E(\ln X) + E(\ln Y) + 0.5[\text{var}(\ln X) + \text{var}(\ln Y) + 2\text{cov}(\ln X, \ln Y)]$$

if $X$, $Y$, and $XY$ are lognormally distributed. Using this and equation (3), equation (9) can be solved for $E(r_j)$:

$$\ln(1) = \ln[e^{\delta}C_0^{-\eta}/E(e^{r_j}C_0e^{g})^{-\eta}]$$

$$0 = \delta - E(r_j) + \eta \mu - 0.5[\text{var}(r_j) + \eta^2 \sigma^2 - 2\eta \text{cov}(r_j, g)]$$

$$\therefore E(r_j) \approx \delta + \eta \mu - 0.5 \eta^2 \sigma^2 + \eta \text{cov}(r_j, g)$$

ignoring $0.5\text{var}(r_j)$ which is relatively small.\(^4\) This can be re-expressed in the form of the consumption capital asset pricing model (CCAPM):

$$E(r_j) = r_F + \beta_j \pi$$

where $\beta_j = \text{cov}(r_j, g)/\sigma^2$ is the consumption beta of asset $j$, and $\pi = \eta \sigma^2$ is the premium for systematic risk. Equation (11) is the same as equation (8), the expression for the risk-free rate, but with the additional term $\eta \text{cov}(r_j, g)$, which quantifies the expected risk premium on the asset, ie $E(r_j) - r_F \approx \eta \text{cov}(r_j, g) = \beta_j \pi$. Equation (11) says that an asset with returns which covary positively with changes in consumption will be priced to give a positive risk premium. This is what we would expect for an asset which tends to pay off most in states in which consumption is highest, and marginal utility is lowest. The risk premium is positively related to $\eta$, the measure of risk aversion, and this is also what we would expect. The higher is $\eta$, the more rapidly marginal utility diminishes as consumption increases. In the consumption-based model, systematic risk means negative covariance of payoff with marginal utility, or positive

\(^4\) The term $0.5\text{var}(r_j)$ arises because $r_j$ is a continuously compounded return: $E(1 + R_j) = E(e^{r_j}) = \exp[E(r_j) + 0.5\text{var}(r_j)]$, where $R_j$ is the uncompounded return.
covariance of payoff with consumption, and unsystematic risk means variance of payoff uncorrelated with consumption. Risk does not mean uncertainty about payoff *per se*, but the belief that the asset will pay least in recession.\(^5\) Equation (11) applies with more than one period if we assume that \(r_{jt}\) and \(g_t\) per period both follow a random walk, so they are jointly normally distributed (Gollier, 2013, pp. 188-93).

### 2.2 Predictions from consumption-based theory

The predictions from the above consumption-based model depend critically on the values assumed for the four parameters. Weitzman (2007a) suggests that the following ‘quartet of twos’ are representative of the numbers that economists accept as reasonable: \(\delta = 2.0\%\); \(\eta = 2.0\%\); \(\mu = 2.0\%\); \(\sigma = 2.0\%\). The mean real growth rate and its standard deviation, \(\mu\) and \(\sigma\), are based on historical evidence from mature economies and on predictions for future growth. The rate of pure time preference and the coefficient of risk aversion, \(\delta\) and \(\eta\), are more debatable. Assume for simplicity that returns on the stock market are perfectly correlated with growth of consumption, and that both have the same volatility. In this case \(\eta \text{cov}(r_j, g) = \eta \sigma^2\) in equation (11).\(^6\) Inserting the quartet of twos into equations (8) and (11) then gives the following predictions:

\[
\begin{align*}
    r_F &= 5.92\% \\
    \text{E}(r_E) &= 6.00\%
\end{align*}
\]

where \(\text{E}(r_E)\) is the expected return on equity. So the predicted equity premium is 0.08% per year. These predictions compare with the following empirical numbers for mature economies, based on historical arithmetic averages over the last 100 years or so:

\[
\begin{align*}
    r_F &\approx 1\% \\
    r_E &\approx 7\%
\end{align*}
\]

and so the historic premium is approximately 6% per year. Many would regard a figure of 3% or 4% as more realistic for the equity premium looking forwards.

The two puzzles that arise from comparing the predicted with the observed numbers are (i) the observed equity premium is much higher than the model can explain, and (ii) the risk-free rate is much lower. There is an important difference between these puzzles. If \(\delta = 0.1\%\) instead of 2.0%, and \(\eta = 1.0\) instead of 2.0, as in the *Stern Review* (2006), the predicted

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\(^5\) In the derivation of the standard CAPM, the individual’s existing consumption at date 1 comes entirely from the payoff from her holding of the market portfolio, so risk means positive covariance with the market portfolio.

\(^6\) In fact the correlation between returns on the stock market and growth of consumption per head is 0.2 at most, and the standard deviation of annual returns on the market is about 17% (US data; Cochrane, 1997). This gives \(\text{cov}(r_j, g) = (0.2)(0.17)(0.02) = 0.07\%\) instead of \(\sigma^2 = (0.02)^2 = 0.04\%.\)
risk-free rate is only 2.1%, much closer to its historical average, whereas the equity premium puzzle is still just as great. It is possible to predict a low risk-free rate, similar to the observed number, with low but still-plausible values of $\delta$ and $\eta$, whereas it is not possible to predict an equity premium of more than about 0.5% even with very high values of $\eta$. The fundamental empirical problem for the model is that the historic variance of consumption per head is not enough to justify much of an equity premium.

3. Finance in practice: estimation of the cost of capital

Although the theory just outlined is fundamental in finance, it is not used in practice to estimate the cost of capital. The current section provides a very brief outline of the practical finance package of ideas and methods of estimation. This is the package found in finance textbooks below advanced level, and used by companies, regulators of private-sector utility companies, and consultants.

