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A multi-objective approach for optimal prioritization of energy efficiency measures in buildings: Model, software and case studies.

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Abstract

Buildings are responsible for some 40\% of the total final energy consumption in the European Union and about 40\% of the world’s primary energy consumption. Hence, the reduction of primary energy consumption is important for the overall energy chain. The scope of the current work is to assess the energy efficiency measures in the residential and small commercial sector and to develop a methodology and a software tool for their optimal prioritization.

The criteria used for the prioritization of energy efficiency measures in this article are the primary energy consumption and the initial investment cost. The developed methodology used is generic and could be implemented in the case of a new building or retrofitting an existing building. A multi-objective mixed-integer non-linear problem (MINLP) needs to be solved and the weighted sum method is used. Moreover, the novelty of this work is that a software tool has been developed using ‘Matlab\textsuperscript{\textregistered},’ which is generic, very simple and time efficient and can be used by a Decision Maker (DM). Two case studies have been developed, one for a new building and one for retrofitting an existing one, in two cities with different climate characteristics. The building was placed in Edinburgh in the UK and Athens in Greece and the analysis showed that the primary energy consumption and the initial investment cost are inversely proportional.

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1 Introduction

The increase of primary energy consumption and the climate change are amongst the biggest challenges the 21st century faces. In most countries, governments have policies which aim to reduce primary energy consumption by promoting energy efficiency. Specifically, the building sector accounts for some 40% of the total final energy consumption in the European Union and some 40% of the world’s primary energy consumption [1], [2]. The European Commission in order to rationalize the use of energy in buildings and increase energy efficiency has issued the Energy Performance of Buildings (EPBD) Directive 2002/91/EC and its recast 2010/31/EU regarding the European energy policy for the energy performance of buildings and the rational use of energy [3], [4].

The reduction of energy consumption and especially primary energy consumption will contribute to the reduction of energy in the total energy chain and increase sustainability in buildings. Investing in energy efficiency is essential as the overall benefits will outweigh the initial investment cost. The building sector is large, both in terms of energy consumption but also in terms of number and type of buildings available. In general, there are two categories of buildings, namely the existing buildings that might need retrofit actions and the new building that are going to be built.

In order to reduce the primary energy consumption in buildings several efficiency measures can be implemented. These measures can be divided into categories, such as those related to the building envelope, the energy systems that provide heating, cooling and hot water, the electrical appliances and the lighting systems and can be found analytically in [5]. Also, there are energy systems that can provide electricity. Those can be cogeneration units or renewable energy sources (RES) such as biomass, wind energy and solar energy. Energy efficiency measures in each category have a different contribution to the reduction of the final and primary energy consumption and also have an initial investment cost, typically higher than conventional systems. Furthermore, the building’s location plays an important role as the climate and the available RES in an area might provide different solutions for each case.

Therefore, a Decision Maker (DM) needs to make a decision between many alternative choices, which is usually not easy. The DM must take into account several criteria such as financial or environmental in order to find the optimal solution according to his own preferences. Although there are many approaches to tackle such problems, in this article a multi-objective programming approach will be used. The first objective is to minimise the
primary energy consumption and the second objective is to minimise the initial investment cost. A general rule followed states that if the initial investment cost is higher in components with better energy behaviour, then the primary energy consumption will be lower resulting in more energy savings. However, it is often the case that a DM does not have unlimited resources, hence a compromise solution between these two criteria needs to be found. The main concept of this approach is to allow the DM to propose the efficiency measures that he is interested in, so as to allow him to find the optimal approaches for each category.

The paper is structured as follows: Section 2 provides a literature review of research on energy efficiency measures in buildings. In Section 3 the proposed model is described. Decision variables, constraints, parameters and objective functions are presented. Section 4 describes the multi-objective optimization approach used to solve the problem. Section 5 presents the developed software tool, which is explained in detail. In Section 6 the performed case studies for a new building and an existing building under renovation are described and in Section 7 the results are analysed. Section 8 concludes the paper. Moreover, in Appendix “A” the equations of the model are shown analytically, in Appendix “B” the proposed components for the buildings in the case studies are presented and finally, in Appendix “C” the values of the decision variables for the performed case studies are presented in detail.

2 Literature Review

Calculating energy loads in building and assessing energy efficiency measures has been researched extensively in the last years. In 2006 Chung et al. in [6] performed a study regarding benchmarking energy efficiency in commercial buildings using multiple regression analysis. In [7] Wang et al. reviewed the energy performance methods for existing buildings. In their study they quantify energy usage and propose a framework for the categorization of energy quantification methods for existing buildings. Energy quantification methods are divided into three categories, namely the calculation-based, measurement-based and hybrid quantification methods. Regarding calculation-based methods, are further divided into dynamic methods (use of basic simulation or representative simulation tools) and steady-state methods (e.g. forward modelling approach or inverse modelling approach). Typical steady-state methods used for the calculation of thermal performance in buildings are the degree-day (DD) method, bin method and equivalent full-load hour method. Measurement based methods are further divided into energy bill-based methods and monitoring-based methods. Finally, hybrid quantification methods consist of calibrated simulations and dynamic inverse models. A method for assessing buildings’ energy efficiency using dynamic simulation and
experiments has been developed by Pisello et al. in [8]. They proposed a methodology for analysing the thermal performance of buildings using non-dimensional indexes. Another framework for characterizing energy efficiency measures has been developed by Trianni et al. in [9], which is based on several attributes grouped into six categories, namely economic, energy, environmental, production, implementation and interaction with other systems.

The problem of designing low energy buildings and prioritizing the energy efficiency measures has been approached by many researchers throughout the years. There are many methods that can be used in order for a DM to make the optimal choice regarding which energy efficiency measures to choose. Kolokotsa et al. [10] analyse the decision support methodologies that can be used regarding the energy efficiency and management in buildings. The criteria that can be used in order to support a decision are divided in categories such as: (a) energy related: primary or final energy consumption, the heating and cooling load, electricity consumption, embodied energy; (b) cost related: direct cost, initial investment cost, life cycle cost, net present value and internal return rate of the investment; (c) environmental related: annual emissions and global warming potential, life cycle environmental potential; (d) indoor quality related: indoor temperature and humidity, CO₂ concentration, ventilation rate, daylight availability, noise levels and (e) other criteria such as construction duration, security etc.

In [11] Evins performed a review of the computational optimization methods that are applied to sustainable building design. His analysis shows that there is a growth in the use of optimization in sustainable building design, and more particular in the use of multi-objective optimization methods. The dominant optimization method is genetic algorithm. Most of the studies performed have energy as an objective function, followed by construction cost. Regarding area of building design, building envelope is the dominant one. Another review about the simulation-based optimization methods applied to buildings performance was done by Nguyen et al. in [12] and revealed that the major drawbacks in these methods are the complexity of the problems, the high computational cost, the uncertainty of the parameters and the multi-objective design problems. Also, their results point out that the most used software packages for building simulations are EnergyPlus and TRNSYS and the most used optimization platforms are GenOpt and Matlab.

Mavrotas et al. in [13] studied energy planning in buildings taking into account the uncertainty of fuel costs. They developed a linear programming model with fuzzy parameters in order to deal with the uncertainties of fuel costs, which then is transformed into an
equivalent multi-objective problem. Their analysis is mainly applied to larger energy consuming buildings where energy investment decisions may be affected significantly.

Wang et al. [14] in 2005 tried to use genetic algorithms in a multi-objective programming approach for designing green buildings. Their approach was to minimise the life cycle cost and the life cycle environmental impact, by taking into account the building’s design variables of the building’s envelope. In their analysis they used genetic algorithms (GA) but as those are random the resulted Pareto Front was considered to be the values of the external population (final solutions). The study showed that optimal values for some variables change between different Pareto zones. Also, it was shown that the utility structure affects the environmental performance significantly.

Chlela et al. in [15] introduced a methodology regarding the design of new buildings based on parametric analysis. This approach requires a design of experiments in order to perform a statistical analysis on the selected variables, resulting in the modelling of the energy consumption.

In 2008, Diakaki et al. [1] built a generic methodology based on multi-objective programming approach, aiming to minimize the primary energy consumption and the initial cost of acquisition of the materials. The proposed model was limited as the only decision variables were the window types, the insulations materials and the thickness of the wall. Also, different multi-objective optimization techniques have been investigated, such as compromise programming with the Tchebyshev criterion, the global criterion method and the goal programming method.

Moreover, Diakaki et al. in [16] further developed the proposed methodology in [1]. They resulted in a more detailed methodology by taking into account all the decision variables regarding the thermal envelope and the energy systems of the building (except those producing electricity). The model was based on a multi-objective programming approach regarding the prioritization of energy efficiency measures in a new building that will be constructed. The decision criteria that were used were the minimization of the primary energy consumption, the initial investment cost (cost of construction, acquisition and installation) and the CO2 emissions.

A different approach aiming to optimize the thermal comfort and the energy consumption in a residential building has been presented by Magnier and Haghicat in [17]. They proposed an efficient model where the decision variables are related to the thermostat settings, heating,
ventilation and air condition system (HVAC) and passive solar design. Their approach was based on the usage of a multi-objective evolutionary genetic algorithm (NSGA-II) with a simulation-based Artificial Neural Network (ANN) method.

Popescu et al. in [18] studied the impact of energy efficiency measures on the economic value of buildings. They assessed investments in energy efficiency measures by measuring the payback period of investments, which they claim depends on the energy savings and the added value of the property. However, they recommend that this financial analysis should be taken into account when there is reliable evidence to support that the real-estate market reacts to energy performance of the buildings. In [19] Saari et al. investigated the financial viability of energy efficiency measures in a new detached building in Finland. They studied the impact on the construction costs and the financial viability of eight alternative design concepts.

Yao in [20] studied energy optimization of building design in apartment buildings. He introduced EDH index, which measures the energy performance difference between housing units in order to evaluate proposed measures in design options aiming to reach 50% energy efficiency improvement. Kusiak et al. in [21] performed a study about modelling and optimization HVAC energy consumption in a typical office building. They used eight data-mining algorithms to evaluate energy consumption, control settings and a set of parameters and they constructed four models of energy consumption. They used a single objective approach that was solved by the particle swarm optimization algorithm.

Fesanghary et al. in [22] proposed a multi-objective optimization model based on harmony search algorithm. The decision criteria in that methodology were the minimization of the life cycle cost and the minimization of the carbon dioxide equivalent emissions of the building.

Asadi et al. in [23] used a multi-objective optimisation programming problem trying to maximize the energy savings and to minimise the retrofit cost, after the refurbishment of a semi-detached building in Portugal. However, despite the fact that their approach was based on the Portuguese regulations of building design, it could be transferred and used for other countries as well.

In [24] Chantrelle et al. developed a multi-criteria optimization tool (MultiOpt) for the renovation of buildings. MultiOpt has a graphical user interface and has a set of four criteria, namely energy consumption, thermal comfort, cost and environmental impact. It takes into account parameters related to control strategies and building envelope. For the optimization procedure genetic algorithm NSGA-II is used.
A more recent study was made by Malatji et al. in [25] using a multi-objective model aiming to maximize the energy savings after retrofitting a building and minimise the payback period of the investment. In this approach the energy savings were not calculated but where taken from the manufacturers’ data. They used compromise programming technique with two objectives and a genetic algorithm was used to solve the problem. Also, a sensitivity analysis was performed to investigate uncertainties in parameters such as auditing error of the facilities, variability of electricity prices, wrong calculation of energy savings, increase of the initial investment cost, and change of the interest or discount rate.

Moreover, Hamdy et al. in [26] presented an efficient and time-saving simulation-based optimization method. Their methodology was referring to the nearly-zero-energy building and cost-optimal solutions of a single-family building in Finland, following the EPBD recast of 2010 [4]. They tried to minimize the primary energy consumption and the difference of the life-cycle cost between a design option and a reference design for the specific climate zone.

### 3 Model Building

Diakaki et al. in [16] developed a multi–objective decision model for the improvement of energy efficiency in buildings. In the current work we expand the model presented in [16] by taking into account the lighting systems, electrical appliances and RES. Also, it is further expanded to include the case of retrofitting an existing building. Another difference between our work and [16] is that all the decision variables are considered to be binary. In other words, we assume predetermined discrete values for the continuous variables of the model in [16] which is in most cases more realistic (e.g. the thickness of insulation has predetermined values). In this way we obtain a discretization of the decision space which is appropriately modelled using binary variables. The basic characteristics of the model are given briefly below while the full model with all the equations is presented in Appendix “A”.

#### 3.1 Decision variables

The current approach consists of decision variables related to: (1) the building envelope; (2) the building’s energy system; (3) the lighting system and (4) the electrical appliances. Regarding the building envelope we have decision variables for door type, window type, wall type with different layers of materials of different type. In other words, each wall type consists of a number of known layers. The materials of these layers have specific thermal
conductivity and thickness. The same holds for the decision variables expressing the ceiling and the floor type.

The building’s energy system related decision variables describe the following issues:

- **Heating systems**: Provide only heating and can be electrical or non-electrical systems which are further categorized according to their input fuel;
- **Cooling systems**: Provide only cooling (in this approach only electrical systems are assumed to be available);
- **DHW systems**: Provide only hot water. They can be electrical or non-electrical, which are further categorized according to their input fuel;
- **Heating – cooling systems**: Provide both space heating and cooling (only electrical systems are assumed to be available);
- **Heating – DHW systems**: Provide both space heating and DHW supply. They can be electrical or non-electrical which are further categorized according to their input fuel;
- **Solar collector systems**: Supply DHW by utilizing solar energy;
- **Electricity generation systems**: Provide electricity using RES.

