SecPAL: Design and Semantics of a Decentralized Authorization Language

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Abstract

We present a declarative authorization language. Policies and credentials are expressed using predicates defined by logical clauses, in the style of constraint logic programming. Access requests are mapped to logical authorization queries, consisting of predicates and constraints combined by conjunctions, disjunctions, and negations. Access is granted if the query succeeds against the current database of clauses. Predicates ascribe rights to particular principals, with flexible support for delegation and revocation. At the discretion of the delegator, delegated rights can be further delegated, either to a fixed depth, or arbitrarily deeply.

Our language strikes a careful balance between syntactic and semantic simplicity, policy expressiveness, and execution efficiency. The syntax is close to natural language, and the semantics consists of just three deduction rules. The language can express many common policy idioms using constraints, controlled delegation, recursive predicates, and negated queries. We describe an execution strategy based on translation to Datalog with Constraints, and table-based resolution. We show that this execution strategy is sound, complete, and always terminates, despite recursion and negation, as long as simple syntactic conditions are met.

Keywords authorization language, access control, policy, trust management

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1 Introduction

Many applications depend on complex and changing authorization criteria. Some domains, such as electronic health records or eGovernment, require that authorization complies with evolving legislation. Distributed systems, such as web services or shared grid computations, involve frequent ad hoc collaborations between entities with no pre-established trust relation, each with their own authorization policies. Hence, these policies must be phrased in terms of principal attributes, asserted by adequate delegation chains, as well as traditional identities. To deploy and maintain such applications, it is essential that all mundane authorization decisions be automated, according to some human readable policy that can be refined and updated, without the need to change (and re-validate) application code.

To this end, several declarative authorization management systems have been proposed; they feature high-level languages dedicated to authorization policies; they aim at improving scalability, maintenance, and availability by separating policy-based access control decisions from their implementation mechanisms. Despite their advantages, these systems are not much used. We conjecture that the poor usability of policy languages remains a major obstacle to their adoption.

In this paper, we describe the design and semantics of SecPAL, a new authorization language that improves on usability in several respects. The following is an overview of the main technical contributions and features of SecPAL.

Expressiveness  Our design is a careful composition of three features for expressing decentralized authorization policies: delegation, constraints, and negation.

• Flexible delegation of authority is the essence of decentralized management.

We employ a delegation primitive (“can say”) that covers a wider spectrum of delegation variants than existing authorization languages, including those specifically designed for flexible delegation such as XrML [24], SPKI/SDSI [31] and Delegation Logic (DL) [46]. The semantics of “can say” is close to the “controls” operator in the original logical treatment of authorization, the ABLP logic [2].

• Support for domain-specific constraints is also important, but existing solutions only consider a specific class of constraints (e.g. temporal constraints [16], periodicity [15], set constraints [64]) or
are very restrictive to preserve decidability and tractability (e.g. unary constraints [48], constraint-compact domains [13]), and disallow constraints required for expressing idioms commonly used in practice.

We provide a set of mild, purely syntactic safety conditions that allow an open choice of constraints without loss of efficiency. SecPAL can thus express a wide range of idioms, including policies with parameterized roles and role hierarchies, separation of duties and threshold constraints, expiration requirements, temporal and periodicity constraints, policies on structured resources such as file systems, and revocation.

- Negation is useful for expressing idioms such as separation of duties, but its liberal adoption can make policies hard to understand, and its combination with recursion can cause intractability and semantic ambiguity [63].

We introduce a syntax for authorization queries, separate from policy assertions. We permit negation within queries (even universally quantified negation), but not within assertions. This separation avoids intractability and ambiguity, and simplifies the task of authoring policies with negation.

**Clear, readable syntax**  The syntax of some policy languages, such as XACML [54] and XrML, is defined only via an XML schema; policies expressed directly in XML are verbose and hard to read and write. On the other hand, policy authors are usually unfamiliar with formal logic, and would find it hard to learn the syntax of most logic-based policy languages (e.g. [46, 43, 26, 49, 37, 48, 13]). SecPAL has a concrete syntax consisting of simple statements close to natural language. (It also has an XML schema for exchanging statements between implementations.)

**Succinct, unambiguous semantics**  Languages such as XACML, XrML, or SPKI/SDSI [31] are specified by a combination of lengthy descriptions and algorithms that are ambiguous and, in some cases, inconsistent. Post-hoc attempts to formalise these languages are difficult and reveal their semantic ambiguities and complexities (e.g. [39, 38, 1]).

For example, it was recently proved that the evaluation algorithms of XrML and the related MPEG REL are not guaranteed to terminate.\(^1\) Moreover, the analysis in [47] shows that the algorithm for

\(^1\)Personal communication, Vicky Weissman.
SPKI/SDSI is incomplete; in fact, the language is likely to be undecidable due to the complex structure of SPKI's authorization tags.

Logic-based languages have a formal semantics and, thus, are unambiguous. In many cases, however, the semantics is specified only indirectly, by translation to another language with a formal semantics, such as Datalog [43, 26, 49], Datalog with Constraints [48, 13] or Prolog [46]. Instead, for the purpose of succinct specification, we define three deduction rules that directly specify the meaning of SecPAL assertions, independently of any other logic.

**Effective decision procedures** We show that SecPAL query evaluation is decidable and tractable (with polynomial data complexity) by translation into Datalog with Constraints. We describe a deterministic tabling resolution algorithm tailored to efficient evaluation of SecPAL authorization queries with constraints and negation, and present correctness and complexity theorems for the evaluation of policies that meet our syntactic safety conditions.

**Extensibility** SecPAL builds on the notion of tunable expressiveness introduced in Cassandra [12] and defines several extension points at which functionality can be added in a modular and orthogonal way. For example, the parameterized verbs, the environment functions, and the language of constraints can all be extended by the user without affecting our results.

In combination, we believe that SecPAL achieves a good balance between syntactic and semantic simplicity, policy expressiveness, and execution efficiency for decentralized authorization. Although system implementation is not the subject of this paper, SecPAL has been implemented and deployed as the core authorization mechanism of a large system-development project, initially targeted at grid applications [29]. The system provides a PKI-based, SOAP-encoded infrastructure for exchanging policy assertions. It also includes a policy-editing tool and support for invoking authorization queries from C#. It relies on an instance of our evaluation algorithm specialized for some fixed, domain-specific verbs and constraints. The development of a formal semantics for SecPAL, in parallel with its experimental use for access control within a distributed computing environment, has led to many improvements in its design.

**Contents** The rest of the paper is organized as follows. Section 2 illustrates SecPAL on a simple example. Section 3 defines syntax, semantics and safety of SecPAL assertions. Section 4 defines SecPAL
authorization queries, built as conjunctions, disjunctions and negations of facts and constraints, and their usage in authorization query tables. Section 5 explains how assertions are filtered to remove expired or revoked assertions. Section 6 shows how to express a variety of authorization policy idioms in SecPAL. Sections 7 and 8 give our algorithm for evaluating authorization queries and establish its formal soundness and completeness. SecPAL assertions are first translated into Datalog with Constraints (Section 7); the resulting program is then evaluated for a series of Datalog queries obtained from the SecPAL query (Section 8). Section 9 discusses the prototype implementation, summarizes related works and concludes. Appendix A provides auxiliary definitions and all proofs.

Earlier versions of this paper are published as a technical report [8] and, abridged, in a conference proceedings [9].

2 A simple example

To introduce the main features of SecPAL, we consider an example in the context of a simplified grid system. Access control in grids typically involves interaction between several administrative domains with individual policies and requires attribute-based authorization and delegation [66, 21, 65].

Assume that Alice wishes to perform some data mining on a computation cluster. To this end, the cluster needs to fetch Alice’s dataset from her file server. A priori, the cluster may not know of Alice, and the cluster and the file server may not share any trust relationship.

We identify principals by names Alice, Cluster, FileServer,...; these names stand for public signature-verification keys in the SecPAL implementation.

Alice sends to the cluster a request to run the command `dbgrep file://project/data` plus a collection of tokens for the request, expressed as three SecPAL assertions:

\[
\begin{align*}
\text{STS says Alice is a researcher} & \quad (1) \\
\text{FileServer says Alice can read file://project} & \quad (2) \\
\text{Alice says Cluster can read file://project/data if } \text{currentTime()} \leq 07/09/2006 & \quad (3)
\end{align*}
\]

Every assertion is XML-encoded and signed by its issuer. Assertion (1) is an identity token issued
by STS, some security token server trusted by the cluster. Assertion (2) is a capability for Alice to read her files, issued by FileServer. Assertion (3) delegates to Cluster the right to access a specific file on that server, for a limited period of time; it is specifically issued by Alice to support her request.

Before processing the request, the cluster authenticates Alice as the requester, validates her tokens, and runs the query Cluster says Alice can execute dbgrep against the set of assertions formed by its local policy plus these tokens. (In practice, an authorization query table on the cluster maps user requests to corresponding queries.) Assume the local policy of the cluster includes the assertions:

\[
\text{Cluster says STS can say}_{0} x \text{ is a researcher} \tag{4} \\
\text{Cluster says } x \text{ can execute } \text{dbgrep if } x \text{ is a researcher} \tag{5}
\]

Assertions (4) and (5) state that Cluster defers to STS to say who is a researcher, and that any researcher may run dbgrep. (More realistic assertions may well include more complex conditions.) Here, we deduce that Cluster says Alice is a researcher by (1) and (4), then deduce the target assertion by (5).

The cluster then executes the task, which involves requesting chunks of file://project/data hosted on the file server. To support its requests, the cluster forwards Alice’s credentials. Before granting access to the data, the file server runs the query Cluster can read file://project/data against its local policy plus Alice’s tokens. Assume the local policy of the server includes the assertion

\[
\text{FileServer says } x \text{ can say}_{m} y \text{ can read file if } \\
\begin{align*}
& x \text{ can read dir, file } \leq \text{ dir,} \\
& \text{markedConfidential(file) } \neq \text{Yes}
\end{align*}
\tag{6}
\]

Assertion (6) is a constrained delegation rule; it states that any principal \(x\) may delegate the right to read a file, provided \(x\) can read a directory \(dir\) that includes the file and the file is not marked as confidential. The first condition is a conditional fact (that can be derived from other assertions), whereas the last two conditions are constraints. Here, by (3) and (6) with \(x = \text{Alice}\) and \(y = \text{Cluster}\), the first condition follows from (2) and we obtain that FileServer says Cluster can read file://project/data provided that FileServer successfully checks the two constraints \(\text{currentTime()} \leq 07/09/2006\).
In the delegation rules (4) and (6), the “can say” assertions have different subscripts: in (4), can say\(_0\) prevents STS from re-delegating the delegated fact; conversely, in (6), can say\(_\infty\) indicates that \(y\) can re-delegate read access to file by issuing adequate can say tokens.

Assume now that the cluster distributes the task to several computation nodes, such as Node23. In order for Node23 to gain access to the data, Cluster may issue its own delegation token, so that the query FileServer says Node23 can read file://project/data may be satisfied by applying (6) twice, with \(x = \text{Alice}\) then \(x = \text{Cluster}\). Alternatively, FileServer may simply issue the assertion

\[
\text{FileServer says Node23 can act as Cluster} \tag{7}
\]

This means roughly that every fact concerning Cluster also applies to Node23. Every fact in SecPAL takes the form \(e\) \text{ verbphrase} , where \(e\) is the subject of the fact, and \text{ verbphrase} is the remainder. Assertion (7) means that for any \text{ verbphrase} , FileServer says Node23 \text{ verbphrase} follows from FileServer says Cluster \text{ verbphrase}.

### 3 Syntax and semantics

We give a core syntax for SecPAL. (The full SecPAL language provides additional syntax for grouping assertions, for instance to delegate a series of rights in a single assertion; these additions can be reduced to the core syntax. It also enforces a typing discipline for constants, functions, and variables, omitted here as it does not affect the semantics of the language.)

**Assertions** An authorization policy is specified as a set \(\mathcal{A}\) of assertions (called assertion context) of the form

\[
A \text{ says } fact_1, \ldots, fact_n, c
\]

where the facts are sentences that state properties on principals, defined below. In the assertion, \(A\) is the issuer; \(fact_1, \ldots, fact_n\) are the conditional facts; and \(c\) is the constraint. Assertions are similar to Horn
clauses, with the difference that (1) they are qualified by some principal $A$ who issues and vouches for the asserted claim; (2) facts can be nested, using the verb phrase can say, by means of which delegation rights are specified; and (3) variables in the assertion are constrained by $c$, a formula that can express e.g. temporal, inequality, path and regular expression constraints. The following defines the grammar of facts.

$$
e ::= x \quad \text{(variables)}
\quad | \quad A \quad \text{(constants)}
\quad | \quad \text{can read } [-] \quad \text{(predicates)}
\quad | \quad \text{has access from } [-] \text{ till } [-]
\quad | \quad \ldots
\quad | \quad \text{can act as } e \quad \text{(principal aliasing)}
\quad | \quad \text{can say}_{D, \text{fact}} \quad \text{(delegation)}
\quad | \quad 0 \quad \text{(no re-delegation)}
\quad | \quad \infty \quad \text{(with re-delegation)}
\quad | \quad \text{pred } e_1 \ldots e_n \quad \text{for } n = \text{Arity(pred)}
$$

$D$ ::= $0$ (no re-delegation)

Constraints $c$ range over any constraint domain that extends the basic constraint domain shown below. Basic constraints include numerical inequalities (for e.g. expressing temporal constraints), path constraints (for hierarchical file systems), and regular expressions (for ad hoc filtering):

$D$ ::= 0 (no re-delegation)

$\inf$ (with re-delegation)

$verbphrase ::= \text{pred } e_1 \ldots e_n \quad \text{for } n = \text{Arity(pred)}$

$fact ::= e \text{ verbphrase}$

Constants represent data such as IP addresses, URLs, dates, and times. We use $A$, $B$, $C$ as met variables for constants, usually for denoting principals. Variables only range over the domain of constants — not predicates, facts, claims or assertions. Predicates are user-defined, application-specific verb phrases (intended to express capabilities of a subject) of fixed arity with holes for their object parameters; holes may appear at any fixed position in verb phrases, as in e.g. has access from $[-]$ till $[-]$. In the grammar above, $\text{pred } e_1 \ldots e_n$ denotes the verb phrase obtained by inserting the arguments $e_1$ up to $e_n$ into the predicate holes. We say that a fact is nested when it includes a can say, and is flat otherwise. For example, the fact Bob can read $f$ is flat, but Charlie can say$_0$ Bob can read $f$ is nested.
\( f \in \{+, -, \text{CurrentTime}, \ldots\} \) \ (built-in functions)

\[
\begin{align*}
r \ ::= & \quad x \\
& \mid A \\
& \mid f(r_1, \ldots, r_n) & \text{for } n = \text{Arity}(f) \geq 0
\end{align*}
\]

\( \text{pattern} \in \text{RegularExpressions} \)

\[
\begin{align*}
c \ ::= & \quad \text{True} \\
& \mid r_1 = r_2 \\
& \mid r_1 \leq r_2 & \text{(numerical inequality)} \\
& \mid r_1 \preceq r_2 & \text{(path constraint)} \\
& \mid r \text{ matches pattern} & \text{(regular expression)} \\
& \mid \text{not}(c) & \text{(negation)} \\
& \mid c_1, c_2 & \text{(conjunction)}
\end{align*}
\]

Additional constraints can be added without affecting decidability or tractability. The only requirement is that the validity of ground constraints is decidable in polynomial time. (A phrase of syntax is ground when it contains no variables.)