The concept of the cost of capital is that of an opportunity cost. It is the expected rate of return available from the next-best alternative to the project in question. The next-best alternative in this context means an asset with the same risk as the project. The CAPM is the leading model in textbooks and in practice used for estimating the expected return on risky asset or project $j$:

$$E(R_j) = R_F + \beta_j[E(R_M) - R_F]$$

where $R_F$ is the (uncompounded) risk-free rate, $E(R_M)$ is the expected return on the market portfolio of all assets, and $\beta_j = \text{cov}(R_j, R_M)/\text{var}(R_M)$, the expected covariance of the return on the asset with the return on the market, divided by the expected variance of the return on the market. The key point for our purposes is that the ingredients of the CAPM are estimated from market data, that is, data about traded financial assets, such as prices and dividends. The risk-free rate tends to be estimated from current yields on long-dated bonds, or from swap rates, or perhaps from a historical average of yields or swap rates. The beta of a share is estimated by regressing the returns of the share on the returns on the stock market of which it is a constituent. The equity risk premium, $E(R_M) - R_F$, is estimated from a historical average of values of the premium for each year observed ex post, or from a forward-looking estimate.

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7 In addition, Weitzman (2007a, 2007b) shows that, if $g_t$ is assumed to have a distribution with fatter tails than the normal distribution, the possibility of ‘disaster states of the world’ is increased, which reduces the implied risk-free rate.

8 See, for example, Graham et al (2001), for survey evidence of company practice. For much more detail about implementation than appears here or in a textbook, see the documentation published by utility regulators and their advisers (for example, NERA, 2009).
of the expected rate of return on equity. There are ongoing debates about what the best methods are to estimate the ingredients of the CAPM, and also about the possible use of multifactor models of expected returns.

The forward-looking approach to estimating the expected rate of return on equity is to infer this number from observed market values and estimates of future cash flows to equity. For the stock market as a whole, the dividend growth model re-arranged gives

$$E(R_M) = \frac{D_1}{V_M} + \gamma$$

(14)

where $D_1$ is the total dividends paid by listed companies to shareholders over the next year (assumed for simplicity to be paid at the year-end), $V_M$ is the current value of the stock market, and $\gamma$ is the expected annual growth rate in total dividends. The same approach can be applied to individual shares. Refinements include the residual-income model, which has become popular in academic papers for estimating the cost of equity of individual companies.

The historical average of estimates of real risk-free returns is around 1% per year. In a mature economy, with a dividend yield (including share repurchases) of 3% or 4%, and a real growth rate of around 2% per year, the expected real rate of return on equity is 5% or 6% per year. These numbers imply that the equity risk premium from the dividend growth model is around 4% or 5% per year.

If the project’s optimal capital structure includes debt, the debt is treated as part of the capital, and the project’s cost of capital is the tax-adjusted weighted average cost of capital (WACC). The cost of a loan is approximated by the easily-observed promised interest rate (gross of income tax) on the face value of the loan, though this overstates the expected rate of return on risky debt. The WACC is used to discount cash flows from the project that are gross of interest on debt, net of corporation tax ignoring interest on debt, and gross of personal taxes. The expected rate of return on equity as estimated via standard methods is a rate net of corporation tax and gross of personal taxes, which is consistent with the rate required by the WACC as normally used.

Once a discount rate has been estimated, it is applied for each future period of the expected life of the project. Uncertainty about the assumptions underlying the cash flow forecasts, or about the discount rate, is usually captured by means of estimating a range of present values using a range of assumptions and discount rates. There is little in the finance literature on how to estimate a project’s risk, as opposed to the risk of a traded financial asset, nor on how to ‘translate’ that risk into a discount rate. The main method recommended in textbooks is to find a listed firm in a similar business to the business of the project, and use estimates of the cost equity for the listed firm as the basis for the project’s discount rate.
4. Comment on discounting over long horizons

Before considering the SDR, we highlight the interplay between risk and the time horizon, because this is critical to the valuation of long-lived projects. The CAPM-inspired guide to allowing for project risk is to look to its beta, or consumption beta in CCAPM, and beta has become the predominant measure of risk in practice. However, discounting over more than one future period means that the relationship between the risk of the project and its present value (PV) depends on the life of the project as well as on its estimated beta.\(^9\)

The PV of a project \(j\) is usually written as the expected cash flows discounted by the cost of capital:

\[
V_j = \sum_{t=1}^{T} E(Y_{jt})/\left[1 + E(R_j)\right]^t
\]

where \(V_j\) is the PV at date 0, \(E(Y_{jt})\) is the expected value of the uncertain cash flow \(Y_{jt}\) at a future date \(t\), \(T\) is the number of the last date on which a cash flow for the project is forecast to arise, and \(E(R_j)\) is the cost of capital, the expected return per period on an asset of the same risk as the project, assuming an identical cost of capital each period. An alternative to equation (15), which sheds light on the relationship between risk and PV, is to write the PV as a series of certainty-equivalent cash flows discounted by the risk-free rate. Adjustment for risk is then made via the future value to be discounted rather than via the discount rate. The risk premium is represented as the difference between the expected value of an uncertain future cash flow, \(E(Y_{jt})\), and its certainty-equivalent cash flow, \(CE(Y_{jt})\). The certainty-equivalent is the amount such that an individual is indifferent between a claim to receive the uncertain payoff \(Y_{jt}\) and a claim to receive the certain payoff \(CE(Y_{jt})\). The present value of \(E(Y_{jt})\) can be written either as \(E(Y_{jt})/[1 + E(R_j)]^t\) or as \(CE(Y_{jt})/(1 + RF)^t\). So we can write

\[
E(Y_{jt})/CE(Y_{jt}) = \left\{\left[1 + E(R_j)\right]/(1 + RF)\right\}^t
\]

Equation (16) makes a crucial point about the normal practice of discounting cash flows, which was first noted by Robichek and Myers (1966). A constant risk premium involves the implicit assumption that the premium for risk, and the ratio between \(E(Y_{jt})\) and \(CE(Y_{jt})\), increase exponentially the further into the future the cash flow will arise (the larger is \(t\)).