The lighting system and the electric appliances are described by appropriate binary decision variables, each one expressing a specific type.

### 3.2 Constraints

The constraints of the problem are mainly the energy balances, which means the satisfaction of the energy demand for heating, cooling, DHW, lighting and electricity supply. Moreover, in order to satisfy the energy demand the appropriate equipment must be selected. Therefore, there are constraints regarding the selection of one equipment to satisfy the energy demand for the respective category. In addition, there are constraints where one piece of equipment is selected in case the same equipment can be used for multiple purposes (e.g. a heat pump for both heating and cooling). Regarding the investment cost, it is calculated depending the selected equipment for each category. The constraints can be seen in detail in Section A.2 in Appendix “A”.

### 3.3 Parameters

The parameters of the model are in general meteorological data, technical coefficients, demand data, efficiencies, standard dimensions and costs which are required in the model’s constraints and objective functions, and most of which need to be insert by the DM.
In order to calculate the energy demand air temperature, solar radiation, water temperature, number of people leaving in the house and dimensions of the building envelope are necessary. Also, for the calculation of primary energy consumption for lighting and electrical appliances, the number and operational hours of lamps and appliances are required. More technical parameters such as efficiency coefficients of the selected equipment and of the electricity grid are also necessary for the calculations. Moreover, the cost of the components is required for the calculation of the total investment cost. All the parameters are presented analytically in Appendix “A”.

3.4 Objective functions

In this model there are two objective functions: (a) minimization of the total annual primary energy consumption or maximization of total annual primary energy savings and (b) the minimization of the total investment cost for the interventions:

\[ g_1(x) : \text{Total annual primary energy consumption or total annual primary energy savings.} \]
\[ g_2(x) : \text{Total Investment Cost} \]

The primary energy consumption is the sum of energy consumption for heating, cooling, DHW, lighting and electrical appliances. In this work, heating and cooling loads are calculated using the DD method (for more details see [7], [27]. For the case of retrofitting an existing building the methodology is similar to that of a new building. However, in this case the objectives would be to achieve maximum primary energy savings with minimal initial investment cost. Therefore, the primary energy consumption of the existing building before any retrofit action must be calculated. The objective functions are described in more detail in Section A.3 of Appendix “A” for both cases, namely, the case of a new building and the case of retrofitting an existing one.

4 Multi-objective Optimization

As the name suggests, multi-objective (or multi-criteria) optimization involves optimization in the presence of more than one (usually conflicting) objective functions. Multi-objective optimization problems arise in a variety of real word applications and the need for efficient and reliable methods is increasing. The main difference between single and multi-objective optimization is that in the case of latter, there is usually no single optimal solution, but a set of
equally good alternatives with different trade-offs, also known as Pareto-optimal (or non-dominated or efficient) solutions. In the absence of any other information, none of these solutions can be said to be better than the other. Usually a decision maker is needed to provide additional preference information and to identify the “most preferred” solution. Depending on the paradigm used, such knowledge may be introduced before, during or after the optimization process. Multi-objective optimization thus has to combine two aspects: optimization and decision support.

In our case the problems defined in Equation (A.100) and in Equation (A.102) is a multi-objective programming problem which fall into the category of mixed-integer non-linear programming problems (MINLP). For the solution of this kind of problems we will first calculate a representation of the Pareto set and then we will select the most preferred among the Pareto optimal solutions. For the calculation of adequate representations of the Pareto set a straightforward method is the weighting method [28]–[30].

Therefore, equation (A.100) is modified as follows:

$$\min \left[u(g_1(x), g_2(x))\right] = p_1 \left(\frac{g_1(x) - g_{1\text{min}}}{g_{1\text{max}} - g_{1\text{min}}}\right) + p_2 \left(\frac{g_2(x) - g_{2\text{min}}}{g_{2\text{max}} - g_{2\text{min}}}\right)$$  \hfill (1)$$

Subject to

Constraints: (A.1) - (A.99)

Where,

- $x$: a vector with the decision variables.
- $g_1(x)$: Total annual primary energy consumption
- $g_2(x)$: Total investment cost
- $g_{1\text{min}}$ and $g_{2\text{min}}$: are the values of the criteria of (A.100) when they are optimized independently.
- $p_1$ and $p_2$: weight coefficients that reflect the relative importance of the two criteria, allowing the DM to take into account his personal preferences. The following condition for the weights must hold:

$$p_1 + p_2 = 1$$  \hfill (2)$$

- $g_{1\text{max}}$ and $g_{2\text{max}}$: are the “nadir” (=worst) values of the criteria of Equation (A.100) and they are obtained from the payoff table (minimization of $g_1$ provides $g_{2\text{max}}$ and vice versa, minimization of $g_2$ provides $g_{1\text{max}}$). The denominator ($g_{k\text{max}} - g_{k\text{min}}$) is necessary as range equalization factor in order to provide a normalization of the objective functions. In this way
the weight coefficients are more meaningful and they are not influenced by differences in the
objective functions’ scale or by the range of the objective functions.

As we told, for multi-objective optimization problems there is not a single solution. Hence the
concept of Pareto optimality is used which is defined as a set of solutions that belong in a pre-
set classification of an optimal solution. The weighting method is a scalarization method
which combines the two functions in one, allowing a DM to express his preference a priori or
a posteriori and compromise between the two criteria [17]. If the weight coefficients are
greater than zero then Equation (1) is sufficient for Pareto optimality [31].

In the case or retrofitting an existing building where the first objective ($g_1$) is to maximize the
energy savings equation (A.102) is modified as follows:

$$\min\left[u\left(g_1(x), g_2(x)\right)\right] = p_1\left(\frac{g_{1\text{max}} - g_1(x)}{g_{1\text{max}} - g_{1\text{min}}}\right) + p_2\left(\frac{g_{2\text{max}} - g_2(x)}{g_{2\text{max}} - g_{2\text{min}}}\right)$$

Subject to

Constraints: (A.1) - (A.99), (2)

Where,

$g_1(x)$: Total annual primary energy savings.

$g_2(x)$: Total investment cost.

5 Software Tool

The methodology described in Section 3 has been used to develop a software tool for the
optimal prioritization of energy efficiency measures for a new and an existing building. The
software tool has been developed using ‘Matlab®’ and ‘Microsoft Excel®’. The novelty of this
software tool is that it has the advantage of being generic and not depending on the number of
components in a building (e.g. number of doors, number of windows, number of walls etc). A
‘Microsoft Excel®’ spreadsheet contains all the relevant data for the analysis, i.e. the climate
data, building’s characteristics and the proposed energy efficient measures.

In this software tool the following assumptions have been made: (a) only four categories of
electrical appliances have been used, which are: a television, an electric cooker, a refrigerator
and a washing machine; (b) only three alternative choices can be proposed for each decision
variable, hence the total number of decision variables is sixty three (63) and (c) only the case
of solar PV has been examined in the category of RES systems that are used to provide
electricity.

It is noted that the electrical energy output of a photovoltaic system is equal to [32]:

$$Q_{pv} = A_{pv} \cdot n_{pv} \cdot PR_{pv} \cdot F_{s,pv} \cdot I_{SL}$$ (4)

Where,

- $A_{pv}$: the area of the photovoltaic array ($m^2$)
- $n_{pv}$: efficiency of the panel (%)
- $PR$: performance ratio expressing the losses of the system (circuit, battery, inverter) (%)
- $F_{s,pv}$: shading factor (%)

In order to solve this multi-objective problem ‘BONMIN’ algorithm has been used which is
suitable for solving convex MINLP problems [33]. As ‘BONMIN’ is not implemented in
‘Matlab®’, the ‘OPTI TOOLBOX’ has been used, which is an open-source software that can
be implemented in ‘Matlab®’ and has many optimization solvers available [34].

In order to use the software tool a DM must know how to use the necessary script files and
needs to have ‘Matlab®’ and the ‘OPTI TOOLBOX’ installed. The software tool can perform
all the necessary calculations and export the results in a ‘Microsoft Excel®’ file. The weight
factors pairs that are used are fixed and equal to: $p_1=1$ and $p_2=0$ to $p_1=0$ and $p_2=1$ with step
equal to 0.05. The reasons why the weight factors pairs are fixed a priori is to provide the full
Pareto front to a DM, allowing him to examine all the optimal solutions.

The results obtained by using the software tool are all the values of the minimization of
equation (1), the primary energy consumption, the initial investment cost and the values of the
decision variables for each working pair of weight coefficients. Moreover, for further
analysis, the software tool can be used to export the results of energy demand and primary
energy consumption of each category for each month by using the respected script file.

Similarly to the provided software tool for the case of a new building described in the
previous section, and based on the methodology described in Section 3 a software tool has
been developed for the optimal prioritization of energy efficiency measures for the case of
retrofitting an existing building.

Its features are similar to the software tool for the new building. Moreover, due to the
constraints described in A.3.2 (i.e. no proposed wall, floor or ceiling structures) the total
number of decision variables is reduced to fifty four (54). Instructions regarding the usage of
the software tool and the spreadsheet are available within.

An additional assumption for the case of retrofitting an existing building is that the DM is
interested to make changes in all the categories of energy efficiency measures. This means
that the software tool will provide solutions for each set of the proposed components. The
software tool can perform all the necessary calculations and it can export the results in a
‘Microsoft Excel®’ file. The results obtained by using the software tool are all the values of
the minimization of equation (3), the maximization of the primary energy consumption
savings, the minimization of the initial investment cost and the values of the decision
variables for each working pair of weight coefficients. Moreover, for further analysis, the
software tool can be used to export the results of energy demand and primary energy
consumption of each category for each month before and after the retrofit actions.

Also, it is noted that in this software tool some variables are considered to be constant and are
presented in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value [27]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ACH \ (h^{-1})$</td>
<td>1.5</td>
</tr>
<tr>
<td>$\rho_{\text{air}} \ (kg/m^3)$</td>
<td>1.2</td>
</tr>
<tr>
<td>$cp_{\text{air}} \ (kJ/kg K)$</td>
<td>1.0035</td>
</tr>
<tr>
<td>$P_{\text{water}} \ (kg/m^3)$</td>
<td>1000</td>
</tr>
<tr>
<td>$cp_{\text{water}} \ (kJ/kg K)$</td>
<td>4.18</td>
</tr>
<tr>
<td>$Q_{\text{human}} \ (W)$</td>
<td>115</td>
</tr>
</tbody>
</table>

Table 1: Parameters with constant value used in this model

It is noted that this software tool is for academic use. It is not developed in an integrated
software platform, therefore the DM must have basic skills of Excel and Matlab. A flowchart
for the operation of software tool (which is similar in both the case of a new building and an
existing one) is presented in Figure 1.
6 Case Study

In order to evaluate the efficiency and the robustness of the proposed methodology and software tools, two simulations on a typical detached UK house (see Figure 2) have been carried out. The building’s characteristics are presented in Table 2. The proposed energy efficient components are presented in Appendix “B” in Table B.1 up to Table B.18. It is noted that the tables with the proposed components consist also of the data for the existing components of the building, which would be examined in the next section. The values regarding the materials, their efficiency and their corresponding cost are from several sources [5], [16], [27] and from an unofficial internet survey of several UK online retailers.

The building will be considered both as a new building and as an existing building under retrofit actions. Moreover, for purposes of comparison the examined building will be placed and simulated in two different locations where the climate characteristics are very different: (a) Edinburgh in the UK and (b) Athens in Greece. The climate characteristics of Edinburgh and Athens are presented in the Table 3. Also, the variables that the DM has to define and are used for this analysis are presented in Table 4. Moreover, for reasons of simplicity it is assumed that all the temperature correction factors and shading factors are considered to be equal to 1. It is further assumed that the cost of the components is the same in both cities and it will be expressed in Great British Pounds sterling (£).
Figure 2: A typical detached house in the UK (source: [35])

<table>
<thead>
<tr>
<th>Component</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall (m²)</td>
<td>194</td>
</tr>
<tr>
<td>Internal Wall (m²)</td>
<td>99</td>
</tr>
<tr>
<td>First floor &amp; ground floor ceiling (m²)</td>
<td>62</td>
</tr>
<tr>
<td>Ground floor (m²)</td>
<td>65</td>
</tr>
<tr>
<td>Roof (m²)</td>
<td>75</td>
</tr>
<tr>
<td>First floor ceiling (m²)</td>
<td>65</td>
</tr>
<tr>
<td>Windows (m²)</td>
<td>13</td>
</tr>
<tr>
<td>External doors (m²)</td>
<td>3</td>
</tr>
<tr>
<td>Internal Volume V (m³)</td>
<td>344</td>
</tr>
</tbody>
</table>

Table 2: Characteristics of the examined building (source: [35])

Note:

a: it can be calculated from the building’s characteristics after calculating the total volume and subtracting the volume occupied by interior walls

