Long-range antiferromagnetic order in the frustrated XY pyrochlore antiferromagnet Er$_2$Ge$_2$O$_7$

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A cubic Er$_2$Ge$_2$O$_7$ pyrochlore was prepared under high-pressure and high-temperature conditions and its magnetic ground state was investigated by measurements of specific heat, dc and ac magnetic susceptibility as functions of temperature, pressure, and magnetic field. We found that Er$_2$Ge$_2$O$_7$ undergoes a long-range antiferromagnetic transition at $T_N \approx 1.4$ K, which can be further enhanced by applying external physical pressure. On the other hand, application of external magnetic fields suppresses the antiferromagnetic order to zero temperature around $H_c \approx 2.3$ T, where a magnetic-field-induced spin-flop transition was observed. $H_c$ increases accordingly with increasing $T_N$ under external pressure. A comparison of the magnetic ground states and structural variations along the isostructural series Er$_2$B$_2$O$_7$ ($B = Sn, Ti, Ge$) together with the high-pressure study on Er$_2$Ge$_2$O$_7$ indicated that the magnetic properties of these highly frustrated XY pyrochlore antiferromagnets are very sensitive to the minute structural changes that determine the anisotropic exchange interactions and the local crystal-electric-field environments of Er$^{3+}$ ions.

Recent attention has been paid to the pyrochlore compound Er$_2$Ti$_2$O$_7$, which has been regarded as a promising textbook example where the exotic “order-by-disorder” (OBD) mechanism plays a decisive role in selecting the long-range-ordered ground state [1–7]. In Er$_2$Ti$_2$O$_7$, the Er$^{3+}$ ions form a three-dimensional network of corner-sharing tetrahedra, i.e., the so-called pyrochlore lattice, and exhibit strong easy-plane anisotropy with local moments confined to planes orthogonal to the local (111) axis of tetrahedron [2]. These facts make Er$_2$Ti$_2$O$_7$ an ideal experimental realization of a highly frustrated XY antiferromagnet on the pyrochlore lattice [2,8]. Earlier numerical simulations on the classical (111) XY pyrochlore antiferromagnet found a macroscopic degeneracy of the magnetic ground state, which can be lifted by finite-temperature thermal fluctuations, leading the system to a $q = 0$ conventional Néel state via the OBD mechanism [1]. This prediction was later confirmed experimentally in Er$_2$Ti$_2$O$_7$, which was found to develop a long-range antiferromagnetic order below $T_N = 1.173$ K [2]. In sharp contrast to the prediction of a coplanar $\psi_3$ state, however, the spin ordered state under zero field can be described almost entirely with a noncoplanar $\psi_2$ structure [2,9]. This discrepancy has left the OBD effect as the most favorable selection mechanism for the experimentally observed noncoplanar $\psi_2$ ground state. Such an OBD scenario was further reinforced by more recent theoretical investigations based on an effective pseudo-spin-1/2 Hamiltonian with anisotropic exchange interactions [4,5]. By varying the relative magnitude of these anisotropic exchange couplings $J_{xx} \equiv \{ J_{x}, \ J_{xx}, \ J_{z}, \ J_{zz} \}$, Wong et al. [6] further presented comprehensive ground-state phase diagrams of $XY$ pyrochlore magnets, which were found to be very sensitive to the minute changes of $J_{xx}$.