For example, suppose \(E(R_j) = 10\%\) per year, \(RF = 5\%\), and we are considering two payoffs, one with an expected value of $100 after one year and the other with an expected value of $100 after five years. From equation (16), \(CE($100_1) = $95.5\) and \(CE($100_5) = \)

\(^9\) Lind’s (1982) review of the SDR draws attention to this question, but the question has since been somewhat ignored in both the finance and SDR literatures.
$79.2, so the PV of the compensation for risk increases from $4.5/1.10 = $4.1 for the year-one payment to $20.8/1.10^5 = $12.9 for the year-five payment.

Compounding the premium is justified if it is the case that uncertainty about the cash flow does in fact increase with the time horizon. More specifically, compounding the premium involves assuming implicitly that the expected covariance at date $t$ of the possible values of the cash flow with possible values of market value (in the standard CAPM) or consumption (in the CCAPM) increases exponentially with $t$, the time horizon. The reason for the existence of a risk premium is that, with positive covariance and diminishing marginal utility, the expected utility at date $t$ from the uncertain cash flow is less than the expected utility from a risk-free amount equal to the expected value of the uncertain cash flow. In the consumption-based model, an investment at date 0 of $Y_0$ in risky asset $j$ will provide an uncertain cash flow at date $t \geq 1$ of $Y_0e^{r_jt}$. Let $Y_0$ be the PV of a risk-free payoff of one unit at date $t$. The certainty-equivalent value of $E(Y_0e^{r_jt})$ is given by the expected value of the utility from the asset, divided by the expected value of marginal utility at date $t$ (ie the expected utility of the risk-free payoff of one unit):

$$CE(Y_0e^{r_jt}) = \frac{E[(Y_0e^{r_jt})(C_0e^{\sigma^2})^{\gamma}]}{E(C_0e^{\sigma^2})^{\gamma}}$$

Using again equations (7) and (10), this can be written

$$CE(Y_0e^{r_jt}) = Y_0e^{[t[E(r_{jt}) + 0.5\text{var}(r_{jt}) - 2\eta\text{cov}(r_{jt}, g_t)]]}$$

Whereas, the expected value of the cash flow is

$$E(Y_0e^{r_jt}) = Y_0e^{[t[E(r_{jt}) + 0.5\text{var}(r_{jt})]]}$$

Comparing equations (19) and (18), it can be seen that the ratio of the expected value and the certainty-equivalent value increases exponentially as $t$ increases. This is why it is correct to compound the constant risk premium, given in equation (11), over multiple future periods.

However, it is correct only if we accept the assumptions that $r_{jt}$ and $g_t$ follow a random walk. This is really a special case, for both variables. The way in which the forecast risk of the outcomes for a given real-life project increases as the time horizon lengthens deserves thought, because compounding a conventional risk premium, of three per cent or more, makes a first-order difference to PV. For example, suppose that the (real) cash flow $Y_{kt}$ from a project $k$ is forecast with as much as confidence whether the cash flow arises after one year or after ten years. In this case it is clearly not correct to assume that $r_{kt}$ follows a random walk, and compounding a constant risk premium per period will result in undervaluation of the project.
5.0 The social discount rate

5.1 The Ramsey formula

The term social discount rate, or rate of social time preference, is normally taken to mean the rate applicable to long-term projects in the public sector. The modern SDR approach does not take the path of the practical finance package of seeking to infer the discount rate from market data. To some extent this is because public projects are not undertaken by companies. However, a public project could in principle be treated like a private sector project. For example, its cost of capital could be inferred using the (degeared) cost of equity estimated for a listed company thought to have similar business risk to that of the project. Most of the practical finance package is potentially relevant to a public project. Why are the cost of capital and the SDR not, fundamentally, the same thing? We return to this question in Section 5.4, after reviewing the SDR.

The SDR approach is for a public sector executive to use consumption-based theory to help determine what the SDR should be, whereas in finance the theory is used to try to explain observed expected rates of return on financial assets. Much of the literature on the SDR takes as its foundation the Ramsey formula, equation (6). This involves assuming that both the growth rate of consumption and the project’s cash flows are certain. These assumptions appear to be maintained for simplicity, and because of a tradition that public projects should be discounted by the risk-free rate (see Section 5.2).

The expected growth rate of consumption, $\mu$, and its standard deviation, $\sigma$ (if used), are based on empirical evidence in the SDR approach, as in finance. But the rate of pure time preference $\delta$, and the coefficient of risk aversion $\eta$, are based explicitly on ethical judgement, as well as, in the case of $\eta$, empirical evidence.\(^\text{10}\) Dasgupta (2008, p. 150), for example, writes that the SDR ‘has to be derived from an overall conception of intergenerational well-being and the consumption forecast’. Evidence from attempts to infer $\delta$ and $\eta$ ‘from the choices people make as they go about their lives’ (p. 147) is an input to the estimation of those parameters, but it is not the only consideration. If the SDR for a project as calculated by an executive differs from an estimate of its cost of capital based on market data, the SDR takes priority and the discrepancy is not necessarily a major cause for concern.

One reason for rejecting market data is that market interest rates do not exist beyond a horizon of 30 years, which is the longest maturity of most government bonds. Undated

\(^{10}\) The judgement involves ethics in that it directly involves taking an explicit view about how much the welfare of people in the future matters compared with the welfare of people today. The finance approach is to rely on market data, and to accept the ‘view’ about intergenerational welfare that is implicit in market data.
government bonds do exist, but the markets for them are illiquid. Historic data could be used to estimate an expected rate of return over a long horizon from owning a succession of the short-term Treasury bills, but the result would be approximate and dependent on assuming that the future will resemble the past. The focus on government bonds is perhaps a corollary of the view that funding for public projects is risk-free, and so the expected returns on risky assets are not relevant. If this view is rejected, the expected rate of return on equity potentially becomes relevant market-based evidence for risky projects, as in Weitzman (2007a) for example.