We use a sugared notation for constraints that can be derived from the basic ones, e.g. \( \text{False}, r_1 \neq r_2, \) and \( c_1 \) or \( c_2. \) We usually omit the \text{True} constraint, and also omit the \text{if} in assertions with no conditional facts, writing \( A \text{ says fact} \) for \( A \text{ says fact if True}. \) We write keywords, function names and predicates in \textit{sans serif}, constants in \texttt{typewriter} font, and variables in \textit{italics}. We use a vector notation to denote a (possibly empty) list of items, e.g. writing \( f(\vec{r}) \) for \( f(r_1, \ldots, r_n). \)

For a given constraint \( c, \) we write \( \models c \) iff \( c \) is ground and valid. The following defines ground validity within the basic constraint domain. The denotation of a constant \( A \) is simply \( \llbracket A \rrbracket = A. \) The denotation of a function \( f(\vec{r}) \) is defined if \( \vec{r} \) is ground, and is also a constant, but may depend on the system state as well as \( \llbracket \vec{r} \rrbracket. \) For example, \( \llbracket \text{CurrentTime}() \rrbracket \) returns a different constant when called at different times. However, we assume that a single authorization query evaluation is atomic with respect to system state. That is, even though an expression may be evaluated multiple times, we require that its denotation not vary during a single evaluation.
Safety Conditions The expressiveness of an authorization language depends to a large extent on the supported classes of constraints. However, adding a wide range of different constraint classes to a language is nontrivial: even if the constraint classes are tractable on their own, mixing them can result in an intractable or even undecidable language. Therefore, constraints have so far been either excluded or heavily restricted, to an extent that not even our basic constraint domain would be allowed. For example, RT C [48] allows only a subclass of unary constraints, and Cassandra [13] allows only constraint-compact constraint domains. [64] only consider set constraints, and in [15], only temporal periodicity constraints are considered. Furthermore, these systems require complex operations such as unground satisfiability checking or existential quantifier elimination that are hard to implement.

We observe that a wide range of constraints are used in authorization policies, but that evaluation of a constraint may be postponed until all of the constraint’s variables are instantiated. Accordingly, rather than restricting constraints, SecPAL’s safety conditions (Definition 3.1) only ensure that constraints will be ground at runtime, once all conditional facts have been satisfied. This approach facilitates high expressiveness while preserving decidability and tractability, and also simplifies the evaluation algorithm (Section 8), thus making it much easier to implement.

Definition 3.1 (Assertion safety). The assertion $A$ says $\text{fact}_1, \ldots, \text{fact}_n, c$ is safe iff the following conditions hold:

- all conditional facts are flat;
• all variables in $c$ also occur somewhere else in the assertion;

• if $fact$ is flat, all variables in $fact$ also occur in a conditional fact.

Note the similarity to the safety condition in Datalog where all variables in the head must also occur in a conditional literal [20]. SecPAL’s safety conditions are less restrictive, as variables in can say assertions need not occur in any conditional fact, so the assertion $\text{FileServer says } x \text{ can say } y \text{ can read } f$ is safe. Nevertheless, all answers generated from can say assertions are still guaranteed to be ground, because the Datalog translation (Section 7) is designed in such a way that parameters of can say are input parameters, i.e., can say goals are always called with all parameters fully instantiated; furthermore, the safety conditions prevent can say from occurring as a (possibly unground) conditional fact or within a query (see Section 4).

At first sight, the safety conditions seem to rule out blanket permissions such as $\text{FileServer says } x \text{ can read } \text{Foo}$ (everybody can read Foo). However, this is not a problem in practice, because it is possible to make the assertion safe by adding a conditional fact qualifying $x$, for example “if $x$ is a user”. The list of users could either be stored locally, or the server could delegate to a trusted third party, e.g. by $\text{FileServer says TrustedDirectory can say } x \text{ is a user}$. Alternatively, the server may accept self-issued statements: $\text{FileServer says } x \text{ can say } x \text{ is a user}$. The safety conditions guarantee that the evaluation of the Datalog translation, as described in Section 8, is complete and terminates in all cases.

**Semantics** To be practically usable, a policy language should not only have a simple, readable syntax, but also a simple, intuitive semantics. We now describe a formal semantics consisting of only three deduction rules that directly reflect the intuition suggested by the syntax. This proof-theoretic approach enhances simplicity and clarity, far more than if we had instead taken the translation to Datalog with Constraints in Section 7 as the language specification. Let a substitution $\theta$ be a function mapping variables to constants and variables, and let $\varepsilon$ be the empty substitution. Substitutions are extended to constraints, predicates, facts, claims, assertions etc. in the natural way, and are usually written in postfix notation. We write $\text{vars}(X)$ for the set of free variables occurring in a phrase of syntax $X$.

Each deduction rule consists of a set of premises and a single consequence of the form $\text{AC, D } \models A \text{ says fact}$ where $\text{vars(fact)} = \emptyset$ and the delegation flag $D$ is 0 or $\infty$. Intuitively, the deduction relation
holds if the consequence can be derived from the assertion context \( \mathcal{AC} \). Furthermore, if \( D = 0 \), no instance of the rule (can say) can occur in the derivation.

\[
\frac{(A \text{ says } \text{fact}) (A \text{ says } \text{fact}_1, \ldots, \text{fact}_k, c) \in \mathcal{AC}}{\mathcal{AC}, D \models A \text{ says } \text{fact}_i \theta \text{ for all } i \in \{1..k\}}
\]

Furthermore, if \( D = 0 \), no instance of the rule (can say) can occur in the derivation.

\[
\frac{\mathcal{AC}, D \models A \text{ says } \text{fact}_i \theta \ \text{vars}(\text{fact}_i \theta) = \emptyset}{\mathcal{AC}, D \models A \text{ says } \text{fact}_i \theta}
\]

\[
\frac{\mathcal{AC}, \infty \models A \text{ says } B \text{ can say } \text{fact}}{\mathcal{AC}, D \models B \text{ says } \text{fact}}
\]

\[
\frac{\mathcal{AC}, D \models A \text{ says } B \text{ can act as } C}{\mathcal{AC}, \infty \models A \text{ says } C \text{ verbphrase}}
\]

\[
\frac{\mathcal{AC}, D \models A \text{ says } B \text{ can act as } C}{\mathcal{AC}, \infty \models A \text{ says } B \text{ verbphrase}}
\]

Rule (cond) allows the deduction of matching assertions in \( \mathcal{AC} \) with all free variables substituted by constants. All conditional facts must be deducible with the same delegation flag \( D \) as in the conclusion. Furthermore, the substitution must also make the constraint ground and valid.

Rule (can say) deduces an assertion made by \( A \) by combining a can say assertion made by \( A \) and a matching assertion made by \( B \). This rule applies only if the delegation flag in the conclusion is \( \infty \). The matching assertion made by \( B \) must be proved with the delegation flag \( D \) read from \( A \)'s can say assertion. Therefore, if \( D = 0 \), then the matching assertion must be proved without any application of the (can say) rule. If on the other hand \( D = \infty \), then \( B \) can re-delegate. In Section 6, we show that the boolean delegation flag \( D \in \{0, \infty\} \) is sufficient for expressing a wide range of complex delegation policies, including depth-restricted delegation.

Rule (can act as) asserts that all facts applicable to \( C \) also apply to \( B \), when \( B \) can act as \( C \) is derivable. A corollary is that can act as is a transitive relation.

**Corollary 3.2.** If \( \mathcal{AC}, D \models A \text{ says } B \text{ can act as } B' \) and \( \mathcal{AC}, D \models A \text{ says } B' \text{ can act as } B'' \) then \( \mathcal{AC}, D \models A \text{ says } B \text{ can act as } B'' \).

The following propositions state basic properties of the deduction relation; they are established by induction on the rules.

**Proposition 3.3.** If \( \mathcal{AC}, D \models A \text{ says } \text{fact} \) then \( \text{vars}(\text{fact}) = \emptyset \).

**Proposition 3.4.** If \( \mathcal{AC}, 0 \models A \text{ says } \text{fact} \) then \( \mathcal{AC}, \infty \models A \text{ says } \text{fact} \).
Proposition 3.5. If $\mathcal{A}_1, D \models A \text{ says } \text{fact}$ then $\mathcal{A}_1 \cup \mathcal{A}_2, D \models A \text{ says } \text{fact}$.

Proposition 3.6. Let $\mathcal{A}_A$ be the set of all assertions in $\mathcal{A}$ whose issuer is $A$. We have $\mathcal{A}, 0 \models A \text{ says } \text{fact}$ iff $\mathcal{A}_A, 0 \models A \text{ says } \text{fact}$.

Proposition 3.6 implies that if $A \text{ says } \text{fact}$ is deduced from a zero-depth delegation assertion $A \text{ says } B \text{ can say}_0 \text{ fact}$ then the delegation chain is guaranteed to depend only on assertions issued by $B$. XrML and DL [46] also support depth restrictions, but these can be defeated as their constructs for depth-restricted delegation do not guarantee this property. Section 6 discusses this issue in more detail.

4 Authorization queries

A reference monitor is a component, typically software, that given a request to access a sensitive resource, decides whether or not to grant access. A reference monitor based on SecPAL generates a suitable authorization query, and evaluates the query against the current assertion context (containing local as well as imported assertions). In SecPAL, authorization queries consist of atomic queries of the form $A \text{ says } \text{fact}$ and constraints, combined by logical connectives including negation:

$$ q ::= e \text{ says } \text{fact} \quad \text{(atomic query)} $$

$$ | q_1, q_2 \quad \text{(conjunction)} $$

$$ | q_1 \text{ or } q_2 \quad \text{(disjunction)} $$

$$ | \neg(q) \quad \text{(negation)} $$

$$ | c \quad \text{(constraint)} $$

$$ | \exists x(q) \quad \text{(existential quantification)} $$

Negative conditions enable policies such as separation of duties, threshold and prohibition policies (see Section 6). However, coupling negation with a recursive language leads to multiple possible models [63], higher computational complexity, or even undecidability [56]. Our solution is based on the observation that negated conditions can be effectively separated from recursion by allowing negation only in authorization queries. Collecting negations at the level of authorization queries also makes for clearer policies whose consequences are easier to foresee. Indeed, SecPAL authorization queries could be further extended by even more powerful composition operators such as aggregation (as in Cassandra [13]).
or threshold operators (as in RT$^T$ [49]), without changing the assertion semantics and without affecting the complexity results.

**Safety conditions**  The safety condition on queries guarantees that (1) there are only finitely many answers to the query, given that the assertions in the assertion context are safe; and (2) subqueries of the form \( \text{not}(q) \) or \( c \) are always ground when they are evaluated, under the assumption that conjunctive queries are evaluated from left to right (see Section 8).

We first define a deduction relation \( \vdash \) with judgments of the form \( I \vdash q : O \) where \( q \) is a query and \( I, O \) are sets of variables. Intuitively, the set \( I \) represents the variables that are ground before evaluating \( q \), and \( O \) represents the new variables that become ground after evaluating \( q \). The sets \( I \) and \( O \) are thus always disjoint, and the set \( I \cup O \) contains the variables that are guaranteed to be instantiated after evaluating \( q \).

The requirement that only flat facts may occur in a query ensures that, during evaluation, *can* say goals never get called with unground parameters, which could result in unground constraints, because *can* say assertions may have variables that do not occur in a conditional fact. This restriction can be relaxed (at the price of an additional rule) by allowing nested facts within a query as long as the nested variables are grounded by the context.

The rules for constraints and negation require that the variables are contained in \( I \), ensuring groundedness at evaluation time. The rules for disjunction and conjunction deal with groundedness propagation. After evaluating a disjunction, only variables occurring in both disjuncts are guaranteed to be ground. Conjunction, on the other hand, grounds all variables occurring in any of the two conjuncts. Conjunctions are evaluated from left to right, so the variable assignments resulting from the left conjunct are taken into account when evaluating the right conjunct. Finally, an existentially quantified query \( \exists x(q) \) grounds all variables also grounded by \( q \) apart from the bound variable \( x \). The requirement that \( x \) must not already be ground in the context \( I \) can easily be satisfied by alpha-renaming the bound variable.

\[
egin{align*}
\text{fact is flat} & \quad \frac{\vdash e \text{ says fact : vars(e says fact)}}{I \vdash \text{false : vars(c) \subseteq I}} \\
\text{conjuncts} & \quad \frac{I \vdash q_1 : O_1 \quad I \vdash q_2 : O_2}{I \vdash q_1 \text{ or } q_2 : O_1 \cap O_2} \\
\text{negation} & \quad \frac{I \vdash q : O \quad \text{vars}(c) \subseteq I}{I \vdash \text{not}(q) : \emptyset} \\
\text{existential} & \quad \frac{I \vdash q : O \quad x \notin I}{I \vdash \exists x(q) : O \setminus \{x\}}
\end{align*}
\]
Definition 4.1 (Authorization query safety). An authorization query $q$ is safe iff there exists a set of variables $O$ such that $\emptyset \vDash q : O$.

