Correlations between oil and stock markets: A wavelet-based approach

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ABSTRACT
In a global economy, shocks occurring in one market can spill over to other markets. This paper investigates the impact of oil shocks and stock markets crashes on correlations between stock and oil markets. We test changes in correlations for different time scales with non-overlapping confidence intervals based on estimated wavelet correlations. Our results indicate that correlation between oil and stock markets tends to be stable in non-shock periods, around zero, but this changes during oil and financial shocks both at higher and lower frequencies. We find evidence of contagion, in particular during the 2008 and 2011 stock market falls. At low frequencies, the number of correlation breakdowns during oil shocks and stock market crashes is higher and they can be interpreted as shifts in market co-movements.

JEL classification: C40; E32; G15; F30

Keywords: Contagion; Correlations; Financial shocks; Interdependence; International financial markets; Oil shocks; Stock market returns; Wavelets.

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I. Introduction

The keystone of both portfolio allocation and risk management decisions is the correlation structure of asset returns. Therefore, understanding the dynamics of correlations remains an important task not only for financial research but also for applications in the financial industry.

The time-varying nature of correlations has been widely documented (see e.g. Cai et al., 2009; De Santis and Gerard, 1997; Longin and Solnik, 1995, 2001) and stock market crashes or currency crises seem to impact the correlations between international stock markets. Crashes can create contagion in many markets, increasing correlation between them over very short periods of time. Yet, they can also contribute to greater market integration by increasing the interdependence or co-movements between markets over longer periods of time (see Bodart and Candelon, 2009; Gallegati, 2012; Reboredo and Rivera-Castro, 2014).

Recent work has used a frequency domain approach to study this issue where each frequency corresponds to a particular component of the variable. Bodart and Candelon (2009) use the framework of a Vector Autoregressive (VAR) model and propose a contagion test based on a causality measure applied at different frequencies. Orlov (2009) uses the finite Fourier transform without assuming any model for the data. Fourier’s analysis decomposes the covariance into different frequency levels. Contagion is estimated as the change of the high-frequency components of the covariance between crisis and non-crisis periods. Gallegati (2012) advocates the use of the multi-resolution decomposition property of the wavelet transform to identify contagion and interdependence separately by associating each with its corresponding frequency components. He proposes using the information of the high frequency part to test for contagion, and low frequency components to analyze interdependence. Finally, Ftiti et al. (2014) uses evolutionary cospectral analysis (Priestley and Tong, 1973) and wavelets to analyze the co-movements dynamics between OCDE countries, the U.S. and Europe. They find that these two sophisticated techniques are both powerful in distinguishing between contagion and interdependence and specially in accessing co-movements between time series regarding time and frequency.

This study focuses on oil and stock market shocks and on their impact on the correlation
structure between oil and stock markets. Oil is a crucial input in economic activity, and consequently oil price increases are an hindrance to economic growth. Stock markets, as recognized bellwethers of the economy (see e.g. Fama, 1990; Fama and French, 1989; Schwert, 1990), anticipate the fall in corporate cash flow and adjust firm value. Since each country’s degree of oil dependence is different, the impact of oil price increases depends on national oil level dependence (Ramos and Veiga, 2013). Therefore, changes in stock market valuation differ between countries, changing correlation within markets.

Recently, the financialization of commodity markets has increased the links between oil and stock markets. Including commodities as an asset class in portfolios has become increasingly attractive for financial investors, backed by innovation in derivative securities. Many argue that with the opening of commodity markets to financial investors, commodity markets become increasingly driven by flows of financial investors and less by their fundamentals, strengthening the links between these markets.\(^1\) Empirical evidence has documented an increase in correlation between commodity and equity markets (see e.g., Büyüksahin and Robe, 2014; Silvennoinen and Thorp, 2013).

Given the links between markets, we posit that price disruptions in one market are likely to affect other markets and we investigate whether oil price shocks and stock market crashes have an impact on stock market and oil market correlations. Note that the focus of our work is on whether the correlation between those markets changes and not on the direct impact of oil shocks in stock market returns (see e.g. Driesprong et al., 2008; Chen et al., 1986; Huang et al., 1996; Jones and Kaul, 1996; Kilian and Park, 2009; Narayan and Sharma, 2011; Narayan and Gupta, 2015; Ramos and Veiga, 2013).

To analyze this issue, we follow the framework of Gallegati (2012) that uses wavelets\(^2\) to

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\(^1\)We refer the reader to the works of Basak and Pavlova (2014); Büyüksahin and Robe (2014); Domanski and Heath (2007); Hamilton and Wu (2015); Irwin and Sanders (2012); Ramos and Veiga (2014); Silvennoinen and Thorp (2013); Singleton (2013); Tang and Xiong (2012).

\(^2\)Wavelets are an alternative way of analyzing time series that are increasingly popular because they are based on elegant new mathematical results and efficient computational algorithms. By using wavelet methods in the context of multiresolution analysis, one can examine the series on a variety of scales. Different types of behavior, such as trends, cycles or extremes, may become evident at different levels of resolution. Unlike Fourier basis functions, which are only localized in frequency, wavelets are local both in frequency, via dilatations, and in...
distinguish between contagion and interdependence. He uses information of the high frequency part to test for contagion, and low frequency components to analyze interdependence. The tests are graphical, based on non-overlapping confidence intervals of estimated wavelet coefficients calculated in shock and non-shock periods. We present the results with a new visualization tool, where the confidence intervals of different periods are shown above the time line. The plot easily represents changes in correlation over time by visually checking the overlap between two consecutive periods. Other advantages of the wavelets methodology are its ability to handle irregular data without imposing any functional form, to decompose the series by time scale so as to capture relationships between variables that may differ across time–scales, to detect sudden regime changes and isolated shocks (Ramsey, 1999) and to allow the components of a non-stationary series to be analyzed. In comparison to the Fourier transform, the wavelet transform is an improved version of it.