The most important reason for explicit appeal to ethical judgement in the SDR is probably the view that market data, whether from bonds or equities, might not reflect enough concern for the welfare of future generations. The assumed aim of the government under the SDR approach is to maximise the utility of society, including the utility of people yet to be born (intergenerational utility). The assumed aim of an individual is to maximise her own lifetime utility. Some lifetime utility could come from anticipation of the utility of the individual’s heirs, which would explain why people make bequests. Despite this, SDR proponents hold, first, that people alive today might, through their individual behaviour, act to maximise their own lifetime utility rather than intergenerational utility, and second, that they might, nevertheless, elect a government which would act to benefit future generations at the expense of the current generation. This approach to the SDR is an example of what Sen and Williams (1982, p. 16) call Government House utilitarianism (see also Lind, 1982, pp. 55-9, and Dietz, Hepburn, and Stern, 2008, for a recent defence). Since market data are supposed to reflect the revealed preferences of individuals acting to maximise their lifetime utility, rather than intergenerational utility, market data are not to be trusted for the purpose of making decisions intended to maximise intergenerational utility. Put differently, people tend to ignore or undervalue externalities in their individual behaviour, and the effects of behaviour on future generations are externalities.

An important example is carbon emissions. ‘Business as usual’, with no mitigation of carbon emissions, involves an externality via probable global warming and the resulting imposition of possibly huge costs on future generations. So observed expected rates of return on assets such as equity, and therefore estimates of the cost of capital, exceed the social rates of return on such assets (Dasgupta, 2008; Stern, 2008). The social rate of return on an asset is the expected internal rate of return after the externalities attributable to the asset have been valued and ‘translated’ into cash flows to include in the cash flow forecast. Climate change is an example of a negative externality from current economic activity, one that in principle
could be costed and included in cash flow forecasts. But even ignoring such specific and potentially measurable externalities, someone who believes that the behaviour of people alive today does not place enough weight on the welfare of future generations, will believe that the utility from long-term projects is greater than the utility that is implied by the observed expected rates of return on equity and other long-term assets.

Broome (1994), for example, maintains that ‘it is only the disenfranchisement of future generations that gives us the share of the world’s resources that we have’ (p. 152). In support of this, he presents a thought experiment in which a trust fund is set up which would act in the interests of future generations. He regards it as self-evident that ‘from the trust’s point of view… future commodities would be much more valuable than they seem to us who are participating in the market now’, and that the trust would transfer resources from the present to the future. But the trust’s purchases of future commodities would not reduce market interest rates permanently, as he assumes that they are determined by the productivity of the economy’s technology. So if we took proper account of the welfare of future generations, we would use a lower SDR than market interest rates.

There are other possible grounds for incorporating ethical judgement in the SDR. A justification that differs from concern about future utility comes from the concepts of rights and sustainability (for example, Anand and Sen, 2000). This perspective can lead to doubt about the adequacy of the expected utility approach. It can be argued that a fundamental ethical principal is that people have an equal right to well-being. From this follows the view that current decisions and consumption should be sustainable, meaning that they should not damage the prospects for the well-being of future generations. One version of the sustainability argument is that the Earth’s ‘capital’, its potential to sustain life, should not be depleted. Current consumption should therefore be from ‘income’, ie from output that does not deplete the planet’s capital. However, it is not easy to spell out in practical terms what sustainable development consists of.

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11 Dasgupta (2008) and others argue that, in the absence of ‘market imperfections’, society will maximise its lifetime utility and the social discount rate will be equal to the market rate of return on investment. His conception of market imperfections includes the existence of externalities which are not reflected in the rate of return on investment. There are more conventional types of market imperfection which might also make it difficult to infer from market data the revealed preferences of the current population regarding social decisions, including taxes and poor information on the part of the population.

12 Assuming a constant return on investment (ROI). The trust would reduce the interest rate if there were diminishing returns on investment. With a constant ROI, \( r_F = \text{ROI} \), otherwise the rate of saving would not be optimal. The growth rate in the Ramsey model is then \( \mu = (\text{ROI} - \delta) \eta \). The trust’s activities imply a lower \( \delta \) or \( \eta \) for society than would prevail without the trust, and a higher saving rate and growth rate.
There are also various special cases to consider, which invite a low discount rate, or other special treatment of investment decisions, or regulation. Some ‘commodities’, such as fresh air, or more generally a reasonably healthy environment, could be considered especially important to maintain prospects for well-being. This could justify, for example, a very low discount rate for public projects designed to maintain a reasonably healthy environment. Some activities, such as lifesaving, discussed by Broome (1994), provide utility which does not diminish as society becomes richer. Some commodities might be seen as essential for future well-being, and so as not substitutable at all for other commodities, in which case they will be regarded as necessary to have at almost any cost. Some features of the world might be given a special status because once lost they cannot be replaced, such as a species of animal or an archaeological site.

Finally, Stern (2006, 2008) emphasises that the opportunity cost of capital is a marginal concept. That is, it assumes that the project in question is small in relation to the market, meaning that the relevant market prices are not affected by whether or not the project is undertaken. He emphasises that it is a basic mistake to use this marginal concept in the context of climate change. Global warming is likely to affect market prices in future, and efforts to reduce it could affect market prices today. This argument is perhaps superfluous as a justification of the SDR approach as outlined, since the SDR approach does not rely on appeal to market data even for small projects.

We now consider separately the two parameters in equation (6), \( \delta \) and \( \eta \). Many authors support a value for \( \delta \) of very close to zero, though 1.0% is common in the finance literature and some authors suggest higher values (very little is said about \( \delta \) in finance, even in advanced texts). The key argument for \( \delta \approx 0 \) is that it is unethical to weight utility according to when the person is alive. The time a person is alive is, in itself, not a relevant consideration when it comes to weighting utility. The Stern Review, for example, concludes that \( \delta \) ought to exceed zero only to the extent that it reflects the possibility that humanity might not exist after some future date. The utility of future generations then counts the same as the utility of people alive today. The Review sets \( \delta \) at 0.1% per year. Setting \( \delta \) exactly equal to zero raises the problem that, with a future that lasts forever, and a positive rate of return on investment, intergenerational utility would be maximised by investing almost all of current consumption.