Example 4.2. The authorization queries on the left hand side are safe; those on the right are unsafe.

<table>
<thead>
<tr>
<th>Safe</th>
<th>Unsafe</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$ says $C$ can read Foo</td>
<td>$A$ says $B$ can say, $C$ can read Foo</td>
</tr>
<tr>
<td>$x$ says $y$ can read $f$, $x = A$</td>
<td>$x = A$, $x$ says $y$ can read $f$</td>
</tr>
<tr>
<td>$x$ says $A$ can read $f$, $B$ says $y$ can read $f$, $x \neq y$</td>
<td>$x$ says $A$ can read $f$, $B$ says $y$ can read $f$, $x \neq W$</td>
</tr>
<tr>
<td>$x$ says $y$ can read $f$, not($y$ says $x$ can read $f$)</td>
<td>$x$ says $y$ can read $f$, not($y$ says $z$ can read $f$)</td>
</tr>
<tr>
<td>not(\exists x (A says $x$ can read Foo))</td>
<td>$\exists x$ (not(A says $x$ can read Foo))</td>
</tr>
</tbody>
</table>

The first query on the right is unsafe because atomic queries are required to be flat. In the second query, the constraint $x = A$ is evaluated without $x$ being instantiated; this is rectified on the left hand side by changing the order of the conjuncts. The last three examples on the right feature a constraint and two negations whose variables are only partially grounded by the preceding context. The last example on the left is safe because the negated query has no free variables, as $x$ is bound by the existential quantifier. ■

Query semantics We write $\theta_{-x}$ to denote the substitution that has domain $\text{dom}(\theta) - \{x\}$ and is equivalent to $\theta$ on this domain. The semantics of authorization queries is defined by the relation $AC, \theta \vDash q$, as follows:

$AC, \theta \vDash e$ says fact iff $AC, \infty \vDash e \theta$ says fact, and $\text{dom}(\theta) \subseteq \text{vars}(e$ says fact)

$AC, \theta_1 \theta_2 \vDash q_1$, $q_2$ iff $AC, \theta_1 \vDash q_1$ and $AC, \theta_2 \vDash q_2 \theta_1$

$AC, \theta \vDash q_1$ or $q_2$ iff $AC, \theta \vDash q_1$ or $AC, \theta \vDash q_2$

$AC, \varepsilon \vDash \text{not}(q)$ iff $AC, \varepsilon \not\vDash q$ and $\text{vars}(q) = \emptyset$

$AC, \varepsilon \vDash c$ iff $\vDash c$

$AC, \theta_{-x} \vDash \exists x(q)$ iff $AC, \theta \vDash q$

One can easily verify that $AC, \theta \vDash q$ implies $\text{dom}(\theta) \subseteq \text{vars}(q)$. (Note that $\text{vars}(\exists x(q))$ is defined as $\text{vars}(q) - \{x\}$.)
Given a query $q$ and an authorization context $AC$, an authorization algorithm should return the answer set of all substitutions $\theta$ such that $AC, \theta \vdash q$. If the query is ground, the answer set is either empty (meaning “no”) or a singleton set containing the empty substitution $\varepsilon$ (meaning “yes”). In the general case, i.e. if the query contains variables, the substitutions in the answer set are all the variable assignments that make the query true. For example, the answer set for the query Alice says $x$ can read $Foo$ contains all assignments to $x$ of principals who can read $Foo$ according to Alice. This returns more information than just “yes, the query can be satisfied for some $x$”. Section 8 gives an algorithm for finding this set of substitutions.

**Authorization query tables** In most trust management systems it is the reference monitor’s responsibility to construct an appropriate authorization query upon an access request. This approach works well if only basic queries are allowed, for example if all queries are predicates of the form $\text{permits}(\text{user}, \text{action})$ as in Cassandra or Lithium [37]. In SecPAL, however, authorization queries are more expressive and thus also more complex. Generally, a policy author writing a local assertion context will also have a set of intended queries in mind. Therefore, authorization queries are conceptually part of the local policy and should be kept separate from the reference monitor’s code.

For this reason, SecPAL introduces the notion of *authorization query table* which collects all authorization queries intended for use with the local assertion context. An authorization query table is a finite set of mappings $r \mapsto q$ from parameterized access requests $r$ to authorization queries $q$.

Upon a request, the reference monitor issues an instantiated request lookup (instead of issuing a query directly) that gets mapped by the table to the corresponding authorization query, which is then used to query the assertion context.

**Example 4.3.** The task of policy authoring could be simplified by automatically generating simple default mappings:

- $\text{read}(x, f) \mapsto \text{FileServer says } x \text{ can read } f$
- $\text{write}(x, f) \mapsto \text{FileServer says } x \text{ can write } f$
- $\text{execute}(x, f) \mapsto \text{FileServer says } x \text{ can execute } f$
- :
Example 4.4. More complex policies may require the policy author to specify a compound query. For example, the request $\text{login}(x)$ could be mapped to the following authorization query:

$$
\exists t_1, t_2 (\text{FileServer says } x \text{ can login } t_1 \text{ till } t_2, \quad t_1 \leq \text{currentTime}() \leq t_2),
$$

$$\text{not } \exists t_3, t_4 (\text{FileServer says } x \text{ cannot login } t_3 \text{ till } t_4, \quad t_3 \leq \text{currentTime}() \leq t_4)
$$

When $\text{login}(x)$ is called, the above authorization query (with $x$ instantiated) is evaluated against the assertion context, and the answer is returned to the reference monitor, which can then enforce the policy. This example features a universally quantified negated statement, encoded by a negated existential quantifier. It also illustrates how a time-dependent prohibition policy with a Deny-Override conflict resolution rule can be written in SecPAL. More elaborate conflict resolution rules such as assertions with different priorities can also be encoded on the level of authorization queries. Just as with negative conditions, prohibition makes policies less comprehensible and should be used sparingly, if at all [32, 27].

The safety condition for queries can be extended to authorization query tables:

**Definition 4.5 (Authorization query table safety).** A mapping $r \mapsto q$ from a request $r$ to an authorization query $q$ is safe iff there exists a set of variables $O$ such that $\text{vars}(r) \models q : O$, and $\text{vars}(q) \subseteq \text{vars}(r)$. An authorization query table is safe iff it only consists of safe mappings.

This condition can be checked statically. If the authorization query table is safe, all queries executed at runtime are guaranteed to be safe as well.
5 Assertion expiration and revocation

This section describes how expiration and revocation can be expressed in SecPAL. Expiration dates can be specified as ordinary verb phrase parameters:

UCambridge says Alice is a student till 31/12/2007 if
currentTime() ≤ 31/12/2007

Sometimes it should be up to the acceptor to specify an expiration date or set their own recency requirements [58]. In this case, the assertion could just contain the date without enforcing it:

UCambridge says Alice is a student till 31/12/2007

An acceptor can then use the date to enforce their own recency requirements:

Admin says x is entitled to discount if
x is a student till date,
currentTime() ≤ date,
date − currentTime() ≤ 1 year

Assertions may have to be revoked before their scheduled expiration date. To deal with compromise of an issuer’s key, we can use existing key revocation mechanisms. But sometimes the issuer needs to revoke their own assertions. For instance, the assertion in the example above has to be revoked if Alice drops out of her university. Historically, revocation is most commonly associated with X.509-based PKIs, which only support revocation of certificates for signing keys. Revoking an issuer’s signing key revokes all the assertions signed by that key. SecPAL includes a simple mechanism for finer-grained revocation, at the level of an individual assertion; one may revoke one or more of an issuer’s assertions without revoking all of the issuer’s assertions.

We assume that every assertion $M$ is associated with an identifier (e.g., a serial number) $ID_M$. (The identifier is kept implicit in the concrete syntax for assertions.) Revocation, and delegation of authority to revoke, can then be expressed in SecPAL by revocation assertions with the verb phrase revokes $ID_M$. 
For example, the revocation assertion

\[ A \text{ says } A \text{ revokes } ID \text{ if } \text{currentTime}() > 31/7/2007 \]

revokes all assertions that are issued by \( A \) and have identifier \( ID \), but only after 31 July 2007.

**Definition 5.1. (revocation assertion)** An assertion is a revocation assertion if it is safe and of the form

\[ A \text{ says } A \text{ revokes } ID \text{ if } c, \text{ or } A \text{ says } B_1 \text{ can say } D_1 \cdots B_n \text{ can say } D_n \text{ A revokes } ID \text{ if } c. \]

Given an assertion context \( AC \) and a set of revocation assertions \( AC_{rev} \) where \( AC \cap AC_{rev} = \emptyset \), we remove all assertions revoked by \( AC_{rev} \) in \( AC \) before evaluating authorization queries. The filtered assertion context is defined by

\[ AC - \{ M \mid M \in AC, A \text{ is the issuer of } M, \text{ and } AC_{rev}, \infty \models A \text{ says } A \text{ revokes } ID_M \} \]

The condition that \( AC \) and \( AC_{rev} \) must be disjoint means that revocation assertions cannot be revoked (at least not within the language). Allowing revocation assertions to be revoked by each other causes the same problems and semantic ambiguities as negated body predicates in logic programming. Although these problems could be formally overcome, for example by only allowing stratifiable revocation sets or by computing the well-founded model, these approaches are not simple enough for users to cope with in practice.

Implementing a system with a working revocation mechanism is not straightforward. For example, as with certificate revocation lists, revocation in SecPAL depends on the availability of revocation assertions. Depending on the application, the distribution of revocation assertions can be managed by pull or push schemes. This and other revocation-related issues, which are orthogonal to the specification mechanisms, are beyond the scope of this paper.
6 Policy idioms

In this section, we give examples of assertions and queries to show how SecPAL can express a wide range of policy idioms, in comparison with other authorization languages.

Discretionary Access Control (DAC) The following assertion specifies that users can pass on their access rights to other users at their own discretion.

\[
\text{FileServer says user can say}_\omega \ x \text{ can access resource if } \text{user can access resource}
\]

For example, if it follows from the assertion context that Bob says Alice can read file://docs/ and FileServer says Bob can read file://docs/, then FileServer says Alice can read file://docs/.

Languages with restricted or no recursion such as XACML [54] or Lithium [37] cannot express such a policy.

Mandatory Access Control (MAC) We assume a finite set of users \( U \) and a finite set of files \( S \), characterised by the verb phrases is a user and is a file. Additionally, every such user and file is associated with a label from an ordered set of security levels. The constraint domain uses the function \( \text{level} \) to retrieve these labels, and the relation \( \leq \) to represent the ordering. Assertions (9) and (10) below implement the Simple Security Property and the \(*\)-Property from the Bell-LaPadula model [14], respectively.

\[
\text{FileServer says } x \text{ can read } f \text{ if } x \text{ is a user, } f \text{ is a file, } \text{level}(x) \geq \text{level}(f)
\]

\[
\text{FileServer says } x \text{ can write } f \text{ if } x \text{ is a user, } f \text{ is a file, } \text{level}(x) \leq \text{level}(f)
\]

Role hierarchies The can act as verb phrase can express role membership as well as role hierarchies in which roles inherit all privileges of less senior roles. The following assertions model a part of the
hierarchy of medical careers in the UK National Health Service (NHS).

\[ \text{NHS says FoundationTrainee can read file://docs/} \] (11)

\[ \text{NHS says SpecialistTrainee can act as FoundationTrainee} \] (12)

\[ \text{NHS says SeniorMedPractitioner can act as SpecialistTrainee} \] (13)

\[ \text{NHS says Alice can act as SeniorMedPractitioner} \] (14)

The first assertion assigns a privilege to a role; the second and third establish seniority relations between roles; and the last assertion assigns Alice the role of a Senior Medical Practitioner. From these assertions it follows that \text{NHS says Alice can read file://docs/}. This example illustrates that SecPAL principals can represent roles as well as individuals; the principal \text{FoundationTrainee} is a role, while the principal Alice is an individual.

**Parameterized attributes** Parameterized roles can add significant expressiveness to a role-based system and reduce the number of roles [35, 50]. In SecPAL, parameterized roles, attributes and privileges can be encoded by introducing verb phrases with arguments that correspond to the parameters, as in Assertion (15). The following example uses the verb phrases can access health record of [−] and is a treating clinician of [−].

\[ \text{NHS says } x \text{ can access health record of patient if } x \text{ is a treating clinician of patient} \] (15)

**Separation of duties** In this simple example of separation of duties, a payment transaction proceeds in two phases, initiation and authorization, which are required to be executed by two distinct bank managers. The following shows a fragment of the authorization query table. The query associated with \text{initPay}(U, P) is issued when a principal \( U \) attempts to initialize a payment \( P \). If this is successful, the reference monitor adds \text{Bank says } U \text{ has initiated } P \text{ to the local assertion context. The operation}
authPay is called when a principal attempts to authorize a payment.

\[
\text{initPay}(x, \text{payment}) \mapsto \begin{align*}
\text{Bank says } x \text{ is a manager,} \\
\not(\exists y (\text{Bank says } y \text{ has initiated payment}))
\end{align*}
\quad (16)
\]

\[
\text{authPay}(x, \text{payment}) \mapsto \begin{align*}
\text{Bank says } x \text{ is a manager,} \\
\exists y (\text{Bank says } y \text{ has initiated payment, } y \neq x)
\end{align*}
\quad (17)
\]

Note that the requirement that a successful execution of an \text{initPay} request should add a new fact to the assertion context is not specified here. However, SecPAL’s authorization query table can be extended in such a way that requests are mapped not just to queries, but also to a sequence of fact insertions and deletions that is executed when the query is successful. Such an extended query table is also useful for RBAC policies with role activation and deactivation within sessions, and can be given a semantics based on transaction logic as in [10].