In an influential study, Forbes and Rigobon (2002) note that heteroskedasticity biases contagion tests based on correlation coefficients. They show that it is not appropriate to look at unadjusted correlation coefficients, as the computed correlation coefficient is an increasing function of the variance of the underlying asset return, so that when coefficients between a tranquil period and a crisis period are compared, the coefficient in the crisis period is biased upwards, as volatility rises substantially. However, Corsetti et al. (2005) argue that this finding is a result of an assumed underlying unrealistic model and Bartram and Wang (2005) report that these biases come from assumptions about the stochastic process of stock returns. Furthermore, using multivariate GARCH models might also lead to bias estimation of correlations if trends are not accommodated for. Narayan and Liu (2015) show that a missing trend in a GARCH(1,1) model in the context of energy variables leads to model misspecification and the majority of the energy time, via translations. Additionally, many classes of functions can be represented via fewer terms with wavelet transforms than with Fourier transforms. Functions with discontinuities and sharp spikes usually require fewer wavelet basis functions than Fourier basis functions. This sparse representation makes wavelets an excellent tool for data compression and statistical applications, such as the study of contagion and interdependence between financial markets. Finally, wavelet algorithms can also be implemented quickly, which is especially important with large amounts of data. For a review of the literature and discussion on the application of wavelets to economics and finance see Crowley (2007); Ramsey (1999, 2002).
series including the oil price series show a positive trend (see Narayan et al., 2014). The solution to the problem could be to extend the trend-GARCH(1,1) unit root model proposed by Narayan and Liu (2015) to the multivariate framework. Although, as the authors argue, this could be computationally demanding.

Hence, when model-free correlation estimators are used, adjustments and trends are not needed. In this context, wavelets are a useful tool to compute correlations and the methodology is model-free.

So far, studies using wavelets to address changes on correlations due to shocks have focused on 2007-2008 crisis (see, e.g., Alaoui et al., 2015; Dajcman et al., 2012; Gallegati, 2012; Reboredo and Rivera-Castro, 2014). In this paper, we study the impact of sharp oil increases during the 1990-2011 period (for example the Kuwait and Iraq wars, the OPEC cutback in 1999 and the July 2008 oil peak) on four stock markets indexes: Germany, Japan, the U.K. and the U.S. An oil shock might change the correlation between country’s stock market and oil prices. For instance, oil prices might increase, whilst the stock market value plummets or, on the other hand, oil prices might decrease, whilst stock market value remains unchanged or even soars, such as oil-exporting countries (Ramos and Veiga, 2013).

Our results show that in non-shock periods, correlation between oil and stock markets tends to be close to zero or slightly positive except in the period after 2008. At high frequencies, i.e., short periods of time, our results indicate changes in correlation between the U.S. stock market and oil in two periods; the Kuwait war and the July 2008 oil peak.

As we go from lower detail coefficients to higher detail coefficients, changes in the correlation between stock markets and oil are more frequent during oil shocks. For instance, the impact of the OPEC cutback period is only captured at low frequencies. We note also that during the July 2008 oil peak, at high frequencies the correlation between stock and oil markets drops, while at low frequencies it increases, indicating higher market integration.

Next, we test for changes in correlation between stock markets caused by an oil price shock, that is, we test whether correlation between international stock markets changes significantly
during a period of oil market turbulence. Correlation between international stock markets might change because oil shocks might influence stock markets differently. For instance, the level of oil dependency can be different, as a country can be a net oil importer or a net exporter. Our results show that wavelet correlations between international stock markets are also different in shock and non-shock periods. Changes are visible at high frequencies but also at low frequencies, mainly between the U.S., the U.K. and German stock markets.

Finally, we analyze the impact of stock market shocks on the correlation structure of oil and stock markets. A financial shock might also change the correlation between oil and stock markets, as stock markets can plummet without impacting the price of oil. However, there is also the perception that with stronger links between markets, contagion can rise. Our results indicate that shock and non-shock periods tend to have statistically different correlations, that is, correlation changes during or after the crisis. The evidence on contagion is stronger for the period following the stock market instability of 2008 and 2011, while interdependence between international stock and oil markets shifts frequently during financial shocks.

Our paper contributes to the literature on financial contagion\(^3\) and oil shocks, first by providing new evidence on the disruption of correlations between stock and oil markets due to oil and financial shocks, using recent methodological developments that overcome heteroscedasticity biases. In this context, we apply the wavelet-based methodology used by Gallegati (2012) but we jointly visualize the confidence intervals of the estimated wavelet correlations, calculated in periods of turbulence and non-turbulence, at a certain scale, for all the periods because this facilitates the analysis of the results. Second, our paper furthers the literature by taking a broader perspective on the impact of oil shocks on the correlation structure between oil and stock markets. The transmission between stock and oil markets has not being analyzed separately in terms of oil and financial shocks, and the transmission within stock markets due to oil shocks has not so far been analyzed.

Third, it extends the paper by Reboredo and Rivera-Castro (2014) because we analyze several

\(^3\)See Forbes (2012) for a recent review on the contagion literature and methodologies.
oil shocks, and not only the 2008 peak. A single shock might not be sufficient to understand the dynamics between the two markets, in particular, when the oil peak operates in tandem with a global financial crisis. The results indeed suggest a difference between the pattern of the 2008 peak and previous oil shocks. After previous oil shocks, correlation bounced back to pre-crisis level, i.e., close to zero. This did not occur after the 2008 peak.

Our results have useful implications for investors that make asset allocation strategies or diversify internationally as we analyze how shocks affect correlations between oil and stock markets and also among stock markets. Correlations are key for achieving meaningful risk reduction in investment strategies and enhancing investment performance. Thus, changes in correlations imply changes in portfolio weights along with loss of performance efficiency in investment strategies. Our results show that contagion in the periods of shock implies a significant reduction in the gains of diversification. In particular the results suggest that investors that are exposed to the U.S. stock market are more vulnerable to oil shocks as the U.S. stock market emerges as the stock market most prone to contagion from oil markets, both during oil and stock market shocks. Moreover, the finding that the correlation with the U.S. stock market changes both in higher and lower frequencies has consequences for portfolios that trade with different rebalancing horizons.

The remainder of the paper is organized as follows. Section II reviews the literature. Section III explains the details of the methodology used to test changes in correlations. Section IV describes the data and Section V presents and analyzes the results. Results of alternative measures of oil and also for other countries are presented in Section VI. Finally, Section VII concludes.