The parameter \( \eta \) provokes further questions. With the CRRA utility function, \( \eta \) measures both the rate at which the utility from marginal consumption declines as consumption grows over time, and aversion to uncertainty about consumption at a given date, i.e., aversion to risk. In the first of these roles, a higher \( \eta \) implies a desire for less inequality in
levels of consumption over time (as does a higher $\delta$). Marginal consumption today provides high utility compared with marginal consumption in the future. A person with a high $\eta$ chooses higher consumption now and in the near future, and slower growth of consumption, compared with a person with a low $\eta$. A lower $\eta$ implies greater concern about future welfare, and a lower SDR. A consequence of lower $\eta$ is less consumption today, i.e., higher saving. Dasgupta (2008) and Nordhaus (2007) argue that a value of $\eta$ of, say, 1.0, as in the *Stern Review*, implies that the proportion of income which would be saved is very high; 40% or more. This is uncomfortable because it is unrealistic that such high savings ratios will arise, and because it suggests that the current generation should consume much less than it actually does, for the benefit of the future, even though people in the future are forecast to be substantially richer than are people today.

In its second role of reflecting aversion to risk, we have seen that lower $\eta$ implies a lower risk premium, which exacerbates the equity premium puzzle. However, for plausible values of $\eta$ of up to about 4.0, differences in $\eta$ make very little difference to the predicted premium in equation (11).

Perhaps more troubling is a comparison with a third role for $\eta$, that of reflecting concern about cross-sectional income inequality at a given date. Lower $\eta$ in this role implies less concern about the welfare of the poor, because the marginal utility from increasing income for the poor increases with $\eta$. We have the awkward conclusion that a public sector executive concerned about current income inequality should apply a high $\eta$ in her project appraisals, whereas she should apply a low $\eta$ if she is concerned about the welfare of future generations. One answer is to use a utility function in which aversion to risk, and to income inequality at a given date, are separate from aversion to inequality of consumption over time. A number of papers explore such a utility function, including Gollier (2002a).

A general question about the SDR approach is, how do we agree on values of $\delta$ and $\eta$, and hence set the SDR? Or if we are using a declining discount rate (below), how should the decline be determined? The values of $\delta$ and $\eta$ are based partly on individual reflection under the SDR approach, so the values proposed will differ across SDR users. The published responses to the *Stern Review* show how much disagreement there is about the SDR, as does the range of discount rates applied by different governments to public projects. Appeal to the

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13 There is no established method of estimation from private sector data, i.e., no equivalent to the practical finance package. Earlier literature does consider which private sector rate of return, or average of such rates, to use in estimating the SDR. The main issue was not risk, but the fact that the (pre-taxes) rate of return on private investment is higher than the (after-taxes) rate on individual saving. Recent reviews, for example Gollier (2013), Dasgupta (2008), and Weitzman (2007a), almost ignore tax.
evidence from clinical studies is rather inconclusive. Salient features about this evidence are as follows (Frederick, Lowenstein, and O’Donoghue, 2002). Measurement of the discount rates used by individuals is fraught with difficulties. The rates reported are highly heterogeneous across studies, and they are generally much larger than the rates of a few per cent used by governments for their SDRs. People apply high discount rates over short horizons, of up to about one year, and lower discount rates for longer future periods. With the important exception of this step reduction after one year, there is little evidence that people apply declining discount rates.

The above discussion risks presenting use of the SDR-by-judgement approach as more firmly rooted than it actually is. The SDR was a rate based on market data in Lind (1982). Portney and Weyant (1999) summarise the deliberations of 20 leading public economists on the SDR in the context of climate change. There was agreement among them that a cost of capital based on market data should be used for projects with a life of up to 40 years, but that the SDR approach should be used for longer-term projects, because of ‘discomfort’ with the cost-of-capital approach for long-term projects. Weitzman (2007a) believes that a distinct SDR approach is not that of mainstream economics.

5.2 The SDR and risk

It has been ‘commonly thought that the risk-free rate of return is appropriate for the appraisal of public projects due to the risk pooling available to governments’ (Groom et al, 2005, p. 452; Arrow and Lind, 1970). That is, the fact that a project is funded by the government means that project risk and adjustment for risk in the discount rate can be disregarded. This view has long been disputed, and the consumption-based theory outlined in Section 2.1 shows that, in general, it is not correct to ignore risk. A public project with uncertain future payoffs should only be discounted at the risk-free rate if the payoffs are uncorrelated with consumption per head, as is in fact assumed by Arrow and Lind (1970), and as is re-iterated by Lind (1982, p. 69). Neither the Stern Review (2006) nor any reviews of Stern take the view that government investment to alleviate climate change should be treated as risk-free by virtue of being funded by the government.14

The finance perspective on the risk of public projects can be summarised as follows. Both portfolio theory and consumption-based theory show that risk-averse investors demand a

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14 Stern (2008, pp. 13-14) does argue that a close-to-risk-free rate is relevant for projects to reduce carbon emissions, but on the grounds that they are ‘likely to be financed via the diversion of resources from consumption (via pricing) rather than from investment’.
risk premium for exposure to systematic risk; that is, risk which is not eliminated through diversification by means of holding a portfolio of assets, whether financial assets or projects. Each public project has its own discount rate, that depends on its systematic risk, as for private projects. Taxpayers bear the risk of public projects. ‘Investing’ in public projects via paying taxes does not appear to offer greater elimination of risk than does investing in financial markets. Most unsystematic risk is eliminated by means of holding about 30 randomly selected shares, so a mature stock market provides more than enough opportunities for diversification to eliminate unsystematic risk. It is true that a government viewed as stable and dependable (ie, with a secure AAA credit rating) can borrow at a lower cost of capital than the WACC of even the safest private-sector company. However, a sufficient reason for this is that a stable government can raise funds via taxation, which is a coercive method not available to a company. The coercive nature of taxation does not mean that the government has reduced project risk for the taxpayer.