Despite these significant achievements in understanding the magnetically ordered state of Er$_2$Ti$_2$O$_7$, it remains elusive why the isostructural Er$_2$Sn$_2$O$_7$ with a nearly identical Er$^{3+}$ pyrochlore lattice does not form a long-range-ordered state down to 100 mK [10–13]. A very recent neutron study on Er$_2$Sn$_2$O$_7$ revealed the existence of short-range correlations, in which the magnetic moments are arranged in the Palmer-Chalker configurations favored by the dipolar interactions [12,14]. A comparison of the crystal-electric-field (CEF) and the anisotropic exchange parameters between Er$_2$Sn$_2$O$_7$ and Er$_2$Ti$_2$O$_7$ indicated that the former compound has a stronger planar character of Er$^{3+}$ moment, but less anisotropic exchange couplings [12]. Variations of these microscopic parameters that influence the ground states of Er$_2$B$_2$O$_7$ ($B = Ti, Sn$) should have a deep root on the minor structural changes in these closely related compounds. An intuitive expectation would connect with their different nearest-neighbor distance $R_{nn}$ between Er$^{3+}$ ions. Therefore, a systematic study on the magnetic ground states of Er$_2$B$_2$O$_7$ by spanning $R_{nn}$ to a large extent is indispensable in order to achieve a thorough understanding.

For this purpose, we have chosen to replace Ti$^{4+}$ (0.605 Å) and Sn$^{4+}$ (0.69 Å) with the much smaller Ge$^{4+}$ ion (0.53 Å) so as to exert a larger chemical pressure on the Er$^{3+}$ pyrochlore lattice. As reported earlier [15], high-pressure and

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high-temperature (HPHT) synthesis is required in order to stabilize the cubic pyrochlore structure for \( \text{Er}_2\text{Ge}_2\text{O}_7 \). In this paper, we report on the HPHT synthesis and the structural analysis of the cubic pyrochlore \( \text{Er}_2\text{Ge}_2\text{O}_7 \), followed by detailed characterizations on the low-temperature magnetic properties of this \( XY \) pyrochlore antiferromagnet. As expected, the application of chemical pressure results in a global, uniform reduction of lattice dimension or \( R_{nn} \). Structural refinement on \( \text{Er}_2\text{Ge}_2\text{O}_7 \) enables one to see subtle modifications of the local CEF environments of \( \text{Er}^{3+} \) ions along the series \( \text{Er}_2\text{B}_2\text{O}_7 \) \((B = \text{Sn, Ti, Ge})\). Measurements of low-temperature specific heat and dc magnetic susceptibility revealed that like \( \text{Er}_2\text{Ti}_2\text{O}_7 \) [2], \( \text{Er}_2\text{Ge}_2\text{O}_7 \) also develops a long-range antiferromagnetic order below \( T_N \approx 1.40 \) K, which is higher than the reported \( T_N = 1.173 \) K of \( \text{Er}_2\text{Ti}_2\text{O}_7 \). In addition, we found that \( T_N \) of \( \text{Er}_2\text{Ge}_2\text{O}_7 \) can be further enhanced by applying external pressure, but \( T_N \) tends to saturate with pressure. These results indicate that the magnetic ground state of \( XY \) pyrochlore antiferromagnets \( \text{Er}_2\text{B}_2\text{O}_7 \) is very sensitive to structural changes. Furthermore, measurements of dc and ac magnetic susceptibility under different dc magnetic fields \( H_{dc} \), revealed the existence of quantum critical point around \( H_c \approx 2.3 \) T, where the long-range ordered state gives way to a highly polarized state. The critical magnetic field \( H_c \) was found to increase slightly with increasing \( T_N \) under pressure.

The cubic \( \text{Er}_2\text{Ge}_2\text{O}_7 \) pyrochlore in the present study was prepared under 7 GPa and 1000 °C by using a Walker-type multianvil module (Rockland Research Co.) in the Institute of Physics, Chinese Academy of Sciences. Another \( \text{Er}_2\text{Ge}_2\text{O}_7 \) sample obtained under 8 GPa and 1100 °C at the University of Edinburgh shows similar results (not shown here). Phase purity of the obtained high-pressure products was examined by powder x-ray diffraction (XRD) at room temperature. Direct current magnetization was measured with a commercial Superconducting Quantum Interference Device (SQUID) magnetometer (Quantum Design) equipped with a home-made \( ^3 \)He insert [16]. Specific-heat data in the temperature range 0.5–30 K were collected using a \( ^3 \)He option of the Physical Property Measurement System (PPMS, Quantum Design). Alternating current magnetic susceptibility under pressures up to \( \sim 20 \) kbar was measured by employing a primary-secondary-coil technique inside a self-clamped piston-cylinder cell, which was attached to commercial Heliox insert (Oxford Instruments). An excitation current of about 2 mA with a frequency of 317 Hz was applied to the primary coil during the measurements. The pressure inside the high-pressure cell was monitored by measuring the superconducting transition temperature of lead (Pb). Glycerin was used as the pressure transmitting medium.