II. Literature Review

This paper is related to several branches of the literature. First, it is related to the literature that uses a wide variety of econometric techniques studying co-movements between financial markets and, in particular, changes in correlation due to financial turmoil (see a review of methodologies in Dungey et al., 2005). Several papers in the literature interpret statistically significant positive
changes in stock market correlations as evidence of contagion (Baig and Goldfajn, 1999; Ellis and Lewis, 2000; Forbes and Rigobon, 2002; King and Wadhwanii, 1990), and conclude that international stock market correlations increase during bear markets. Shawky et al. (1997) find that during the economic recession of the early 1990’s correlations between the U.S. and European markets were higher than when the economies recovered in the years following the recession. De Santis and Gerard (1997) examine the correlations between the G-7 countries over the 1970-1994 period, finding that the largest three correlations are associated with the most severe market declines. Longin and Solnik (2001) find that correlations increase substantially in bear markets and that negative returns have higher correlations than positive returns. Karolyi and Stulz (1996) and Ramchand and Susmel (1998) show that the correlations between the U.S. and other markets are higher when markets are more volatile. King and Wadhwanii (1990), Bertero and Mayer (1990) and Lee (2004) find evidence that correlation coefficients for stock returns between the U.S. and the other countries increased significantly after the 1987 crash.

Second, our work is related to studies that analyze how oil shocks affect stock markets. The literature has been inspired by macroeconomics work investigating the role of oil price shocks in causing recessions. As stock markets anticipate economic cycles (see e.g. Fama, 1990; Fama and French, 1989; Schwert, 1990), and in particular, their effect on firm valuation, the literature has also analyzed the link between oil shocks and stock market changes. The results have been mixed. Huang et al. (1996), Chen et al. (1986) and Ferson and Harvey (1994) find that oil futures returns do not have much impact on market indices such as the S&P 500, and that there is no risk premium for oil price risk in stock markets. However, Jones and Kaul (1996), provide evidence that aggregate stock market returns in the U.S., Canada, Japan and the U.K. are negatively sensitive to the adverse impact of oil price shocks on their economies. Kilian and Park (2009) use Kilian (2009)’ structural VAR decomposition to show that the response of U.S. real stock returns depends on whether the change in the price of oil is driven by demand or supply shocks. More recently, Ramos and Veiga (2013) reconcile the evidence, finding that the effects on oil-

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4 For a recent review of the literature on the link between economic recessions and oil prices we refer the reader to Kilian (2008).
importing and oil-exporting countries run in opposite directions. Oil price hikes have a negative effect on the stock markets of oil-importing countries, while the impact is positive for the stock markets of oil-exporting countries.

Several works have also analyzed the effect of oil price changes on firms returns (see e.g., Narayan and Sharma, 2011; Narayan and Narayan, 2014) and on firm return variance (see e.g., Narayan and Sharma, 2014; Oberndorfer, 2009). Others focus on whether the returns of the oil sector firms change with oil returns (see e.g., Boyer and Filion, 2007; Faff and Brailsford, 1999; Faff and Chan, 1998; Park and Ratti, 2008; Ramos and Veiga, 2011; Sadorsky, 2001) and tend to find a positive relation between oil price changes and oil firms returns. The literature has also distinguished the effect of oil price changes on oil producers and other oil related sectors (see e.g., Boyer and Filion, 2007; Elyasiani et al., 2011; Hammoudeh et al., 2004; Ramos et al., 2014; Phan et al., 2015; Scholtens and Wang, 2008) and find differences. For instance, based on the differences, Phan et al. (2015) devise simple trading strategies and provide evidence that the profits made by investors in oil producer sectors are higher than those made by investors in oil consumer sectors.

A recent strand of literature studies the effects of commodity market financialization. Financial investors have increasingly turned to commodities as an asset class. At the same time, financial innovation has provided a simple way to make financial investments in commodities, and empirical evidence has documented increasing involvement of financial investors in oil futures markets from 2003 (see e.g., Alquist and Kilian, 2010; Hamilton and Wu, 2015; Tang and Xiong, 2012). With the opening of commodity markets to financial investors the perception has also increased that these markets have become increasingly driven by financial flows of investors and less by their fundamentals. Silvennoinen and Thorp (2013) find that the conditional correlation between commodity futures returns and U.S. stock index returns has increased in recent years, especially in periods of high volatility. Büyüksahin and Robe (2014) report increased equity-energy price co-movements, which appear related to the entry of hedge funds that take positions in both equity and energy markets.
Our work is related to the recent strand of literature that analyzes co-movement between variables using different frequency levels, in particular those that address the relation between oil and economic variables (see e.g., Aguiar-Conraria and Soares, 2011; Benhmad, 2013b) and the relation between oil and financial market variables (see, e.g., Jammazi and Aloui, 2010; McCarthy and Orlov, 2012). Gallegati (2012) tests for contagion between international stock markets during the subprime crisis. Reboredo and Rivera-Castro (2014) uses Gallegati (2012)’s test to study the relationship between oil and stock markets before and after the 2008 crisis. They do not find evidence that oil price changes influenced stock market returns in the pre-crisis period at either the aggregate or sectoral level (with the exception of oil and gas company stocks). However, after the crisis they found contagion between these markets.

Frequency domain analysis has also provided new insights. Results seem to question the traditional negative relation between oil price increases and stock markets. Ciner (2013), using a frequency domain regression method developed by Ashley and Verbrugge (2008), finds that although oil price changes impact U.S. stock returns, there is variation across the frequency spectrum. Oil price changes that are expected to last longer do not have the same impact as shocks that are expected to disappear after a shorter period of time. He finds that both aggregate market indexes exhibit a negative reaction to oil price shocks with less than 12-month persistency. On the other hand, oil price shocks that persist for between 12 and 36 months have a positive impact on the stock market. Permanent oil price shocks at zero frequency have a negative impact on the stock market; the findings of McCarthy and Orlov (2012) cast doubt on the negative relation between oil and stock markets. Moreover, studies seem to concur over the importance of analyzing correlation for different frequencies. Gallegati (2012) finds evidence that contagion effects are scale dependent, in the sense that they do not display their effects uniformly across scales. Dajcman et al. (2012), using rolling window correlation, find that co-movements between stock market returns are time varying and scale dependent and a financial crisis in the

observed period does not uniformly increase co-movement between stock market returns across all scales. Vacha and Barunik (2012) find that dependence between energy commodities is high and more stable for longer dependencies, while within a short-term investment horizon they vary over time more quickly.