Despite the importance of risk as a determinant of private sector discount rates, governments apply a single discount rate to all public projects (Spackman, 2008). They do not attempt to set different discount rates to reflect the different risks of projects. The fact that a single rate is used by a given government for all projects constitutes an important difference from the cost-of-capital approach. The practical finance package directs attention to differences in risk across companies and projects. The investment appraisal process prescribed in the package results in large differences in the cost of capital, of several percentage points, which arise because of differences in risk across projects.

Risk looms much less large for the SDR, for a cluster reasons. One is the mistaken tradition, just mentioned, that government funding in itself implies that the discount rate for all public projects is low. Second, the weight given to the welfare of future people, rather than risk, is seen as the primary determinant of the SDR. Guided by the Ramsey formula, future payoffs are discounted because society will be richer, and, if δ is non-negligible, because utility in the future counts for less than utility today. The payoffs are not discounted because they are risky.

Third, the measurement of risk will often be more problematic for public than for private projects. Public projects typically involve non-commercial objectives, and provide ‘payoffs’ that do not arise via cash flows. For example, what is the risk of investment to alleviate climate change? The cost of global warming is usually modelled as a fixed proportion of future output or consumption. In this case the size of the payoff, which is a reduction in that cost, is proportional to output, and the consumption beta is approximately
one. On the other hand, global warming might be sufficiently destructive as to have a major impact on welfare. Weitzman (2007a) argues that expenditure to reduce carbon emissions is akin to the purchase of insurance. There might be a global catastrophe if emissions are not reduced, i.e., a large fall in consumption per head, with a probability that is non-negligible but impossible to quantify. On this view, the main benefit of the ‘insurance’ of reducing emissions is to reduce the probability of climate-induced catastrophe. If it were to turn out that there would be economic collapse under business as usual, and if our investment to reduce emissions were to succeed in averting the collapse, the payoff would be very large, and society’s marginal utility without the payoff would have been higher than today’s. This perspective implies a low or negative consumption beta for investment to alleviate climate change.

Finally, the relevant conception of systematic risk is not settled. If the SDR is a rate determined by judgement in the setting of consumption-based theory, the systematic risk for a public project is measured by the correlation of the payoffs with consumption per head. But, then, the premium for risk is predicted to be small in consumption-based theory, as we have seen. The choice of consumption beta makes very little difference to the SDR in the standard model of Section 2.1, assuming that \( \eta \) and \( \sigma^2 \) are chosen from the range of conventional values. This potentially provides a justification for ignoring risk, though it is not a justification which has actually been used much in the SDR literature.

In contrast, if the SDR is estimated from market data, presumably risk is measured by beta in the CAPM, or by the betas in a multifactor model, by techniques that form part of the practical finance package. Use of market data implies an estimate of a premium for systematic risk of at least three per cent per year for a beta of one, and as a result the cost of capital is highly sensitive to the estimate of beta.

Our discussion here also raises the question of the relationship in practice between consumption and CAPM betas. The normal method of estimating the CAPM beta of a project, which is ultimately from the correlation of a share price with a stock market index, is probably not an adequate method of estimating the consumption beta. There is no evidence about how estimates of the CAPM beta for private sector projects, using the practical finance package, compare with estimates of the consumption beta for the same projects. This is probably because consumption betas normally are not estimated for private projects. A second point is that the practical finance package is suited to estimating the risk and cost of capital of a project with cash flows. If a public project is undertaken by the private sector, there have to be forecast cash flows to provide payoffs to private finance. But the characteristics of the cash
flows to private investors are determined, or at least influenced, by the government. The risks faced by private investors, affecting the cost of capital set by investors, may well differ from the underlying risks of the project itself.

None of the above reasons mean that it is correct to assume that all projects are risk free, or that they all have the same risk. At the same time, the measurement of the risk of real investment projects is a serious challenge for both public and private sector agents. A good deal of judgement is involved, and, for the cost of capital, the judgement made regarding choice of beta has a major impact on the discount rate. Although SDRs used in practice are not explicitly adjusted for risk, evidence from market data is likely to affect one’s view of the SDR that is chosen. Plausible numbers, used by governments for the SDR, range from the Stern Report’s 1.4% per year up to 6.0% or more. If a public sector executive chooses a low SDR, of around 1% to 2%, someone used to the private sector approach to the cost of capital, in which risk matters, would see this SDR as implying a belief that public projects are close to risk-free. If the executive chooses a high SDR, of around 5% to 6%, this would imply a belief that public projects have a risk similar to investment in the stock market.

5.3 Declining discount rate

A further difference between the cost of capital and the SDR approach is that a declining SDR is sometimes proposed. This is another aspect of the use of consumption-based theory to determine what the SDR should be. The cost of capital in practice and in finance texts is always a flat rate, although in principle a term structure of discount rates could be used, based on the observed term structure of government bond yields (proxying for risk-free rates).

A declining discount rate can mean either that the rate applicable to a given future period declines with distance into the future, or that there is a flat discount rate, the same for all periods, but the flat discount rate is lower for longer-lived projects. Conventional discounting, with a flat rate, is sometimes referred to as exponential discounting, and discounting with a declining rate as hyperbolic discounting.

There are a number of arguments for, or models that result in, a declining discount rate. First, if the expected growth rate of consumption per period is assumed to diminish with time into the future, ie the expected $g_t$ for periods in the far future is lower than $g_t$ in the near future, the discount rate per period predicted by equations (6) or (8) will diminish. If a $g_t$ is negative in a period $t$, marginal utility increases, and the discount rate for the period could be negative.
Second, the analysis under uncertainty in Section 2.1 assumes that $g_t$ shows random variation over time. A plausible type of modification is that the changes in $g_t$ show some persistence. This can result in a declining risk-free rate, since persistence implies that the dispersion of possible future consumption increases with the time horizon by more than is the case under random variation. A greater dispersion of possible future consumption implies a lower discount rate, since it increases expected marginal utility, assuming that marginal utility diminishes with consumption (see equation (8)). Gollier (2013) reviews several types of modification which feature persistence in future growth.