The powder XRD pattern shown in Fig. 1(a) confirmed that the as-obtained \( \text{Er}_2\text{Ge}_2\text{O}_7 \) sample is single phase with the cubic pyrochlore structure. To extract more structural information, we have refined the XRD pattern in a cubic \( Fd-3m \) (No. 227) space group with the Er atom at 16d (1/2, 1/2, 1/2), the Ge atom at 16c (0, 0, 0), the O1 atom at 48f \((x, 1/8, 1/8)\), and the O2 atom at 8b \((3/8, 3/8, 3/8)\) site, respectively. The structural parameters after refinements are given in Table I together with those of \( \text{Er}_2\text{Ti}_2\text{O}_7 \) (Ref. [17]) and \( \text{Er}_2\text{Sn}_2\text{O}_7 \) (Ref. [18]) for comparison. The obtained lattice parameter \( a = 9.8782(2) \) Å is consistent with the value reported previously [15], and is much smaller than that of \( \text{Er}_2\text{Ti}_2\text{O}_7 \) \((a = 10.074 \) Å) [17] and \( \text{Er}_2\text{Sn}_2\text{O}_7 \) \((a = 10.35 \) Å) [18]. As shown in Fig. 1(b) and Table I, with decreasing the ionic radius (IR) of \( B^{4+} \) ions along the \( \text{Er}_2\text{B}_2\text{O}_7 \) series, the cubic lattice parameter \( a \) and the corresponding \( R_{nn} \equiv (\sqrt{2}/4)a \) decrease linearly, signaling a progressive increment of chemical pressure on the \( \text{Er}^{3+} \) pyrochlore lattice by reducing the \( B^{4+} \)-ion size.

In addition to the global, uniform reduction of lattice dimension upon the application of chemical pressure along the \( \text{Er}_2\text{B}_2\text{O}_7 \) series, the local coordination around \( \text{Er}^{3+} \) ions also experiences subtle modifications, which should have a nontrivial impact on the CEF and in turn on the anisotropic magnetic degree of freedom of \( \text{Er}^{3+} \) ions. In the \( \text{Er}_2\text{B}_2\text{O}_7 \) pyrochlores, the Er atoms are coordinated by six O1 and two O2 atoms; the much shorter Er-O2 bonds produce a pronounced axial distortion along the local \( [111] \) direction. In general, this distortion increases with decreasing \( x \) [8], which is indeed observed along this series \( \text{Er}_2\text{B}_2\text{O}_7 \) \((B = \text{Sn, Ti, Ge})\). As shown in Table I, however, variation of \( x \)

### Table I. Comparison on the structural parameters of cubic pyrochlores \( \text{Er}_2\text{B}_2\text{O}_7 \) \((B = \text{Ge, Ti, Sn})\).