III. Methodology

The methodology is based on wavelets, which allows the study of time series on a variety of time scales, to obtain correlation estimates for different frequencies and consequently to investigate for contagion and interdependence between financial markets. Contrary to tests performed at different arbitrary time scales (such as daily, weekly or monthly), wavelet decomposition allows us to avoid information loss (what is not captured at one scale will be capture at another) and overlapping of the information used in one scale with respect to the other (thanks to the orthonormality property of the wavelets Daubechies (1992)).

We start by noting that different definitions of contagion have been used in the literature when studying stock markets.6 We adopt the framework of Gallegati (2012) that considers the existence of contagion when there are changes in the wavelet correlations at high frequencies (smaller scales) and interdependence when these changes are for low frequencies (large scales) that corresponds to longer periods of time. Then he studies contagion using a test on the equality of wavelet correlations.

A. Wavelet series decomposition

A time series of financial returns can be decomposed into orthogonal components: the wavelet details ($D_1, D_2, ..., D_J$) and the wavelet smooth ($S_J$). Let $r_{it}$ be a time series $i$ of financial returns at time $t$. $r_{it}$ can be approximated using the orthogonal wavelet series approximation

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6See Table I of Forbes (2012) for a list of definitions. For discussions on contagion and interdependence we refer the reader to Bodart and Candelon (2009); Corsetti et al. (2005); Dungey et al. (2005); Gallegati (2012); Pesaran and Pick (2007).
which contains the wavelet smooth coefficients \( v_{r,J,k} \) and the wavelet detail coefficients \( w_{r,J,k} \), such that:

\[
    r_t = S_{r,J}^r(t) + D_{r,J}^r(t) + D_{r,J-1}^r(t) + \ldots + D_{r,1}^r(t),
\]

where 

\[
    S_{r,J}^r(t) = \sum_k v_{r,J,k} \phi_{J,k}(t),
\]

\[
    D_{r,J}^r(t) = \sum_k w_{r,J,k} \psi_{J,k}(t),
\]

\[
    \phi_{J,k}(t) = 2^{-J/2} \phi\left(\frac{t-2^Jk}{2^J}\right)
\]

and

\[
    \psi_{J,k}(t) = 2^{-j/2} \psi\left(\frac{t-2^j k}{2^j}\right)
\]

for \( j = 1, 2, \ldots, J \). Equation (1) represents the wavelet decomposition of \( r_t \). As an example, the wavelet decomposition of \( r_t \), for a level 6 of multiresolution, consists of 6 wavelet details \( (D_6(t), D_5(t), \ldots, D_1(t)) \) and a single wavelet smooth \( (S_6(t)) \). The wavelet smooth captures the low frequency dynamics and the wavelet details the high frequency characteristics of \( r_t \). The maximum number of scales in this case is \( 2^6 \) which must satisfy \( 2^6 \leq T \), where \( T \) is the number of observations in the sample.

Many mother wavelets can be used to compute the wavelet transform and the corresponding coefficients. Following Gallegati (2012), we use the Daubechies extremal phase orthogonal wavelets with symmetric-padding boundary conditions (Daubechies, 1992) with length eight since filters with moderate lengths, such as eight, are adequate to capture the main features of financial time series (see Gençay et al., 2001). Daubechies are a family of wavelets that form an orthonormal basis via a multiresolution analysis and have finite vanishing moments. This property insures that the number of non-zero coefficients in the associated filter is finite. This is important because it reduces the number of distortions (see Khare and Tiwary, 2005). Moreover, Daubechies wavelets are optimal in the sense that they have the minimal support for a prescribed number of vanishing moments. We also use a modification of the Discrete Wavelet Transform (DWT) known as maximal-overlap DWT (MODWT), a stationary wavelet transform, designed to avoid the lack of translation-invariance of DWT (Percival and Walden, 2000).

In the literature, there is evidence that suggests that financial markets’ shock transmissions caused by contagion are quick and last only few days. Consequently, the correlations die out after five days at most (see Ait-Sahalia et al., 2015; Baig and Goldfajn, 1999; Gallegati, 2012). Following the literature and given that the frequency of the data is daily, contagion corresponds to the wavelet details of level 1 (1 day), level 2 (2 days) and level 3 (4 days) that represent the
weekly effect, and the wavelet details of levels 4 (8 days) to 6 (32 days) correspond to variations related with interdependence or co-movements. Computations have been performed using the \textit{waveslim} package developed by Whitcher for the R statistical package of R Core Team (2012) and the \textit{wavelet toolbox} of MATLAB (2010).

\section*{B. Wavelet–based correlations}

In this paper, we are interested in testing for significant changes in the wavelet correlations between international stock markets and oil market and also for changes between pairs of international stock markets. We do this separately for each scale \( j \) with \( j = 1, \ldots, 6 \). Consider two periods, for instance the Kuwait War (\( I_1 \)) and the period from the end of the Kuwait War until the OPEC agreement (\( I_2 \)). Let \( \rho_j(X,Y)^{I_1} \) and \( \rho_j(X,Y)^{I_2} \) be the wavelet correlations of two random variables \((X,Y)\) in these two periods \( I_1 \) and \( I_2 \), respectively. The null hypothesis of the test

\[ H_0 : \rho_j(X,Y)^{I_1} = \rho_j(X,Y)^{I_2} \]

is rejected at a significance level of 5\% if the two confidence intervals for \( \rho_j(X,Y)^{I_1} \) and \( \rho_j(X,Y)^{I_2} \) at confidence level of 95\% are non-overlapping (see Gallegati, 2012; Gençay et al., 2001, 2002). We use the intervals estimators proposed by Whitcher et al. (2000) because they are robust to non-Gaussianity.\footnote{According to Gençay et al. (2001, page 253), the approximate confidence interval for the estimated wavelet correlation needs no adjustment regarding the distribution of the incoming wavelet coefficients, they may be Gaussian or non-Gaussian.}