Third, if $g_t$ follows a random walk, but the utility function exhibits diminishing relative risk aversion (DRRA) instead of CRRA, there will be a declining discount rate for a risk-free project (Gollier, 2002a). DRRA means that as an individual’s wealth increases, she would pay a smaller proportion of her wealth to avoid a risk given in absolute terms. Both DRRA and CRRA imply aversion to risk (diminishing marginal utility of consumption), but DRRA implies less aversion than CRRA. Intuitively, there are two opposing forces at work. The expected variance of $C_t$ increases with the time horizon $t$, which reduces expected utility. This means that the expected utility from a risk-free asset – that is, expected marginal utility – increases with $t$, which implies a lower discount rate. On the other hand, if $g_t$ per period is zero or positive, consumption increases over time, which reduces marginal utility, and implies a higher discount rate. DRRA is necessary for the first effect to dominate the second effect. If negative values of $g_t$ are possible, then DRRA does not guarantee a declining discount rate, and more is involved in specifying the set of utility functions that result in a declining discount rate (Gollier, 2002b).

Fourth, if the discount rate for each future period is fixed but uncertain ex ante, there is a declining discount rate in the sense that, for a given collection of possible discount rates, the single, fixed discount rate that represents the range of possible rates declines as the number of future periods increases (Weitzman, 1998). This argument is easiest to understand by means of an example (Guo et al, 2006). Let the possible discount rates be 1%, 3% and 5% per year, each with equal probability, and let $T$ be number of years into the future over which the discounting is to be made. What Weitzman calls the certainty-equivalent discount rate for an interval of $T$ years, $CEDR_T$, is the discount rate which results in the same PV as the average of the PVs which arise from using each of the possible discount rates. $CEDR_T$ is calculated from the certainty-equivalent discount factor, $CEDF_T$, which is the weighted average of the discount factors for the possible discount rates:

$$CEDF_T = 1/(1 + CEDR_T) = (1/3)[(1/1.01^T) + (1/1.03^T) + (1/1.05^T)]$$

(20)
For $T = 10$ years, $CED_{T}^F = 0.754$ and $CED_{T}^R = 2.86\%$; for $T = 100$ years, $CED_{T}^F = 0.143$ and $CED_{T}^R = 1.96\%$. The mechanism at work here is that, as the future horizon recedes, the lowest discount rate explains an increasing proportion of the PV. The argument only makes a difference if at least one of the possible discount rates is sufficiently low that PV is non-negligible. If the time horizon is 100 years or more, and the lowest of the possible discount rates is around 4%, PV is approximately zero even using the lowest possible rate.

Gollier (2004) criticises this argument on the grounds that there is an implicit assumption of risk neutrality, and that the current generation is exposed to the risk (see also Guo et al., 2005). The future payoff is viewed as certain, while the PV is uncertain until the uncertainty about the discount rate is resolved. The decision about whether to invest in the project is made before this uncertainty is resolved, by comparing the expected NPV with the cost. Risk neutrality comes in because of the expected NPV calculation: each possible PV is weighted by its probability, with no adjustment for risk (or different marginal utilities in different states of the world at the time when uncertainty about the discount rate is resolved). The current generation bears the risk because the PV once uncertainty is resolved could turn out to be higher or lower than its expected value. Gollier (2004) notes that the conclusion is reversed if, in contrast, the aim is to calculate the expected future value, given the cost of the project. For long horizons the expected future value becomes dominated by the outcome in which the rate of return is the highest of the possible rates, and so the rate to assume in calculating the expected future value tends towards the highest rate, not the lowest, as the future lengthens. From this perspective the generation at the terminal date is exposed to the risk, as the future value could turn out to be lower than the expected future value.

The debate can be resolved by considering the question in the consumption-based model. We assume that the uncertain discount rate is the rate of return on saving (Gollier and Weitzman, 2010, with a linear production function) or the marginal rate of return on saving (Gollier, 2013, pp. 103-5). In this case it can be shown that the certainty-equivalent discount rate declines as $T$ increases. The reason is the same as the reason why persistence over time of the growth rate can result in a declining discount rate. The assumption that the uncertain return on saving is fixed (very persistent) once the uncertainty is resolved means that, from the perspective of date 0, when the discount rate is unknown, the dispersion of possible future consumption outcomes increases with the time horizon by more than is the case under random variation of $g_t$, for a given variance of $g_t$ equal to the variance of uncertain discount rate. Since the term structure is flat if $g_t$ follows a random walk, the greater dispersion of
consumption, compared with under a random walk, implies that the discount rate diminishes as $T$ increases (assuming that $\eta > 1$).

This analysis re-establishes the case for a declining discount rate under the Weitzman assumption that the discount rate is uncertain now but will in future become known and fixed in perpetuity thereafter. However, the analysis is quite different from the original Weitzman (1998) argument. The latter is a point about the calculation of present value, and it is implicit in standard valuation procedures. The standard method of allowing for uncertainty about the discount rate is to calculate a range of possible PVs using several different discount rates. It is natural for the average of the possible PVs to be taken as the best-guess PV, so the Weitzman (1998) argument is merely highlighting an aspect of standard procedure. The Gollier-Weitzman analysis is about how the discount rate is determined in the consumption-based model of Section 2.1, given the Weitzman (1998) assumption.

We have outlined several of the conditions under which a consumption-based theory results in a declining discount rate. These ideas have probably been developed in the SDR literature, rather than in finance, because of the view that a discount rate based on market data should not be applied to long-term projects, a point to which we now return.