<table>
<thead>
<tr>
<th>( \text{Er}_2\text{B}_2\text{O}_7 )</th>
<th>( B = \text{Ge} )</th>
<th>( B = \text{Ti} )</th>
<th>( B = \text{Sn} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IR(( B^{4+} )) (Å)</td>
<td>0.53</td>
<td>0.605</td>
<td>0.69</td>
</tr>
<tr>
<td>( a ) (Å)</td>
<td>9.8782(2)</td>
<td>10.074(1)</td>
<td>10.3504(1)</td>
</tr>
<tr>
<td>( R_{nn} \equiv (\sqrt{2}/4)a ) (Å)</td>
<td>3.492</td>
<td>3.562</td>
<td>3.659</td>
</tr>
<tr>
<td>( x ) of O1 at 48f (x, 1/8, 1/8)</td>
<td>0.3292(6)</td>
<td>0.331</td>
<td>0.3375</td>
</tr>
<tr>
<td>( \text{Er}-\text{O2} \times 2 ) (Å)</td>
<td>2.1387(1)</td>
<td>2.1811</td>
<td>2.2409</td>
</tr>
<tr>
<td>( \text{Er}-\text{O1} \times 6 ) (Å)</td>
<td>4.283</td>
<td>2.4637</td>
<td>2.4853</td>
</tr>
<tr>
<td>( \rho \equiv (\text{Er}-\text{O2})/(\text{Er}-\text{O1}) )</td>
<td>0.881(1)</td>
<td>0.8853</td>
<td>0.9017</td>
</tr>
<tr>
<td>Reference</td>
<td>this work</td>
<td>[17]</td>
<td>[18]</td>
</tr>
</tbody>
</table>
values as a function of IR is not uniform: Δχ/IR is 0.076/Å from Er$_2$Sn$_2$O$_7$ to Er$_2$Ti$_2$O$_7$, but it is only 0.024/Å from Er$_2$Ti$_2$O$_7$ to Er$_2$Ge$_2$O$_7$. Here, we introduce an Er-O bond-length ratio ρ = (Er-O2)/(Er-O1) to further characterize this axial distortion; the smaller ρ, the stronger axial distortion. As seen in Fig. 1(b), with reducing IR$(B^2)$ in this series, ρ does not decrease linearly as the lattice constant a does, but tends to level off, which implies that the local CEF environments might change only slightly with further applying chemical or physical pressure. It is worth noting that ρ scales linearly with x.

The availability of Er$_2$Ge$_2$O$_7$ enables us to check the magnetic ground state of XY pyrochlore antiferromagnet with a much reduced lattice dimension and a stronger local structural distortion. The main panel of Fig. 2 displays the temperature dependence of the dc magnetic susceptibility $\chi(T)$ of Er$_2$Ge$_2$O$_7$ between 0.7 and 30 K measured under $H = 0.1$ T after zero-field cooling (ZFC) from room temperature. A cusplike anomaly at about 1.4 K was clearly observed, 0.1 T after zero-field cooling (ZFC) from room temperature. As can be seen, $\chi(T)$ with a much reduced lattice dimension and a stronger local magnetic ground state of XY pyrochlore antiferromagnet.

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Figure 3(a) shows the temperature dependence of specific heat $C(T)$ of Er$_2$Ge$_2$O$_7$. Below the temperature dependence of specific heat $C(T)$ of Er$_2$Ge$_2$O$_7$. See the text for the analysis. (b) Magnetic specific heat $C_m$ and entropy $S$ associated with the long-range antiferromagnetic transition. (c) A plot of $C_m$ vs $T^3$. The solid line is a linear fitting curve.