Let \( h(\rho) = \tanh^{-1}(\rho) \), then an approximate \( 100(1-2p)\% \) Confidence Interval for \( \rho_j(X,Y) \) for time period \( I \) is

\[
\left[ \tanh \left\{ h^{-1}(\hat{\rho}_j) - \frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j - 3}} \right\}, \tanh \left\{ h^{-1}(\hat{\rho}_j) + \frac{\Phi^{-1}(1-p)}{\sqrt{\hat{N}_j - 3}} \right\} \right],
\]

where \( \hat{N}_j = N_j - L_j \) and \( L_j = \lceil (L - 2)(1 - 2^{-j}) \rceil \) is the number of MODWT wavelet coefficients associated with scale \( j \), \( \Phi^{-1}(p) \) is the \( p \times 100 \) percentage point for the standard normal distribution.
and \( \hat{\rho}_j \) is the following unbiased estimation of the wavelet correlation at scale \( j \):

\[
\hat{\rho}_j = \frac{\hat{\gamma}_{X,Y}^j}{\hat{\sigma}_X^j, \hat{\sigma}_Y^j}.
\]

The wavelet covariance \( \hat{\gamma}_j \) and the wavelet variances \( \hat{\sigma}_j \) for time period \( I \) can be estimated as

\[
\hat{\gamma}_{X,Y}^j = \tilde{N}^{-1} \sum_{k \in \tilde{I}} \tilde{w}_{X,j,k} \tilde{w}_{Y,j,k}
\]

\[
\hat{\sigma}_X^j = \tilde{N}^{-1} \sum_{k \in \tilde{I}} (\tilde{w}_{X,j,k})^2,
\]

where \( \tilde{I} \) is the time period \( I \) after removing the times \( t \) that are affected by the boundary conditions, \( \tilde{N} \) is the length of \( \tilde{I} \), and \( \tilde{w}_{X,j,k} \) (respectively, \( \tilde{w}_{Y,j,k} \)) are the detail coefficients of the MODWT decomposition of \( r_X \) (resp. \( r_Y \)) at scale \( j \).

In order to simplify the visualization of the different tests, for each pair of series of interest (i.e., for each stock market and oil return, and for each pair of stock market returns) we plot the confidence interval of the wavelet correlation at each scale level \( j \) in a set of periods of interest. Let \( I_1, I_2, \ldots, I_K \) denote the periods of interest, we propose to jointly visualize the confidence intervals of the wavelet correlation at certain scale \( j \) for all the \( K \) periods. Each interval is located along the horizontal edge according to the date in the middle of the time period. In this way, testing if the correlation in period \( I_r \) is significatively different from that in period \( I_s \) would correspond to compare the intervals obtained in these periods. If intervals do not overlap, then the correlations are significatively different at that scale \( j \). Plotting the intervals over time becomes a useful tool for summarizing and interpreting the test results.
IV. Data

The data are the stock market indexes of Germany, Japan, the United Kingdom (U.K.) and the United States (U.S.).\textsuperscript{8} Oil prices are settlement prices of the continuous oil futures contract\textsuperscript{9} of the New York Mercantile Exchange (NYMEX), the most widely traded futures contract on oil. The underlying asset is the West Texas Intermediate oil, a light crude oil widely used as a current benchmark for U.S. crude production. All data are drawn from Datastream.

Indexes are in U.S. dollars and oil prices are in U.S. dollars per barrel ($U/BBL). The sample period runs from February 27, 1990 to November 22, 2011 comprising 5665 daily observations. The choice of daily data fits better the purpose of studying contagion, a short lived phenomenon (see e.g. Benhmad, 2013a; Cai et al., 2009; Gallegati, 2012; Huang, 2011; Reboredo and Rivera-Castro, 2014).\textsuperscript{10} As it is customary in the financial literature, returns are computed as \( r_{it} = [\ln(I_{it}) - \ln(I_{it-1})] \), where \( I_{it} \) is the stock market index of country \( i \) at time \( t \).

We define periods where there are oil shocks versus periods without oil shocks. We consider the following oil shocks: the Kuwait war in 1990, the OPEC cutback starting in March 1999, the Iraq war in March 2003 and the July 2008 peak of oil.\textsuperscript{11} We choose these events because in all these periods there were dramatic changes in the price of oil.

Figures 1 and 2 depict the series of prices and returns for our sample, respectively. Moreover, Figure 3 shows oil prices together with historical oil events. Oil prices peaked in 1990 with the

\textsuperscript{8} For reasons of space, in the supplementary appendix we also present the analysis for other countries: France, Italy and Canada.

\textsuperscript{9} Continuous futures prices are perpetual series of futures prices derived from individual futures contracts. They start at the nearest available contract month, which forms the first values for the continuous series, with subsequent switchovers depending on the continuous series methodology requested. Unlike individual futures contracts, continuous series do not expire until the actual future contract ceases to exist. We use the type 0, that means that it switches over on 1st day of new month trading.

\textsuperscript{10} The use of daily data to study contagion is common on the literature on contagion. Reboredo and Rivera-Castro (2014) advocates that "Using daily data is more appropriate for our purpose of testing contagion effects since shock transmission due to contagion is very fast and dies out quickly after a few days, so correlation disappears in less than a week (...). Therefore, we can obtain a more realistic measure of contagion using daily data than using data sampled at lower frequencies (weekly or monthly)." Also Narayan et al. (2013) state that "the information contents increases with an increase in data frequency" (page 3883).

\textsuperscript{11} See Hamilton (2013) for a reference in oil shocks. A table with the list and dates of shocks is presented in the supplementary appendix. The exact dates are chosen according to key historical events, such as the start and end of Iraq war.
invasion of Kuwait by Iraq, and then dropped. After that, the price of oil did not fluctuate very much until around 2002. The price of $40/BBL was only reached again in October 2004. Then a period of price escalation started. Oil prices went from $50/BBL in 2005 to $100/BBL in 2007, to reach almost $150/BBL in July 2008. As many countries entered in recession, prices continued to slide until the end of 2008, to increase again during 2009. The value in December 2009 was again close to $80/BBL and increased during 2011.