5.4 The nature of the SDR

In view of the long-standing debate about the fundamental nature of the SDR, it is worth being as clear as possible about how an SDR not derived from market data differs from the cost of capital. Consider a risky project, which has a single payoff with an expected value of $100 in real terms in 100 years’ time. The project is estimated to have a conventional CAPM beta of one. The discount rate derived from market data – the expected real rate of return on the stock market – is 5.0% per year, and with this discount rate $PV = \$0.8$. A public sector executive estimates the SDR to be 1.4% per year, and with this rate $PV = \$24.9$. The cost of the project is $10$, so it is acceptable using the SDR but not using the cost of capital. This example reflects the essence of the evidence and debate highlighted by the Stern Review. There are many projects available today which would result in reduced carbon emissions. They would provide uncertain gains, in the form of reduced costs arising from global warming, that will arise after around 100 years. Given the estimated future costs of global warming, society should invest much more to reduce carbon emissions than is currently being invested, if a discount rate of 1.4% is used to value the benefits of reduced emissions.

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Estimation of the (systematic) risk of carbon-reducing investment is problematic, as we have noted, and the market-based cost of capital is sensitive to estimated risk. If private investors judge such investment to be low- or zero-beta, this would approximately eliminate the discrepancy between the cost of capital and the SDR, assuming that the real risk-free rate is between 1% to 2% per year. However, the reason the executive in the example chooses a lower SDR than the cost of capital cannot be because the executive estimates the risk of carbon-reducing investment to be lower than the market’s estimate of the risk. This is because the estimate of the consumption beta makes very little difference to the SDR in the standard model of Section 2.1, assuming that $\eta$ and $\sigma^2$ are chosen from the range of conventional values. Since disagreement about risk cannot be the reason for the discrepancy between the two rates, let us assume that the executive agrees that CAPM beta for the project is one, and that the expected rate of return on equity is about 5% per year – yet still chooses a much lower SDR.

The executive would almost certainly make society in 100 years’ time better off by not undertaking the project, and instead investing the $10 cost for 100 years in the stock market. So it seems that the correct discount rate is the market rate. The public and private sectors face the same opportunities for investment in the capital markets. It follows that the (opportunity) cost of capital is the same for both. When funds ‘can be invested at the market rate of interest [or in the stock market], whether by the private sector or the public sector, it is clearly undesirable to divert them to an investment that will return the lower social discount rate’ (Brealey, Cooper and Habib, 1997, p. 20.

The reason the SDR for the project is 1.4% rather than 5.0% is that the executive thinks that society’s utility from the uncertain payoff to be received in 100 years time exceeds its utility from $0.8 today. The market, which sets the expected market rate, does not ‘think’ in the same way, otherwise the market rate would be lower than it is. The SDR is a device to reflect judgements about the utility derived from payoffs at different future dates. An SDR which is lower than the market-based cost of capital is a means of indicating that, for reasons that depend on the judgement of the relevant executive(s), more resources should be transferred from the present to the future than would be the case were discount rates based on market data employed.

We might think that use of an SDR confuses things; that it would be clearer always to use a cost of capital based on market data, and to keep separate the question of what a given project’s discount rate is from the question of whether the government should invest in the project. It could offer a rate of return that is less than its cost of capital, but the executive
might still want to undertake the project if she believes that somehow the benefits from it exceed the payoffs as estimated. There are two possible reasons why this might be the case. First, the project could provide externalities that are hard to measure and are not fully reflected in the payoffs. For example, the project might reduce poverty in a particular region, and we might want to put a value on that benefit which would not appear in a conventional cash flow forecast. This is a problem of measuring the costs and benefits comprehensively. In principle, it is a problem about the cash flow forecast for the project, not about the discount rate. The SDR could be used to correct for perceived omissions of externalities from the cash flow forecast. Positive (negative) net externalities would result in a lower (higher) SDR. But it would probably be more transparent and rigorous, and certainly more correct conceptually, to try to capture the externalities in the cash flow forecast, and not to adjust the discount rate for externalities that could be captured in the cash flows.

Suppose the costs and benefits of the project are measured comprehensively; the externalities are translated into cash flow equivalents, and included in the forecast cash flows. There remains a second reason why the executive might want to apply an SDR that differs from the project’s cost of capital. The reason is that the executive estimates the utility of the comprehensively measured payoffs to be different from the utility estimate that is implicit in the cost of capital. Usually this would mean that the executive places more weight on the utility of future generations than is implicit in the cost of capital, and so she applies a lower SDR. That is, \( \delta \) or \( \eta \) or both are set by the executive so that the discount rates produced by equations (6), (8) or (11) are below the discount rate estimated by the practical finance package.

The impact of long-lived projects on future utility is a type of (positive) externality, if we make the ethical judgement that a market-based cost of capital involves placing too little weight on future utility. It is a special type of externality, one that requires adjustment to the discount rate rather than the cash flow forecast, because it is an omission of some of the utility derived from the forecast comprehensive cash flows from the project. The higher weight for future utility implied by the SDR could be captured by increasing the amounts of the cash flows to be discounted, and then using the higher market-based cost of capital to discount these augmented cash flows. But then, we would not be using the SDR as the discount rate. If we are using the SDR as the discount rate, and it is different from the cost of capital, we cannot at the same time make an adjustment to the cash flows to capture the difference between the two approaches. Use of the SDR, rather than the cost of capital, means that we capture the different weighting of future utility in the discount rate.
6.0 Conclusion

This paper has compared two rather separate approaches to the discount rate for long-term projects, many of which are in the public sector. The message from the finance literature is that there is nothing special about long-term projects, or about public projects. All projects are in principle valued in the same manner, which we have summarised as the practical finance package. The resulting discount rates are high enough that a payoff arising beyond a few decades into the future has a negligible PV, compared with its undiscounted value. Many economists are uneasy about this, and the paper identifies some possible reasons why. Consumption-based theory, which has been developed to explain how the expected rates of return on financial assets are determined, has given rise to a huge academic literature, but has so far made little impact on the practical finance package.

The message from the current SDR literature is that it is probably not appropriate to try to apply the practical finance package to public projects, especially those which affect the welfare of future generations. A different approach is called for, one which involves explicit ethical judgement, and which is informed by the consumption-based model. At the time of writing there is still much doubt about how to arrive at a specific number for the SDR, and about how relevant market-based evidence is to the SDR.
References