**Threshold policies and compound principals** SPKI/SDSI has the concept of $k$-of-$n$ threshold subjects (at least $k$ out of $n$ given principals must sign a request) to provide a fault tolerance mechanism. $RT^T$ has the language construct of “threshold structures” for similar purposes [49]. There is no need for a dedicated threshold construct in SecPAL, because threshold constraints can be expressed directly.

In the following example, Alice trusts a principal if that principal is trusted by at least three distinct, trusted principals. We assume a function “distinct” that takes as argument a list of constants and returns \text{Yes} if the list contains no duplicates, and \text{No} otherwise. Since the assertion is safe, the list will be fully
instantiated at the time the function is called.

\[
\text{Alice says } x \text{ is trusted by Alice if} \\
\quad x \text{ is trusted by } a, x \text{ is trusted by } b, x \text{ is trusted by } c, \\
\quad \text{distinct}([a, b, c]) = \text{Yes}
\]

\[
\text{Alice says } x \text{ can say } y \text{ is trusted by } x \text{ if} \\
\quad x \text{ is trusted by Alice}
\]

A similar encoding combined with the authorization query table can be used to express compound principal policies in SecPAL, without the need of dedicated language constructs as in ABLP [2] or XrML [24]. Recall that the authorization query table is written by the policy author to provide an interface to the assertions. The table entry below specifies that the request to authorize a payment takes three parameters \(x, y\) and \(z\), and maps the request to an authorization query that checks if \(x, y, z\) are three distinct managers. Upon a user request, the reference monitor forwards the request to the SecPAL engine, which evaluates the corresponding authorization query and returns the results back to the reference monitor, which can then act based on the results.

\[
\text{authPay}(x, y, z) \mapsto \\
\quad \text{Bank says } x \text{ is a manager,} \\
\quad \text{Bank says } y \text{ is a manager,} \\
\quad \text{Bank says } z \text{ is a manager,} \\
\quad \text{distinct}([x, y, z])
\]

**Hierarchical resources**  
Suppose the assertion

\[
\text{FileServer says Alice can read file://docs/}
\]

is supposed to mean that Alice has read access to the path \(\text{/docs/}\) as well as to all descendants of that path. For example, Alice’s request to read \(\text{/docs/foo/bar.txt}\) should be granted. To encode this, one
might try to write the following assertion instead:

\[
\text{FileServer says Alice can read path if } path \preceq \text{file://docs/}
\] (21)

Unfortunately, this assertion is not safe. Instead, we can stick with the original assertion and put the constraint into the authorization query table:

\[
\text{canRead}(x, path) \iff \exists path2 (\text{FileServer says } x \text{ can read } path2, \text{ path } \preceq \text{ path2})
\] (22)

The same technique can be used in conditional facts. With the following assertion, users can not only pass on their access rights, but also access rights to specific descendants; Alice could then for example delegate read access for file://docs/foo/.

\[
\text{FileServer says } x \text{ can say } \infty y \text{ can read path if } \\
\begin{align*}
    x & \text{ can read path2,} \\
    \text{path} & \preceq \text{path2}
\end{align*}
\] (23)

The support of hierarchical resources is a very common requirement in practice, but existing policy languages cannot express the example shown above. For example, in RT^C [48], the \preceq relation cannot take two variable arguments, because it only allows unary constraints in order to preserve tractability. Again, it is SecPAL’s safety conditions that allow us to use such expressive constraint domains without losing efficiency.

**Attribute-based delegation**  Attribute-based (as opposed to identity-based) authorization enables collaboration between parties whose identities are initially unknown to each other. The authority to assert that a subject holds an attribute (such as being a student) may then be delegated to other parties, who in turn may be characterised by attributes rather than identity.

In the example below, a shop gives a discount to students every Friday. Both this temporal periodicity requirement and the expiration date of the student attribute can be expressed by a constraint. The authority over the student attribute is delegated to holders of the university attribute, and authority over
the university attribute is delegated to a known principal, the Board of Education.

\[
\text{Shop says } x \text{ is entitled to discount if}
\]
\[
x \text{ is a student till } date,
\]
\[
currentTime() \leq date, currentDay() = \text{Friday}
\]

\[
\text{Shop says } \text{univ can say}_x x \text{ is a student till } date \text{ if}
\]
\[
\text{univ is a university}
\]

\[
\text{Shop says } \text{BoardOfEducation can say}_x
\]
\[
\text{univ is a university}
\]

SPKI/SDSI [31], DL [46], Binder [26], RT [49] and Cassandra [13] can all express attribute-based delegation and linked local name spaces. SecPAL makes the delegation step explicit and thus allows for more fine-grained delegation control, as demonstrated in the following examples of various sorts of delegation. There are other general techniques to constrain delegation; for example, Bandmann, Firozabadi and Dam [4] propose the use of regular expressions to constrain the shape of delegation trees.

**Constrained delegation** Delegators may wish to restrict the parameters of the delegated fact. Such policies typically require domain-specific constraints that are not supported by previous languages for the sake of tractability. In the example below, a Security Token Server (STS) is given the right to issue tickets for accessing some resource for a specified validity period of no longer than eight hours.

\[
\text{FileServer says } \text{STS can say}_x
\]
\[
x \text{ has access from } t1 \text{ till } t2 \text{ if}
\]
\[
t2 - t1 \leq 8 \text{ hours}
\]

The delegation depth in Assertion (27) is unlimited, so STS can in turn delegate the same right to some STS2, possibly with additional constraints. For example, with Assertion (28) issued by STS, FileServer accepts tickets issued by STS2 with a validity period of at most eight hours, where the start date
is not before 01/01/2007 (but STS2 may not re-delegate).

\[ \text{STS says STS2 can say}_0 x \text{ has access from } t_1 \text{ till } t_2 \text{ if } \]
\[ t_1 \geq 01/01/2007 \]  \hspace{1cm} (28)

**Depth-bounded delegation**  The verb phrase can say\(_{0} \) fact allows no further delegation of fact, while can say\(_{\infty} \) fact allows arbitrary further delegation. This dichotomy may seem restrictive at first sight. However, SecPAL can express any fixed integer delegation depth by nesting can say\(_{0}\). In the following example, Alice delegates the authority over is a friend facts to Bob and allows Bob to re-delegate at most one level further.

Alice says Bob can say\(_{0} x \) is a friend  \hspace{1cm} (29)

Alice says Bob can say\(_{0} x \) can say\(_{0} y \) is a friend  \hspace{1cm} (30)

Suppose Bob re-delegates to Charlie with the assertion Bob says Charlie can say\(_{\infty} x \) is a friend. Now, Alice says Eve is a friend follows from Charlie says Eve is a friend. Since Alice does not accept any longer delegation chains, Alice (in contrast to Bob) does not allow Charlie to re-delegate with Charlie says Doris can say\(_{0} x \) is a friend.

SPKI/SDSI has a boolean delegation depth flag that corresponds to the 0 or \( \infty \) subscript in can say but cannot express any other integer delegation depths. In XrML [24] and DL, the delegation depth can be specified, and can be either an integer or \( \infty \). However, in both languages, the depth restrictions can be defeated by Charlie:

Charlie says \( x \) is a friend if \( x \) is a friend\(_2\)  \hspace{1cm} (31)

Charlie says Doris can say\(_{0} x \) is a friend\(_2\)  \hspace{1cm} (32)

In XrML and DL, Charlie can then re-delegate to Doris via is a friend\(_2\), thereby circumventing the depth specification. The SecPAL semantics prevents this by threading the depth restriction through the entire branch of the proof; this is a corollary of Proposition 3.6. It would be much harder to design a semantics with this guaranteed property if the depth restriction could be any arbitrary integer; this is also why
XrML and DL cannot be easily “fixed” to support integer delegation depth that is immune to this kind of attack.

**Width-bounded delegation** Suppose Alice wants to delegate authority over is a friend facts to Bob. She does not care about the length of the delegation chain, but she requires every delegator in the chain to satisfy some property, e.g. to possess an email address from fabrikam.com. The following assertions implement this policy by encoding constrained transitive delegation using the can say verb phrase with a 0 subscript. Principals with the is a delegator attribute are authorized by Alice to assert is a friend facts, and to transitively re-delegate this attribute, but only amongst principals with a matching email address.

\[
\text{Alice says } x \text{ can say}_0 y \text{ is a friend if } x \text{ is a delegator} \quad (33)
\]

\[
\text{Alice says } \text{Bob is a delegator} \quad (34)
\]

\[
\text{Alice says } x \text{ can say}_0 y \text{ is a delegator if } x \text{ is a delegator, } y \text{ possesses Email email, email matches *@fabrikam.com } \quad (35)
\]

If these are the only assertions by Alice that mention the predicate is a friend or is a delegator, then any derivation of Alice says x is a friend can only depend on Bob or principals with a matching email address. As with depth-bounded delegation, this property cannot be enforced in SPKI/SDSI, DL or XrML.

**Local namespaces** In a distributed system, name clashes can occur if two collaborating principals use the same local name for some property (such as a role name), and these names are imported via can say-credentials into a common assertion context. One way to address this problem without requiring all names to be globally unique is to require all such properties to have an extra parameter specifying the namespace it originated from.

Suppose, for example, that both Bob and Charlie define assertions containing facts of the form A is a friend. Alice wants to write a policy where all of Bob’s friends are also her friends, but Charlie’s friends
should only enjoy the privileges of “acquaintances”. To distinguish Bob’s friends from Charlie’s friends, the verb phrase “is a friend” is required to have a namespace parameter, e.g.

Bob says Doris is a friend (Bob’s namespace).

(Note that this example is concerned with name clashes regarding verb phrases rather than principals; indeed, we assume here that principals are uniquely identified by their public keys.) Then Alice could implement her policy as follows:

Alice says Bob can say $x$ is a friend (Bob’s namespace) (36)

Alice says $x$ is a friend (Alice’s namespace) if $x$ is a friend (Bob’s namespace) (37)

Alice says Charlie can say $x$ is a friend (Charlie’s namespace) (38)

Alice says $x$ is an acquaintance (Alice’s namespace) if $x$ is a friend (Charlie’s namespace) (39)

7 Translation into Datalog with Constraints

We now give a translation from SecPAL assertion contexts into equivalent programs in Datalog with Constraints. In Section 8, we then exploit Datalog’s computational complexity properties (polynomial data complexity) and use the translated Datalog program for query evaluation.

Our terminology for Datalog with Constraints is as follows. (See [20] or [3] for a detailed introduction to Datalog and [57, 56] for Datalog with Constraints.) An atom, $P$, consists of a predicate name plus an ordered list of parameters, each of which is either a variable or a constant. An atom is ground if and only if it contains no variables. A clause, written $P_0 \leftarrow P_1, \ldots, P_n, c$, consists of a head atom, a list of body atoms, and a constraint. We assume the sets of variables, constants, and constraints are the same as for SecPAL. A Datalog program, $\mathcal{P}$, is a finite set of clauses. The semantics of a program $\mathcal{P}$ is the least fixed point of the standard immediate consequence operator $T_\mathcal{P}$.

Definition 7.1. (Consequence operator) The immediate consequence operator $T_\mathcal{P}$ is a function be-
tween sets of ground atoms and is defined as:

\[ T_P(I) = \{ P_0\theta \mid (P_0 \leftarrow P_1, \ldots, P_n, c) \in \mathcal{P}, \]
\[ P_i\theta \in I \text{ for } i \in \{1..n\}, \]
\[ c\theta \text{ is ground and valid}, \]
\[ P_0\theta \text{ is ground} \} \]

The operator \( T_P \) is monotonic and continuous, and its least fixed point \( T_\omega^P(\emptyset) \) contains all ground atoms deducible from \( \mathcal{P} \).

We treat expressions of the form \( e_1 \text{ says}_k \text{ fact} \) as Datalog atoms, where \( k \) is either a variable or 0 or \( \infty \). This can be seen as a sugared notation for a atom where the predicate name is the string concatenation of all infix operators (\text{say}, \text{can say}, \text{can act as}, and predicates) occurring in the expression, including subscripts for \text{can say}. The arguments of the atom are the collected expressions between these infix operators. For example, the expression \( A \text{ says}_k x \text{ can say}_0 \text{ y can say}_\infty \text{ B can act as } z \) is shorthand for \( \text{says can say infinity can say zero can act as}(A, k, x, y, B, z) \).

**Algorithm 7.2.** The translation of an assertion context \( \mathcal{AC} \) proceeds as follows:

1. If \( \text{fact}_0 \) is flat, then an assertion \( A \text{ says}_k \text{ fact}_0 \text{ if } \text{fact}_1, \ldots, \text{fact}_m, c \) is translated into the clause \( A \text{ says}_k \text{ fact}_0 \leftarrow A \text{ says}_k \text{ fact}_1, \ldots, A \text{ says}_k \text{ fact}_m, c \) where \( k \) is a fresh variable.

2. Otherwise, \( \text{fact}_0 \) is of the form \( e_0 \text{ can say}_{k_0} \ldots e_{n-1} \text{ can say}_{k_{n-1}} \text{ fact} \), for some \( n \geq 1 \), where \( \text{fact} \) is flat. Let \( \text{fact}_n = \text{fact} \) and \( \text{fact}_i = e_i \text{ can say}_{k_i} \text{ fact}_{i+1} \), for \( i \in \{0..n-1\} \). Note that \( \text{fact}_0 = \hat{\text{fact}}_0 \). Then the assertion

\[ A \text{ says}_k \text{ fact}_0 \text{ if } \text{fact}_1, \ldots, \text{fact}_m, c \]

is translated into a set of \( n + 1 \) Datalog clauses as follows.

(a) We add the Datalog clause

\[ A \text{ says}_k \hat{\text{fact}}_0 \leftarrow A \text{ says}_k \text{ fact}_1, \ldots, A \text{ says}_k \text{ fact}_m, c \]
where \( k \) is a fresh variable.