<table>
<thead>
<tr>
<th>Month</th>
<th>Air Temperature (°C)</th>
<th>Daily Solar Radiation (kwh/m²/day)</th>
<th>Water Temperature (°C)</th>
<th>Relative Humidity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EDI a</td>
<td>ATH a</td>
<td>EDI a,b</td>
<td>ATH a,b</td>
</tr>
<tr>
<td>January</td>
<td>3.9</td>
<td>7.4</td>
<td>0.57</td>
<td>1.39</td>
</tr>
<tr>
<td>February</td>
<td>4.2</td>
<td>7.8</td>
<td>1.28</td>
<td>1.91</td>
</tr>
<tr>
<td>March</td>
<td>5.6</td>
<td>10.8</td>
<td>2.19</td>
<td>2.78</td>
</tr>
<tr>
<td>April</td>
<td>7.3</td>
<td>15.8</td>
<td>3.32</td>
<td>3.85</td>
</tr>
<tr>
<td>May</td>
<td>10.1</td>
<td>21.5</td>
<td>4.58</td>
<td>5.01</td>
</tr>
<tr>
<td>June</td>
<td>12.9</td>
<td>26.4</td>
<td>4.56</td>
<td>5.27</td>
</tr>
<tr>
<td>July</td>
<td>14.9</td>
<td>28.6</td>
<td>4.31</td>
<td>4.93</td>
</tr>
<tr>
<td>August</td>
<td>14.7</td>
<td>28.0</td>
<td>3.68</td>
<td>4.62</td>
</tr>
<tr>
<td>September</td>
<td>12.5</td>
<td>24.2</td>
<td>2.54</td>
<td>3.93</td>
</tr>
<tr>
<td>October</td>
<td>9.5</td>
<td>18.9</td>
<td>1.45</td>
<td>2.49</td>
</tr>
<tr>
<td>November</td>
<td>6.4</td>
<td>13.1</td>
<td>0.74</td>
<td>1.54</td>
</tr>
<tr>
<td>December</td>
<td>4.5</td>
<td>8.7</td>
<td>0.44</td>
<td>1.22</td>
</tr>
</tbody>
</table>

Table 3: Climate Characteristics of Edinburgh (EDI) and Athens (ATH)

Notes:

a: source [36]
b: Daily solar radiation is assumed to be falling at the optimal angle of the area and is calculated with the methodology described in [37]
c: source [38]
de: source [39]
7 Results

7.1 The Case of a New Building

The obtained results from the simulations are presented in Table 5. The values of the decision variables for each working pair of weight coefficients can be seen analytically in Appendix “C” from Table C.1 up to Table C.4. It is noted that the software tool took 4 and 4.1 minutes in a ‘Windows 8.1” operating system, supported by a 3.07GHz i7 processor and 12GB RAM, to run the simulations for the case of Edinburgh and Athens respectively. This time includes the input of the necessary data, all the optimizations and the exportation of the results to the Excel spreadsheet file; hence it can be seen that the proposed method and software tool can be time efficient.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{IH}$ (°C)</td>
<td>18 [32]</td>
<td>$n_w$ (l/day)</td>
<td>60 [5]</td>
</tr>
<tr>
<td>$T_{IC}$ (°C)</td>
<td>24 [32]</td>
<td>Number of People</td>
<td>4</td>
</tr>
<tr>
<td>$T_{DHW}$ (°C)</td>
<td>60 [32]</td>
<td>$n_{grd}$ (%)</td>
<td>35 [16]</td>
</tr>
<tr>
<td>$h_1$ (W/m²K)</td>
<td>8.3 [27]</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$h_2$ (W/m²K)</td>
<td>28 [27]</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Parameters used in the case studies that are set by the DM
From Table C.1 up to Table C.4 it can be seen that when the primary energy consumption criterion is independently minimized the components which have the best energy behaviour are selected. This means the components of the building’s envelope (doors, windows, wall structure, floors structure and ceilings structure) with the lowest $U_{value}$ are selected, and the energy systems with the higher generation efficiency. For instance, it can be seen that the Door 2 has been selected, the window type 2 which is a low-e window and so forth. However, it is observed that there are differences at each city. In Athens the window 3 that has lower SHGC is selected for all the working pairs of weight coefficients. This happens as in Athens the solar radiation is higher than in Edinburgh which causes a significant increase in cooling demand. By contrast, in Edinburgh the window with the lowest $U_{value}$ is most frequently selected (when the primary energy consumption criterion is more important) in order to minimize heating demand.

In the category of the building’s energy systems, in both cities the heating-DHW system 3 is selected which is a highly efficient heat pump with a COP 4 and the electrical cooling system 3 that has is an air-condition system with a COP 3. Also, the LED proposed lamps are chosen to provide lighting as they have the lowest power. The same applies for all the electrical appliances. In both cities solar collector 1 and photovoltaics 3 have been chosen as they can produce more hot water and electricity respectively.

On the other hand, when the cost criterion is minimized independently, the components with the lowest investment cost are selected. As shown in Table 5 the solution is the same for both cities. The building’s envelopes components with the highest $U_{value}$ and the energy systems with the lowest efficiency are selected. A low efficiency heating-cooling system has been selected and a low efficiency oil-based boiler to provide hot water. In addition, a fluorescent lamp and electrical appliances with the lowest cost that have the highest power have been selected.
The Pareto frontier that includes the values of primary energy consumption and total investment cost for all the working weighting pairs is shown in Figure 3 and it represents all the optimal solutions. It can be seen clearly that the initial investment cost and primary energy consumption of a building are inversely proportional. The higher the total investment cost, the lower the primary energy consumption. When the primary energy consumption criterion is more important the components with the best energy behaviour are selected, but as the cost criterion gets more important cheaper components are selected, which confirms the general hypothesis and shows that the methodology and the developed software are robust. It is suggested that the most preferred solutions for the DM are those indicated in the diagram of the Pareto frontier, because for these cases a small reduction of the investment cost does not increase primary energy consumption dramatically.

The minimal and maximum total initial investment cost comes to £28,509 and £53,006 respectively in Edinburgh and £28,509 and £52,031 respectively in Athens. The minimal and the maximum primary energy consumption in Edinburgh is 58,499 MJ/year and 126,899 MJ/year respectively; while the minimal and the maximum primary energy consumption in Athens is 59,147 MJ/year and 116,093 MJ/year respectively. Furthermore, it can be noticed that the primary energy consumption in Athens and Edinburgh is similar although the climate characteristics are different, however there are major differences between the energy categories.
The importance of the climate characteristics can be seen in more detail by comparing the energy demand and primary energy consumption of the building in each city. In Figure 4 the contribution of each energy category to the total annual energy demand is presented for the case of weight coefficients \((p_1, p_2) = (0.35, 0.65)\). As previously mentioned, the DM can use the software tool to obtain the energy demand and energy consumption analytically for any working pair of weight coefficients, according to his own preferences. In this analysis, the particular working pair of weight coefficients has been chosen because is in the area of the most preferred optimal solutions.

It is shown that if the building is located in Edinburgh the heating energy demand is the dominant category, whilst when the building is located in Athens the cooling energy demand is higher because of the difference in Degree-days in the two cities. In Figure 5 the primary energy consumption share of each category is presented. It is observed that in Edinburgh the primary energy consumption for heating has the highest contribution to the total primary energy, whilst in Athens the primary consumption for the electrical appliances is the highest. The importance of the chosen components is significant as they can have a major impact on primary energy consumption. For instance, although in Athens the DHW demand is lower than in Edinburgh the primary consumption is higher due to the choice of a less efficient component for the hot water.

![Figure 4: Annual Energy Demand (MJ/year) for the case of a new building (a) Edinburgh and (b) Athens, for the case of weight coefficients \((p_1,p_2) = (0.35, 0.65)\).](image-url)
Moreover, solar photovoltaics are chosen for both cities for this case. Electricity generation from RES is important as it can reduce the primary energy consumption significantly. The annual generation from PV in Athens is some 3,856 MJ/year while in Edinburgh is only 2,938 MJ/year which shows that Athens has much higher potential for utilizing solar energy.

### 7.2 The Case of Retrofitting an Existing Building

In this case, the examined building that has been used in the previous section is assumed to be an old existing building in the broader area of Edinburgh and Athens respectively. As mentioned before, the proposed components for each category and the components of the existing buildings are presented from Table B.1 up to Table B.18. It is noted that the existing building has low energy efficient components and is uninsulated.

Using the developed software tool for the case of retrofitting an existing building, the primary energy consumption of the existing building has first been calculated. The total annual primary energy consumption savings for the existing building when is located in Edinburgh and Athens is calculated to be 600,369 MJ/year and 290,801 MJ/year respectively.

The obtained results from the simulations are presented in Table 6. The values of the decision variables for each working pair calculated after the optimizations can be seen analytically in Appendix “C” in the Table C.5 up to the Table C.8. It is noted that the software tool took 4.9 and 4.2 minutes to run the simulations for the case of Edinburgh and Athens respectively. For comparison purposes with the case of a new building in Table 6 is also presented the primary energy consumption after the retrofit actions for the building placed in the city of Edinburgh and Athens respectively.
Table 6: Values of the primary energy consumption savings and the initial investment cost for each working pair of weight coefficients for the case of retrofitting an existing building in Edinburgh (EDI) and Athens (ATH)

<table>
<thead>
<tr>
<th>$\min[u(g_1(x),g_2(x))]$</th>
<th>$p_1$</th>
<th>$p_2$</th>
<th>Primary Energy Consumption Savings (MJ/year)</th>
<th>Primary Energy Consumption (MJ/year)</th>
<th>Initial Investment Cost (£)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EDI</td>
<td>ATH</td>
<td>EDI</td>
<td>ATH</td>
<td>EDI</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>540,687</td>
</tr>
<tr>
<td>0.029</td>
<td>0.039</td>
<td>0.950</td>
<td>0.050</td>
<td>0.050</td>
<td>535,593</td>
</tr>
<tr>
<td>0.040</td>
<td>0.053</td>
<td>0.900</td>
<td>0.100</td>
<td>0.100</td>
<td>513,246</td>
</tr>
<tr>
<td>0.041</td>
<td>0.067</td>
<td>0.850</td>
<td>0.150</td>
<td>0.150</td>
<td>497,410</td>
</tr>
<tr>
<td>0.040</td>
<td>0.072</td>
<td>0.800</td>
<td>0.200</td>
<td>0.200</td>
<td>495,933</td>
</tr>
<tr>
<td>0.039</td>
<td>0.070</td>
<td>0.750</td>
<td>0.250</td>
<td>0.250</td>
<td>495,933</td>
</tr>
<tr>
<td>0.038</td>
<td>0.068</td>
<td>0.700</td>
<td>0.300</td>
<td>0.300</td>
<td>495,933</td>
</tr>
<tr>
<td>0.037</td>
<td>0.066</td>
<td>0.650</td>
<td>0.350</td>
<td>0.350</td>
<td>495,933</td>
</tr>
<tr>
<td>0.036</td>
<td>0.063</td>
<td>0.600</td>
<td>0.400</td>
<td>0.400</td>
<td>495,933</td>
</tr>
<tr>
<td>0.035</td>
<td>0.060</td>
<td>0.550</td>
<td>0.450</td>
<td>0.450</td>
<td>488,830</td>
</tr>
<tr>
<td>0.033</td>
<td>0.056</td>
<td>0.500</td>
<td>0.500</td>
<td>0.500</td>
<td>480,872</td>
</tr>
<tr>
<td>0.028</td>
<td>0.050</td>
<td>0.450</td>
<td>0.550</td>
<td>0.550</td>
<td>470,262</td>
</tr>
<tr>
<td>0.025</td>
<td>0.047</td>
<td>0.400</td>
<td>0.600</td>
<td>0.600</td>
<td>470,262</td>
</tr>
<tr>
<td>0.021</td>
<td>0.042</td>
<td>0.350</td>
<td>0.650</td>
<td>0.650</td>
<td>470,262</td>
</tr>
<tr>
<td>0.017</td>
<td>0.036</td>
<td>0.300</td>
<td>0.700</td>
<td>0.700</td>
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<tr>
<td>0.014</td>
<td>0.029</td>
<td>0.250</td>
<td>0.750</td>
<td>0.750</td>
<td>470,262</td>
</tr>
<tr>
<td>0.010</td>
<td>0.022</td>
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<td>0.800</td>
<td>0.800</td>
<td>470,262</td>
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<tr>
<td>0.007</td>
<td>0.015</td>
<td>0.150</td>
<td>0.850</td>
<td>0.850</td>
<td>470,262</td>
</tr>
<tr>
<td>0.003</td>
<td>0.007</td>
<td>0.100</td>
<td>0.900</td>
<td>0.900</td>
<td>470,262</td>
</tr>
<tr>
<td>0.000</td>
<td>0.000</td>
<td>0.050</td>
<td>0.950</td>
<td>0.950</td>
<td>470,262</td>
</tr>
</tbody>
</table>

From Table C.5 up to the Table C.8 it is shown that when the primary energy consumption criterion is independently minimized the components with the best energy behaviour are selected, which is similar to the analysis presented in the previous chapter. In this case the initial investment cost is lower as the wall structure, floor structure and ceiling structure are not included in the retrofit actions. It is noted that the differences between Edinburgh and Athens that existed in the previous chapter still apply. For instance, when the energy criterion is independently minimized the components of the building envelope with the lowest $U_{value}$ e.g. in Edinburgh the door number 2, window number 2, insulation material number 1 and so forth are selected, whilst in Athens window number 3 is again selected in all cases. The same energy systems as in the case of a new building have also been selected.

On the other hand, when the cost criterion is minimized independently, the components with the lowest initial investment cost are selected (e.g. door number 1, insulation number 3 etc.), and the energy systems that were chosen in the case of the new building, which are the same for both cities. Solar photovoltaics and solar collector systems are not selected for this case. It is observed that when the primary energy consumption savings criterion is more important the components with the best energy behaviour are selected, but as the cost criterion gets more

...
important the cheaper components are selected, which confirms the general hypothesis and
showing that the methodology and the developed software is robust. This means that the more
you invest in energy efficient measures the higher the energy savings are and the lower the
primary energy consumption becomes, which is similar to the case of a new building
presented in the previous section.

