Increasing the Expressivity of Conditional Functional Dependencies without Extra Complexity

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Abstract—The paper proposes an extension of CFDs [1], referred to as extended Conditional Functional Dependencies (eCFDs). In contrast to CFDs, eCFDs specify patterns of semantically related values in terms of disjunction and inequality, and are capable of catching inconsistencies that arise in practice but cannot be detected by CFDs. The increase in expressive power does not incur extra complexity: we show that the satisfiability and implication analyses of eCFDs remain NP-complete and coNP-complete, respectively, the same as their CFDs counterparts. In light of the intractability, we present an algorithm that approximates the maximum number of eCFDs that are satisfiable. In addition, we revise SQL techniques for detecting CFD violations, and show that violations of multiple eCFDs can be captured via a single pair of SQL queries. We also introduce an incremental SQL technique for detecting eCFD violations in response to database updates. We experimentally verify the effectiveness and efficiency of our SQL-based detection methods.

I. INTRODUCTION

Conditional functional dependencies (CFDs) have recently been introduced in [1] for data cleaning. CFDs extend functional dependencies (FDs) by enforcing patterns of semantically related values, and have proved more effective in catching data inconsistencies than FDs, which were currently the basis of many data-cleaning tools [2], [3], [4], [5], [6].

To capture inconsistencies that commonly arise in real-life data, however, one often needs to use more expressive constraints, as illustrated by the following example.

Example 1.1: Let us consider a schema similar to the one used in [1]: cust(AC, PN, NM, STR, CT, ZIP). It specifies a customer in New York State in terms of the customer's phone number (AC), phone number (PN), name (NM), and address (street (STR), city (CT), zip code (ZIP)). An instance $D_0$ of cust is shown in Fig. 1.

One may want to specify the CFD below on cust:

$$\phi_1: \ (CT \rightarrow AC, \ \{(Albany \mid 518), (Troy \mid 518), (Colonie \mid 518)\})$$

The CFD is a pair consisting of an embedded standard FD and a pattern tableau. It states that the FD CT $\rightarrow$ AC (city uniquely determines area code) holds if CT is Albany, Troy or Colonie; and in addition, the pattern tableau refines the FD by enforcing bindings between cities and area codes: if CT is one of these cities, then AC must be 518. This CFD identifies tuple $t_1$ in Fig. 1 as an error: CT is Albany but AC is not 518. This error cannot be caught by traditional FDs.

A cursory examination of New York area codes reveals that most cities in the state have a unique area code, except NYC and LI (Long Island). Such situations commonly arise in practice, and it is useful to capture this as constraints when checking inconsistencies of the data. Unfortunately, it cannot be defined as a standard FD or even a CFD.

This, however, can be expressed as the constraint below:

$$\phi_2: \ CT \not\in \{NYC, LI\} \rightarrow AC$$

which assures that the FD CT $\rightarrow$ AC holds if CT is not in the set \{NYC, LI\}, instead of on the entire cust database.

For NYC, one can write the following constraint:

$$\phi_3: \ CT \in \{NYC\} \rightarrow AC$$

This asserts that when CT is NYC, AC must be either 212, 718, 646, 347, 917. That is, here CT is associated with a disjunction of options rather than with a single value, and chooses one from the multiple choices. Again this is common in practice. With $\phi_3$ we can identify tuple $t_4$ of Fig. 1 as an error: 100 is not one of the area codes associated with NYC. Similarly one can specify the area codes for LI.

However, these constraints cannot be defined as CFDs: they are specified with inequality ($\phi_2$) and disjunction ($\phi_3$), which are beyond the expressive power of CFDs.

A more powerful language is necessary to express these constraints. However, increased expressive power often comes with the extra charge for complexity.

Contribution. The first contribution of the paper is a CFD extension, well-balanced between expressiveness and complexity, referred to extended Conditional Functional Dependencies (eCFDs). eCFDs support disjunction and inequality, and can

<table>
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<tr>
<th>ID</th>
<th>AC</th>
<th>PN</th>
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<th>ZIP</th>
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<tbody>
<tr>
<td>t1</td>
<td>718</td>
<td>111111</td>
<td>Mike</td>
<td>Tree Ave.</td>
<td>Albany</td>
<td>12328</td>
</tr>
<tr>
<td>t2</td>
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<td>2222222</td>
<td>Joe</td>
<td>Elm St.</td>
<td>Colonie</td>
<td>12205</td>
</tr>
<tr>
<td>t3</td>
<td>518</td>
<td>2222222</td>
<td>Jim</td>
<td>Oak Ave.</td>
<td>Troy</td>
<td>12181</td>
</tr>
<tr>
<td>t4</td>
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<td>1111111</td>
<td>Rick</td>
<td>8th Ave.</td>
<td>NYC</td>
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<tr>
<td>t5</td>
<td>212</td>
<td>3333333</td>
<td>Ben</td>
<td>5th Ave.</td>
<td>NYC</td>
<td>10016</td>
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<tr>
<td>t6</td>
<td>645</td>
<td>4444444</td>
<td>Ian</td>
<td>High St.</td>
<td>NYC</td>
<td>10011</td>
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Fig. 1. Instance $D_0$ of the cust relation
express all the constraints we have encountered so far. They include CFDs as a special case, and can catch inconsistencies, as violations of CFDs, that commonly arise in real-life data but cannot be detected by CFDs.

Our second contribution consists of complexity bounds of two central technical problems associated with CFDs: the satisfiability problem is to determine whether or not an input set of CFDs makes sense, i.e., whether there exists a nonempty database that satisfies the CFDs; and the implication problem is to determine whether or not an CFD is entailed by a given set of CFDs. These are important in validation and optimization of data-cleaning processes in practice. We show that despite the increased expressive power, CFDs do not make our lives harder: for CFDs these problems remain NP-complete and coNP-complete, respectively, the same as their CFD counterparts.

Our third contribution is an approximation-preserving reduction from the maximum satisfiability problem of CFDs to the maximum generalized satisfiability problem (MAXSAT) [7]. As a result we can apply existing approximation algorithms for MAXSAT to find an approximation of the largest possible subset of satisfiable CFDs in a given input set \( \Sigma \). This not only allows to efficiently detect the unsatisfiability of \( \Sigma \) with some certainty, but also provides the user with a set of satisfiable CFDs that can serve as a starting point to inspect the remaining CFDs in \( \Sigma \).

Our fourth contribution consists of two algorithms for detecting data inconsistencies. The first one is a batch algorithm that, given a dataset \( D \) and a set \( \Sigma \) of CFDs, finds all tuples that violate some CFDs in \( \Sigma \). We rewrite the SQL detection technique developed in [1] and show that a single pair of SQL queries suffices to find all violations of \( \Sigma \) despite that \( \Sigma \) may consist of multiple CFDs. The second one is a novel incremental algorithm: in response to updates \( \Delta D \) to the database \( D \), the algorithm efficiently finds all violations of \( \Sigma \) in the updated dataset \( D \oplus \Delta D \), by updating violations of \( \Sigma \) in \( D \) and minimizing unnecessary recomputation. This again uses SQL queries only.