In order to extract the magnetic contribution $C_m$ and to estimate the entropy associated with this transition, we measured $C(T)$ of an isostructural, nonmagnetic Lu$_2$Ge$_2$O$_7$ pyrochlore as the lattice standard $C_{lat}$. After subtracting $C_{lat}$ from the measured $C_{total}$, we obtained the specific-heat contribution from the magnetic Er$^{3+}$ ions, $C_{Er}$. As shown in Fig. 3(a), besides the low-temperature peak, there exists a substantial contribution above 10 K associated with the Er$^{3+}$ CEF levels, $C_{CEF}$. As the 16-fold degeneracy of Er$^{3+}$ ($J = 15/2$) is expected to split into eight doublets in the $D_{3d}$ symmetry, we have tried to simulate the $C_{CEF}$ by considering the first two doublets (two doublets) or the first three doublets (three doublets), respectively. As can be seen, the latter gives a better description to $C_{Er}$ between 15 and 30 K, but both underestimate the measured $C(T)$ below 15 K. From the best fitting, the first and second excited doublets are found to be located at 73(6) K and 150(70) K, respectively. The large uncertainty of the second excited doublet arises from the limited fitting range, but the estimated value for the first excited doublet, $\sim 6.3$ meV, agrees in general with that of Er$_2$Ti$_2$O$_7$ and Er$_2$Sn$_2$O$_7$ determined from neutron scattering measurements [12,13].
Finally, \( C_m \) is obtained by \( C_m = C_{\text{total}} - C_{\text{lat}} - C_{\text{CEF}} \), and is shown in Fig. 3(b). The magnetic entropy \( S \) obtained by integrating \( C_m/T \) almost saturates to its ideal value of \( R \ln(2S+1) \) with \( S = 1/2 \). This points to the effective \( S = 1/2 \) pseudo-spin character of the magnetic Er\(^{3+} \) ions with a Kramers doublet ground state [20]. In addition, greater than 50% of the entropy is found to release above \( T_N \), suggesting the existence of profound short-range spin correlations before finally establishing the long-range order. On the other hand, \( C_m \) below \( T_N \) follows nicely the \( T^3 \) dependence at least up to 1.1 K, as shown in Fig. 3(c). A linear fitting to \( C_m \approx \sigma T^3 \) yields a coefficient \( \sigma = 3.14 \text{ J/(mol Er K}^3) \). Similar behavior has also been observed in Er\(_2\)Ti\(_2\)O\(_7\) [2,20].

The observation of a higher \( T_N = 1.4 \text{ K} \) in Er\(_2\)Ge\(_2\)O\(_7\) than that of 1.17 K in Er\(_2\)Ti\(_2\)O\(_7\) indicated that the chemical pressure applied on the Er\(^{3+} \) pyrochlore lattice can promote the long-range antiferromagnetic ordering. This finding motivated us to check if the \( T_N \) of Er\(_2\)Ge\(_2\)O\(_7\) can be further enhanced by applying external physical pressure. Figure 4 displays the ac magnetic susceptibility \( \chi'(T) \) of Er\(_2\)Ge\(_2\)O\(_7\) measured below 2 K under zero magnetic field in the presence of different pressures up to 19.7 kbar. As can be seen, the overall feature of \( \chi'(T) \) resembles that of dc \( \chi(T) \) data shown in Fig. 2. We defined \( T_N \) as the maximum of \( \chi'(T) \). At ambient pressure, the measured \( T_N = 1.376 \text{ K} \) agrees well with that determined from \( \chi(T) \) and \( C(T) \). With increasing pressure, \( T_N \) shifts gradually to higher temperatures and reaches 1.461 K at 19.7 kbar. The pressure dependence of \( T_N(P) \) is shown in the main panel of Fig. 4(b). As can be seen, \( T_N \) does not increase linearly with pressure, but seems to saturate with further applying pressure. Thus, the combined chemical and physical pressure study indicated that the long-range antiferromagnetic transition temperature is sensitive to structural changes, but exhibits a complex behavior. We will return to this point below.