The minimal and the maximum total initial investment cost of the components is £6,335 and
£19,085 respectively in Edinburgh and £6,335 and £18,310 respectively in Athens. The
minimal and the maximum primary energy consumption savings in Edinburgh are 470,262
MJ/year (78%) and 540,687 MJ/year (90%) respectively which means that the primary energy
consumption is between 59,682 MJ/year to 130.107 MJ/year. In Athens the minimal and the
maximum primary energy consumption savings are 171,035 MJ/year (59%) and 229,875
MJ/year (79%) respectively, resulting in primary energy consumption between 60,927
MJ/year to 119,766 MJ/year. In the case of retrofitting an existing building the initial
investment cost is lower than the case of the new building presented in the previous chapter
because the wall structure, floor structure and ceiling structure are not included in the retrofit
actions.

The results for the other weight coefficients working pairs are in-between those values. The
Pareto frontier diagram shown in Figure 6 for the case of the retrofitting an existing building
in Edinburgh and Athens respectively represents all the optimal solutions. It can be indicated
that also in this case the initial investment cost and primary energy consumption of a building
are inversely proportional.
The possibility of primary energy consumption savings is high in both cities but is greater in Edinburgh than in Athens due to the differences in climate characteristics. This is indicated with further analysis of the energy demand and of the primary energy consumption of each category. Figure 7 and Figure 8 present the energy demand and the primary energy consumption of the existing building before and after the retrofit actions in Edinburgh and Athens for the case of working weight coefficients $(p_1, p_2)=(0.95, 0.05)$ and $(p_1, p_2)=(0.85, 0.15)$ respectively. Those working pairs have been chosen as they belong in the area with the most preferred optimal solutions. It can be seen that when the best energy efficient measures are selected the energy demand is reduced significantly, resulting in high primary energy savings.

![Figure 6: The Pareto Frontier using the weighted sum method for the case study of retrofitting an existing building in Edinburgh](image)

![Figure 7: Annual Energy Demand before and after the retrofit for an existing building in (a) Edinburgh and (b) Athens, for the case of working weight coefficient $(p_1,p_2)=(0.55, 0.45)$](image)
Figure 8: Annual primary energy consumption before and after the retrofit for an existing building in (a) Edinburgh and (b) Athens, for the case of working weight coefficient \((p_1, p_2) = (0.55, 0.45)\) and \((p_1, p_2) = (0.65, 0.35)\) for Edinburgh and Athens respectively.

8 Conclusions

8.1 Main Findings

The minimization of energy demand and primary energy consumption in the building sector is essential in order to reduce the energy consumption in the overall energy supply chain and lead to sustainability in buildings. The reduction of primary energy in buildings will also contribute in the achievement of the policy goals set by the UK Government and the European Commission by the EPBD. Moreover, if less primary energy is used from fossil fuels then the carbon dioxide emissions would also get reduced.

The scope of the present article is to expand a previously developed methodology to optimally prioritize energy efficiency measures in terms of their energy behaviour and the initial cost and also develop a software tool to be used by a DM. The methodology is generic and can be used in order to optimally prioritize the energy efficiency measures for the case of a new building and for the case of retrofitting an existing one. As described in Section 3, the proposed methodology is depended on previous work with a more limited number of energy efficiency measures and it was further expanded to take into account more categories of energy efficiency measures, and also to analyse the case of retrofitting an existing building. Many criteria exist to assess the energy efficiency measures but in the current article only the primary energy consumption and the initial investment cost have been used, resulting in a multi-objective optimisation problem.

Moreover, two software tools have been developed to allow the DM to propose energy efficiency measures and prioritize them according to his own preferences, in the case of a new building and of retrofitting an existing one. In order to solve the MINLP multi-objective
problem the weighted sum method has been used and the ‘bonmin’ algorithm has been chosen. The software tools have been examined in two case studies, each for a new and existing building, and they have been proven to be robust and time efficient. The analysis showed that the more someone invests in energy efficiency the lower the primary energy consumption becomes. Hence, a DM according to his own preferences can find the most preferable solution from the provided Pareto front.

8.2 Proposals for Future Work

As previously mentioned, the two decision criteria used in this methodology are the primary energy consumption and the initial investment cost. However, there are also many other criteria that refer to energy efficiency measures (e.g. life cycle cost or operating cost). A DM would probably be more interested in reducing the operating costs and his bills. Moreover, environmental criteria could be also used such as the carbon dioxide emissions. As the climate is one of the major challenges the planet faces a software tool that takes into account the life cycle cost of the components and the carbon dioxide emissions or the global warming potential might be preferable. It must be noted that our software tool could be expanded to being capable of dealing with more than two objective functions (criteria).

Another constraint of the developed methodology is that it assumes that the loads are constant, i.e. it is a steady-state approach. A methodology that would examine the energy demand variations on a time basis would provide more accurate results but it would be more difficult to solve. Also, the software tool can be further expanded to include wind energy or CHP units and more categories of electrical appliances. Moreover, the software tool can be further developed and become a software package in a more compact form that could be executed independently without the need of a DM having ‘Matlab®’ installed.

9 References


## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACH</td>
<td>Air changes per hour</td>
<td>h(^{-1})</td>
</tr>
<tr>
<td>ASLC</td>
<td>Area of a solar collector</td>
<td>m(^2)</td>
</tr>
<tr>
<td>A(_{win})</td>
<td>Area of a window</td>
<td>m(^2)</td>
</tr>
<tr>
<td>BLC</td>
<td>Building load factor</td>
<td>W/K</td>
</tr>
<tr>
<td>COSTCEIL</td>
<td>Initial investment cost for ceilings</td>
<td>£</td>
</tr>
<tr>
<td>COSTCS</td>
<td>Initial investment cost for a cooling system</td>
<td>£</td>
</tr>
<tr>
<td>COSTDOR</td>
<td>Initial investment cost for doors</td>
<td>£</td>
</tr>
<tr>
<td>COSTEA</td>
<td>Initial investment cost for electrical appliances</td>
<td>£</td>
</tr>
<tr>
<td>COSTFLO</td>
<td>Initial investment cost for floors</td>
<td>£</td>
</tr>
<tr>
<td>COSTHCS</td>
<td>Initial investment cost for a heating-cooling system</td>
<td>£</td>
</tr>
<tr>
<td>COSTHS</td>
<td>Initial investment cost for a heating system</td>
<td>£</td>
</tr>
<tr>
<td>COSTHWS</td>
<td>Initial investment cost for a heating-DHW system</td>
<td>£</td>
</tr>
<tr>
<td>COSTLIGHT</td>
<td>Initial investment cost for lamps</td>
<td>£</td>
</tr>
<tr>
<td>COSTRES</td>
<td>Initial investment cost for a RES power system</td>
<td>£</td>
</tr>
<tr>
<td>COSTSLC</td>
<td>Initial investment cost for a solar collector</td>
<td>£</td>
</tr>
<tr>
<td>COSTWAL</td>
<td>Initial investment cost for walls</td>
<td>£</td>
</tr>
<tr>
<td>COSTWIN</td>
<td>Initial investment cost for windows</td>
<td>£</td>
</tr>
<tr>
<td>c(_{air})</td>
<td>Specific heat of air at constant pressure</td>
<td>kJ/kg/K</td>
</tr>
<tr>
<td>c(_{wat})</td>
<td>Specific heat of water at constant pressure</td>
<td>kJ/kg/K</td>
</tr>
<tr>
<td>CS(_{m})</td>
<td>Indicator for cooling demand each month</td>
<td>-</td>
</tr>
<tr>
<td>F(_{cm,wn})</td>
<td>Window correction factor for movable devices</td>
<td>%</td>
</tr>
<tr>
<td>F(_{F,wn})</td>
<td>Frame factor of a window</td>
<td>%</td>
</tr>
<tr>
<td>f(_{grid})</td>
<td>Percentage of electricity supply from the grid</td>
<td>%</td>
</tr>
<tr>
<td>F(_{s})</td>
<td>Shading factor of a solar collector</td>
<td>%</td>
</tr>
<tr>
<td>F(_{s,wn})</td>
<td>Shading factor of a window</td>
<td>%</td>
</tr>
<tr>
<td>f(_{use})</td>
<td>Factor indicating the usage of a device each day</td>
<td>h/day</td>
</tr>
<tr>
<td>h(_{i})</td>
<td>Indoors combined convection-radiation coefficient</td>
<td>W/m(^2)K</td>
</tr>
<tr>
<td>h(_{o})</td>
<td>Outdoors combined convection-radiation coefficient</td>
<td>W/m(^2)K</td>
</tr>
<tr>
<td>HS(_{m})</td>
<td>Indicator for heating demand each month</td>
<td>-</td>
</tr>
<tr>
<td>I(_{SL})</td>
<td>Solar radiation</td>
<td>kWh/m(^2)/day</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>k</td>
<td>Thermal conductivity</td>
<td>W/mK</td>
</tr>
<tr>
<td>l</td>
<td>Thickness of a material</td>
<td>M</td>
</tr>
<tr>
<td>$\dot{m}_w$</td>
<td>Daily need of hot water</td>
<td>L/day</td>
</tr>
<tr>
<td>$n_{ECS,ecs}^{EHS}$</td>
<td>Efficiency of an electric system $ecsj$ of category $ecsi$</td>
<td>%</td>
</tr>
<tr>
<td>$n_{ehs,ehsj}^{EHS}$</td>
<td>Efficiency of an electric system $ehsj$ of category $ehsi$ for heating</td>
<td>%</td>
</tr>
<tr>
<td>$n_{ehcs,ehcsj}^{EHC}$</td>
<td>Efficiency of an electric system $ehcsj$ of category $ehcsi$ for heating-cooling</td>
<td>%</td>
</tr>
<tr>
<td>$n_{ehws,ehwsj}^{EWS}$</td>
<td>Efficiency of an electric system $ehwj$ of category $ehwi$ used for heating-DHW</td>
<td>%</td>
</tr>
<tr>
<td>$n_{ews,ewsj}^{EWS}$</td>
<td>Efficiency of an electrical system $ewj$ of category $ewi$ for DHW</td>
<td>%</td>
</tr>
<tr>
<td>$n_{grid}$</td>
<td>Average efficiency of power generation of the grid</td>
<td>%</td>
</tr>
<tr>
<td>$n_{NEHS,nehs}^{NEHS}$</td>
<td>Efficiency of a non-electric system $nehsj$ of category $nehsi$ used for heating</td>
<td>%</td>
</tr>
<tr>
<td>$n_{nehs,nehsj}^{NEHS}$</td>
<td>Efficiency of a non-electric system $nehsj$ of category $nehsi$ used for heating-DHW</td>
<td>%</td>
</tr>
<tr>
<td>$n_{ews,ewsj}^{NEWS}$</td>
<td>Efficiency of a non-electric system $ewsj$ of category $ewsi$ used for DHW</td>
<td>%</td>
</tr>
<tr>
<td>$n_{stc,stcj}^{SLC}$</td>
<td>Efficiency of a non-electric system $stcj$ of category $stci$ used for heating-DHW</td>
<td>%</td>
</tr>
<tr>
<td>$n_{tot}$</td>
<td>Total efficiency of a CHP unit</td>
<td>%</td>
</tr>
<tr>
<td>$P_L$</td>
<td>Power Rate of a Lamp</td>
<td>W</td>
</tr>
<tr>
<td>$q'$</td>
<td>Heat Flux</td>
<td>W/m²</td>
</tr>
<tr>
<td>$Q_C$</td>
<td>Annual primary energy consumption for cooling</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q^{CD}$</td>
<td>Annual cooling demand</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{el}^{C}$</td>
<td>Annual primary energy consumption for cooling consumed by an electrical system</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{DHW}$</td>
<td>Annual primary energy consumption for DHW</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{dSL,C,m}$</td>
<td>Energy provided by a solar collector for DHW</td>
<td>MJ/month</td>
</tr>
<tr>
<td>$Q_{ea}$</td>
<td>Heat emitted from appliances</td>
<td>W</td>
</tr>
<tr>
<td>$Q^H$</td>
<td>Annual primary energy consumption for heating</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{net,fuel}^H$</td>
<td>Annual primary energy consumption for heating from a non-electrical system using a specific fuel</td>
<td>MJ/year</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>$Q_{el}^{H}$</td>
<td>Annual primary energy consumption for heating from an electrical system</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q^{HD}$</td>
<td>Annual heating energy demand</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{human}$</td>
<td>Heat emitted from people</td>
<td>W</td>
</tr>
<tr>
<td>$Q_{INHG,m}$</td>
<td>Internal heat gain each month</td>
<td>kWh/month</td>
</tr>
<tr>
<td>$Q_{lamp}$</td>
<td>Electricity consumption of a lamp</td>
<td>kWh/day</td>
</tr>
<tr>
<td>$Q_{L}$</td>
<td>Annual primary energy consumption for lighting</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{LD}$</td>
<td>Annual energy demand for lighting</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{Sl,m}$</td>
<td>Solar heat gain each month</td>
<td>kWh/month</td>
</tr>
<tr>
<td>$Q_{TM,m}$</td>
<td>Heat transmission losses each month</td>
<td>kWh/month</td>
</tr>
<tr>
<td>$Q_{vm,m}$</td>
<td>Ventilation losses each month</td>
<td>kWh/month</td>
</tr>
<tr>
<td>$Q_{el}^{W}$</td>
<td>Annual primary energy consumption of an electrical system for DHW</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q_{net,fuel}^{W}$</td>
<td>Annual primary energy consumption for DHW by a non-electrical system</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$Q^{WD}$</td>
<td>Annual energy demand for DHW</td>
<td>MJ/year</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>°C</td>
</tr>
<tr>
<td>$t_{d}$</td>
<td>Duration of a month in days</td>
<td>days/month</td>
</tr>
<tr>
<td>$T_{DCW}$</td>
<td>Temperature of cold water inlet to the DHW system</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{DHW}$</td>
<td>Supply temperature of hot water by the DHW system</td>
<td>°C</td>
</tr>
<tr>
<td>$T_{HI}$</td>
<td>Internal design temperature for heating season</td>
<td>°C</td>
</tr>
<tr>
<td>$t_{L}$</td>
<td>Operation time of a lamp</td>
<td>h/day</td>
</tr>
<tr>
<td>$t_{m}$</td>
<td>Month duration in hours</td>
<td>h/month</td>
</tr>
<tr>
<td>$T_{o,m}$</td>
<td>Average air temperature of each month</td>
<td>°C</td>
</tr>
<tr>
<td>$U$</td>
<td>Overall heat transfer coefficient</td>
<td>W/m²K</td>
</tr>
<tr>
<td>$V$</td>
<td>Internal volume of the building</td>
<td>m³</td>
</tr>
<tr>
<td>$WS_{m}$</td>
<td>Indicator for DHW demand each month</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{d}^{DOOR}$</td>
<td>Decision variable for doors</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{ea_l,ea_j}^{EA}$</td>
<td>Decision variable of electric appliance $ea_l$ of category $ea_i$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{ecs_l,ecs_j}^{ECS}$</td>
<td>Decision variable for an electrical cooling system $ecs_j$ of categories $ecs_i$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{ehs_l,ehs_j}^{EHS}$</td>
<td>Decision variable for an electrical heating system $ehs_j$ of categories $ehs_i$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{ehcsl,ehcs_j}^{EHCS}$</td>
<td>Decision variable for an electrical heating-cooling system</td>
<td>binary</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td>Unit</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>$x_{EHWS}$</td>
<td>Decision variable for an electrical heating-DHW system $ehwsj$ of category $ehws$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{h}$</td>
<td>Decision variable for floor structure $h$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{L_{i,j}}$</td>
<td>Decision variable of lamp $lj$ of category $li$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{NEHS}$</td>
<td>Decision variable for a non-electrical heating system $nehsj$ of categories $nehi$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{NEHWS}$</td>
<td>Decision variable for a non-electrical heating-DHW system $nehwsj$ of category $nehwi$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{CEIL}$</td>
<td>Decision variable for ceiling structure $r$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{RES}$</td>
<td>Decision variable for a RES energy system $resj$ of category $resi$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{SLC}$</td>
<td>Decision variable for a solar collector system $slcj$ of category $slci$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{WIN}$</td>
<td>Decision variable for windows type $z$</td>
<td>binary</td>
</tr>
<tr>
<td>$x_{WALL}$</td>
<td>Decision variable for wall structure $w$</td>
<td>binary</td>
</tr>
<tr>
<td>$\rho_{air}$</td>
<td>Air density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_{wat}$</td>
<td>Water density</td>
<td>kg/m$^3$</td>
</tr>
</tbody>
</table>
Appendix “A”: Equations of the model