Our fifth contribution is an experimental study of the performance of our batch and incremental detection methods. We find that our methods scale well when the data size gets large and CFDs (pattern tuples in the CFDs) get complicated; in addition, the incremental method outperforms the batch one in response to reasonably-sized database updates.

**Organization.** Section II formally defines CFDs. The satisfiability and implication problems of CFDs are studied in Section III, followed by our approximation analysis of the satisfiability problem in Section IV. Next, Section V presents the batch and incremental detection techniques, followed by their experimental study in Section VI. Related work is discussed in Section VII and Section VIII presents some concluding remarks.

\[ \psi_1 = \text{cust:} [\text{CT}] \to [\text{AC}], \; \emptyset, \; T_1 \}, \text{ where the pattern tableau } T_1 \text{ is} \]

<table>
<thead>
<tr>
<th>CT</th>
<th>AC</th>
<th>( \emptyset )</th>
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<tr>
<td>{NYC, LI}</td>
<td></td>
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<tr>
<td>{Albany, Troy, Colonie}</td>
<td>{518}</td>
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\[ \psi_2 = \text{cust:} [\text{CT}] \to \emptyset, \; \text{AC}, \; T_2 \}, \text{ where the pattern tableau } T_2 \text{ is} \]

<table>
<thead>
<tr>
<th>CT</th>
<th>( \emptyset )</th>
<th>AC</th>
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<tbody>
<tr>
<td>{NYC}</td>
<td>212, 718, 646, 347, 917</td>
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**II. EXTENDED CFDs**

We now define extended Conditional Functional Dependencies (CFD). Consider a relation schema \( R \) defined over a finite set of attributes, denoted by \( \text{attr}(R) \). For each \( A \) in \( \text{attr}(R) \) we denote by \( \text{dom}(A) \) the domain of attribute \( A \), which can be infinite or finite (with at least two elements).

**Syntax.** An CFD \( \varphi \) is a triple \( (R : X \to Y, \; Y, \; T_p) \), where (1) \( X, Y, Y_p \subseteq \text{attr}(R) \); and \( Y \cap Y_p = \emptyset \); (2) \( X \to Y \) is a standard FD, referred to as the embedded functional dependency of \( \varphi \); and (3) \( T_p \) is a pattern tableau consisting of a finite number of pattern tuples over the attributes in \( X \cup Y \cup Y_p \), such that for any tuple \( t_p \in T_p \) and for each attribute \( A \) in \( X \cup Y \cup Y_p \), \( t_p[A] \) is either an unnamed variable \( 't' \), a set \( S \) or a complement set \( \bar{S} \), where \( S \) is a finite subset of \( \text{dom}(A) \). If \( A \) appears in both \( X \) and \( Y \cup Y_p \), we use \( t_p[A] \) and \( t_p[A] \) to indicate the field of \( t_p \) corresponding to \( A \) in \( X \) and \( Y \cup Y_p \), respectively. We denote \( X \) by \( \text{LHS}(\varphi) \) and \( Y \cup Y_p \) by \( \text{RHS}(\varphi) \).

**Example 2.1:** Constraints \( \phi_1 = \phi_2 \) of Example 1.1 can be expressed as the CFDs shown in Fig. 2. In \( \psi_1, X = \{\text{CT}\} \), \( Y = \{\text{AC}\} \) and \( Y_p = \emptyset \). In \( \psi_2, X = \{\text{CT}\}, Y = \emptyset \) and \( Y_p = \{\text{AC}\} \). Here \( \psi_1 \) expresses \( \phi_1 \) and \( \phi_2 \), while \( \psi_2 \) represents \( \phi_3 \). We use \( \emptyset \) to separate \( X, Y \) and \( Y_p \).

Observe that each pattern tuple is actually a constraint that enforces binding of semantically related values, and is referred to as a pattern constraint in the CFD.

**Semantics.** Let us consider \( Z \subseteq X \cup Y \cup Y_p \), a tuple \( t \) in an instance \( I \) of \( R \), and a pattern tuple \( t_p \). We say that the data tuple \( t[Z] \) matches the pattern tuple \( t_p[Z] \), denoted by \( t[Z] \prec t_p[Z] \), if for each \( A \in Z \), (1) if \( t_p[A] = 't' \), then \( t[A] \in \text{dom}(A) \), i.e., an arbitrary value; (2) if \( t_p[A] = S \), then \( t[A] \in S \); and (3) if \( t_p[A] = \bar{S} \), then \( t[A] \not\in S \).

For example, consider \( t_1, t_2 \) of Fig. 1 and the first pattern tuple \( t_p \) of \( \psi_1 \) in Fig. 2. Then \( t_1[\text{CT, AC}] = t_p[\text{CT, AC}] \) since \( t_1[\text{CT}] \not\in \{\text{NYC, LI}\} \) and \( t_1[\text{AC}] \not= 't' \). However, \( t_1[\text{CT, AC}] \not\in t_2[\text{CT, AC}] \) since \( t_2[\text{CT}] \in \{\text{NYC, LI}\} \).

A relation \( I \) of \( R \) satisfies CFD \( \varphi \), denoted by \( I \models \varphi \), if for each pattern tuple \( t_p \), \( t_p \) in \( T_p \), the following holds. Let \( I(t_p) = \{ t \in I \mid t[Z] \prec t_p[Z] \} \), which is the set of tuples \( t \) in \( I \) such that \( t[Z] \) matches \( t_p[Z] \). Then (1) \( I(t_p) \) must satisfy the embedded FD \( X \to Y \), i.e., for any two tuples \( t_1, t_2 \) in
I(\{t_p\}), if t_1[X] = t_2[X], then t_1[Y] = t_2[Y]; and furthermore, (2) for each tuple t \in I(\{t_p\}), \{t[Y, Y_p]\} \times t_p[Y, Y_p], i.e., all tuples t in I(\{t_p\}) must match the pattern t_p[Y, Y_p].

Intuitively, I(\{t_p\}) identifies the set of tuples on which the constraint \(t_p\) is defined, i.e., the constraint only applies to the tuples in I that match the pattern \(t_p[X]\). Both, the embedded FD \(X \rightarrow Y\) and the pattern \(t_p[Y, Y_p]\), are enforced on the tuples in I(\{t_p\}).

**Example 2.2:** Consider the database \(D_0\) of Fig. 1 and the first pattern tuple \(t_p\) in \(\psi_1\). Here \(D_0(\{t_p\}) = \{t_1, t_2, t_3\}\), i.e., the tuples whose CT attribute is neither NYC nor LI. In other words, the constraint specified by \(t_p\) does not apply to the entire \(D_0\); it holds conditionally on \(D_0\), i.e., only on \(D_0(\{t_p\})\).