Table I reports the summary statistics of stock market indexes and oil returns, the autocorrelations of orders one and fifteen of returns and the p-values of the Ljung-Box test statistics. Stock market indexes register positive mean returns during the period, with the exception of Japan. Volatility is lower for the U.S. stock indexes. Oil and stock market returns other than these of Japan display negative skewness. Therefore, the Jarque–Bera test rejects the assumption of Gaussian returns for all stock and oil returns. Regarding the autocorrelation of returns, it seems that the returns are correlated although the values of these autocorrelations are quite close to zero.

In order to compute the correlations and test for contagion and interdependence, we adjust the data for different time zones\textsuperscript{12} by matching the return series of U.S. at time $t$ to the daily return series of Germany, the U.K. and Japan at time $t + 1$. We consider that most of the news comes from the U.S., as it is the largest stock market and one of the world’s largest oil producers.

V. Empirical Results

In this section we calculate the wavelet multiscale correlations between stock market indexes of different countries, and between stock market indexes and oil returns. Then, we test for changes in the correlations at different frequencies. We follow Gallegati (2012)’s framework, where changes in wavelet correlations at high frequencies (levels 1-3) are considered evidence of contagion and interdependence is studied by the changes in correlations at low frequencies\textsuperscript{12}Martens and Poon (2001) state that the use of non-synchronous closing prices has led to a downward bias in correlation estimates.
We proceed first to analyze the impact of oil shocks in the correlations between stock and oil markets; then, we analyze whether oil shocks impact correlations between stock markets; and finally, we investigate whether there is contagion or interdependence between oil and stock markets if stock markets crash.

A. Do oil shocks change wavelet correlations?

Figures 4 and 5 depict correlations and the estimated confidence intervals for high and low frequencies. The red dotted line connects the estimated coefficients, while the bars represent the estimated confidence intervals for the shock and non-shock periods (bars for shock and non-shock periods are differentiated with different colours). For two consecutive shock and non-shock periods, a statistically significant change in correlation happens if the estimated confidence intervals for the correlations between the series of wavelet details of oil returns and the series of wavelet details of stock market returns \( i \), where \( i \in \{\text{Germany, Japan, U.K., U.S.}\} \), do not overlap.

A.1. High–frequency wavelet correlations (wavelet details 1–3)

Figure 4 presents the results of testing changes in the correlations between the oil market and international stock markets for three frequencies (1, 2 and 4 days).

Looking briefly at columns, we see a common pattern: correlations in calm periods tend to be around zero, whereas the correlations in shock periods are different. This pattern is more evident in the third column, which depicts the estimated confidence intervals for correlations at the frequency of 4 days. For this frequency, the changes between calm and shock period are quite pronounced and the line pattern is constant over the different countries. There is a noticeable bump in correlations in 2008 oil peak, that rebounds significantly after 2008.

We address now the change in correlations comparing the bars. We start by the analysis of one day impact on the correlations (first column of Figure 4). For this frequency, we observe three significant changes in correlations: the first in the Kuwait war, between the U.S. and the
oil markets; the second and the third in the oil peak in July 2008, between the Japanese stock market and oil, and between the U.S. stock market and oil. For these three cases, the surge in oil prices leads to negative correlations between stock and oil markets. After the shocks, the correlations become positive.

The panels of column two of the same figure depict the estimated confidence intervals for the correlations at the frequency of two days. For this frequency, we observe three statistically significant changes in the correlation between stock and oil markets. The first and second in the Kuwait war, between the Japanese stock market and oil, and the U.S. stock market and the oil; the third, during the 2008 oil peak, between the U.S. stock market and oil. During the Kuwait war shock, the correlations between the series of wavelet details of the major stock and oil markets are quite negative. For both cases, the estimated confidence intervals do not overlap with the confidence intervals of the period after the shock, where the correlations are almost zero, i.e., the change in correlations is statistically significant. For the latter case, correlation has a statistically significant increase after the shock.

Column three of Figure 4 depicts the estimated confidence intervals for correlations at the frequency of 4 days. For this frequency, we observe several changes in the correlation between oil and the stock markets returns that correspond to three oil shocks: the Kuwait war, the Iraq war and the 2008 oil peak. For the first shock, correlations of stock markets with oil drop, but after the shock they increase for values close to zero and the increase is statistically significant. In the Iraq war shock, correlation changes between the German, the U.K. and the U.S. stock markets and the oil. Again, the correlations drop during the Iraq war and after the shock they register a statistically significant increase. Finally, in the 2008 oil peak all the correlations between stock markets and oil are negative and statistically different from both stable periods, before and after the shock, where correlations are positive.

Overall, estimated correlations between stock markets and oil returns are close to zero in the non-shock periods as confidence interval bars tend to lie on the main axe. The exception is the period after the 2008 oil peak, but we note that it overlies with the aftermath of the subprime
financial crisis where stock market returns have severe drops. During oil shocks, correlations between stock markets and oil returns tend to decrease. The U.S. stock market appears as the stock market with greater contagion which can be explained by being a leading and an efficient stock market that reacts quickly to the arrival of new information. As we go from lower wavelet details to higher details (i.e., from higher to lower frequencies), the correlation between stock markets and oil changes more frequently. Inspecting shocks, the Kuwait war and the 2008 oil peak are the events where changes in correlations are more noticeable at higher frequencies. In contrast, the Iraq war changes correlation significantly only at 4-days frequency and the oil shock related to OPEC cutback does not seem to lead to changes in correlations at these high frequencies. In the next sub-section, we proceed with the study of the low-frequency correlations.

A.2. Low–frequency wavelet correlations (wavelet details 4–6)

Figure 5 displays estimated correlations between stock and oil markets for three frequencies (8, 16 and 32 days). If the estimated confidence intervals for the wavelet correlations do not overlap that suggests changes in market interdependence.

A snapshot at the red dotted line that connects the estimated correlations shows that the pattern of stock market correlation with oil overtime is very similar across countries. Detail coefficient 4 depicts correlations around zero in calm periods and a substantial increase after 2008. For detail coefficient 5, it is noticeable in all countries the peak of the OPEC cutback.