We further explored the stability of the long-range antiferromagnetic order under external magnetic fields at each pressure. Figures 5(a)–5(d) present the ambient-pressure ac \( \chi'(T) \) data of Er\(_2\)Ge\(_2\)O\(_7\) under different magnetic fields \( H_{dc} \) up to 6 T. As can be seen in Fig. 5(a), the \( \chi'(T) \) under \( H_{dc} = 0.1 \text{ T} \) resembles the dc \( \chi(T) \) curve shown in Fig. 2, showing a relatively broad cusplike anomaly around \( T_N = 1.4 \text{ K} \) followed by a low-temperature upturn. This comparison thus confirms our ac susceptibility data obtained by using a home-made primary-secondary coil. The small peaklike feature in the zero-field \( \chi'(T) \) curve smears out immediately upon the application of external fields. With increasing \( H_{dc} \), the evolution of \( \chi'(T) \)
can be divided into four regions: (1) for 0 ≤ \( H_{dc} \) ≤ 0.2 T, \( T_N \) remains essentially constant; \( \chi'(T) \) is nearly unchanged for \( T > T_N \), but is enhanced dramatically for \( T < T_N \); (2) for 0.3 ≤ \( H_{dc} \) ≤ 1.0 T, \( T_N \) decreases slightly and the low-temperature upturn in \( \chi'(T) \) is suppressed gradually; (3) for 1.0 ≤ \( H_{dc} \) ≤ 2.3 T, the cusplike anomaly of \( \chi'(T) \) at \( T_N \) transforms to a broad maximum, which shifts progressively to lower temperatures with increasing \( H_{dc} \) and completely disappears around 2.3 T; (4) for 2.3 < \( H_{dc} \) ≤ 6.0 T, no anomaly can be discerned in \( \chi'(T) \) and the magnitude of \( \chi'(T) \) in the measured temperature region drops quickly. Similar features were observed for \( \chi'(T) \) data under different high pressures. The evolution of \( \chi'(T) \) in these four regions can be tracked by monitoring the field dependence of \( \chi'(T) \) at \( T = 0.35 \) K, the lowest temperature of our present study. As shown in Fig. 5(e), \( \chi'(H) \) curves at different pressures all exhibit a double-peak structure: the first peak corresponds to the initial enhancement of \( \chi'(T) \) below \( T_N \) under low \( H_{dc} \), and the second peak to the critical magnetic field \( H_c \), where \( T_N \) is suppressed to below 0.35 K. As can be seen, upon application of external pressure, the first peak does not exhibit noticeable change, whereas the second peak moves gradually to higher temperatures, consistent with the enhancement of \( H_c \) with pressure. Finally, the field dependencies of \( T_N(H) \) determined from \( \chi'(T, H_{dc}) \) curves at different pressures are summarized in Fig. 5(f). It is clear that both \( T_N \) and \( H_c \) increases with pressure. These results demonstrated that, on the one hand, the application of external magnetic field destabilizes the long-range antiferromagnetic order, leading to a zero-temperature quantum critical point around \( H_c = 2.3 \) T at ambient pressure, and on the other hand, the application of physical pressure stabilizes the long-range antiferromagnetic order, leading to an enhancement of both \( T_N \) and \( H_c \).

In order to gain insights into the high-field phase above \( H_c \), we turn to the temperature and field dependence of dc magnetization. As shown in Fig. 6(a), the dc \( \chi(T) \) data under different magnetic fields resemble those of ac \( \chi'(T) \) data shown in Figs. 5(a)–5(d). The \( \chi(T) \) curve measured under \( H = 4 \) T > \( H_c \) exhibits a ferromagnetic like saturation behavior, which suggested that the sample enters a highly polarized state under \( H > H_c \). This is confirmed by the \( M(H) \) measurement below \( T_N \). As shown in Fig. 6(b), the \( M(H) \) curve measured at \( T = 0.6 \) K displays a clear slope change around \( H_c = 2.3 \) T, which corresponds to a sharp peak in the \( dM/dH \) curve and signals a field-induced spin-flip transition from an antiferromagnetic state for \( H < H_c \) to a highly polarized state for \( H > H_c \). The weak peak located around 0.2 T in both \( dM/dH \) and \( \chi'(H) \) curves might be associated with the alignment of magnetic domains.