The database \(D_0\) does not satisfy \(t_p\) or \(\psi_0\). Even though \(t_1[\text{CT}] \neq t_p[\text{CT}]\) and \(t_1\) does not violate the FD \(\text{CT} \rightarrow \text{AC}\), where \(t_p^{\prime}\) is the second pattern tuple of \(\psi_1\), \(t_1\) violates \(\psi_1\) since \(t_1[\text{AC}] = t_p[\text{AC}]\). The tuple \(t_3\) violates \(\psi_2\) since \(t_1[\text{AC}] \neq t_p[\text{AC}]\) although \(t_1[\text{CT}] = t_p[\text{CT}]\), where \(t_p^{\prime}\) is the pattern tuple of \(\psi_2\). This tells us that a single tuple may violate an eCFD while it takes two tuples to violate a standard FD.

An instance I of R satisfies a set \(\Sigma\) of eCFDs, denoted by I = |\(\Sigma\), if I = \(\varphi\) for each \(\varphi \in \Sigma\).

**Remarks.** (1) eCFDs support inequality (\(\rangle\), e.g., the first pattern tuple of \(\psi_1\)) and disjunction (\(\lor\), e.g., \(\psi_2\)) in which the area code for NYC is specified as either 212, 718, 646, 347 or 917. (2) Conditional functional dependencies (CFDs) introduced in [1] are a special case of eCFDs. Recall that a CFD is of the form \((R: X \rightarrow Y, T_p)\), in which each pattern tuple consists of either = or \(\langle\rangle\) or a single constant value. Hence, a CFD can be written as an eCFD \(\varphi = (R: X \rightarrow Y, \emptyset, T_p^{\prime})\), where \(T_p^{\prime}\) is identical to \(T_p\) except that each constant \(a\) in \(T_p\) is replaced with \(\{\alpha\}\) in \(T_p^{\prime}\). That is, a CFD is an eCFD with neither inequality nor disjunction. Since CFDs extend standard FDs, so do eCFDs.

**III. SATISFIABILITY AND IMPLICATION OF eCFDS**

In this section we investigate the satisfiability and implication analyses of eCFDs. These are classical decision problems associated with any constraint language.

The **satisfiability problem** for eCFDs is to decide, given a set \(\Sigma\) of eCFDs on a relation schema \(R\), whether or not there exists a nonempty instance I of R such that I = \(\Sigma\).

The **implication problem** for eCFDs is to determine, given a set \(\Sigma\) of eCFDs and another eCFD \(\varphi\) defined on the same relation schema \(R\), whether or not I = \(\Sigma\) \(\leftarrow \varphi\), i.e., whether for every instance I of R, if I = \(\Sigma\) then also I = \(\varphi\).

The main result of this section is that despite the increased expressive power of eCFDs, they retain the same complexity bounds for these static analyses as CFDs.

**Satisfiability.** As shown in [1], CFDs may not be satisfiable. It is thus not surprising that the same holds for eCFDs.

**Example 3.1:** Consider an eCFD \(\psi_3\) on cust databases: \{cust: [CT] \rightarrow [CT], \emptyset, \{NYC\} \rightleftharpoons \{NYC\} \rightleftharpoons \emptyset, \{NYC\} \rightleftharpoons \{LI\} \rightleftharpoons \emptyset\}). This eCFD is not satisfiable. Indeed, for any cust instance I and any tuple t in I, if \([CT] = \text{NYC}\), then \(\psi_3\) requires it to be \(\text{LI}\); but \(\psi_3\) forces it to be \(\text{NYC}\) again.

This highlights the need for the satisfiability analysis of eCFDs; it is necessary to determine whether or not the given eCFDs are not dirty themselves before one uses the eCFDs to detect inconsistencies in a database, which is typically much larger than the set of constraints.

It is known that the satisfiability problem for CFDs is NP-complete [1]. The result below tells us that eCFDs do not make the satisfiability analysis more complicated.

**Proposition 3.1:** The satisfiability problem for eCFDs is NP-complete.

**Proof:** The lower bound follows from the NP hardness of the CFD counterpart, since CFDs are a special case of eCFDs. For the upper bound, the satisfiability problem has the following small model property: For any given set \(\Sigma\) of eCFDs, if \(\Sigma\) is satisfiable, then there exists a database consisting of a single tuple that satisfies \(\Sigma\). Thus an NP algorithm for checking eCFD satisfiability (i) guesses a database I with a single tuple and (ii) then checks whether I = \(\Sigma\); the latter can be done in PTIME.

**Implication.** Due to the presence of pattern tuples in eCFDs, one expects the number of eCFDs to be larger than their FD counterparts. A natural optimization strategy for cleaning data with eCFDs is by removing redundancies in a given set of eCFDs, i.e., by removing eCFDs and pattern tuples that are entailed by other eCFDs. This calls for the implication analysis of eCFDs.

The implication problem is coNP-complete for CFDs [1].

The complexity remains unchanged for eCFDs:

**Proposition 3.2:** The implication problem for eCFDs is coNP-complete.

**Proof:** The coNP-hardness follows from the lower bound for CFD implication. For the upper bound, the complement of the implication problem has the following small model property: For any given set \(\Sigma\) of eCFDs and a single eCFD \(\varphi\) on a schema R, \(\Sigma \not\vDash \varphi\) iff there exists an instance I of R such that I = \(\Sigma\) but I \(\not\vDash \varphi\), and moreover, I consists of at most two tuples. Thus an NP algorithm for checking the complement of eCFD implication (i) guesses a database I with two tuples and (ii) checks whether I = \(\Sigma\) and I \(\not\vDash \varphi\). The latter can be done in PTIME.

**Special case.** A tractable special case is identified in [1]: if the given CFDs involve no attributes that have a finite domain, then the satisfiability and implication analyses are in PTIME. This is not longer the case for eCFDs, since we can enforce, via eCFDs, an attribute A to draw values from a finite set only, no matter whether \(\text{dom}(A)\) is infinite or not.

**Proposition 3.3:** The satisfiability and implication problems for eCFDs remain NP-complete and coNP-complete, respectively, in the absence of finite-domain attributes.
Proof: For the satisfiability problem, the NP-hardness is by reduction from CFD satisfiability with finite-domain attributes. Given any CFDs $\Sigma$ on schema $R$, we define another schema $R'$ with only infinite-domain attributes: for each $A \in \text{attr}(R)$ with a finite domain we replace $A$ with $A'$ of an infinite domain; in addition, we define $\Sigma'$ to be the set consisting of CFDs in $\Sigma$ and an eCFD $\psi_A = (R': [A'] \rightarrow \emptyset, A', \{(\neg \bot \| \text{dom}(A))\})$ for each such attribute $A$. The eCFD $\psi_A$ forces $A'$ to take value only from dom$(A)$. Clearly, $\Sigma'$ is satisfiable iff $\Sigma$ is. Similarly we can show the coNP-hardness of the implication of eCFDs with infinite-domain attributes only.