Looking at changes in correlations, regarding the analysis of 8-day impact on the wavelet correlations (first column of Figure 5), we observe that the Kuwait war affects negatively the correlations between three stock markets (Germany, Japan and the U.S.) and oil market, that is, they become strongly negative. After the shock, correlation rebounds to values close to zero. In the Iraq war, wavelet correlations decrease again, and the differences are statistically significant for two stock markets: the U.K. and the U.S. After the shock, correlations are again close to zero and then plunge during the July 2008 peak of oil. In this period, they are extremely negative between the Japanese stock market and oil, and between the U.S. stock market and oil, but after
2008 they have a statistical significant increase.

The second column of the same figure shows the estimated confidence intervals for the correlations at the frequency of 16 days. The correlation between the Japanese stock market and the oil presents always changes in the major events. It drops in case of the Kuwait and Iraq wars and rises in the other events. The OPEC cutback, and the Iraq and Kuwait wars shift the correlation between the U.S. stock market and the oil and between the German stock market and oil. Finally, the correlation between the U.K. stock market and oil is the least affected in this frequency.

The third column of Figure 5 depicts the estimated confidence intervals for correlations at the frequency of 32 days. At this frequency, the line indicates an upward trend in correlation, and the wavelet correlations change frequently for all stock markets, except for Japan. In the Kuwait war, the OPEC cutback and the Iraq war the correlation between the U.S. stock market and the oil decrease, and it soars during the July 2008 peak of oil. Similar happens to the U.K. stock and oil markets correlation; although, in this case, the correlation in the OPEC cutback period is not statistically different from that of the following period, maybe reflecting the U.K. autonomy in oil. Finally, the OPEC cutback, the Iraq war and the July 2008 peak of oil affect the correlation between the German stock and oil markets, it drops for the first two oil shocks and surges in case of the July 2008 peak of oil.

Comparing the high and low frequency results, it is noticeable that the July 2008 peak of oil has a different effect depending on the scale, which can be explained by the contemporaneous turmoil in stock markets. For wavelet correlations of level 3, the correlation is negative in all countries, and statistically different from that of the following period. However, if we look at Figure 5, this correlation is positive (level 6) and statistically different from that of the next period for all stock markets, with the exception of the U.S. Therefore, the impact is negative at higher frequencies but the interdependence seems strengthened according to the results at lower frequencies. It is also noteworthy that the impact of OPEC cutback period is only captured at lower frequencies.
A.3. Contagion and interdependence between stock markets

The next step is to study whether correlation between stock markets changes with an oil shock. If all stock market returns drop sharply then correlation is expected to increase.

We report results for the wavelet detail series of levels 3 and 6 that are the frequencies for which we observed the majority of the changes in correlation in the previous analysis.\(^\text{13}\)

Looking at the red dotted lines at Figure 6 that connect the estimated wavelet correlations, we note first that the estimated correlation coefficients are largely positive, in some cases close to one as in the case of the correlation between the U.K. and German stock markets. It is also visible the trend of increasing correlations in 90’s, quite notorious among the group of the U.S., the U.K. and German stock markets.\(^\text{14}\) The pattern is in line with our expectation that during oil shocks stock market correlations tend to increase.

The small size of the bars indicates that the coefficients are estimated with high precision, with the exception of the 2008 period where several systemic events affected stock markets.\(^\text{15}\) The results indicate five changes in the correlation (level 3) that correspond to three oil shocks: the Kuwait war, the OPEC cutback period and the Iraq war. For the first shock, we find that correlation has a statistically significant increase, between the U.K. and the Japanese stock markets, and the U.S. and the Japanese stock markets. During, the OPEC cutback period, correlation increases between European stock markets. Finally, correlation increases significantly between the two European stock markets and Japan in the Iraq war period.

Figure 7 depicts correlations for detail level 6. Again we note the trend of increasing correlation between stock markets in the 90’s and the high values of correlation between the German, European stock markets.

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\(^\text{13}\) We made the analysis for the frequencies 1 and 2 days but we did not observe any significant change in the correlation between stock markets. Moreover, the analysis for the frequencies 8 and 16 days was also done but did not reveal interesting conclusions. The results are available from authors upon request.

\(^\text{14}\) Rua and Nunes (2009) use wavelets to study co-movements of stock markets. They find that among all the country pairs considered, the U.S. and U.K. stock markets seem to present the highest co-movement across time and frequencies while the Japanese market shows a low degree of co-movement with any other major stock market in the time-frequency space. Regarding Germany, they find a high degree of co-movement at lower frequencies with U.S. and U.K. over the whole sample period and since the end of the 90s this is also observed for all the other frequencies.

\(^\text{15}\) The first signs of the crisis started in 2007 with the reported losses of Bearn Sterns and in September 15, 2008 there was the major event of the Lehman Brothers collapse.
the U.K. and the U.S. stock markets, with values close to one. The bars are also quite small meaning that the precision of the estimates is high. In general, the correlation with Japanese stock market falls after the OPEC cutback period, but after 2008 it rebounds again reaching high values. In the Kuwait war, correlation changes significatively between the pairs U.K., German and the Japanese stock markets, and it increases for the U.K. and the Japanese stock markets during the shock, but for the other pairs of countries it drops. In the OPEC cutback period, correlation increases are statistically different from those of the calm periods that precede or/and follow the oil shock. In the Iraq war, we observe drops in correlation between the U.K. and the Japanese stock markets, between the U.S. and the Japanese stock markets, but the correlation between the U.K. and U.S. stock markets increases. Finally, the July 2008 oil peak precedes strong positive correlations for all pairs of stock markets denoting the spillover effects of the global financial crisis in international stock markets.

Summing up, wavelet correlations among international stock markets increase in oil shocks periods both at higher and lower frequencies, being more frequent at lower frequencies mainly among the U.S., the U.K. and German stock markets.

B. Do correlations change with stock market shocks?

In this subsection we consider the impact on the correlation between oil and stock markets given a crash in stock markets. We focus on daily drops in stock market return larger than 5%, which allows important market events to be captured such as the Asian Crisis in 1997, the Russian Crisis and the bankruptcy of the 'Long Term Capital Management' in 1998, the dot.com bubble bursting in 2000, the aftermaths of the terrorist attack to U.S. on September 11, 2001, the bankruptcy of 'Lehman Brothers' in September 2008 and the 'Sovereign Crisis' in 2011.