A.1 Decision Variables

A.1.1 Building Envelope

a. Doors

Let \( D \) be the available number of alternative type of doors. A decision variable \( x^D_{d,DOOR} \) where, \( d = 1, ..., D \), is defined such as:

\[
    x^D_{d,DOOR} = \begin{cases} 
    1 , & \text{if door type } d \text{ is selected} \\
    0 , & \text{else}
    \end{cases}
\] (A.1)

It is assumed that the available proposed doors are of the same type and only one can be selected, which leads to the following constraint:

\[
    \sum_{d=1}^{D} x^D_{d,DOOR} = 1
\] (A.2)

b. Windows

Let \( Z \) be the available number of alternative type of windows (e.g. double glaze, low-e) where each consists of \( T_z \) sub-types (e.g. xenon-filled, vacuum-filled). A decision variable \( x^W_{zt,WIN} \) where \( z = 1, ..., Z \) and \( t = 1, ..., T_z \) is defined such as:

\[
    x^W_{zt,WIN} = \begin{cases} 
    1 , & \text{if window sub-type of type } z \text{ is selected} \\
    0 , & \text{else}
    \end{cases}
\] (A.3)

It is assumed that the available window types are of the same type and only one may be selected, which leads to the following constraint:

\[
    \sum_{z=1}^{Z} \sum_{t=1}^{T_z} x^W_{zt,WIN} = 1
\] (A.4)

c. Walls
Let $W$ be the available number of alternative types of structures of wall structures. A decision variable $x_w^{\text{WALL}}$ where $w = 1, ..., W$ is defined such as:

$$
x_w^{\text{WALL}} = \begin{cases} 1, & \text{if wall structure } w \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.5)
$$

It is assumed that from the available wall structures only one may be selected, which leads to the following constraint:

$$
\sum_{w=1}^{W} x_w^{\text{WALL}} = 1 
$$

(A.6)

Furthermore, each wall structure consists of $NWL_w$ number of known layers (with $nwl = 1, ..., NWL_w$). The materials of these layers have specific thermal conductivities $k_{wk,nwl}^{\text{WALL}} (W/mK)$ and thicknesses $t_{w,nwl}^{\text{WALL}} (m)$.

Also let $Y_w$ (with $y = 1, ..., Y_w$) be the number of unknown layers (e.g. insulation) layer where their materials have to be chosen between the available ones. For each unknown layer $y$ of structure $w$ there are $P_{wy}$ (with $p=1, ..., P_{wy}$) alternative materials available and only one is allowed to be chosen for the respected structure. Therefore, the following decision variable and constraint are defined:

$$
x_{wy}^{mWALL} = \begin{cases} 1, & \text{if material } p \text{ is selected for layer } y \text{ of wall structure } w \\ 0, & \text{else} \end{cases} \quad (A.7)
$$

$$
\sum_{p=1}^{P_{wy}} x_{wy}^{mWALL} = x_w^{\text{WALL}} \quad \forall \left( y = 1, \ldots, Y_w \quad \forall \quad w = 1, \ldots, W \right) \quad (A.8)
$$

The thickness of the unknown layers of materials $Y_w$ is considered to be predefined. Also, each of material $c$ of layer $y$ of wall structure $w$ has $k_{wy,yc}^{mWALL} (W/mK)$ thermal conductivity and $t_{w,nc}^{WALL}$ thickness.

d. Ceilings

Similarly to walls, let $R$ be the number of available alternative structures of ceilings. A binary decision variable $x_r^{\text{CEIL}}$ where $r = 1, ..., R$ is defined such as:

$$
x_r^{\text{CEIL}} = \begin{cases} 1, & \text{if ceiling structure } r \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.9)
$$
Also, it is assumed that only one ceiling structure may be selected from all the proposed ceiling structures, which leads to the following constraint:

\[
\sum_{r=1}^{R} x_{r}^{\text{CEIL}} = 1
\]  

(A.10)

Also, let \( NCL_r \) be the number of the known layers of the ceiling structure \( r \) with \( ncl = 1, ..., NCL_r \). The materials of these layers have specific thermal conductivities \( k_{r,ncl}^{\text{CEIL}} \) (\( W/m/K \)) and thicknesses \( t_{r,ncl}^{\text{CEIL}} \) (\( m \)) which are already known.

Also there is a number \( F_r \) (with \( f = 1, ..., F_r \)) of unknown layers where their materials have to be chosen between the available ones. For each unknown layer \( f \) of structure \( r \) there are \( A_{rf} \) (with \( a = 1, ..., A_{rf} \)) alternative materials available and one can be selected for the chosen structure. Therefore, the following decision variable and constraint are defined:

\[
x_{r,f,a}^{\text{mCEIL}} = \begin{cases} 1, & \text{if material } \alpha \text{ is selected for layer } f \text{ of ceiling structure } r \\ 0, & \text{else} \end{cases}
\]  

(A.11)

\[
\sum_{a=1}^{A_{rf}} x_{r,f,a}^{\text{mCEIL}} = x_{r}^{\text{CEIL}} \quad \forall \left( f = 1, ..., F_r \quad \forall \quad r = 1, ..., R \right)
\]  

(A.12)

The thickness of the unknown layers of materials \( F_r \) is considered to be predefined. Also, each of material \( a \) of layer \( f \) of ceiling structure \( r \) has \( k_{rf,a}^{\text{mCEIL}} \) (\( W/m/K \)) thermal conductivity and \( t_{df,a}^{\text{mCEIL}} \) thickness.

e. Floors

Similarly to the approach for walls and ceilings, let \( H \) be the number of available alternative structures of floors, which leads to the decision variable \( x_{h}^{\text{FLO}} \) where \( h = 1, ..., H \) is defined as:

\[
x_{h}^{\text{FLO}} = \begin{cases} 1, & \text{if floor structure } h \text{ is selected} \\ 0, & \text{else} \end{cases}
\]  

(A.13)

Also, it is assumed that only one floor structure may be selected from all the proposed floor structures, leading to the following constraint:
Each floor structure \( h \) consists of \( NFL_h \) layers (with \( nfl = 1, ..., NFL_h \)). The materials of these layers have specific thermal conductivities \( k k_{h,nfl}^{IFLO} \ (W/m \ K) \) and thicknesses \( l_{h,nfl}^{IFLO} \ (m) \) are already known.

Also there is a number \( E_h \) (with \( e = 1, ..., E_h \)) of unknown layers where their materials have to be chosen between the available ones. For each unknown layer \( e \) of structure \( h \) there are \( G_{he} \) alternative materials available and one can be selected for the chosen structure. Therefore, the following decision variable and constraint are defined:

\[
X_{mFLO}^{hheg} = \begin{cases} 
1, & \text{if material } g \text{ is selected for layer } e \text{ of floor structure } h \\
0, & \text{else} 
\end{cases} \tag{A.15}
\]

\[
\sum_{g=1}^{G_{he}} X_{mFLO}^{hheg} = X_{FLO}^{h} \forall (e = 1, ..., E_h) \forall (h = 1, ..., H) \tag{A.16}
\]

The thickness of the unknown layers of materials \( E_h \) is considered to be predefined. Also, each of material \( g \) of layer \( e \) of floor structure \( h \) has \( k_{mFLO}^{mFLO} \ (W/m \ K) \) thermal conductivity and \( l_{mFLO}^{mFLO} \) thickness.

### A.1.2 Building’s energy systems

The energy systems categories that are assumed to be available in this methodology are:

- Heating systems: Provide only heating and can be electrical or non-electrical systems which are further categorized according to their input fuel;
- Cooling systems: Provide only cooling (in this approach only electrical systems are assumed to be available);
- DHW systems: Provide only hot water. They can be electrical or non-electrical, which are further categorized according to their input fuel;
- Heating – cooling systems: Provide both space heating and cooling (only electrical systems are assumed to be available);
- Heating – DHW systems: Provide both space heating and DHW supply. They can be electrical or non-electrical which are further categorized according to their input fuel;
- Solar collector systems: Supply DHW by utilizing solar energy;
Electricity generation systems: Provide electricity using RES.