IV. APPROXIMATION ALGORITHM FOR SATISFIABILITY

In light of the intractability of the eCFD satisfiability analysis, it is beyond reach to find an efficient algorithm that, given a set $\Sigma$ of eCFDs, returns true iff $\Sigma$ is satisfiable. This motivates us to consider the maximum satisfiable subset problem (MAXSS): given a set $\Sigma$ of eCFDs, it is to find a maximum subset of $\Sigma$ that is satisfiable. Although this problem is also intractable, we develop an approximation factor preserving reduction to a well-studied NP-complete problem, called Maximum Generalized Satisfiability (MAXGSAT), for which a number of approximation algorithms are already in place (see, e.g., [7]). The MAXSAT problem is, given a set $\Phi = \{\phi_1, \ldots, \phi_n\}$ of Boolean expressions, to find a truth assignment that satisfies the maximum number of expressions in $\Phi$.

Given this reduction and an $\epsilon$-approximation algorithm for MAXGSAT, we obtain an $\epsilon$-approximation algorithm for MAXSS that, given $\Sigma$, finds a satisfiable subset $\Sigma_m$ of $\Sigma$ such that $\text{card}(\Sigma_m) \geq (1 - \epsilon) \cdot \text{card}(\text{OPT}_{\text{maxss}}(\Sigma))$, where $\text{OPT}_{\text{maxss}}(\Sigma)$ denotes a maximum satisfiable subset of $\Sigma$, and $\text{card}(S)$ denotes the cardinality of $S$.

Clearly, if $|\Sigma_m| = |\Sigma|$ then we may conclude that $\Sigma$ is satisfiable. Moreover, if $|\Sigma_m| < (1 - \epsilon) |\Sigma|$ then $\Sigma$ is unsatisfiable. Therefore, only when $|\Sigma| > |\Sigma_m| \geq (1 - \epsilon) |\Sigma|$ we cannot conclude whether $\Sigma$ is satisfiable or not.

Approximation factor preserving reduction. An approximation factor preserving reduction from MAXSS to MAXGSAT consists of two PTIME functions $f$ and $g$ such that for any set $\Sigma$ of eCFDs,

1) $f(\Sigma)$ is an MAXGSAT instance $\Phi_\Sigma$, and $g(\Phi_m)$ is a satisfiable set in $\Sigma$ if $\Phi_m$ is a set of satisfied formulas in $\Phi_\Sigma$;
2) $\text{card}(\text{OPT}_{\text{maxgsat}}(f(\Sigma))) \geq \text{card}(\text{OPT}_{\text{maxss}}(\Sigma))$;
3) $\text{card}(g(\Phi_m)) \geq \text{card}(\Phi_m)$, where $\text{OPT}_{\text{maxgsat}}(f(\Sigma))$ is the maximum set of satisfied expressions in $\Phi_m$. Thus $g$ is guaranteed to return a feasible MAXSS solution for $\Sigma$.

Such a reduction ensures that MAXSS has an $\epsilon$-factor approximation algorithm if MAXGSAT has one. Indeed, if $\text{card}(\Phi_m) \geq (1 - \epsilon) \cdot \text{card}(\text{OPT}_{\text{maxgsat}}(f(\Sigma)))$, then from (2) and (3) above, it follows that $\text{card}(g(\Phi_m)) \geq (1 - \epsilon) \cdot \text{card}(\text{OPT}_{\text{maxss}}(\Sigma))$. In particular, one can verify that if $\Phi_m$ is the optimal solution of MAXGSAT for $f(\Sigma)$, then $g(\Phi_m)$ is the optimal solution of MAXSS for $\Sigma$.

Reduction. We give a reduction by leveraging the small model property shown in the proof of Proposition 3.1. We define an $\epsilon$-factor algorithm $t$ such that $\Sigma$ is satisfiable iff there is a valuation $\rho$ for variables in $t$ with $\{\rho(t)\} \models \Sigma$.

We first introduce variables used in the reduction. Consider a set $\Sigma$ of eCFDs defined on a relation schema $R$, where $\text{attr}(R) = \{A_1, \ldots, A_n\}$. For each $i \in [1, n]$, we define the active domain $\text{dom}(A_i)$ to be the set consisting of (1) all the constants appearing in $t_B[A_i]$ for some pattern tuple $t_B$ in $\Sigma$; (2) a value in $\text{dom}(A_i)$ that is not yet in $\text{dom}(A_i)$ if there exists any (if $\text{dom}(A_i)$ is a finite domain, there may not exist such a value). Let $k$ be the size of $\Sigma$. Then $\text{dom}(A_i)$ has at most $k+1$ values. For each $i \in [1, n]$ and $a \in \text{dom}(A_i)$, we introduce a Boolean variable $x(i, a)$ such that $x(i, a) = true$ if $t[A_i] = a$. We use the Boolean expression below to ensure that $t[A_i]$ has a unique value: $\forall_i \exists_a (x(i, a) \land x(i, a' \neg))$

Proof: First, functions $f$ and $g$ can be computed in PTIME in $k$ and $n$. Second, $\text{card}(\text{OPT}_{\text{maxgsat}}(f(\Sigma))) = \text{card}(\text{OPT}_{\text{maxss}}(\Sigma))$. Third, for any truth assignment $\rho$ for $\Phi_\Sigma$, if $\Phi_m$ is the set of formulas in $\Phi_\Sigma$ satisfied by $\rho$, then $\text{card}(\Phi_m) = \text{card}(g(\Phi_m))$. Taken together, the reduction is indeed approximation factor preserving.
V. Detecting eCFD Violations

In this section, we develop techniques for detecting violations in a database $D$ with respect to a given set $\Sigma$ of eCFDs. We consider static and dynamic settings, as stated as follows:

Given a database $D$ and a set $\Sigma$ of eCFDs, a batch detection algorithm is to find the violation set $\text{vio}(D)$ with respect to $\Sigma$, i.e., the set of all tuples in $D$ that violate some eCFDs in $\Sigma$.

Given $D$, $\Sigma$, the violation set $\text{vio}(D)$ of $D$ with respect to $\Sigma$, and updates $\Delta D$ to the database $D$, an incremental detection algorithm is to find the set $\Delta \text{vio}(D)$ such that $\Delta \text{vio}(D) \cup \text{vio}(D)$ is the violation set $\text{vio}(\Delta D \cup D)$ with respect to $\Sigma$, where $\Delta S \cup S$ denotes the set of updates $\Delta S$ to the set $S$. Here the updates $\Delta D$ can either be a set of tuple insertions or deletions, denoted by $\Delta D^+$ and $\Delta D^-$, respectively.