Our detailed characterization on the cubic Er\(_2\)Ge\(_2\)O\(_7\) pyrochlore thus confirm that it develops into long-range antiferromagnetic order below \( T_N \approx 1.4 \) K, with several similar characteristics as its sister compound, Er\(_2\)Ti\(_2\)O\(_7\), as detailed below: (1) the \( \lambda \)-shaped specific-heat anomaly around \( T_N \) evidences a continuous, second-order phase transition; (2) the magnetic specific heat follows the linear \( T \) dependence below \( T_N \); (3) the magnetic entropy saturates to the expected value of \( R \ln 2 \) for \( S = \frac{1}{2} \); (4) a quantum critical point (QCP) can be reached by suppressing the \( T_N \) smoothly to zero temperature via the application of external magnetic fields. It has been well known that the first two observations contradict classical theories [2], which predicted a first-order transition with a constant density of low-lying excitations. Very recent studies have achieved significant progress on this problem. Zhitomirsky et al. [4] calculated the low-energy excitation spectrum of Er\(_2\)Ti\(_2\)O\(_7\) based on an effective “pseudodipolar” spin-1/2 model, and found a gapless mode with linear dispersion, which can account for the experimentally observed \( C_m \approx \sigma T^3 \) dependence as well as the inelastic neutron spectrum. Savary et al. [5] proposed that the slope \( \sigma \) is related to the mean spin-wave velocity \( v \) of the pseudo-Goldstone mode, i.e., \( \sigma = k_B^2 \pi^2 a^3/120 \rho^3 \). By using the experimental values of \( \sigma_{(Er_2Ge_2O_7)} = 3.14 \) J/(mol Er K\(^4\)) obtained in the present study and the \( \sigma_{(Er_2Ti_2O_7)} = 4.6 \) J/(mol Er K\(^4\)) [5], as well as the lattice constants shown in Table I, we can estimate that \( v \) of Er\(_2\)Ge\(_2\)O\(_7\) is about 1.11 times that of Er\(_2\)Ti\(_2\)O\(_7\), roughly consistent with the ratio of \( T_N \), i.e., 1.19, since the velocity \( v \) is roughly proportional to the overall exchange interactions (see Supplemental Material of Ref. [5]). Based on an effective pseudo-spin-1/2 Hamiltonian with anisotropic exchange interactions, the experimentally observed noncoplanar \( \psi_2 \) state for Er\(_2\)Ti\(_2\)O\(_7\) has recently been confirmed to be selected by the quantum fluctuations via the OBD mechanism [4–6]. The importance of quantum fluctuations is consistent with the observation (3), which points to an effective \( S = 1/2 \) pseudo-spin character of the ground state Kramers doublet of Er\(^{3+}\) ions. By means of single-crystal neutron scattering measurements, Ruff et al. [3] have shown explicitly that the magnetic-field-induced QCP in Er\(_2\)Ti\(_2\)O\(_7\) corresponds to a smooth transformation from the noncoplanar \( \psi_2 \) structure to a highly polarized \( XY \) state. Although neutron scattering measurements are needed to determine if Er\(_2\)Ge\(_2\)O\(_7\) adopts a similar magnetic structure as Er\(_2\)Ti\(_2\)O\(_7\), our magnetization curves shown in Fig. 6 as well as the field-induced QCP [3] suggest that a similar process should take place in both compounds.