The decision variables regarding the above systems are defined as follows:

Let $EHSI$ be the available categories of electrical heating systems which include $EHS_{ehs_i}$ systems, and let $NEHSI$ be the available categories of non-electrical heating systems including $NEHS_{nehs_i}$ different systems, where $ehs_i = 1, ..., EHSI$, $ehs_j = 1, ..., EHS_{ehs_i}$, $nehs_i = 1, ..., NEHSI$, $nehs_j = 1, ..., NEHS_{nehs_i}$. Then the binary decision variables defined are:

$$
x_{EHS, ehs_i, ehs_j} = \begin{cases} 1, & \text{if an electrical heating system } ehs_j \text{ of category } ehs_i \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.17)
$$

$$
x_{NEHS, nehs_i, nehs_j} = \begin{cases} 1, & \text{if a non-electrical heating system } nehs_j \text{ of category } nehs_i \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.18)
$$

Let $ECSI$ be the available categories of electrical cooling systems which include $ECS_{ecsi}$ systems where $ecsi = 1, ..., ECSI$ and $ecsj = 1, ..., ECS_{ecsi}$:

$$
x_{ECS, ecsi, ecsj} = \begin{cases} 1, & \text{if an electrical cooling system } ecsj \text{ of category } ecsi \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.19)
$$

Let $EWSI$ be the available categories of electrical DHW systems categories which includes $EWS_{ewsi}$ systems and the let $NEWSI$ be the available categories of non-electrical DHW systems consisting of $NEWS_{newsi}$ different systems, where $ewsi = 1, ..., EWSI$, $ewsj = 1, ..., EWS_{ewsi}$, $newsi = 1, ..., NEWSI$ and $newsj = 1, ..., NEWS_{newsi}$:

$$
x_{EWS, ewsi, ewsj} = \begin{cases} 1, & \text{if a DHW system } ewsj \text{ of category } ewsi \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.20)
$$

$$
x_{NEWS, newsi, newsj} = \begin{cases} 1, & \text{if a non-electrical DHW system } newsj \text{ of category } newsi \text{ is selected} \\ 0, & \text{else} \end{cases} \quad (A.22)
$$
Let $EHCSI$ be the available categories of electrical heating - cooling systems consisting of $EHCSI_{ehcsi}$ systems, where $ehcsi = 1, ..., EHCSI$ and $ehcsj = 1, ..., EHCSI_{ehcsj}$:

$$
EHCS_{x_{ehcsi, ehcsj}} = \begin{cases} 
1, & \text{if an electrical heating-cooling system of category $ehcsi$ is selected} \\
0, & \text{else}
\end{cases}
$$

(A.23)

Let $EHWSI$ be the available categories of electrical heating – DHW systems consisting of $EHWSI_{ehwsi}$ systems and let $NEHWSI$ be the available categories of non-electrical heating–DHW systems consisting of $NEHWSI_{nehwsi}$ different systems, where $ehwsi = 1, ..., EHWSI$, $ehwsj = 1, ..., EHWSI_{ehwsi}$, $nehwsi = 1, ..., NEHWSI$ and $nehwsj = 1, ..., NEHWSI_{nehwsi}$:

$$
EHWS_{x_{ehwsi, ehwsj}} = \begin{cases} 
1, & \text{if an electrical heating – DHW system of category $ehwsi$ is selected} \\
0, & \text{else}
\end{cases}
$$

(A.24)

$$
NEHWS_{x_{nehwsi, nehwsj}} = \begin{cases} 
1, & \text{if a non-electrical heating – DHW system of category $nehwsi$ is selected} \\
0, & \text{else}
\end{cases}
$$

(A.25)

Let $SLCI$ be the available categories of solar collector systems consisting of $SLCI$ different systems, where $slci = 1, ..., SLCI$ and $slcj = 1, ..., SLCJ$:

$$
SLC_{x_{slci, slcj}} = \begin{cases} 
1, & \text{if solar collector of category $slci$ is selected} \\
0, & \text{else}
\end{cases}
$$

(A.26)

Let $RESI$ be the available categories of RES electricity generation systems consisting of $RESI$ different systems, where $resi = 1, ..., RESI$ and $resj = 1, ..., RESI$:

$$
RES_{x_{resi, resj}} = \begin{cases} 
1, & \text{if RES power generation system of category $resi$ is selected} \\
0, & \text{else}
\end{cases}
$$

(A.27)

Some of the systems described above could belong into more than one categories. Therefore, some additional constraints are required in order to allow for the selection of only one system for each purpose:
Space heating system amongst those available:

\[
\begin{align*}
&\text{EHS} \sum_{ehzi=1}^{EH} x_{ehzi, ehzj} + \sum_{nehzi=1}^{NEHS} x_{nehzi, nehzj} + \sum_{ehczi=1}^{EHCS} x_{ehczi, ehczej} \\
&\text{EHSJS} \sum_{ehwsi=1}^{EHWS} x_{ehwsi, ehwsj} + \sum_{nehwsi=1}^{NEHWS} x_{nehwsi, nehwjsj} = 1
\end{align*}
\] (A.28)

Space cooling system amongst those available:

\[
\begin{align*}
&\text{ECS} \sum_{eczzi=1}^{EC} x_{eczzi, eczj} + \sum_{nehwsi=1}^{NEHWS} x_{nehwsi, nehwjsj} = 1
\end{align*}
\] (A.29)

DHW system amongst those available:

\[
\begin{align*}
&\text{EWS} \sum_{ewzi=1}^{EW} x_{ewzi, ewzj} + \sum_{newzi=1}^{NEWS} x_{newzi, newzj} + \sum_{ehwsi=1}^{EHWS} x_{ehwsi, ehwsj} \\
&\text{NEHWS} \sum_{nehwsi=1}^{NEHWS} x_{nehwsi, nehwjsj} = 1
\end{align*}
\] (A.30)

Solar collector system to provide DHW amongst those available if would be beneficial to choose one:

\[
\sum_{slzi=1}^{SCL} \sum_{slzj=1}^{SCL} x_{slzi, slzj} \leq 1
\] (A.31)

RES electricity system amongst those available if one would be beneficial. It is noted that it is assumed that the building would be connected to the grid:

\[
\sum_{rezi=1}^{RE} x_{rezi, rezj} \leq 1
\] (A.32)

A.1.3 Lighting systems

Let \(LI\) be the number of available categories of lighting systems, consisting of \(LJL\) types of lamps. Then the decision variable \(x_{li, lj}^L\), where \(li = 1, \ldots, LI\) and \(lj = 1, \ldots, LJL\) is defined such as:

\[
x_{li, lj}^L = \begin{cases} 
1, & \text{if lamp type } lj \text{ of category } li \text{ is selected} \\
0, & \text{else}
\end{cases}
\] (A.33)
Assuming that from the available lamps only one can be selected the following constraint is defined:

$$\sum_{li=1}^{L} \sum_{lj=1}^{L} x_{li, lj} = 1$$  \hspace{1cm} (A.34)

### A.1.4 Electrical appliances

Let $EAI$ be the number of available categories electrical appliances consisting $EAI$ types of appliances available. Then the decision variable $x^{EA}_{eai, eaj}$, where $eai = 1, \ldots, EAI$ and $eaj = 1, \ldots, EAI_{eai}$, is defined such as:

$$x^{EA}_{eai, eaj} = \begin{cases} 1, & \text{if the electric appliance } eaj \text{ is selected of category } eai \\ 0, & \text{else} \end{cases}$$  \hspace{1cm} (A.35)

Assuming that from the available electrical appliances only one can be selected for each category the following constraints are defined:

$$\sum_{eaj=1}^{EAI_{eai}} x^{EA}_{eai, eaj} = 1$$  \hspace{1cm} (A.36)

$$\sum_{eai=1}^{EAI} \sum_{eaj=1}^{EAI_{eai}} x^{EA}_{eai, eaj} = EAI, \ \forall (eaj = 1, \ldots, EAI_{eai} \ \forall eai = 1, \ldots, EAI)$$  \hspace{1cm} (A.37)

### A.2 Constraints

#### A.2.1 Primary Energy consumption

The total annual primary energy consumption in a building is the primary energy used for heating, cooling, DHW, lighting and electrical appliances [32]:

$$Q_T = Q_H + Q_C + Q_{DHW} + Q_L + Q_A$$  \hspace{1cm} (A.38)

#### A.2.1.1 Primary Energy Consumption for Heating

Annually, the total annual primary energy consumption for heating would be equal to:
\[ Q_H = \frac{Q_{grid}^H f_{grid}}{n_{grid}} + \sum_{fuel=1}^{FUEL} Q_{fuel,fuel}^H \]  

(A.39)

where:

- \( Q_{el}^H \): Energy consumed by an electrical system for heating purposes (MJ/year)
- \( f_{grid} \): Percentage of electricity supply from the grid (RES electricity supply does not contribute to primary energy consumption).
- \( n_{el} \): The average efficiency for the electricity supply from the grid to the building (it is assumed to be 0.35 [16])
- \( Q_{fuel,fuel}^H \): Energy consumed by a non-electrical system using a fuel (where fuel = 1, ..., FUEL) (MJ/year)

The energy consumed by an electrical and a non-electrical system can be calculated as:

\[ Q_{el}^H = Q^{HD} SEH_{el} \]  

(A.40)

\[ Q_{fuel,fuel}^H = Q^{HD} SEH_{fuel,fuel} \]  

(A.41)

where:

- \( Q^{HD} \): The total annual heating energy demand (MJ/year)

\[ SEH_{el} = \sum_{ehsi=1}^{EHSI} \sum_{ehsj=1}^{EHSJ} \left( \frac{X_{ehsi, ehsj}}{n_{ehsi, ehsj}} \right) + \sum_{ehci=1}^{ECHSI} \sum_{ehcj=1}^{ECHSJ} \left( \frac{X_{ehci, ehcj}}{n_{ehci, ehcj}} \right) + \sum_{ehwi=1}^{EHWISI} \sum_{ehwj=1}^{EHWWSJ} \left( \frac{X_{ehwi, ehwj}}{n_{ehwi, ehwj}} \right) \]  

(A.42)

\[ SEH_{fuel,fuel} = \sum_{nehsi=1}^{NEHSI} \sum_{nehsj=1}^{NEHSJ} \left( \frac{X_{nehsi, nehsj}}{n_{nehsi, nehsj}} \right) + \sum_{nehwi=1}^{NEHWISI} \sum_{nehwj=1}^{NEHWWSJ} \left( \frac{X_{nehwi, nehwj}}{n_{nehwi, nehwj}} \right) \]  

(A.43)

\( SEH_{el} \) and \( SEH_{fuel,fuel} \): The efficiency of the chosen system for heating

- \( n_{ehsi, ehsj}^{EHS} \), \( n_{ehci, ehcj}^{ECH} \), \( n_{ehwi, ehwj}^{EHW} \): The efficiency (%) of the electrical systems \( ehsi, ehci \) and \( ehwi \) of the respected categories \( ehsi, ehcsi \) and \( ehwi \)

- \( n_{nehsi, nehsj}^{NEHS} \), \( n_{nehwi, nehwj}^{NEHW} \): The efficiency (%) of the non-electrical systems \( nehsj \) and \( nehwj \) of the respected categories \( nehsi \) and \( nehwi \)

The annual heating demand can be calculated by summing the demand of each month:
The heating demand for each month is equal to the sum of heat losses, i.e. monthly transmission \( Q_{T,m} \) (kWh/month) and ventilation losses \( Q_{VEN,m} \) (kWh/month), minus internal heat gains \( Q_{INHG,m} \) (kWh/month) and solar gains \( Q_{SL,m} \) (kWh/month) [16]. Regarding the solar gains only the direct solar gains from window are taken into account and not the indirect (such the absorbance of solar radiation of the walls) despite they might offer a small heat gain [40]:

\[
Q_{ID}^m = \begin{cases} 
H S_m F_{conv} \left( Q_{T,m} + Q_{VEN,m} - Q_{INHG,m} - Q_{SL,m} \right), & \text{if positive} \\
0, & \text{else}
\end{cases} 
\]  

(A.45)

\[
Q_{T,m} = B L C \left( T_{IH} - T_{o,m} \right) t_m 
\]  

(A.46)

\[
Q_{VEN,m} = \rho_{air} c_{pair} ACH \cdot V \cdot (T_{IH} - T_{o,m}) t_m / 3600 
\]  

(A.47)

\[
Q_{INHG,m} = \left( n_{people} Q_{people,m} + Q_{each,m} \right) t_m 
\]  

(A.48)

\[
Q_{SL,m} = \sum_{wn=1}^{WN} \left( a^{WIN}_{wn} F_{F,wn} F_S,wn F_{CM,wn} I_{SL,wn,m} t_m \right) \sum_{z=1}^{Z} \sum_{i=1}^{Z} \left( x^{WIN}_{zI} g^{WIN}_{zI} \right) 
\]  

(A.49)

where:

- \( H S_m \): Parameter indicating if heating is required for month \( m \) (binary variable with values 1 or 0)
- \( F_{conv} \): conversion factor (MJ/kWh)
- \( B L C \): Building load coefficient (W/K)
- \( T_{IH} \): Internal design temperature for heating season (K)
- \( T_{o,m} \): Average external temperature of month \( m \) (K)
- \( t_m \): Month duration in hours (h/month)
- \( t_d \): Month duration in days (days/month)
- \( \rho_{air} \): Air density (kg/m\(^3\))
- \( c_{pair} \): Specific heat of air (kJ/kg K)
- \( ACH \): Air changes per hour (h\(^{-1}\))
- \( V \): Internal Volume of the Building (m\(^3\))
Moreover the BLC of a building can be calculated as:

\[
BLC = \sum_{com} A_{com} U_{com} b_{com}
\]

(A.50)

Where

com: is a building envelope component

\(A_{com}\): surface area \((m^2)\)

\(U_{com}\): total heat transfer coefficient \((W/m^2K)\)

\(b_{com}\): temperature correction factor, between 0 for unheated surfaces (e.g. floors or basements) and 1 for components that face outside air

In detail the BLC is equal to:

\[
BLC = \sum_{dr=1}^{DR} \left( A_{DOOR_{dr}} b_{DOOR_{dr}} \right) \sum_{d=1}^{D} \left( x_{DOOR_{d_{d}}} U_{DOOR_{d_{d}}} \right) + \sum_{wn=1}^{WN} \left( A_{WIN_{wn}} b_{WIN_{wn}} \right) \sum_{z=1}^{Z} \sum_{r=1}^{R} \left( x_{WIN_{wn}} U_{WIN_{wn}} \right)
\]

\[
+ \sum_{n=1}^{WL} \left( A_{WALL_{w_n}} b_{WALL_{w_n}} \right) \sum_{w=1}^{W} \left( x_{WALL_{w_{w}}} U_{WALL_{w_{w}}} \right) + \sum_{ce=1}^{CE} \left( A_{CELL_{c_e}} b_{CELL_{c_e}} \right) \sum_{r=1}^{R} \left( x_{CELL_{c_e}} U_{CELL_{c_e}} \right)
\]

\[
+ \sum_{fr=1}^{FL} \left( A_{FLO_{f_h}} b_{FLO_{f_h}} \right) \sum_{h=1}^{H} \left( x_{FLO_{f_h}} U_{FLO_{f_h}} \right)
\]

(A.51)