In Sections V-A and V-B, we develop a batch and an incremental detection algorithm, referred to as $\text{BATCHDETECT}$ and $\text{INCDetect}$, respectively. Both algorithms only generate SQL queries to detect violations. This is important since eCFD violation detection can then be directly implemented on top of RDBMS, and we can therefore benefit from existing optimization techniques of RDBMS. Better still, in both settings, only a fixed number of SQL queries are needed, no matter how many eCFDs are in $\Sigma$, how many pattern tuples are in the eCFDs, and how large the sets are in each pattern-tuple attribute. The key idea is to treat pattern tableaux in $\Sigma$ as data tables, rather than as meta-data.

Before we present our detection algorithms, we decide on a uniform way of representing the set of violations. Instead of simply returning the tuples in $D$ that violate some eCFD in $\Sigma$, we explicitly store whether a tuple in $D$ is a violation or not. More precisely, we extend the schema $R$ of $D$ with two Boolean attributes: $SV$ (for “Single tuple Violation”) and $MV$ (for “Multiple tuple Violation”). That is, $t.SV = 1$ if $t$ violates an eCFD in $\Sigma$ all by itself; and $t.SV = 0$ otherwise. Similarly, $t.MV = 1$ if $t$ violates an embedded FD for an eCFD in $\Sigma$; and $t.MV = 0$ otherwise. Hence, $t \in \text{vio}(D)$ if and only if $t.SV = 1$ or $t.MV = 1$.

A. A Batch Algorithm

We first consider the static case and outline algorithm $\text{BATCHDETECT}$. Here we extend the approach proposed in [1] and generate a pair of SQL queries for violation detection and corresponding update statements:

1) Query $Q_{sv}$ finds all single-tuple violations caused by eCFDs in $\Sigma$;
2) Query $Q_{mv}$ identifies multiple-tuple violations caused by a violation of an FD embedded in some eCFD in $\Sigma$.
3) Given these two queries, $\text{BATCHDETECT}$ performs update statements to $D$ and sets the $SV$ (resp. $MV$) attribute to 1 for those tuples returned by $Q_s$ (resp. $Q_v$).

**Encoding of eCFDs.** To achieve this, we encode $\Sigma$ with several auxiliary relations. This encoding is also used in the incremental detection algorithm and therefore we explain it in detail. We start by encoding the attribute structure of the eCFDs with a relation $\text{enc}$, and the patterns in the eCFDs in terms of separate relations, as will be explained below. The relation $\text{enc}$ is a relation of arity $2|\text{attr}(R)| + 1$ consisting of (i) an attribute $\text{CID}$ that stores an identifier of the eCFDs in $\Sigma$; and (ii) attributes $\text{AT}_{\text{L}}$ (for “left”) and $\text{AT}_{\text{R}}$ (for “right”) for each attribute in $A \in \text{attr}(R)$. We may assume that the eCFDs in $\Sigma$ all contain a single pattern tuple only. Indeed, we can always split an eCFD with multiple patterns into a set of eCFDs with only a single pattern tuple. Consider a single eCFD $\varphi = (R : X \rightarrow Y, Y_p, T_p = \{t_p\})$ in $\Sigma$. We now encode whether an attribute is part of $X$, $Y$ or $Y_p$ and whether the pattern tuple for that attribute is of the form $S$, $\overline{S}$, ‘-’, using integers in $\{-3,-2,-1,0,1,2,3\}$ as follows: we add a tuple $t_{\varphi}$ to $\text{enc}$ such that $t_{\varphi}[\text{CID}]$ is an identifier for $\varphi$; the attributes of $t_{\varphi}$ are defined as follows: $t_{\varphi}[\text{AT}_{\text{L}}] = 0$ (resp. $t_{\varphi}[\text{AT}_{\text{R}}] = 0$) in case that $A$ does not appear on the LHS($\varphi$) (resp. RHS($\varphi$)); $t_{\varphi}[\text{AT}_{\text{L}}] = 1$ (resp. $t_{\varphi}[\text{AT}_{\text{R}}] = 1$) in case that $A$ does appear in $X$ (resp. $Y$) and moreover $t_{\varphi}[A] = S$. Similarly, $t_{\varphi}[\text{AT}_{\text{L}}] = -2$ (resp. $t_{\varphi}[\text{AT}_{\text{R}}] = 2$) in case that $A$ does appear in $X$ (resp. $Y$) and moreover $t_{\varphi}[A] = \overline{S}$. Moreover, $t_{\varphi}[\text{AT}_{\text{L}}] = 3$ (resp. $t_{\varphi}[\text{AT}_{\text{R}}] = 3$) in case that $A$ does appear in $X$ (resp. $Y$) and moreover $t_{\varphi}[A] = \cdot$. Finally, for attributes $A_R$ that appear in $Y_p$, we use a similar encoding but use the negative integers $-1$, $-2$, and $-3$ instead. We illustrate the relation $\text{enc}$ encoding of $\psi_1$ and $\psi_2$ of Example 2.1 in Fig. 3 (top). We only show some attributes relevant for these eCFDs. The tuples have zeroes for all other attributes. Note that $\text{enc}$ encodes all eCFDs in $\Sigma$ uniformly in the single relation, one tuple for each pattern tuple in $\Sigma$.

We also need to store the tuples in the pattern tableaux of the eCFDs in $\Sigma$. For this, we create $2|\text{attr}(R)|$ binary relations as follows: For each attribute $A \in \text{attr}(R)$, we define $T_{\text{AL}}$ (resp. $T_{\text{AR}}$) as the relation consisting of pairs (cid,a) where cid is an identifier of an eCFD in $\Sigma$ and a $\in S$ or a $\in \overline{S}$, where $S$ is such that $t_{\varphi}[\text{AT}_{\text{L}}] = S$ or $t_{\varphi}[\text{AT}_{\text{L}}] = \overline{S}$ (resp. $t_{\varphi}[\text{AT}_{\text{R}}] = S$ or $t_{\varphi}[\text{AT}_{\text{R}}] = \overline{S}$). Figure 3 (bottom) shows these relations for the eCFDs of Example 2.1.

**Algorithm $\text{BATCHDETECT}$**. The following SQL queries are employed by $\text{BATCHDETECT}$.

1) We first detect single-tuple violations that are caused by

<table>
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<tr>
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<table>
<thead>
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</tr>
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<td>T_AR</td>
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<td>646</td>
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<tr>
<td>2</td>
<td>Colony</td>
<td>3</td>
<td>347</td>
</tr>
<tr>
<td>3</td>
<td>NYC</td>
<td>3</td>
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pattern constraints, 
\textit{i.e.}, tuples in $D$ that satisfy the pattern constraints of the LHS of an eCFD in $\Sigma$ but do not satisfy those of its RHS. The encoding is similar to the one for CFDS presented in [1], by literally expressing pattern-constraint violation in SQL. In contrast to CFDS, patterns are now sets (or the complement thereof). For this, we need to express the fact that an element is in (resp. not in) a set by means of \textsc{exists} (resp. \textsc{not exists}). Figure 4 (top) shows the query $Q_{\text{en}}$ for the example eCFDs given in Fig. 2. In the queries in Fig. 4, $\mu$ stands for a mapping defined by $\mu(A_L) = A$ and $\mu(A_R) = A$ for all $A \in \text{attr}(R)$. We omit parts of the queries addressing irrelevant attributes.