(b) For each \( i \in \{1..n\} \), we add a Datalog clause

\[
\begin{align*}
A \text{ says}_\infty \mathbf{fact}_i & \leftarrow x \text{ says}_{K_{i-1}} \mathbf{fact}_i, \\
A \text{ says}_\infty x \text{ can say}_{K_{i-1}} \mathbf{fact}_i
\end{align*}
\]

where \( x \) is a fresh variable.

3. Finally, for each Datalog clause created above with head \( A \text{ says}_k e \text{ verbphrase} \) we add a clause

\[
\begin{align*}
A \text{ says}_k x \text{ verbphrase} & \leftarrow A \text{ says}_k x \text{ can act as } e, \\
A \text{ says}_k e \text{ verbphrase}
\end{align*}
\]

where \( x \) is a fresh variable.

**Example 7.3.** For example, the assertion

\[
A \text{ says} B \text{ can say}_\infty y \text{ can say}_0 C \text{ can read } z \text{ if } y \text{ can read } \text{Foo}
\]

is translated into

\[
\begin{align*}
A \text{ says}_k B \text{ can say}_\infty y \text{ can say}_0 C \text{ can read } z & \leftarrow A \text{ says}_k y \text{ can read } \text{Foo} \\
A \text{ says}_\infty y \text{ can say}_0 C \text{ can read } z & \leftarrow \xspace \\
A \text{ says}_\infty x \text{ can say}_\infty y \text{ can say}_0 C \text{ can read } z
\end{align*}
\]

\[
\begin{align*}
A \text{ says}_\infty x \text{ can say}_0 C \text{ can read } z & \leftarrow \xspace \\
A \text{ says}_\infty x \text{ can say}_0 C \text{ can read } z
\end{align*}
\]
\textbf{RESOLVE-CLAUSE}(\langle P \rangle)
\begin{align*}
\text{Ans}(P) & := \emptyset; \\
\text{foreach} & (Q \leftarrow \vec{Q}, c) \in \mathcal{P} \text{ do} \\
& \quad \text{if } nd = \text{resolve}(\langle P; Q; \vec{Q}; Q; \emptyset; C \rangle, P) \text{ exists then} \\
& \quad \quad \text{PROCESS-NODE}(nd)
\end{align*}
\textbf{PROCESS-NODE}(nd)
\begin{align*}
\text{match } nd & \text{ with } \langle P; \vec{Q}; c; \emptyset \rangle \text{ in} \\
& \quad \text{if } nd \notin \text{Ans}(P) \text{ then} \\
& \quad \quad \text{Ans}(P) := \text{Ans}(P) \cup \{nd\}; \\
\text{foreach} & nd' \in \text{Wait}(P) \text{ do} \\
& \quad \text{if } nd'' = \text{resolve}(nd', nd) \text{ exists then} \\
& \quad \quad \text{PROCESS-NODE}(nd'')
\end{align*}
\textbf{PROCESS-ANSWER}(nd)
\begin{align*}
\text{match } nd & \text{ with } \{nd; \vec{Q}::\emptyset; c\} \text{ in} \\
& \quad \text{if } \vec{Q} = \emptyset \text{ then} \\
& \quad \quad \text{PROCESS-ANSWER}(nd)
\end{align*}
\begin{align*}
\text{else match } & \vec{Q} \text{ with } Q \text{ in} \\
& \quad \text{if there exists } Q_0 \in \text{dom}(\text{Ans}) \text{ such that } Q_0 \not\leq Q \text{ then} \\
& \quad \quad \text{Wait}(Q_0) := \text{Wait}(Q_0) \cup \{nd\}; \\
\text{foreach} & nd'' \in \text{Ans}(Q_0) \text{ do} \\
& \quad \text{if } nd'' = \text{resolve}(nd, nd') \text{ exists then} \\
& \quad \quad \text{PROCESS-NODE}(nd'')
\end{align*}
\textbf{RESOLVE-CLAUSE}(\langle Q_0 \rangle)

Figure 1: A tabled resolution algorithm for evaluating Datalog queries.

in Steps 2a and 2b. Finally, in Step 3, the following clauses are also added:

\begin{align*}
A \text{ says}_k x & \text{ can say}_\infty y \text{ can say}_0 C \text{ can read } z \leftarrow \\
A \text{ says}_k x & \text{ can act as } B, \\
A \text{ says}_k B & \text{ can say}_\infty y \text{ can say}_0 C \text{ can read } z
\end{align*}

\begin{align*}
A \text{ says}_\infty x & \text{ can say}_0 C \text{ can read } z \leftarrow \\
A \text{ says}_k x & \text{ can act as } y, \\
A \text{ says}_\infty y & \text{ can say}_0 C \text{ can read } z
\end{align*}

\begin{align*}
A \text{ says}_\infty x & \text{ can read } z \leftarrow \\
A \text{ says}_k x & \text{ can act as } C, \\
A \text{ says}_\infty C & \text{ can read } z
\end{align*}

Intuitively, the \textit{says} subscript keeps track of the delegation depth, just like the \textit{D} in the three semantic rules in Section 3. This correspondence is reflected in the following theorem that relates the Datalog translation to the SecPAL semantics.

\textbf{Theorem 7.4 (Soundness and completeness).} Let \mathcal{P} be the Datalog translation of a safe assertion context \mathcal{AC}. We have \text{A says}_D \text{ fact } \in T^\omega_D(\emptyset) \text{ iff } \mathcal{AC}, D \models \text{A says} \text{ fact}.

\textbf{\textit{\square}}

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8 Evaluation of authorization queries

This section describes an algorithm for evaluating authorization queries (Section 4) against a SecPAL assertion context.

8.1 Atomic Datalog queries

The first step is to evaluate atomic Datalog queries of the form \( e \text{ says}_a \text{ fact} \) (i.e., computing all query instances that are in \( T^a_P(\emptyset) \)) against the Datalog program \( P \) obtained by translation. The naive bottom-up approach [3], where the fixed-point model is precomputed for all queries, is not suitable, as the assertion context may be completely different between different requests. Furthermore, top-down resolution algorithms are usually more efficient in computing fully or partially instantiated goals. However, standard SLD resolution (as used in e.g. Prolog) may run into loops even for simple assertion contexts. Tabling, or memoing, is an efficient approach for guaranteeing termination by incorporating some bottom-up techniques into a top-down resolution strategy [60, 28, 23]. Tabling has also been applied to Datalog with Constraints, but requires complex constraint solving procedures [62].

Our tabling algorithm is a simplified and deterministic version that is tailored to the clauses produced by the translation of a safe assertion context (as described in Section 7). It does not require constraint solving and is thus simpler to implement. A node is either a root node of the form \( \langle P \rangle \), where the index \( P \) is an atom, or a sextuple \( \langle P; Q; c; S; nd; Cl \rangle \), where \( Q \) is a list of atoms (the subgoals), \( c \) a constraint, \( S \) an atom (the partial answer), \( nd \) a list of sextuple nodes (the child nodes), and \( Cl \) a clause.

The algorithm makes use of two tables. The answer table \( \text{Ans} \) maps atoms to sets of answer nodes (i.e., nodes where \( Q \) is empty and \( c = \text{True} \)). The set \( \text{Ans}(P) \) is used to store all the found answer nodes pertaining to a query \( \langle P \rangle \). The wait table \( \text{Wait} \) maps atoms to sets of nodes with nonempty lists of subgoals. \( \text{Wait}(P) \) is a list of all those nodes whose current subgoal (i.e., the left-most subgoal) is waiting for answers from \( \langle P \rangle \). Whenever a new answer for \( \langle P \rangle \) is produced, the computation of these waiting nodes is resumed.

Before presenting the algorithm in detail, we define a number of terms. The function \( \text{simplify} \) is a function on constraints whose return value is always an equivalent constraint, and if the argument is ground, the return value is either \text{True} or \text{False}.
The infix operators :: and @ denote the cons and append operations on lists, respectively.

Let $P$ and $Q$ range over atoms. A variable renaming for $P$ is an injective substitution whose range consists only of variables. A fresh renaming of $P$ is a variable renaming for $P$ such that the variables in its range have not appeared anywhere else. A substitution $\theta$ is no less general than $\theta'$ iff there exists a substitution $\rho$ such that $\theta' = \theta \rho$.

A substitution $\theta$ is a unifier of $P$ and $Q$ iff $P \theta = Q \theta$. A substitution $\theta$ is a most general unifier of $P$ and $Q$ iff it is no less general than any other unifier of $P$ and $Q$. When $P$ and $Q$ are unifiable, they also have a most general unifier that is unique up to variable renaming. We denote it by $\text{mgu}(P, Q)$. Finding the most general unifier is relatively simple (see [44] for an overview) but there are more intricate algorithms that run in linear time, see e.g. [55, 51].

Let $P$ be an instance of $Q$ iff $P = P \theta$ for some substitution $\theta$, in which case we write $P \equiv Q$.

A node $nd \equiv (P; Q :: \bar{Q}; c; S; \bar{nd}; Cl)$ and an atom $Q'$ are resolvable iff some $Q''$ is a fresh variable renaming of $Q'$, $\theta \equiv \text{mgu}(Q, Q'')$ exists and $d \equiv \text{simplify}(c \theta) \neq \text{False}$. Their resolvent is $nd'' \equiv (P; \bar{Q} \theta; d; S \theta; \bar{nd}; Cl)$, and $\theta$ is their resolution unifier. We write $\text{resolve}(nd, Q') = nd''$ if $nd$ and $Q'$ are resolvable. By extension, a node $nd \equiv (P; Q :: \bar{Q}; c; S; \bar{nd}; Cl)$ and an answer node $nd' \equiv (\text{[True]}; Q'; \bar{\cdot})$ are resolvable iff $nd$ and $Q'$ are resolvable with resolution unifier $\theta$, and their resolvent is $nd'' \equiv (P; \bar{Q} \theta; d; S \theta; \bar{nd}@[\bar{nd}']; Cl)$. We write $\text{resolve}(nd, nd') = nd''$ if $nd$ and $nd'$ are resolvable.

Figure 1 shows the pseudocode of our Datalog evaluation algorithm. Let $P$ be an atom and $\text{Ans}$ be an answer table. Then $\text{Answers}_P(P, \text{Ans})$ is defined as

$$\{\theta : (\ldots : \bar{S}; \bar{\cdot}) \in \text{Ans}(P'), S = P \theta, \text{dom}(\theta) \subseteq \text{vars}(P)\}$$

if there exists an atom $P' \in \text{dom}(\text{Ans})$ such that $P \equiv P'$. In other words, if the supplied answer table already contains a suitable answer set, we can just return the existing answers. If no such atom exists in the domain of $\text{Ans}$ and if the execution of $\text{RESOLVE-CLAUSE}(\bar{P})$ terminates with initial answer table $\text{Ans}$ and an initially empty wait table, then $\text{Answers}_P(P, \text{Ans})$ is defined as

$$\{\theta : (\ldots : \bar{S}; \bar{\cdot}) \in \text{Ans}'(P), S = P \theta, \text{dom}(\theta) \subseteq \text{vars}(P)\}$$

where $\text{Ans}'$ is the modified answer table after the call. In all other cases $\text{Answers}_P(P, \text{Ans})$ is undefined.
Theorem 8.4 states the termination, soundness and completeness properties of $\text{Answers}_P$. These properties will be exploited in Section 8 where we present an algorithm for evaluating composite authorization queries.

At first sight, completeness with respect to $T^ω_P(\emptyset)$ depends on $P$ being Datalog-safe, i.e., all variables in the head atom must occur in a body atom. However, the translation of a safe SecPAL assertion context does not always result in a Datalog-safe program. We define an alternative notion of safety for Datalog programs that is satisfied by the SecPAL translation and that still preserves completeness.

**Definition 8.1.** Every parameter position of a predicate is associated with either IN or OUT. A Datalog clause is IN/OUT-safe iff any OUT-variable in the head also occurs as an OUT-variable in the body, and any IN-variable in a body atom also occurs as IN-variable in the head or as OUT-variable in a preceding body atom. A Datalog program $P$ is IN/OUT-safe iff all its clauses are IN/OUT-safe. A query $P$ is IN/OUT-safe iff all its IN-parameters are ground.

**Lemma 8.2.** If $\mathcal{AC}$ is a safe assertion context and $P$ its Datalog translation then there exists an IN/OUT assignment to predicate parameters in $P$ such that $P$ is IN/OUT-safe.

**Definition 8.3.** An answer table $\text{Ans}$ is sound (with respect to some program $P$) iff

\[
\text{for all } P \in \text{dom}(\text{Ans}) : \quad (P',[\cdot],[\cdot]_\text{True};\vec{\cdot}) \in \text{Ans}(P) \implies P = P', S \preceq P, \text{ and } S \in T^ω_P(\emptyset).
\]

$\text{Ans}$ is complete (with respect to $P$) iff for all $P \in \text{dom}(\text{Ans})$: $S \in T^ω_P(\emptyset)$ and $S \preceq P$ implies that $S$ is the answer of an answer node in $\text{Ans}(P)$. Note, in particular, that the empty answer table is sound and complete.

**Theorem 8.4. (soundness, completeness, termination)** Let $\text{Ans}$ be a sound and complete answer table, $\mathcal{P}$ an IN/OUT-safe program and $P$ an IN/OUT-safe query. Then $\text{Answers}_P(P,\text{Ans})$ is defined, finite and equal to \{ $\theta : P\theta \in T^ω_P(\emptyset), \text{dom}(\theta) \subseteq \text{vars}(P)$\}.