In this methodology the overall heat transfer coefficient \(U_{total}\) is used in order to take into account the phenomenon of heat transfer by convection and radiation mechanisms. For doors and windows the manufacturers usually provide the \(U_{value}\) instead of the thermal conductivity and the thickness. For multi-layer components the calculation of the total heat transfer coefficient \((W/m^2K)\) takes into account the thickness of each layer, the thermal
conductivity and the inside and outside heat convection coefficient to air. Therefore the following equations are used:

$$U_{\text{Door}}^\text{DOOR} = \left( \frac{1}{h_i} + \frac{1}{U_{\text{value, door}}} + \frac{1}{h_o} \right)^{-1}$$  \hspace{1cm} (A.52)$$

$$U_{\text{Win}}^\text{WIN} = \left( \frac{1}{h_i} + \frac{1}{U_{\text{value, win}}} + \frac{1}{h_o} \right)^{-1}$$  \hspace{1cm} (A.53)$$

$$U_{\text{W}}^{\text{WALL}} = \left( \frac{1}{h_i} + \sum_{m=1}^{N_{\text{WALL}}} \left( \frac{F_{\text{WALL}}}{k_{\text{WALL}, m}} \right) + \sum_{y=1}^{N_{\text{WALL}}} \sum_{p=1}^{P_{\text{WALL}}} \left( \frac{F_{\text{WALL}}}{k_{\text{WALL}}^{\text{WALL}, y,p}} \right) + \frac{1}{h_o} \right)^{-1}$$  \hspace{1cm} (A.54)$$

$$U_{\text{Ceil}}^{\text{CEIL}} = \left( \frac{1}{h_i} + \sum_{nc=1}^{N_{\text{CEIL}}} \left( \frac{F_{\text{CEIL}}}{k_{\text{CEIL}, nc}} \right) + \sum_{f=1}^{F_{\text{CEIL}}} \sum_{a=1}^{A_{\text{CEIL}}} \left( \frac{F_{\text{CEIL}}}{k_{\text{CEIL}}^{\text{CEIL}, f,a}} \right) + \frac{1}{h_o} \right)^{-1}$$  \hspace{1cm} (A.55)$$

$$U_{\text{Flo}}^{\text{FLO}} = \left( \frac{1}{h_i} + \sum_{nf=1}^{N_{\text{FLO}}} \left( \frac{F_{\text{FLO}}}{k_{\text{FLO}, nf}} \right) + \sum_{e=1}^{E_{\text{FLO}}} \sum_{g=1}^{G_{\text{FLO}}} \left( \frac{F_{\text{FLO}}}{k_{\text{FLO}}^{\text{FLO}, e,g}} \right) + \frac{1}{h_o} \right)^{-1}$$  \hspace{1cm} (A.56)$$

where $h_i$ and $h_o$ represent the combined convection radiation coefficients ($W/m^2K$)

A.2.1.2  Primary Energy Consumption for Cooling

Similarly to the heating energy consumption calculations the total annual primary energy consumption for cooling can be calculated as:

$$Q_C = \frac{Q_{el}^C \cdot f_{\text{grid}}}{n_{\text{grid}}}$$  \hspace{1cm} (A.57)$$

Where:

$Q_{el}^C$: Energy consumed by an electrical system used for cooling (MJ/year)

The energy consumed by an electrical system can be calculated as:

$$Q_{el}^C = Q^{CD}SEC_{el}$$  \hspace{1cm} (A.58)$$

where:

$Q^{CD}$: The total annual cooling energy demand (MJ/year)
The efficiency of the chosen system providing cooling energy is denoted by $\eta_{ECSi, ECSj}$, $\eta_{EHCSI, EHCSE}$:

$$\eta_{ECSi, ECSj} \times \eta_{EHCSI, EHCSE}$$

The total annual cooling energy demand can be calculated by summing the cooling energy demand of each month:

$$Q^{CD} = \sum_{m=1}^{12} Q_{m}^{CD}$$

The cooling energy demand for each month is equal to the sum of heat losses, i.e. monthly transmission $Q_{TM, m}$ (kWh/month) and ventilation losses $Q_{VEN, m}$ (kWh/month), minus internal heat gains $Q_{INHG, m}$ (kWh/month) and solar gains $Q_{SL, m}$ (kWh/month). The calculation of cooling energy demand is similar to the one for heating energy demand, but in this case the solar-air temperature is used which takes into account the effect of solar radiation on the outside temperature [27]:

$$Q_{m}^{CD} = \left\{ \begin{array}{ll}
CS_{m} F_{com} \left( Q_{INHG, m} + Q_{SL, m} - Q_{TM, m} - Q_{VEN, m} \right), & \text{if positive} \\
0, & \text{else}
\end{array} \right.$$  \hspace{1cm} (A.61)

$$Q_{VEN, m} = \rho_{air} c_{pair} ACH \cdot V \cdot (T_{IC} - T_{o, m}) t_{m} / 3600$$

$$+ \rho_{air} h_{fg} ACH \cdot V \cdot (w_{i} - w_{o, m}) t_{m} / 3600$$  \hspace{1cm} (A.62)

$$Q_{TM, m} = BLC \left( T_{IC} - T_{sol-air, m} \right) t_{m}$$  \hspace{1cm} (A.63)

$$T_{sol-air, m} = T_{o, m} + \frac{a \cdot \dot{q}_{sol}}{h_{o}}$$  \hspace{1cm} (A.64)

$CS_{m}$: Parameter indicating if heating is required for month $m$ (binary variable)

$T_{IC}$: Internal design temperature for cooling season (K)

$h_{fg}$: latent heat of vaporization (usually 2340 kJ/kg)

$w_{i}$: Specific humidity indoors ($\text{kg}_{\text{wat}}/\text{kg}_{\text{air}}$)
Specific humidity outdoors (kg\text{wat}/kg\text{air})

Sol-air temperature (K)

Absorptivity of the material

Solar radiation (W/m²)

A.2.1.3 Primary Energy Consumption for Domestic Hot Water

The total annual primary energy consumption for DHW supply would be equal to:

\[ Q_{\text{DHW}} = \frac{Q_{\text{grid}}^W}{n_{\text{grid}}} + \sum_{\text{fuel}=1}^{\text{FUEL}} Q_{\text{nel, fuel}}^W \]  

(A.65)

where:

\( Q_{\text{el}}^W \): Energy consumed by a DHW system using electricity (MJ/year)

\( Q_{\text{nel, fuel}}^W \): Energy consumed by a DHW system using a fuel, \( f_{\text{uel}} = 1,...,\text{FUEL} \) (MJ/year)

The energy consumption of an electrical and a non-electrical system can be calculated as:

\[ Q_{\text{el}}^W = Q_{\text{WD SEW}_{\text{el}}}^W \]  

(A.66)

\[ Q_{\text{nel, fuel}}^W = Q_{\text{WD SEH}_{\text{nel, fuel}}}^W \]  

(A.67)

where:

\( Q_{\text{WD}}^W \): The total annual energy demand for DHW (MJ/year)

\[ SEW_{\text{el}} = \sum_{\text{ews}=1}^{\text{ews}} \sum_{\text{ewsj}=1}^{\text{ewsj}} \left( \frac{x_{\text{ews}, \text{ewsj}}^{\text{EWS}}}{n_{\text{ews}, \text{ewsj}}^{\text{EWS}}} + \sum_{\text{ehw}=1}^{\text{ehw}} \sum_{\text{ehwj}=1}^{\text{ehwj}} \left( \frac{x_{\text{ehw}, \text{ehwj}}^{\text{EHWS}}}{n_{\text{ehw}, \text{ehwj}}^{\text{EHWS}}} \right) \right) \]  

(A.68)

\[ SEW_{\text{nel, fuel}} = \sum_{\text{news}=1}^{\text{news}} \sum_{\text{newsj}=1}^{\text{newsj}} \left( \frac{x_{\text{news}, \text{newsj}}^{\text{NEWS}}}{n_{\text{news}, \text{newsj}}^{\text{NEWS}}} \right) + \sum_{\text{nehw}=1}^{\text{nehw}} \sum_{\text{nehwj}=1}^{\text{nehwj}} \left( \frac{x_{\text{nehw}, \text{nehwj}}^{\text{NEHWS}}}{n_{\text{nehw}, \text{nehwj}}^{\text{NEHWS}}} \right) \]  

(A.69)

\( SEW_{\text{el}} \) and \( SEW_{\text{nel, fuel}} \): The efficiency of the chosen system providing hot water

\( n_{\text{ews}, \text{ewsj}}^{\text{EWS}} \), \( n_{\text{ehw}, \text{ehwj}}^{\text{EHWS}} \): The efficiency (%) of the electrical systems \( \text{ewsj} \) and \( \text{ehwj} \) of the respected categories \( \text{ews} \) and \( \text{ehw} \)
$n^{NEWS}_{newsi, newsj}$, $n^{NEHW}_{nehwi, nehwj}$: Denotes the generation efficiency (%) of the non-electrical systems newsj and nehwsj of the respected categories newsi and nehwi

The annual DHW energy demand can be calculated by summing the demand of each month:

\[ Q^{WD} = \sum_{m=1}^{12} (DQ^{DHW}_m) \quad (A.70) \]

The net DHW demand for each month is equal to the average monthly hot water demand minus the energy a solar collector system provides (in case one is selected):

\[ DQ^{DHW}_m = \begin{cases} \text{WS}_m F_{\text{conv}} (Q_{\text{dhwu}, m} - Q_{\text{dSLC}, m}), & \text{if } Q_{\text{dhwu}, m} \geq Q_{\text{dSLC}, m} \\ 0, & \text{else} \end{cases} \quad (A.71) \]

Where:

$\text{WS}_m$: Parameter indicating if DHW is required for month $m$ (binary variable)

$Q_{\text{dhwu}, m}$: Average monthly demand for DHW supply (MJ/month) calculated as:

\[ Q_{\text{dhwu}, m} = \dot{m}_w \rho_w c_{p_w} (T_{\text{DHW}} - T_{\text{DCW}, m}) t_m \quad (A.72) \]

$\dot{m}_w$: Rate of consumption of hot water at each day ($m^3/s$)

$T_{\text{DHW}}$: The base temperature set for the DHW system (K)

$T_{\text{DCW}, m}$: The temperature of the cold water supply at month $m$ (K)

$\rho_w$: The water density ($kg/m^3$)

$c_{p_w}$: Specific heat of water ($kJ/kg K$)

$Q_{\text{dSLC}, m}$: The monthly hot water demand (MJ/month) provided from a solar collector system (in case one is selected)

\[ Q_{\text{dSLC}, m} = F_{\text{conv}} A_{\text{SLC}} F_s \cdot \sum_{\text{SLC}} I_{\text{SLC}, \text{SLC}, m} \cdot \sum_{\text{SCI}} I_{\text{SCI}, \text{SCI}, m} \cdot n_{\text{SCI}, \text{SCI}}^{\text{SLC}} \cdot n_{\text{SLC}, \text{SLC}}^{\text{SCI}} \quad (A.73) \]

$A_{\text{SLC}}$: Area of solar collector ($m^2$)

$F_s, \text{SLC}$: Correction factor for shading (%)

$I_{\text{SLC}, \text{SLC}, m}$: Solar radiation incident on a solar collector type $slcj$ of category $slci$, under a specific tilt and orientation (kwh/m²/day)

$n_{\text{SCI}, \text{SCI}}^{\text{SLC}}$: Efficiency of a solar collector type $slcj$ of the category $slci$ (%)
A.2.1.4 Primary Energy Consumption for Lighting

The total annual primary energy consumption for lighting purposes is calculated as:

\[ Q_{el} = \frac{Q_{grid}^L}{n_{grid}} \]  

(A.74)

where:

- \( Q_{el}^L \): is the annual electrical energy consumed for lighting (MJ/year)

The electrical energy consumption for providing lighting is equal to:

\[ Q_{el}^L = Q_{LD}^D \mathrm{SEL}_{el} \]  

(A.75)

where:

- \( Q_{LD}^D \): Total annual demand for electricity for lighting (MJ/year)
- \( \mathrm{SEL}_{el} = 1 \), assuming no losses of electricity from supply to consumption

The annual energy demand for lighting can be calculated by summing the demand of each month:

\[ Q_{LD}^D = \sum_{m=1}^{12} Q_{LD}^{ID_m} \]  

(A.76)

It is assumed that the lamps would be operating the same number of hours each day and consequently all the months of the year. The energy consumption of lamps can be calculated as:

\[ Q_{m}^{LD} = F_{cont} t_d \sum_{l=1}^{L} \left( P_{L, l \_use} f_{use, l} \right) \sum_{ll=1}^{L} \sum_{lj=1}^{L} x_{li, lj} \]  

(A.77)

\( l = 1, ..., L \): Number of lamps
\( P_{L, l \_use} \): Lamp power rating (kW)
\( f_{use, l} \): Time that the device is used (h/day)

A.2.1.5 Primary Energy Consumption for Electrical appliances

The total annual primary energy consumption for the operation of the electrical appliances is calculated as:
Where:

\[ Q_{d} = \frac{Q_{e}^{\text{grid}}}{n_{\text{grid}}} \]  
(A.78)

\[ Q_{d}^{\text{AD}} \] \text{ is the annual energy (electricity) consumed for operation of electrical appliances (MJ/year)}

The electrical energy consumed for operation of electrical appliances is:

\[ Q_{d}^{\text{AD}} = Q_{d}^{\text{AD}} \times \text{SEA}_{d} \]  
(A.79)