(2) We next detect the multiple-tuple violations that are caused by violations of the embedded FDs in the eCFDs in $\Sigma$. Similar to [1], detection of such violations can be readily expressed using \textsc{group by} in SQL. However, we have to group by different attributes depending on the eCFD under consideration. This can be achieved by blanking out (using a constant "@" not appearing in any database) those attributes that are not relevant. Attributes irrelevant to the embedded FD have non-positive entries in the relation \textsc{enc}. We use the case construct in the \textsc{select} statement to replace the attributes values of tuples in $D$ by "@" if the attribute is irrelevant to the embedded FD; otherwise we return the attribute value of the tuple instead. We provide an example query $Q_{\text{en}}$ in Fig. 4 (bottom). Note that $Q_{\text{en}}$ returns tuples of the form $(cid,p)$, where $cid$ is an identifier for an eCFD (as given by \textsc{enc}) and $p$ is a tuple consisting of constant values and "@". Intuitively, if a tuple $t \in D$ matches $p$ for some $(cid,p) \in Q_{\text{en}}(D)$ then it violates the embedded FD of the eCFD identified by $cid$.

(3) We set the SV attribute to "1" for tuples returned by $Q_{\text{en}}$. For the MV-attribute, note that a tuple $t$ in $D$ is involved in a multiple tuple violation iff there exists a $(cid,p) \in Q_{\text{en}}(D)$ such that $t$ matches $p$. An additional SQL query identifies these tuples and updates their MV-attribute to "1".

Putting these together, given schema $R$ and set $\Sigma$ of eCFDs defined on $R$, algorithm \textsc{batchdetect} generates SQL queries and update statements for detecting pattern-constraint violations and embedded FD violations, respectively, by capitalizing on the encoding given above.

\textbf{Remarks.} (1) The schema of the encoding relations, namely, \textsc{enc} and the binary relations $T_A$, is determined by the schema $R$ rather than $\Sigma$. (2) The entire encoding relations are \textit{linear} in the size of the input eCFDs $\Sigma$. (3) The detection SQL queries conduct two passes of the database $D$, regardless of the number of eCFDs and the size of pattern tuples in $\Sigma$. That is, they have the \textit{same data complexity} as detection queries for CFDS [1]. Note that these queries necessarily use \textsc{exists} and \textsc{not exists}; but these operations are only applied to auxiliary relations that encode the sets of constants mentioned in the eCFD patterns, rather than to the underlying database. Indeed, for each data tuple $Q_{\text{en}}$ conducts a linear scan of $\Sigma$, the \textit{same as} its CFD counterpart; similarly for $Q_{\text{en}}$. It is also worth remarking that the coding of eCFDs for algorithm \textsc{batchdetect} is more involved than that of [1],

\[ Q_{\text{en}} = (\text{SELECT } t.A, t.PN, t.NM, t.STR, t.CT, t.ZIP \text{ FROM cust } t, \text{enc } c \text{ WHERE } (\text{c.CT} \neq 1 \text{ OR } ((\text{EXISTS } Q_{\text{en}}^{c.CT} \text{ AND } c.CT \leq 1)) \text{ AND } (c.CT \neq 2 \text{ OR } ((\text{NOT EXISTS } Q_{\text{en}}^{c.CT} \text{ AND } c.CT \leq 2)) \text{ AND } ((\text{NOT EXISTS } Q^{c.AC}) \text{ AND } c.CT \leq 1) \text{ OR } ((\text{EXISTS } Q^{c.AC}) \text{ AND } c.CT \leq 2))) \]

where for any attribute $A$, $Q^A$ stands for

\[ \text{SELECT } T_A, A \text{ FROM } T_A \text{ WHERE } c.CID = T_A, CID \text{ AND } t.\mu(A) = T_A, A \]

\[ Q_{\text{en}} = (\text{SELECT } m.CID, m.CT, m.QMV \text{ COUNT(*) FROM macro } m \text{ GROUP BY } m.CID, m.CT, m.QMV \text{ HAVING COUNT(*) > 1}) \]

in order to cope with the set elements in pattern tuples. A direct extension of the technique of [1] may lead to excessive space overhead, as opposed to the linear space taken by \textsc{batchdetect}.

\textbf{B. An Incremental Algorithm}

We next present incremental algorithm \textsc{incdetect} in response to database updates $\Delta D$. Of course, \textsc{batchdetect} can be directly applied to the new database obtained by updating the database $D$ with $\Delta D$. We want to \textit{incrementally} detect violations because the deletion or insertion of a small number of tuples only affects a small part of $D$ and as a result, one only needs to identify violations in the affected part rather than inspect the \textit{entire} database. Algorithm \textsc{incdetect} aims to minimize unnecessary recomputation conducted for finding violations.

Like \textsc{batchdetect}, Algorithm \textsc{incdetect} also generates SQL queries to identify changes to the violations of pattern constraints and changes to the violations of embedded FDs in $\Sigma$. In addition, it maintains an auxiliary relation in order to reuse previous computations. Observe that tuple deletions $\Delta D^-$ may remove violations from $D$ but do not introduce new violations; on the other hand, tuples insertions $\Delta D^+$ may add new violations introduced by inserted tuples alone or together with tuples in $D$.

\textbf{Auxiliary relation.} We maintain an auxiliary relation $\text{Aux}(D)$, initialized by storing the query result of $Q_{\text{en}}$ from \textsc{batchdetect} on $D$. Recall that $\text{Aux}(D)$ consists of tuples of the form $(cid,p)$ where $cid$ is an eCFD identifier and $p$ is a tuple consisting of constants and "@". As noted above, each $(cid,p)$ corresponds to the set of tuples that are involved in a multiple-tuple violation of the eCFD identified by $cid$ and that match $p$.

We next describe how $\text{Aux}(D)$ is maintained during updates on $D$ and how it can be used to incrementally compute the updated set of violations.
**Algorithm** IncDetect. Due to space limitations, we only provide a high-level description of IncDetect. Initially, we are given (i) $D$ in which the SV and MV attributes correctly indicate the violations of $\Sigma$ (this can be obtained by running algorithm BatchDetect); (ii) the set of updates $\Delta D$; and (iii) the auxiliary relation $Aux(D)$ (initialized as described above).

Algorithm IncDetect needs to perform several tasks: it needs to compute $D \oplus \Delta D$, correctly update the SV and MV attributes for the tuples in $D \oplus \Delta D$, and update $Aux(D)$ to $Aux(D \oplus \Delta D)$. Moreover, IncDetect performs these tasks using SQL statements only. Since deletions and insertions are dealt with in different ways, we treat them separately. We first consider the case of deletions.