Actually, the modified answer table after evaluation is still sound and complete and contains enough information to reconstruct the complete proof graph for each answer: an answer node $(\vec{\cdot};\vec{\cdot};S;\vec{n}d;\text{Cl})$ is interpreted to have edges pointing to each of its child nodes $\vec{n}d$ and an edge pointing to the clause $\text{Cl}$.
\[
\begin{align*}
\text{AuthAns}_{AC}(e \text{ says fact}) &= \text{Answers}_P(e \text{ says}_\omega \text{ fact, } \emptyset) \\
\text{AuthAns}_{AC}(q_1, q_2) &= \{ \theta_1 \theta_2 \mid \theta_1 \in \text{AuthAns}_{AC}(q_1) \text{ and } \theta_2 \in \text{AuthAns}_{AC}(q_2) \} \\
\text{AuthAns}_{AC}(q_1 \text{ or } q_2) &= \text{AuthAns}_{AC}(q_1) \cup \text{AuthAns}_{AC}(q_2) \\
\text{AuthAns}_{AC}(\text{not}(q)) &= \begin{cases} 
\{ \varepsilon \} & \text{if } \text{vars}(q) = \emptyset \text{ and } \text{AuthAns}_{AC}(q) = \emptyset \\
\emptyset & \text{if } \text{vars}(q) = \emptyset \text{ and } \text{AuthAns}_{AC}(q) \neq \emptyset \\
\text{undefined} & \text{otherwise}
\end{cases} \\
\text{AuthAns}_{AC}(\exists x(q)) &= \{ \theta_{-x} \mid \theta \in \text{AuthAns}_{AC}(q) \}
\end{align*}
\]

Figure 2: SecPAL evaluation algorithm

Section 8.3 shows how this Datalog proof graph can be converted back into a corresponding SecPAL proof graph.

8.2 Complex authorization queries

Based on this algorithm for evaluating atomic queries, we can now show how to evaluate complex authorization queries as defined in Section 4. Let \(\mathcal{A}_C\) be an assertion context and \(\mathcal{P}\) its Datalog translation. The function \(\text{AuthAns}_{AC}\) on authorization queries is defined in Figure 2. The following theorem shows that \(\text{AuthAns}_{AC}\) is an algorithm for evaluating safe queries.

**Theorem 8.5 (Finiteness, soundness, and completeness of query evaluation).** For all safe assertion contexts \(\mathcal{A}_C\) and safe authorization queries \(q\),

1. \(\text{AuthAns}_{AC}(q)\) is defined and finite, and
2. \(\mathcal{A}_C, \theta \vdash q\) iff \(\theta \in \text{AuthAns}_{AC}(q)\).

The evaluation of the base case \(e \text{ says fact}\) calls the function \(\text{Answers}_P\) with an empty answer table. But since the answer table after each call remains sound and complete with respect to its domain (it will just have a larger domain), an efficient implementation could initialize an empty table only for the first call in the evaluation of an authorization query, and then reuse the existing, and increasingly populated, answer table for each subsequent call to \(\text{Answers}_P\).
Finally, the following theorem states that SecPAL has polynomial data complexity. Data complexity [3, 25] is a measure of the computation time for evaluating a fixed query with fixed intensional database (IDB) but variable extensional database (EDB). This measure is most often used for policy languages, as the size of the EDB (the number of “plain facts”) typically exceeds the size of the IDB (the number of “rules”) by several orders of magnitude.

**Theorem 8.6.** Let $M$ be the number of flat atomic assertions (i.e., those without conditional facts) in a safe assertion context $\mathcal{AC}$ and let $N$ be the maximum length of constants occurring in these assertions. The time complexity of computing $\text{AuthAns}_{\mathcal{AC}}$ is polynomial in $M$ and $N$.

### 8.3 Proof graph generation

When testing and troubleshooting policies, it is useful to be able to see a justification of an authorization decision. This could be some visual or textual representation of a corresponding proof graph constructed according to the rule system in Section 3.

Given a Datalog program $\mathcal{P}$, a *proof graph* for $\mathcal{P}$ is a directed acyclic graph with the following properties. Leaf nodes are either Datalog clauses in $\mathcal{P}$ or ground constraints that are valid. Every non-leaf node is a ground instance $P^\theta$ of the head of a clause $P' \leftarrow P_1, \ldots, P_n, c$, where $\theta$ substitutes a constant for each variable occurring in the clause; the node has as child nodes the clause, the ground instances $P_1^\theta, \ldots, P_n^\theta$ of the body atoms, and the ground instance $c^\theta$ of the body constraint. A ground atom $P$ occurs in $T^{\theta}_{\mathcal{P}}(\emptyset)$ if and only if there is a proof graph for $\mathcal{P}$ with $P$ as a root node. The algorithm in Figure 1 constructs such a Datalog proof graph during query evaluation of the Datalog program $\mathcal{P}$ obtained by translating an assertion context $\mathcal{AC}$. Each answer to a query is a root node of the graph. Every non-leaf node is a ground Datalog atom of the form $A \text{ asserts fact}$. Leaf nodes are either Datalog clauses in the program $\mathcal{P}$, or ground constraints that are valid. (See left panels of Figures 3, 4 and 5.)

Similarly, we can define a notion of proof graph for SecPAL such that there is a derivation of $\mathcal{AC}, \infty \vdash A \text{ says fact}$ according to the three deduction rules of Section 3 if and only if there is a SecPAL proof graph with $\mathcal{AC}, \infty \vdash A \text{ says fact}$ as a root node.

If during execution of Algorithm 7.2, each generated Datalog clause is labelled with the algorithm step at which it was generated (i.e., 1, 2a, 2b, or 3), the Datalog proof graph contains enough information to be easily converted into the corresponding SecPAL proof graph. The conversion is illustrated in
Figures 3, 4 and 5.

9 Discussion

Implementation A prototype of SecPAL [52] has been implemented as part of a project investigating access control solutions for multi-domain grid computing environments [29].

Assertions are XML-encoded then digitally signed and encrypted for distribution over the network. Apart from the basic syntactic elements presented in this paper, the SecPAL XML schema [30] also pre-defines a set of standard verb phrases and attributes that provide sufficient expressiveness for the majority of grid applications. All verb phrases are augmented by a common set of environmental parameters for specifying validity time spans, principal locations, and revocation freshness requirements.

The implementation provides tools for conversion between assertions represented in a text-based format (almost identical to the concrete syntax presented here), in the XML schema, and in a .NET object model. The latter provides an interface to the query evaluation engine for developing SecPAL-based reference monitor implementations. Partial auditing of access decisions is performed according to an XML audit policy. An audit log tool allows security administrators to view the details of past access decisions and visualize answers to queries as SecPAL proof graphs. The current prototype includes code samples for several grid-related, distributed delegation scenarios in which the assertion contexts are assembled from native SecPAL assertions as well as Kerberos, X.509 and Active Directory tokens that are translated into SecPAL at run time.

The prototype has been evaluated by Humphrey et al. [40] in the context of authorization for the GridFTP protocol. Current implementation work includes tools for explaining and analyzing access denials, based on the techniques described in [11].

Related work The ABLP logic [2, 45] introduced the “says” modality and the use of logic rules for expressing decentralized authorization policies. SecPAL’s delegation operators can say and can act as resemble the “controls” and “speaks for” operators, respectively, of ABLP, but we have not established precise connections. As more is learnt about the semantics of access control logics like ABLP, such as their relation to classical modal logics [34], we hope for a better understanding of the formal connections between SecPAL, ABLP, and related logics.
Figure 3: Left: Datalog proof node with parent from Translation Step 1 or 2a. Right: corresponding SecPAL proof node using Rule (cond).

Figure 4: Left: Datalog proof node with parent from Translation Step 2b. Right: corresponding SecPAL proof node using Rule (can say).

Figure 5: Left: Datalog proof node with parent from Translation Step 3. Right: corresponding SecPAL proof node using Rule (can act as).
PolicyMaker and Keynote [19, 18] introduced the notion of decentralized trust management. Quite a few other authorization languages have been developed since. SPKI/SDSI [31] is an experimental IETF standard using certificates to specify decentralized authorization. Authorization certificates grant permissions to subjects specified either as public keys, or as names defined via linked local name spaces [59], or as $k$-out-of-$n$ threshold subjects. Grants can have validity restrictions and indicate whether they may be delegated.

XrML [24] (and its offspring, MPEG REL) is an XML-based language targeted at specifying licenses for Digital Rights Management. Grants may have validity restrictions and can be conditioned on other existing or deducible grants. A grant can also indicate under which conditions it may be delegated to others. XACML [54] is another XML-based language for describing access control policies. A policy grants capabilities to subjects that satisfy the specified conditions. Deny policies explicitly state prohibitions. XACML defines policy combination rules for resolving conflicts between permitting and denying policies such as First-Applicable, Deny-Override or Permit-Override. XACML does not support delegation and is thus not well suited for decentralized authorization.

The OASIS SAML [53] standards define XML formats and protocols for exchanging authenticated user identities, attributes, and authorization decisions, such as whether a particular subject has access to a particular object. SAML messages do not themselves express authorization rules or policies.

The original Globus security architecture [33] for grid computing defines a general security policy for subjects and objects belonging to multiple trust domains, with cross-domain delegation of access rights. More recent computational grids rely on specific languages, such as Akenti [61], Permis [22], and XACML, to define fine-grained policies, and exchange SAML or X.509 certificates to convey identity, attribute, and role information. Version 1.1 of XACML has a formal semantics [39] via a purely functional implementation in Haskell.

Policy languages such as Binder [26], SD3 [43], Delegation Logic (DL) [46] and the RT family of languages [49] use Datalog as basis for both syntax and semantics. To support attribute-based delegation, these languages allow predicates to be qualified by an issuing principal. Cassandra [13, 12] and RT$^C$ [48] are based on Datalog with Constraints [41, 56] for higher expressiveness. The Cassandra framework also defines a transition system for evolving policies and supports automated credential retrieval and automated trust negotiation.
DKAL [36] is an authorization logic that extends SecPAL with constructs for specifying and reasoning about localized knowledge and targeted communication of authorization statements. DKAL can thus express policies that cannot be easily encoded in SecPAL and other previous languages. In SecPAL, if $A$ says $\text{fact}$, then there is nothing to prevent $B$ from importing this statement using $B$ says $A$ can say $\text{fact}$. Importing knowledge can be controlled more finely in DKAL, where the says operator is parameterized by the intended target of the communication: only if $A$ says $\text{fact}$ to $B$ can it be deduced that $B$ knows $A$ said $\text{fact}$.

Dynamic updates to the authorization environment occur in history-dependent authorization policies such as dynamic separation of duties and RBAC-style role activation and deactivation. Becker and Nanz [10] have shown how Datalog-based authorization languages can be extended to support such policies and be given a semantics based on transaction logic. SecPAL could be easily extended according to that scheme: authorization query tables would map requests not only to queries, but also to a sequence of insertions (and retractions) of basic assertions into (and from) the assertion context.

Much research has been done on logic-based access control languages for single administrative domains that do not require decentralized delegation of authority. Many of these are also based on Datalog or Datalog with Constraints, e.g. [15, 17, 42, 5, 64]. Lithium [37] is a language for reasoning about digital rights and is based on a different fragment of first order logic. It is the only language allowing real logical negation in the conclusion as well as in the premises of policy rules. This is useful for analyzing merged policies, but Lithium restricts recursion and cannot easily express delegation.

**Conclusions** We have designed an authorization language that supports fine-grained delegation control for decentralized systems, highly expressive constraints and negative conditions that are needed in practice but cannot be expressed in other languages. Combining all these features in a single language without sacrificing decidability and tractability is nontrivial. If authorization queries are extended by an aggregation operator (which can be easily done without modifying the assertion semantics and without sacrificing polynomial data complexity), SecPAL can (safely) express the entire benchmark policy described in [6] and [7], one of the largest and most complex examples of a formal authorization policy to date. Despite its expressiveness, we argue that SecPAL is relatively simple and intuitive, due to the resemblance of its syntax to natural language, its small semantic specification and its purely syntactic
safety conditions. Future work includes tools for policy authoring, deployment, and formal analysis.

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A Auxiliary definitions and proofs

This appendix contains proofs of all theorems stated in the main part of the paper as well as supporting lemmas and definitions.

A.1 Authorization queries

Lemma A.1. Let $\mathcal{AC}$ be safe. If $\mathcal{AC}, \theta \vdash q$ then $\text{dom}(\theta) \subseteq \text{vars}(q)$, and $\theta$ grounds all $x \in \text{dom}(\theta)$.

Proof. By induction on the definition of $\vdash$.

Lemma A.2. Let $\mathcal{AC}$ be safe. If $\mathcal{AC}, \theta \vdash e \text{ says fact}$ then $\text{dom}(\theta) = \text{vars}(e \text{ says fact})$.

Proof. Follows immediately from the definitions of $\vdash$ and $|=.$

Lemma A.3. If $I \models q : O$ then for all substitutions $\theta$ that map variables to constants, $I - \text{dom}(\theta) \models q\theta : O - \text{dom}(\theta)$.

Proof. By induction on $q$.

Corollary A.4. If $I \models q : O$ and $I \subseteq \text{dom}(\theta)$ then $q\theta$ is safe, for all substitutions $\theta$ that map variables to constants.

Lemma A.5. If $\emptyset \models q : O$ and $\mathcal{AC}, \theta \vdash q$ then $O \subseteq \text{dom}(\theta)$.

Proof. By induction on $q$.

Suppose $q \equiv e \text{ says fact}$. Then $O = \text{vars}(e \text{ says fact}) = \text{dom}(\theta)$, by Lemma A.2.

Suppose $q \equiv q_1, q_2$ and $\theta \equiv \theta_1 \theta_2$. By the induction hypothesis, $O_1 \subseteq \text{dom}(\theta_1)$. Therefore, by Lemma A.3, $\emptyset \models q_2 \theta_1 : O_2 - \text{dom}(\theta_1)$. Then by the induction hypothesis, $O_2 - \text{dom}(\theta_1) \subseteq \text{dom}(\theta_2)$.

Therefore, $O_1 \cup O_2 \subseteq \text{dom}(\theta_1 \theta_2)$.

The other cases are straightforward.

Lemma A.6. If $\mathcal{AC}$ and $q_1, q_2$ are safe and $\mathcal{AC}, \theta_1 \vdash q_1$ then $q_2 \theta_1$ is safe.