Where:

\[ Q_{d}^{\text{AD}} \] : Total annual demand for electricity for operation of electrical appliances (MJ/year)

\[ \text{SEA}_{d} = 1, \text{ assuming no losses of electricity from supply to consumption} \]

The annual energy demand for the operation of electrical appliances can be calculated by summing the demand of each month:

\[ Q_{\text{AD}}^{\text{AD}} = \sum_{m=1}^{12} Q_{m}^{\text{AD}} \]  
(A.80)

It is assumed that the electrical appliances would be operating the same number of hours each day and consequently all the months of the year. The energy consumption of electrical appliances can be calculated as:

\[ Q_{m}^{\text{AD}} = F_{\text{com}} \times t_{d, m} \sum_{e_{d}}^{E_{d}} P_{A, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}, e_{d}} \]  
(A.81)

\[ P_{A,a} \] : Electric appliance power rate (W)

\[ f_{\text{use},a} \] : Time that the device is used (h/day)

\[ f_{\text{load},a} \] : Load factor of the device (%)

\[ A.2.1.6 \quad \text{Electricity supply} \]

The total annual demand for electrical energy is equal to the electricity consumption for heating, cooling, DHW, lighting, and operation of the electrical appliances:
The annual electricity demand of the electrical systems consists of the average demand for electricity supply from the grid $Q_{el,grid}$, reduced by the electricity provided by a RES system $Q_{el,alt}$, in case one is selected and is operating:

$$Q_{el,alt} = \sum_{resi=1}^{RES} \sum_{resj=1}^{RES_{resi}} Q_{el, resi, resj} x^{RES}_{resi, resj}$$  \hspace{1cm} (A.83)

Where

$Q_{el,rest, resj}$: electricity generation from a RES system $resj$ of category $resi$ (MJ/year)

The renewable sources that could be used to provide electricity are solar energy (photovoltaic systems) or wind energy (wind turbines). Moreover, it is further assumed that all the electricity generated from RES would be either used in the building or exported to the grid [32]. Therefore, the total supply from the grid would be equal to:

$$Q_{el, grid} = \begin{cases} (Q_{EL}^{D} - Q_{el, alt}) & \text{if } Q_{EL}^{D} > Q_{el, alt} \\ 0, \text{else} \end{cases}$$  \hspace{1cm} (A.84)

A.2.2 Initial Investment Cost

As it was mentioned before, several approaches regarding the cost have been made in such models. Similarly to [16] in this model the initial investment cost is used which is defined as the initial cost of acquisition of the components and the cost of installation. The initial investment cost for the proposed components would be equal to:

$$\text{INVCOST} = \text{COST}_{DOR} + \text{COST}_{WIN} + \text{COST}_{WAL}$$  
$$+ \text{COST}_{CEIL} + \text{COST}_{FLO} + \text{COST}_{HS}$$  
$$+ \text{COST}_{CS} + \text{COST}_{WS} + \text{COST}_{HCS}$$  
$$+ \text{COST}_{HWS} + \text{COST}_{SLC} + \text{COST}_{RES}$$  
$$+ \text{COST}_{LIGHT} + \text{COST}_{EA}$$  \hspace{1cm} (A.85)

Independently, the cost for each component can be calculated as:
\[
COST_{DOR} = \sum_{dr=1}^{D} \left( A_{DOR}^{dr} \sum_{d=1}^{D} (x_d^{DOR} C_d^{DOR}) \right) \quad (A.86)
\]

\[
COST_{WIN} = \sum_{wn=1}^{WN} \left( A_{WIN}^{wn} \sum_{t=1}^{T_n} (x_{st}^{WIN} C_{st}^{WIN}) \right) \quad (A.87)
\]

\[
COST_{WAL} = \sum_{wl=1}^{WAL} \left( A_{WAL}^{wl} \sum_{w=1}^{W} \left( \sum_{wn=1}^{NW} (x_{wn}^{WAL} C_{wn}^{WAL}) \right) + \sum_{y=1}^{W} \left( \sum_{p=1}^{P_y} (x_{mWALL}^{WAL} C_{mWALL}^{WAL}) \right) \right) \quad (A.88)
\]

\[
COST_{CEIL} = \sum_{ce=1}^{CEIL} \left( A_{CEIL}^{ce} \sum_{d=1}^{D} \left( \sum_{ncl=1}^{NCL} (x_{ncl}^{CEIL} C_{ncl}^{CEIL}) + \sum_{f=1}^{F_x} \left( \sum_{g=1}^{G_y} (x_{rfja}^{CEIL} C_{rfja}^{CEIL}) \right) \right) \right) \quad (A.89)
\]

\[
COST_{FLO} = \sum_{fl=1}^{FL} \left( A_{FLO}^{fl} \sum_{h=1}^{H} \left( \sum_{nfl=1}^{NFL} (x_{nfl}^{FLO} C_{nfl}^{FLO}) + \sum_{e=1}^{E_x} \left( \sum_{g=1}^{G_y} (x_{mFLO}^{FLO} C_{mFLO}^{FLO}) \right) \right) \right) \quad (A.90)
\]

\[
COST_{HS} = \sum_{ehsi=1}^{EHS} \sum_{ehsj=1}^{EHS} \left( x_{ehsi,ehsj}^{EHS} C_{ehsi,ehsj}^{EHS} \right) + \sum_{nehs=1}^{NEHS} \sum_{nehsj=1}^{NEHS} \left( x_{nehs,nehsj}^{NEHS} C_{nehs,nehsj}^{NEHS} \right) \quad (A.91)
\]

\[
COST_{CS} = \sum_{ecsi=1}^{ECS} \sum_{ectj=1}^{ECS} \left( x_{ecsi,ectj}^{ECS} C_{ecsi,ectj}^{ECS} \right) \quad (A.92)
\]

\[
COST_{WS} = \sum_{ewsi=1}^{EWS} \sum_{ewsj=1}^{EWS} \left( x_{ewsi,ewsj}^{EWS} C_{ewsi,ewsj}^{EWS} \right) + \sum_{newsi=1}^{NEWS} \sum_{newsj=1}^{NEWS} \left( x_{newsi,newsj}^{NEWS} C_{newsi,newsj}^{NEWS} \right) \quad (A.93)
\]

\[
COST_{HCS} = \sum_{ehcsi=1}^{EHCS} \sum_{ehcsj=1}^{EHCS} \left( x_{ehcsi,ehcsj}^{EHCS} C_{ehcsi,ehcsj}^{EHCS} \right) \quad (A.94)
\]

\[
COST_{HWS} = \sum_{ehwsi=1}^{EHWS} \sum_{ehwsj=1}^{EHWS} \left( x_{ehwsi,ehwsj}^{EHWS} C_{ehwsi,ehwsj}^{EHWS} \right) + \sum_{neshws=1}^{NEHWS} \sum_{neshwsj=1}^{NEHWS} \left( x_{neshws,neshwsj}^{NEHWS} C_{neshws,neshwsj}^{NEHWS} \right) \quad (A.95)
\]
\[
\text{COST}_{\text{SLC}} = \sum_{\text{slc}1}^{\text{SLC}} \sum_{\text{slj}1}^{\text{SLC}} \left( x_{\text{slc1,slj1}}^{\text{SLC}} \times \text{CST}^{\text{SLC}}_{\text{slc1,slj1}} \right) \tag{A.96}
\]

\[
\text{COST}_{\text{RES}} = \sum_{\text{resi}=1}^{\text{RES}} \sum_{\text{resj}=1}^{\text{RES}} \left( x_{\text{resi,rezj}}^{\text{RES}} \times \text{CST}^{\text{RES}}_{\text{resi,rezj}} \right) \tag{A.97}
\]

\[
\text{COST}_{\text{LIGHT}} = 1 \sum_{\text{l1}=1}^{\text{L}} \sum_{\text{l2}=1}^{\text{L}} \left( x_{\text{l1,l2}}^{\text{L}} \times \text{CST}^{\text{L}}_{\text{l1,l2}} \right) \tag{A.98}
\]

\[
\text{COST}_{\text{EA}} = \sum_{\text{eai}=1}^{\text{EA}} \sum_{\text{eaj}=1}^{\text{EA}} \left( x_{\text{eai,eaj}}^{\text{EA}} \times \text{CST}^{\text{EA}}_{\text{eai,eaj}} \right) \tag{A.99}
\]

Where,

- \( C^\text{DOOR}_v \): the initial investment cost for a door of type \( d \) (£/m²)
- \( C^\text{WIN}_{zt} \): the initial investment cost for a window of sub-type \( t \) of type \( z \) (£/m²)
- \( CK^\text{mWALL}_{w,\text{mwl}} \), \( CK^\text{mCEIL}_{r,\text{ncl}} \), \( CK^\text{mFLO}_{h,\text{nfl}} \): the initial investment costs for the materials used in the known layers \( \text{mwl} \), of wall structure \( w \), \( \text{ncl} \) of ceiling structure \( r \) and \( \text{nfl} \) layers of floor structure \( h \) (£/m²)
- \( C^\text{mWALL}_{wp} \), \( C^\text{mCEIL}_{af} \), \( C^\text{mFLO}_{eg} \): the initial investment costs for the material \( p \) that is used in the unknown layer \( y \) of wall structure \( w \), the material \( a \) that is used in the unknown layer \( f \) of ceiling structure \( r \) and the material \( g \) that is used in the unknown layer \( e \) of floor structure \( h \) (£/m²)
- \( \text{CST}^{\text{EHS}}_{\text{ehsi},\text{ehsj}} \), \( \text{CST}^{\text{NEHS}}_{\text{ehsi},\text{ehsj}} \): the initial investment cost for the electrical heating system \( \text{ehsj} \) of category \( \text{ehsi} \) and the non-electrical heating system \( \text{nehsj} \) of category \( \text{nehsi} \) (£)
- \( \text{CST}^{\text{ECS}}_{\text{ecsi},\text{ecsj}} \): the initial investment cost for the electrical cooling system \( \text{ecsj} \) of category \( \text{ecsi} \) (£)
- \( \text{CST}^{\text{EWS}}_{\text{ewsi},\text{ewsj}} \), \( \text{CST}^{\text{NEWS}}_{\text{newsi},\text{newsj}} \): the initial investment cost for the electrical DHW system \( \text{ewsj} \) of category \( \text{ewsi} \) and the non-electrical DWH system \( \text{newsj} \) of category \( \text{newsi} \) (£)
- \( \text{CST}^{\text{EHCS}}_{\text{ehcisi},\text{ehcjsj}} \): the initial investment cost for the electrical heating-cooling system \( \text{ehcsj} \) of category \( \text{ehcsi} \) (£)
A.3 Objective Functions

A.3.1 The Case of a New Building

In order to determine the optimal prioritization of the energy efficiency measures in a new building, the primary energy consumption and the initial investment cost criteria must be minimized according to the procedure described in subsections A.2.1 and A.2.2 respectively:

\[
\begin{align*}
\min g_1(x) &= Q_T \\
\min g_2(x) &= \text{INVCOST}
\end{align*}
\]  
(A.100)

Subject to

Constraints: (A.1) - (A.99)

A.3.2 The Case of Retrofitting an Existing Building

For the case of retrofitting an existing building the methodology is similar to that of a new building. However, in this case the objectives would be to achieve maximum primary energy savings with minimal initial investment cost. Therefore, the primary energy consumption of the existing building before any retrofit action must be calculated.

The primary energy consumption of an existing building is calculated with the methodology described in subsection A.2.1. However, in this case there are no decision variables. Also, the constraints regarding the components of the building’s envelope as were set in subsection A.2.1 might not apply as an existing building theoretically might have more than one type of doors, windows etc. The procedure used in Section A.2.1 is followed similarly:
The total annual primary energy consumption of an existing building can be calculated using equation (A.38) and is equal to:

$$Q_{T_{\text{pre}}} = Q_H + Q_C + Q_{DHW} + Q_L + Q_A$$  \hspace{1cm} (A.101)

Where:

- $Q_{T_{\text{pre}}}$: The annual primary energy consumption before any retrofit action.

Moreover, it is assumed that an existing building before retrofit would not have RES to provide electrical energy.

To calculate the primary energy consumption after the retrofit actions ($Q_{T_{\text{post}}}$) on a building the procedure in subsection A.2.1 is followed again. Moreover, it is assumed that in the case of retrofitting an existing building the wall, ceilings and floor structures would not be changed. Hence, the decision variables (A.5), (A.9) and (A.10) have already value equal to 1. Insulation layers may exist in some components but also they could be applied to the other components.

The initial investment cost of the components for retrofitting a building, represents the cost acquisition and installation of the proposed components and can be calculated similarly to subsection A.2.2. The variables $CK^{mWALL}_{w,kel}$, $CK^{mCEIL}_{d,kcl}$ and $CK^{mFLO}_{d,kfl}$ are equal to 0 as they already exist.

The criteria to find the best solution are the energy savings and the initial cost of the investment. In this case energy savings must be maximized and the investment cost must be minimized:

$$\max \left[g_1(x)\right] = Q_{T_{\text{pre}}} - Q_{T_{\text{post}}}$$  

$$\min \left[g_2(x)\right] = \text{INVCOST}$$  \hspace{1cm} (A.102)

Subject to

Constraints: (A.1) - (A.99), except those excluded in this subsection.

Given that $Q_{T_{\text{pre}}}$ is a constant parameter of the model, the first objective function of (101) is actually equivalent to the first objective function of (99).

Appendix “B”

Available online.
Appendix “C”

Available online.