**Tuple deletions.** Let $\Delta D^-$ be the set of tuples that are to be deleted from $D$. We first explain how $Aux(D)$ is updated and then show how it is used to correctly update the multiple violation attribute MV in $D$. Because deletions do not eliminate single tuple violations (except those that are in $\Delta D^-$), we do not need to update the SV attribute.

1. To update $Aux(D)$, observe the following: a tuple $(cid, p)$ can be removed from $Aux(D)$ if it either does not match any tuple in $D \oplus \Delta D^-$, or all matching tuples in $D \oplus \Delta D^-$ do not violate the embedded FD of the eCFD identified by $cid$. It suffices to only consider $(cid, p)$’s that are potentially affected by the update $\Delta D^-$, i.e., those $(cid, p)$’s that match a tuple in $\Delta D^-$, and thus avoid unnecessary computations. After removing these $(cid, p)$’s from $Aux(D)$, we obtain the updated $Aux(D \oplus \Delta D^-)$.

2. In order to update the MV attribute, we first observe that it is sufficient to only consider tuples $t$ in $D$ with $t.MV = 1$. For each such $t$, we check whether it does not match any $p$ in $(cid, p) \in Aux(D \oplus \Delta D^-)$, and if so, update $t.MV$ to 0.

**Tuple insertions.** Let $\Delta D^+$ be the set of tuples to be inserted into $D$. We perform the following steps:

1. We first detect the single-tuple violations in $\Delta D^+$. That is, we apply $Q_{sv}$ of BatchDetect on $\Delta D^+$ and update the SV-attribute in $\Delta D^+$ accordingly.

2. Next, we identify new multiple-tuple violations in $D \oplus \Delta D^+$ by performing the following steps:

   2.a) Update the MV attribute of tuples in $\Delta D^+$ that violate an eCFD together with a tuple in $D$. These can be easily found by matching tuples in $Aux(D)$ with tuples in $\Delta D^+$.

   2.b) Update $Aux(D)$. Denote by $D_{clean}$ the set of tuples in $D$ satisfying $\Sigma$, which can be easily identified using the MV attribute. We insert tuples $(cid, p)$ into $Aux(D)$ that correspond to violations between (previously clean) tuples in $D$ and tuples in $\Delta D^+$.

   2.c) We then update the MV attribute for tuples in $D_{clean} \oplus \Delta D^+$ that match some tuple $(cid, p)$ in $Aux(D)$.

   2.d) To account for multiple-tuple violations caused by tuples in $\Delta D^+$ alone, we have to update $Aux(D)$ again. For this, we run $Q_{sv}$ on $\Delta D^+$ and insert the result tuples into $Aux(D)$. After this step, $Aux(D)$ becomes $Aux(D \oplus \Delta D^+)$.

   2.e) Finally, we add $\Delta D^+$ to $D$ and update the MV-attribute of tuples in $\Delta D^+$ that match a $(cid, p)$ tuple in $Aux(D \oplus \Delta D^+)$.

It is easily verified that the above steps correctly maintain both the auxiliary relation and violation set for both tuple deletions and insertions. Moreover, they can all be performed using SQL statements.

**Remarks.** (1) Algorithm IncDetect uniformly employs an auxiliary relation and SQL queries to handle multiple tuple deletions and insertions, for the entire set $\Sigma$ of eCFDs. This is the first SQL-based technique for incrementally detecting violations of multiple eCFDs. (2) Recomputation is avoided by only considering relevant tuples in $D$ using both the auxiliary relation and the update set.

**VI. Experimental Study**

Our experimental study focuses on the SQL-based algorithms BatchDetect and IncDetect for detecting data inconsistencies. We evaluate (1) the scalability of BatchDetect and IncDetect w.r.t. the size of databases, the complexity of eCFDs and the error rate in the databases, and (2) the performance of IncDetect versus BatchDetect in response to database updates.

**Experimental setting.** Our experiments are based on an extension of the cust relation shown in Fig. 1, that adds information about items bought by different customers. We scraped real-life CT, AC, ZIP data for cities and towns in the US and different items, such as books, CDs and DVDs, from online stores. Using this, we wrote a program to generate synthetic datasets, denoted by $D$. We considered two parameters of the datasets $D$: $|D|$ for the number of tuples in $D$, ranging from 10k to 100k, and noise% for the percentage of tuples in $D$ that were modified to violate an eCFD, ranging from 0% to 9%. The modification consists of changing tuples in $D$ in attributes in the right-hand side of some eCFDs from a correct to an incorrect value.

We used a set $\Sigma$ consisting of 10 eCFDs to express real-life semantics of the real-life data, including the two eCFDs of Fig. 2. We measured the complexity of the eCFDs in terms of the $|T_p|$, i.e., the number of tuples in the pattern tableaux $T_p$, ranging from 10 to 500 pattern tables. Note that each tuple itself is a constraint. The number of wildcards (""), positive domain constraints ($S$) and negative domain constraints ($\overline{S}$) in the pattern tuples are uniformly distributed.

Our experiments were conducted on an Apple Xserve with 2.3GHz PowerPC dual CPU and 4GB of memory, and with a commercial DBMS installed. Each experiment was run five times and the average is reported here.

**Experiment 1: Scalability.** In the first set of experiments we evaluated the scalability of BatchDetect.

We first set $|T_p| = 10$ and investigated the effect of varying $|D|$ and noise% on the performance of BatchDetect. Fixing
noise% = 5%, we varied |D| from 10k to 100k in 10k increments. Moreover, fixing |D| = 100k, we varied noise% from 0% to 9% in 1% increments. The results are presented in Figs. 5(a) and 5(b). As expected, BatchDetect scales well w.r.t. the size of the datasets and the error rate.

We then set |D| = 100k and noise% = 5%, and studied the impact of varying the complexity of eCFDs of Σ on the cost of BatchDetect. We selected an eCFD from Σ and varied its |Tp| from 50 to 500 in 50 increments. As shown in Fig. 5(c) BatchDetect scales linearly in |Tp|.

**Experiment 2: Incremental vs. Batch.** In the second set of experiments we compared the cost of incremental detection vs. its batch counterpart in response to database updates. We use |ΔD+| and |ΔD−| to indicate the number of tuples to be inserted into and deleted from D, respectively. We always ensure that ΔD+ and ΔD− do not overlap. As opposed to the first set of experiments, here BatchDetect was applied to the data after database updates are executed.

First, we fixed |ΔD+| = 10k and |ΔD−| = 10k and repeated the same set of experiment sets as above, i.e., we investigated...
the scalability of INCDETECT in response to the size of datasets, the error rate and the complexity of eCFDs. The results are shown in Figs. 6(a), 6(b), and 6(c). In each figure we show the running time of INCDETECT and BATCHDETECT, in response to both tuple insertions and deletions. The results tell us that INCDETECT scales well w.r.t. |D|, noise% and |Tp| and more importantly, performs better than BATCHDETECT. We note that the running time, reported in Figs. 6(a) and 6(b), not only depend on the size of the updates but also on which tuples are part of the update. This explains why the curves show some irregular behavior, although we averaged over different updates.