Proof. From the definition of safety and $\models$ it follows that $\emptyset \models q_1, q_2 : O_1 \cup O_2$ where $\emptyset \models q_1 : O_1$. By Lemma A.5, $O_1 \subseteq \text{dom}(\theta_1)$. Then by Corollary A.4, $q_2 \theta_1$ is safe.
A.2 Translation into Datalog with Constraints

Lemma A.7. (Soundness) Let $\mathcal{A}C$ be safe and let $\mathcal{P}$ be its Datalog translation. If $A \text{ says}_D \text{ fact } \in T^n_{\mathcal{P}}(\emptyset)$ then $\mathcal{A}C, D \models A \text{ says } \text{ fact }$.

Proof. We assume $A \text{ says}_D \text{ fact } \in T^n_{\mathcal{P}}(\emptyset)$ and prove the statement by induction on stages of $T^n_{\mathcal{P}}$.

Case Step 1 and 2a If $A \text{ says}_D \text{ fact }$ is added based on a clause produced by Step 1 or 2a, then by the inductive hypothesis, $\mathcal{A}C, D \models A \text{ says } \text{ fact } \theta$ for $i = 1...n$. Furthermore, $c\theta$ is ground and valid, so by Rule (cond), $\mathcal{A}C, D \models A \text{ says } \text{ fact }$.

Case Step 2b If $A \text{ says}_\infty \text{ fact }$ is added based on a clause produced by Step 2b, then by the inductive hypothesis, $\mathcal{A}C, K \models B \text{ says } \text{ fact }$ and $\mathcal{A}C, \infty \models A \text{ says } B \text{ can say } \text{ fact }$, for some $B$ and $K$. By Rule (can say), $\mathcal{A}C, \infty \models A \text{ says } \text{ fact }$.

Case Step 3 If $A \text{ says}_D B \text{ verbphrase }$ is added based on a clause produced by Step 3, then by the inductive hypothesis, $\mathcal{A}C, D \models A \text{ says } B \text{ can act as } C$ and $\mathcal{A}C, D \models A \text{ says } C \text{ verbphrase }$, for some $C$. By Rule (can act as), $\mathcal{A}C, D \models B \text{ says } C \text{ verbphrase }$.

Lemma A.8. Let $\mathcal{A}C$ be safe. If $\mathcal{A}C, D \models A \text{ says } B \text{ verbphrase }$ then there exists an assertion in $\mathcal{A}C$ of the form

$$A \text{ says } e_1 \text{ can say}_{D_1} ... e_n \text{ can say}_{D_n} e \text{ verbphrase}^\prime \text{ if ...}$$

for some $e$ and $e_i$, for $i = 1...n$ where $n \geq 0$, and verbphrase is an instance of verbphrase$^\prime$.

Proof. By induction on the SecPAL rules. If the last rule used in the deduction of $\mathcal{A}C, D \models A \text{ says } B \text{ verbphrase }$ was (cond), there exists an assertion in $\mathcal{A}C$ of the form

$$A \text{ says } e \text{ verbphrase}^\prime \text{ if ...}$$

where $B \text{ verbphrase } = (e \text{ verbphrase}^\prime)\theta$. 

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If the last rule used was (can say), we have $\mathcal{A}C, \infty \models A \text{ says } B \text{ can say}_D B \text{ verbphrase}$. Therefore, by the induction hypothesis, there exists an assertion in $\mathcal{A}C$ of the required form.

If the last rule used was (can act as), we have $\mathcal{A}C, D \models A \text{ says } C \text{ verbphrase}$. Therefore, by the induction hypothesis, there exists an assertion in $\mathcal{A}C$ of the required form.

Lemma A.9. (Completeness) Let $\mathcal{P}$ be the Datalog translation of a safe assertion context $\mathcal{A}C$. If $\mathcal{A}C, D \models A \text{ says } \text{fact}$ then $A \text{ say}_D \text{ fact} \in T^0_\mathcal{P}(\emptyset)$.

Proof. Assume $\mathcal{A}C, D \models A \text{ says } \text{fact}$. We prove the statement by induction on the SecPAL rules:

Case (cond) If the last rule used in the deduction was (cond), $(A \text{ says } \text{fact if } \text{fact}_1, \ldots, \text{fact}_k, c) \in \mathcal{A}C$ is translated in Step 1 or 2a. Also, $\mathcal{A}C, D \models A \text{ say}_D \text{ fact}_i$, and by the induction hypothesis, $A \text{ say}_D \text{ fact}_i \in T^0_\mathcal{P}(\emptyset)$. Furthermore, $S \models c\emptyset$ and $\text{vars}(\text{fact}\emptyset) = \emptyset$, so by definition of $T^0_\mathcal{P}$, $A \text{ say}_D \text{ fact} \in T^0_\mathcal{P}(\emptyset)$.

Case (can say) If Rule (can say) was used last, we assume

1. $D = \infty$,
2. $\mathcal{A}C, K \models B \text{ says } \text{fact}$, and
3. $\mathcal{A}C, \infty \models A \text{ says } B \text{ can say}_K \text{ fact}$.

By Lemma A.8, there is an assertion in $\mathcal{A}C$ of the form

$A \text{ says } e_1 \text{ can say}_{D_1} \ldots e_n \text{ can say}_{D_n} e \text{ can say}_K \text{ fact}'$,

for some $e, e_i, D_i$ with $i = 1 \ldots n, n \geq 0$ and where $\text{fact}$ is an instance of $\text{fact}'$. In Step 2b, this is translated into

$A \text{ say}_\infty \text{ fact } \leftarrow$

$x \text{ say}_K \text{ fact}',$

$A \text{ say}_\infty x \text{ can say}_K \text{ fact}'$
where \( x \) is a fresh variable not occurring anywhere else in the clause, so it can in particular bind to \( B \). By the induction hypothesis, \( B \text{ says}_K \text{ fact} \in T^\omega_P(\emptyset) \) and \( A \text{ says}_\omega \text{ B can say}_K \text{ fact} \in T^\omega_P(\emptyset) \). By definition of \( T^\omega_P \), \( A \text{ says}_\omega \text{ fact} \in T^\omega_P(\emptyset) \).

**Case (can act as)** If Rule (can act as) was used last, we assume \( \mathcal{AC}, D \models A \text{ says } C \text{ verbphrase} \), and \( \mathcal{AC}, D \models A \text{ says } B \text{ can act as } C \), where \( \text{ fact } = B \text{ verbphrase} \). By the induction hypothesis, \( A \text{ says}_D C \text{ verbphrase} \in T^\omega_P(\emptyset) \). This is only possible if there is a clause in \( P \) of the form

\[
A \text{ says}_k e \text{ verbphrase}' \leftarrow \ldots
\]

where \( C \text{ verbphrase} = (e \text{ verbphrase}')\theta \) for some \( \theta \). By Step 3, there must also be a clause in \( P \) of the form

\[
A \text{ says}_k y \text{ verbphrase}' \leftarrow \\
A \text{ says}_k y \text{ can act as } e \\
A \text{ says}_a e \text{ verbphrase}'
\]

where \( y \) is a fresh variable not occurring anywhere else in the clause, so it can in particular bind to \( B \). By the induction hypothesis, we also have \( A \text{ says}_D B \text{ can act as } C \in T^\omega_P(\emptyset) \). Therefore, by definition of \( T^\omega_P \), \( A \text{ says}_D B \text{ verbphrase} \in T^\omega_P(\emptyset) \). \( \square \)

**Restatement of Theorem 7.4.** Let \( P \) be the Datalog translation of a safe assertion context \( \mathcal{AC} \). We have

\[
A \text{ says}_D \text{ fact} \in T^\omega_P(\emptyset) \text{ iff } \mathcal{AC}, D \models A \text{ says } \text{ fact}.
\]

**Proof.** Follows from Lemmas A.7 and A.9. \( \square \)

### A.3 Evaluation of authorization queries

The tabling evaluation algorithm in Section 8 can also be described as a non-deterministic labelled transition system. We present this system here because it is easier to prove properties for it than for the pseudocode in Figure 1. The results for the transition system apply also to the pseudocode, as the latter is a straightforward implementation of the former.
Restatement of Theorem 8.2. If \( \mathcal{A} \mathcal{C} \) is a safe assertion context and \( \mathcal{P} \) its Datalog translation then there exists an IN/OUT assignment to predicate parameters in \( \mathcal{P} \) such that \( \mathcal{P} \) is IN/OUT-safe.

Proof. Literals introduced by Algorithm 7.2 are of the form \( e_1 \text{ say} e_2 \text{ fact} \). We assign OUT to the parameter position of \( e_1 \), and IN to the position of \( e_2 \). The parameter positions in fact are all OUT if it is flat, and IN otherwise. Then by inspection and by assertion safety, all OUT variables in the head of a clause produced by the algorithm also occur in the body, and all IN variables in the body of a clause also occur in its head. \( \square \)

Definition A.10. A state is a triple \((\text{Nodes}, \text{Ans}, \text{Wait})\) where Nodes is a set of nodes, Ans is an answer table, and Wait is a wait table. A path is a series of 0 or more labelled transitions between states, as defined in the labelled transition system below. A state \( S' \) is reachable from a state \( S \) iff there is a path from \( S \) to \( S' \). In the following, let \( A \uplus B \) denote the union of \( A \) and \( B \) with the side condition that the
sets be disjoint. If \( \text{Ans} \) is a function mapping atoms to sets of nodes, then \( \text{Ans}[P \mapsto A] \) is a function that maps atoms \( Q \) to \( \text{Ans}(Q) \) if \( Q \in \text{dom}(\text{Ans}) \) and \( Q \neq P \) and additionally maps \( P \) to \( A \).

A state \( \langle \text{Nodes}, \text{Ans}, \text{Wait} \rangle \) is an initial state iff \( \text{Nodes} = \{\langle P \rangle\} \) for some IN/OUT-safe query \( P \) (with respect to the IN/OUT-safe program \( \mathcal{P} \)), \( \text{Ans} \) is sound and complete, and \( \text{Wait} \) is empty. A state \( S \) is a final state iff there is no state \( S' \) and no label \( \ell \) such that \( S \xrightarrow{\ell} S' \).

**Lemma A.11. (answer groundness)** If \( \langle \text{Nodes}, \text{Ans}, \text{Wait} \rangle \) is reachable from some initial state and \( \langle P; |; c; S; \tilde{nd}; Cl \rangle \in \text{Nodes} \) then \( S \) and \( c \) are ground and \( c \) is valid.

**Proof.** We prove the following, stronger, invariant by induction on the transition rules. If \( \langle P \rangle \in \text{Nodes} \) then all IN-parameters in \( P \) are ground. If \( \langle P; \tilde{Q}; c; S; \tilde{nd}; Cl \rangle \in \text{Nodes} \) then all IN-parameters in \( S \) are ground, and all OUT-parameters in \( S \) are either ground or occur as OUT-variable in \( \tilde{Q} \). If the node has a current subgoal \( Q \) (the head of \( \tilde{Q} \)), all IN-parameters of the current subgoal \( Q \) are ground, hence the new root node \( \langle Q \rangle \) satisfies the required property as well.

Suppose the node is produced by \text{ResolveClause}. The IN-parameters in its partial answer \( S \) are ground because it is resolved with \( P \) which satisfies the same property, by the inductive hypothesis. If an OUT-parameter in \( S \) is a variable, it must occur as an OUT-parameter in \( \tilde{Q} \), as all clauses in \( \mathcal{P} \) are IN/OUT-safe. If the node has a current subgoal, its IN-parameters are either already ground in the original clause, or they also occur as IN-parameters in the head of the clause, \( Q \). But \( Q \) is resolved against \( P \) which grounds its IN-parameters by the inductive hypothesis, therefore all IN-parameters in \( Q \) are also grounded by the resolution unifier, which is also applied to the current subgoal.

In all other cases, the node is the resolvent of an existing node \( \langle P; Q_0 :: \tilde{Q}; \ldots S'; :: Cl \rangle \) with an existing answer node \( \langle P'; |; :: S''; :: \ldots \rangle \), both of which enjoy the stated property by the inductive hypothesis. All IN-parameters of the partial answer \( S \) of the resolvent are ground because \( S \) is the product of applying the resolution unifier to \( S' \) which already has the same property. For the sake of contradiction, assume an OUT-parameter of \( S \) is neither ground nor occurs as an OUT-parameter in \( \tilde{Q} \). Then it must be an OUT-variable in \( S' \) which occurs as an OUT-variable in \( Q_0 \). But the resolution unifier unifies \( Q_0 \) with (a renaming of) \( S'' \), and \( S'' \) is completely ground, by the inductive hypothesis. But then the resolution unifier must also ground that variable, which contradicts the assumption. Finally, if \( \tilde{Q} \) is non-empty and
has a head \( Q \), all its IN-parameters must be ground: if the corresponding parameter in the clause \( Cl \) is a variable, it must be an IN-variable, therefore it must occur as an IN-variable in the head or as an OUT-variable in a preceding body atom. In the former case, the corresponding parameter in \( S \) (which originates from the head of \( Cl \)) is ground, and thus the parameter in \( Q \) is also ground. In the latter case, it the corresponding OUT-parameter in the preceding \( Q_0 \) is either ground or grounded by the resolution unifier, as established before. Either way, the parameter in \( Q \) will therefore also be ground. 

\[ \text{Lemma A.12. (node invariant)} \]

We write \( \bigcup \text{Ans} \) as short hand for \( \bigcup_{P \in \text{dom(Ans)}} \text{Ans}(P) \). If \( (\text{Nodes, Ans, Wait}) \) is reachable from some initial state and \( (P; \tilde{Q}; c; S; \tilde{n}d; Cl) \in \text{Nodes} \) with \( Cl \equiv R \leftarrow \tilde{R}, d \) ∈ \( P \), then:

1. \( S \preceq P \);
2. \( Cl \in P \);
3. \( \tilde{n}d \subseteq \bigcup \text{Ans} \);
4. there is some \( \theta \) such that \( R\theta = S \), and \( \tilde{R}\theta = \tilde{Q} \odot \tilde{Q} \) (where \( \tilde{Q} \) are the answers in \( \tilde{n}d \)), and \( d\theta \) is equivalent to \( c \).

\[ \text{Proof.} \] By induction on the transition rules. The statements follow directly from the definition of the transition rules and from the definition of resolution. \[ \square \]

\[ \text{Lemma A.13. (soundness)} \]

If \( (\text{Nodes, Ans, Wait}) \) is reachable from an initial state \( S_0 \) then \( \text{Ans} \) is sound.

\[ \text{Proof.} \] By induction on transition rules. The statement holds by definition for \( S_0 \).