The availability of Er\(_2\)Ge\(_2\)O\(_7\) offers us an excellent opportunity to look into the critical factors governing the magnetic ground state of the Er\(_2\)B\(_2\)O\(_7\) series, a family of \( XY \) pyrochlore antiferromagnets. As mentioned earlier, although short-range
correlations with the Palmer-Chalker configurations have been detected in Er$_2$Sn$_2$O$_7$,[11,12] it does not show any long-range order down to 100 mK. In contrast, Er$_2$Ti$_2$O$_7$ undergoes an antiferromagnetic transition at $T_N = 1.173$ K [2]. The observed higher $T_N$ in Er$_2$Ge$_2$O$_7$ clearly indicated that the application of chemical pressure plays an important role in determining the magnetic ground state. This conclusion is further supported by our high-pressure $\chi'(T)$ results shown in Fig. 4, where the application of physical pressure can further enhance $T_N$ of Er$_2$Ge$_2$O$_7$. This is understandable since the strength of exchange interactions $J_{\text{ex}}$ including the nearest- and the next-nearest-neighbor ones should be reinforced dramatically with decreasing $R_{\text{nn}}$. Earlier numerical calculations by Bramwell et al. [1] have predicted that the long-range order of XY pyrochlore antiferromagnets takes place at $T_N \sim 0.1 J_{\text{ex}}$. Recent investigations have realized the importance of anisotropic exchange interactions for the XY pyrochlore antiferromagnets; the ground-state phase diagrams were found to be very sensitive to the minute change of the relative magnitude of these anisotropic exchange couplings $J_{\text{ex}} \equiv \{J_1, J_{\text{bex}}, J_{\text{nex}}, J_2\} [6,7]$. As shown in Fig. 1(b), the effect of chemical pressure applied on the Er$^{3+}$ pyrochlore lattice via reducing the IR of B$_4$ex makes $\gamma$ larger, which implies that the nearest-neighbor Er$^{3+}$ distance $R_{\text{nn}}$ is at least twofold: (1) a global, uniform reduction of lattice dimension that reduces the nearest-neighbor Er$^{3+}$ distance $R_{\text{nn}}$ and (2) a strengthening of the local axial distortion of Er$_{6\text{a}}$ polyhedron that can change the local CEF environment of Er$^{3+}$ ions. Both factors are expected to change effectively the relative magnitude of these four independent anisotropy exchange couplings $J_{\text{ex}}$, and thus $T_N$.

Although our combined chemical and physical pressure studies demonstrated unambiguously an intimate correlation between $T_N$ and structural changes, we cannot reach a simple relationship between $T_N$ and $R_{\text{nn}}$. By using a bulk modulus $B_0 = 228$ (4) GPa determined from our high-pressure XRD measurements on Er$_2$Ge$_2$O$_7$, we have determined $R_{\text{nn}}(P)$ and plotted the $T_N$ of Er$_2$Ge$_2$O$_7$ as a function of $R_{\text{nn}}$ in the inset of Fig. 4(b). As can be seen, the ambient-pressure data of Er$_2$Ti$_2$O$_7$ does not fall on the extrapolated curve of $T_N(R_{\text{nn}})$ of Er$_2$Ge$_2$O$_7$. This observation indicated that in addition to $R_{\text{nn}}$, the subtle changes of local CEF environment also influence dramatically $J_{\text{ex}}$ and $T_N$. Although the resolution of our high-pressure XRD does not allow us to extract reliable information regarding the local distortion, the observations that both the Er-O bond-length ratio $\rho \equiv \langle \text{Er-O2} \rangle/\langle \text{Er-O1} \rangle$ in Fig. 1(b) and $T_N(P)$ in Fig. 4(b) tend to saturate with reducing $R_{\text{nn}}$ imply an important correlation between $T_N$ and the local distortions of Er$^{3+}$ ions. To gain a better understanding, future studies on the solid solutions of Er$_2$(Ge$_{1-x}$Ti$_x$)$_2$O$_7$ are planned with a focus on the detailed evolutions of $T_N$ and local structural changes.

In summary, we have prepared the cubic Er$_2$Ge$_2$O$_7$ pyrochlore under 7 GPa and 1000 °C and characterized in details its magnetic ground state by measuring low-temperature specific heat, dc and ac magnetic susceptibility under external pressure and magnetic fields. We found that Er$_2$Ge$_2$O$_7$ forms a long-range antiferromagnetic order below $T_N \approx 1.4$ K, with similar characteristics as the well-studied Er$_2$Ti$_2$O$_7$. The application of external magnetic fields suppresses the antiferromagnetic order continuously to zero temperature and leads to a highly polarized state above $H_c \approx 2.3$ T. On the other hand, the application of physical pressure can stabilize antiferromagnetic order, leading to higher $T_N$ and $H_c$. Neutron scattering experiments are planned to determine the magnetic structure and excitations of this XY pyrochlore antiferromagnet.

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