Second, we fixed |D| = 100K, noise% = 5%, |Tp| = 10 and varied |ΔD⁺| and |ΔD⁻| from 2k to 12k in 2k increments and from 20k to 60k in 20k increments. Note that |D| is indeed fixed, since we delete and insert the same number of tuples. We compared the cost of INCDETECT vs. the cost of BATCHDETECT in response to the number of updates. As shown in Fig. 7(a), INCDETECT outperforms BATCHDETECT when the size of the updates is relatively small, and is slightly better for larger ones. However, as expected, BATCHDETECT outperforms INCDETECT for very large updates. Indeed, in our experiments this happens when around 50% of the data is updated. Overall, we may conclude that INCDETECT works extremely well and scales up in a similar way as BATCHDETECT.

Finally, in Fig. 7(b), we report the growth of the number of single (resp. multiple) tuple violations, denoted by DSV (resp. DMV) in the database before and after updates, for an increasing number of updates. On our datasets, we observed that the number of single-tuple violations grows linearly in the number of updates. However, the number of multiple-tuple violations increases dramatically for large updates. This also explains why BATCHDETECT performs better for large updates (see Fig. 7(a)). Indeed, maintaining the auxiliary information by INCDETECT incurs a large overhead when the number of violations changes too much.

Summary. We may conclude the following from our experimental evaluation. (1) BATCHDETECT and INCDETECT scale well w.r.t. when the dataset size, the error rate and the complexity of eCFDs increase. (2) INCDETECT significantly outperforms BATCHDETECT in response to both tuple insertions and deletions, for reasonably-sized updates. (3) BATCHDETECT performs better than INCDETECT when more than 50% of the data is updated.

VII. RELATED WORK

Conditional functional dependencies (CFDs) were proposed in [1] for data cleaning. In [1], the intractability of the satisfiability and implication problems for CFDs is proved, and a batch algorithm for detecting inconsistencies based on CFDs and SQL is developed. In [8], approximation of the satisfiability analysis and incremental inconsistency detection are addressed for CFDs. This work differs from [1], [8] in that we study eCFDs, a class of dependencies more expressive than CFDs, and show that the complexity bounds for CFDs carry over to these dependencies. In addition, we develop effective techniques to tackle detection issues for eCFDs; in particular, incremental detection methods were not considered in [1], [8] for multiple CFDs. Database repairing techniques were developed in [9] based on CFDs, which we do not consider in this paper.

As observed in [1], previous work on constraint-based data cleaning has mostly focused on two topics, both introduced in [2]: repairing is to find another database that is consistent and minimally differs from the original database (e.g., [3], [4], [5]); and consistent query answering is to find an answer to a given query in every repair of the original database (e.g., [2], [6]). A variety of constraint formalisms have been used in data cleaning, ranging from standard FDs and inclusion dependencies [2], [3], [4], denial constraints [10] to logic programs (see [11] for a survey). To our knowledge, except [1], [9] no prior work on data cleaning has studied pattern tableaux as those embedded in eCFDs.

Closer to eCFDs are dependencies of [12], [13], [14], [15] developed for constraint databases. Constraints of [13], also referred to as conditional functional dependencies, are of the form \((X \rightarrow Y) \rightarrow (Z \rightarrow W)\), where \(X \rightarrow Y\) and \(Z \rightarrow W\) are standard FDs. Constrained dependencies of [14]...
extend [13] by allowing $\xi \rightarrow (Z \rightarrow W)$, where $\xi$ is an arbitrary constraint that is not necessarily an FD. In a nutshell, these dependencies are to apply FD $Z \rightarrow W$ only to the subset of a relation that satisfies $X \rightarrow Y$ or $\xi$. They cannot express even CFDs since $Z \rightarrow W$ does not allow patterns with constants as found in CFDs and eCFDs. More expressive are constraint-generating dependencies (CGDs) of [12] and constrained tuple-generating dependencies (CTGDs) of [15], of the form $\forall x (R_1(x) \land \ldots \land R_k(x) \land \xi \rightarrow \xi'(x))$ and $\forall x (R_1(x) \land \ldots \land R_k(x) \land \xi \rightarrow \exists y (R_1(x, y) \land \ldots \land R_k(x, y) \land \xi'(x, y)))$, respectively, where $R_1, R_2$ are relation symbols, and $\xi, \xi'$ are arbitrary constraints. While both CGDs and CTGDs can express CFDs and eCFDs, little is known about the complexity of their satisfiability and implication analyses, or effective algorithms for checking these dependencies. Indeed, for CGDs, the complexity of these analyses is an open issue in the presence of constants or finite-domain attributes, even when $\xi$ and $\xi'$ are $(-, \neq)$ constraints; for CTGDs the satisfiability and implication problems are already undecidable even in the absence of $\xi, \xi'$ and constants. That is, the expressive power of these dependencies comes with the price of high complexity. Furthermore, none of the prior results applies to CFDs or eCFDs. We are not aware of any applications of these constraints in data cleaning.

Codid tables, variable tables and conditional tables have been studied for incomplete information [16], [17], which also allow both variables and constants in the specifications. As clarified in [1], these formalisms differ from eCFDs and CFDs in that each of these tables is used as a representation of possibly infinitely many relation instances, one instance for each instantiation of variables in the table. No instance represented by these table formalisms can include two tuples that result from different instantiations of a table tuple. In contrast, all pattern tuples in a pattern tableau of an eCFD or CFD constrain a single relation instance, which can contain any number of tuples that are all instantiations of unnamed variables in the same pattern tuple.

The satisfiability problem is not an issue for standard FDs: one can specify arbitrary FDs without worrying about their satisfiability. There has been work on heuristic algorithms for the satisfiability analysis of first-order logic constraints (see, e.g., [18], [19]), but attributes with finite domains were not considered there, and those algorithms do not yield an effective method for eCFD satisfiability checking.

Incremental methods have been studied for checking constraints (see, e.g., [20] for a survey). However, we are not aware of any previous work on incrementally checking multiple constraints via a fixed number of SQL queries.

VIII. CONCLUSIONS

We have proposed eCFDs, an extension of CFDs that can capture more errors in real-life data than CFDs. We have shown that the satisfiability and implication analyses of eCFDs have the same complexity bounds as their CFD counterparts. We have also revised the detection technique for CFD violations [1] such that eCFD violations can also be handled efficiently using SQL. Thus, despite the increased expressive power, eCFDs incur no extra complexity in static analyses or inconsistency detection. Moreover, we have developed an incremental SQL-based technique for eCFD violation detection, as well as an approximation algorithm for the satisfiability analysis of eCFDs, the most important decision problem for eCFDs. Our experimental results show that our batch and incremental detection methods are effective.

One topic for future work is to develop algorithms for eliminating eCFD violations and repairing data. Another practical topic is to find effective methods for automatically discovering eCFDs from data samples; we defer the full treatment of eCFD discovery to another publication.

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