Now assume the state is not an initial state, and let \( \text{Ans}' \) be the answer table of the preceding state. For \( \text{PropagateAnswer} \), we only have to consider the new answer \( nd \equiv (P; []; \text{True}; S; \tilde{n}d; Cl) \). By Lemma A.12, \( S \preceq P \); furthermore, \( Cl \equiv R \leftarrow \tilde{R}, d \in P \), and there exists \( \theta \) such that \( R\theta = S \) and \( d\theta = \text{True} \). Also, \( \tilde{R}\theta \) is equal to the set of answers in \( \tilde{n}d \) which in turn is a subset of \( \bigcup \text{Ans}' \). So by the inductive hypothesis, \( \tilde{R}\theta \subseteq T_p'(\emptyset) \). Therefore, by definition of \( T_p', S \in T_p'(T_p'(\emptyset)) = T_p'(\emptyset) \), as required.

For the other transition rules the statement trivially holds by the inductive hypothesis. \[ \square \]

\[ \text{Lemma A.14. (table monotonicity)} \]

If \( S \equiv (\text{Nodes, Ans, Wait}) \) is reachable from an initial state, and \( S' \equiv (\text{Nodes}', \text{Ans}', \text{Wait}') \) is reachable from \( S \), then \( \text{dom(Ans)} \subseteq \text{dom(Ans')} \), \( \text{dom(Wait)} \subseteq \text{dom(Wait')} \).
and \( \text{dom}(\text{Wait}) \subseteq \text{dom}(\text{Ans}) \). For all \( P \in \text{dom}(\text{Ans}) \): \( \text{Ans}(P) \subseteq \text{Ans}'(P) \). For all \( P \in \text{dom}(\text{Wait}) \): \( \text{Wait}(P) \subseteq \text{Wait}'(P) \).

Proof. By induction on the transition rules. The statements follow from the observation that \text{PropagateAnswer} and \text{RecycleAnswers} only increase \( \text{Ans}(P) \) and \( \text{Wait}(P) \), respectively; \text{SpawnRoot} always increases the domains of \( \text{Ans} \) and \( \text{Wait} \); and \text{ResolveClause} leaves \( \text{Wait} \) unchanged and either increases the domain of \( \text{Ans} \) (that can only happen if in the very first transition from the initial state) or leaves \( \text{Ans} \) unchanged.

\[ \square \]

Lemma A.15. (completeness) If \( S_f \equiv (\text{Nodes}, \text{Ans}, \text{Wait}) \) is a final state reachable from an initial state \( S_0 \) then \( \text{Ans} \) is complete.

Proof. We have to show that for any predicates \( P, S \) such that \( S \in T^P_n(\emptyset) \), \( P \in \text{dom}(\text{Ans}) \) and \( S \preceq P \), \( S \) is the answer of an answer node in \( \text{Ans}(P) \). For any such \( S \), there is an integer \( n \) such that \( S \in T^P_n(\emptyset) \). We prove the lemma by induction on \( n \). The statement vacuously holds for \( n = 0 \). For \( n > 0 \), if \( P \) is already in the domain of \( S_0 \)'s answer table the statement holds by definition of initial state and by monotonicity of the transition rules with respect to the answer table (Lemma A.14). So now assume that \( P \) was added to the domain as a result of a \text{SpawnRoot} transition.

By definition of \( T^P_n \), there exists a clause \( Cl \equiv R \leftarrow R_1, \ldots, R_n, c \in P \) and a substitution \( \theta \) such that \( R\theta = S \), \( R_i\theta \in T^P_{n-1}(\emptyset) \), and \( c\theta \) is valid. By the inductive hypothesis, for all \( R'_i \in \text{dom}(\text{Ans}) \) such that \( R_i\theta \preceq R'_i \) there is an answer node \( nd'_i \) in \( \text{Ans}(R'_i) \) with answer \( R_i\theta \).

Let \( P' \) be a fresh renaming of \( P \), \( \theta_0 = \text{mgu}(R, P') \), and for \( i = 1..n \), let

\[ \theta_i = \theta_{i-1}\text{mgu}(R,R_i,R_i\theta_i). \]

Furthermore, let

\[ nd_i \equiv \langle P; [R_{i+1}\theta_i, \ldots, R_n\theta_i]; c\theta; P'\theta_i; [nd'_1, \ldots, nd'_i]; Cl \rangle \]

for \( i = 0..n \). Note that \( nd_n \) is an answer node with answer \( S \), by Lemma A.11. We will now show that \( nd_n \in \text{Ans}(P) \).

After \( P \) is added to the domain of the answer table in the \text{SpawnRoot} transition, there will eventually
be a ResolveClause transition producing a new set of nodes that contains \( nd_0 \), because \( \langle P; [R, R_1, ..., R_n]; c; R; []; Cl \rangle \) and \( P \) are resolvable with resolution unifier \( \theta_0 \).

Suppose for some \( i = 0..n - 1 \) that \( nd_i \) gets produced as a node along a path leading from \( S_0 \) to \( S_f \). Then there must be a later RecycleAnswers or a SpawnRoot transition where \( nd_i \) is added to the wait table for some \( R_{i+1}' \) where \( R_{i+1} \theta_i \not\prec R_{i+1}' \). Since \( \theta_i \) is no less general than \( \theta \), we also have \( R_{i+1} \theta \not\prec R_{i+1}' \), so \( R_{i+1} \theta \) is the answer of some answer node in \( Ans(R_{i+1}') \), by the inductive hypothesis. Therefore, this answer node is resolved with \( nd_i \) either in a PropagateAnswer or a RecycleAnswers transition, and the set of nodes produced by this transition contains \( nd_{i+1} \).

Therefore, along all paths leading from \( S_0 \) to \( S_f \) the nodes \( nd_0, ..., nd_n \) are produced. Therefore, \( nd_n \) is eventually added to the answer table for \( P \) in a PropagateAnswer transition, and hence it is in \( Ans(P) \) (by Lemma A.14), as required.

**Lemma A.16. (termination and complexity)** All transition paths starting from an initial state are of finite length, and the path lengths are polynomial in the number of facts (i.e., clauses with empty body) in \( \mathcal{P} \).

**Proof.** Let \( S \) be the set of predicate names that occur in the body of a clause in \( \mathcal{P} \), and let \( C \) be the number of constants in \( \mathcal{P} \) that occur as a parameter of a predicate in \( S \). Further, let \( N \) be the number of clauses in \( \mathcal{P} \), \( M \) the maximum number of distinct variables in the head of any clause in \( \mathcal{P} \), and \( V \) the maximum number of distinct variables occurring in the body of any clause in \( \mathcal{P} \). When a root node \( \langle P \rangle \) is produced in a SpawnRoot transition, \( P \) is permanently added to the domain of the answer table. Due to the side conditions of SpawnRoot, such a node is only produced if there is no \( P' \) in the domain of the answer table for which \( P \not\prec P' \). Moreover, the only predicate names and constants that can occur in a path are the ones that also occur in \( \mathcal{P} \), of which there are only finitely many. Therefore, the number of SpawnRoot transitions is bounded by \( C^M \), and thus the number of ResolveClause transitions is bounded by \( C^M N \) which is also an upper bound on the number of nodes produced by ResolveClause. PropagateAnswer and RecycleAnswers both replace a node with a number of nodes whose subgoal lists are strictly shorter until an answer node is produced. From any given node, these two transition rules together produce no more than \( C^V \) new nodes. Thus the number of nodes produced by them is bounded by \( C^{M+V} N \).

It follows that the length of any path is bounded by \( 4C^{M+V} N \). Hence all path lengths are polynomial.
in the number of facts in $P$ as $C$ is proportional to this number. \hfill \square 

**Theorem A.17.** All paths from an initial state $((P_i),\ldots)$ terminate at a final state. The answer table of any such final state is sound and complete, and its domain contains $P$.

**Proof.** This follows immediately from Lemmas A.16, A.13 and A.15. \hfill \square 

**Restatement of Theorem 8.4. (soundness, completeness, termination)** Let $Ans$ be a sound and complete answer table, $P$ an IN/OUT-safe program and $P$ an IN/OUT-safe query. Then $Answers_P(P, Ans)$ is defined, finite and equal to $\{ \theta : P^\theta \in T^\infty_2(\theta), dom(\theta) \subseteq vars(P) \}$.

**Proof.** This follows directly from Theorem A.17, noting that the pseudocode in Figure 1 is a deterministic implementation of the labelled transition system. \hfill \square 

**Restatement of Theorem 8.5.** For all safe assertion contexts $AC$ and safe authorization queries $q$,

1. $AuthAns_{AC}(q)$ is defined and finite, and

2. $AC, \theta \vdash q$ iff $\theta \in AuthAns_{AC}(q)$.

**Proof.** By induction on the structure of $q$.

**Case** $q \equiv e \text{ says } fact$

1. By authorization query safety, $fact$ is flat, hence all parameters in $e \text{ says } fact$ can be assigned OUT as in the proof of Lemma 8.2, hence $q$ is an IN/OUT-safe Datalog query. Therefore, $AuthAns_{AC}(q)$ is defined and finite by Theorem 8.4.

2. Assume $AC, \theta \vdash e \text{ says } fact$. This holds iff $AC, \infty \models e\theta \text{ says } fact\theta$, by definition of $\vdash$. The translated program $P$ is IN/OUT-safe, by Lemma 8.2, and we have already established above that the query $e \text{ says } fact$ is also IN/OUT-safe. So by Theorems 7.4 and 8.4, this holds iff $(e\theta \text{ says } fact\theta) \in Answers_P(e \text{ says } fact)$. Since $dom(\theta) \subseteq vars(e \text{ says } fact)$ (by Lemma A.1) and $vars(e\theta \text{ says } fact\theta) = \emptyset$, this holds iff the most general unifier of the two is $\emptyset$, and iff $\theta \in AuthAns_{AC}(e \text{ says } fact)$.

**Case** $q \equiv q_1, q_2$
1. If \( q \) is safe, then \( q_1 \) must also be safe, so by the induction hypothesis for finiteness, \( \text{AuthAns}_{\mathcal{AC}}(q_1) \) is defined and finite. By the induction hypothesis for soundness, \( \theta_1 \in \text{AuthAns}_{\mathcal{AC}}(q_1) \) implies \( \mathcal{AC}, \theta_1 \models q_1 \). It follows from Lemma A.6 that \( q_2 \theta_1 \) is safe, so by the induction hypothesis for finiteness, \( \text{AuthAns}_{\mathcal{AC}}(q_2 \theta_1) \) is defined and finite, and hence \( \text{AuthAns}_{\mathcal{AC}}(q) \) is defined and finite.

2. Assume \( \mathcal{AC}, \theta \models q_1, q_2 \). This holds iff \( \theta = \theta_1 \theta_2 \) such that \( \mathcal{AC}, \theta_1 \models q_1 \) and \( \mathcal{AC}, \theta_2 \models q_2 \theta_1 \). By the induction hypothesis, this holds iff \( \theta_1 \in \text{AuthAns}_{\mathcal{AC}}(q_1) \) and \( \theta_2 \in \text{AuthAns}_{\mathcal{AC}}(q_2 \theta_1) \) and hence \( \theta \in \text{AuthAns}_{\mathcal{AC}}(q) \).

Case \( q \equiv q_1 \) or \( q_2 \) The statements follow directly from the induction hypotheses.

Case \( q \equiv \text{not}(q_0) \)

1. By definition of authorization query safety, \( q_0 \) must also be safe, hence \( \text{AuthAns}_{\mathcal{AC}}(q) \) is defined and finite.

2. Assume \( \mathcal{AC}, \theta \models \text{not}(q_0) \). By definition of \( \models \), \( q_0 \) must be ground. Lemma A.1 implies that \( \theta = \varepsilon \), therefore \( \mathcal{AC}, \varepsilon \not\models q_0 \). In other words, there exists no \( \sigma \) such that \( \mathcal{AC}, \sigma \not\models q_0 \). By the induction hypothesis, this holds iff \( \text{AuthAns}_{\mathcal{AC}}(q_0) = \emptyset \) and hence \( \text{AuthAns}_{\mathcal{AC}}(\text{not}(q_0)) = \{ \varepsilon \} \).

Case \( q \equiv c \)

1. By definition of authorization query safety, \( \text{vars}(c) \subseteq \emptyset \), hence \( \text{AuthAns}_{\mathcal{AC}}(q) \) is defined and finite.

2. This is similar to the previous case.

Case \( q \equiv \exists x(q_0) \)

1. If \( q \) is safe, then \( q_0 \) is also safe, by definition of authorization query safety. By the inductive hypothesis, \( \text{AuthAns}_{\mathcal{AC}}(q_0) \) is defined and finite, hence \( \text{AuthAns}_{\mathcal{AC}}(q) \) is also defined and finite.

2. This is straightforward by the inductive hypothesis and the definition of \( \models \). \( \square \)

**Restatement of Theorem 8.6.** Let \( M \) be the number of flat atomic assertions (i.e., those without conditional facts) in a safe assertion context \( \mathcal{AC} \) and let \( N \) be the maximum length of constants occurring in these assertions. The time complexity of computing \( \text{AuthAns}_{\mathcal{AC}} \) is polynomial in \( M \) and \( N \).
Proof. The number of clauses with empty body in the translated Datalog program is proportional to $M$. The constants stay unchanged, so the maximum length of any constant occurring in the set of those clauses is $N$. From Lemma A.16 and the fact that each step in the labelled transition system can be computed in time polynomial with respect to $N$ (as the validity of a ground constraint can be checked in polynomial time), we get that $\text{Answers}_P$ is polynomial-time computable with respect to $M$ and $N$. The time complexity of $\text{AuthAns}_{AC}$ for a fixed query is clearly polynomial in the computation time for $\text{Answers}_P$, hence it is also polynomial in $M$ and $N$.  \qed
References