B.1. High–frequency wavelet correlations (wavelet details 1–3)

Figure 8 depicts the estimated wavelet correlations between oil and stock markets at high frequencies. As the red dotted lines indicates, correlation between stock markets and oil markets
tends to be around zero in stable periods, or even slightly negative in early 90’s, but after 2008 it increases.

For the first frequency, we observe a drop in correlation between the German stock and oil markets during the Asian crisis in 1997. At the frequency of 2 days, there are a number of statistically changes of correlation between stock markets and oil. During the Lehman Brothers collapse, correlation with oil increases for the Japanese and the U.S. stock market. In 2011, correlation between the German and the U.S. stock markets with oil also increases significantly.

At the frequency of 4 days (detail coefficient 3), correlation breakdowns can be observed on two different dates. In 2008, there is a statistically significant increase in correlation between the Japanese and the U.K. stock market with oil. In 2011 there is an increase in correlations between the four stock markets and oil; the difference is statistically significant in the cases of Japan, the U.K. and the U.S.

**B.2. Low–frequency wavelet correlations (wavelet details 4–6)**

Figure 9 reports wavelet correlations for wavelet details 4, 5 and 6 that correspond to longer time period variations as co-movements. As before, lower frequencies depict more peaks and troughs between shock and non-shock periods, which might anticipate more statistically differences in estimated correlations.

For detail coefficient 4 (graphs in the first column), the most noteworthy events in leading to statistically changes in correlations between stock markets and oil are the market instabilities in 2008 and 2011; correlation increases significantly in all stock markets. In the Asian crisis in 1997, the correlation between the German stock market and the oil market is the most affected. After the ‘dot.com’ crash there is a statistically significant decrease in correlations with oil for the German, the U.K. and the U.S. stock markets.

Inspecting wavelet correlations for detail 5, we see that during the Asian crisis, correlations for all sample stock markets and oil increase and are statistically different from those of the previous period. During the ‘dot.com’ crash in 2000, the German stock and oil markets, and
the U.K. stock and oil markets see a statistically significant decrease in correlations. After 2001 stock market drop, correlations increase substantially for Germany, the U.K. and the U.S. stock markets and oil. Regarding the 'Sovereign crisis' in 2011 there is only one statistically significant change in correlation, that is the correlation between the U.S. stock market and oil.

A large number of statistically significant changes in correlations are depicted at wavelet detail 6, in periods such as the 1997 Asian crisis, the 'dot.com’ bubble burst and 2008 and 2011 crisis. During the 'dot.com’ bubble burst, all correlations between international stock and oil markets have a statistically significant increase. In Japan, it is noticeable the meander of correlation in the period 1997-2001, correlations in stable periods and in shock periods do not overlap. During shock periods, correlations are close to one and after a shock, they seem to go back to normal values, i.e., slightly above zero. For the U.K. stock market, it is also noticeable several changes in correlation during market turmoils such as the Asian crisis, the 2000 crisis, the 'Lehman Brothers’ bankruptcy in 2008 and the 'Sovereign crisis’ in 2011. For the U.S. is quite visible the change in correlation in 2000 and 2001. Correlations jump to values close to one, and then bounce back to normal values. The correlation between the German stock and oil markets is quite positive in 2011.

VI. Robustness Analysis

In this section we present the results of additional robustness tests. First, we checked if results are robust to other sources of oil prices. We consider the futures price of the Brent oil contract (see e.g. Phan et al., 2015) and also the West Texas Intermediate Spot price.\textsuperscript{16} We confirm that our results are robust to the use of a different oil price series. The figures are presented in the supplementary appendix in order to save space.

Second, we analyze whether there are changes in the correlation between oil prices and Gulf Cooperation Council (GCC) stock markets. We have used the Datastream market index of GCC countries to represent these countries, available from 2003, which means that the only oil

\textsuperscript{16}Brent Crude OIL Continuous from Intercontinental Exchange (ICE) and Crude Oil WTI Cushing US$/BBL.
shock analyzed is the July 2008 oil peak. These results are also presented in the supplementary appendix due to reasons of space.

Regarding oil shocks, the results show that for wavelet details up to D4, the changes in correlation are not statistically significant, but for details D5 and D6 correlation increases statistically during the July 2008 oil peak, but after that it decreases. The result is similar when we use Brent futures price and WTI spot price.

The results for changes in correlation, when financial shocks occur, show that for D1, D2 and D3 there is an increasing trend in correlation during the period analyzed. However, they are not statistically significant for D1 and D2. For D3, correlation has an increase statistically significant in 2011 stock market turmoil. For D4, correlation changes significantly with 2008 stock market crash, but for the other wavelet details, the other changes are not statistically significant. Results do not change if we use the Brent futures price or WTI oil spot price.

VII. Conclusions

The analysis of contagion has long been a topic of interest. We follow recent work that advocates the use of frequency analysis to isolate whether the increase in cross-market linkages is due to co-movements (low frequency) or contagion (high frequency) (see Bodart and Candelon, 2009; Gallegati, 2012; Orlov, 2009; Reboredo and Rivera-Castro, 2014). The paper uses the methodology of Gallegati (2012) based on wavelets and proposes to jointly visualize the confidence intervals of the estimated wavelet correlations, calculated in periods of oil and stock markets turbulence and periods of non-turbulence, at a certain scale, for all the periods. Gallegati (2012) argues that the multi-resolution decomposition property of the wavelet transform can be used to separately identify contagion and interdependence by associating each to its corresponding frequency components.

We test for changes in correlation between oil and four large stock markets, Germany, Japan, the U.K. and the U.S. We distinguish wavelet correlations in stable periods and in periods with
sharp oil price increases like the Kuwait and Iraq wars, the OPEC cutback in 1999-2000 and the July 2008 peak of oil. We find that correlations between oil and stock markets estimated in non-shock periods tend to be close to zero, but during oil and stock market shocks, correlation tends to shift, with the change differing with scales (or frequencies). Oil shocks tend to decrease correlations between oil and stock markets at high frequencies, being the U.S. stock market that had more significant changes in correlations. The oil peaks caused by the Kuwait war and in 2008 also changed significantly correlations. At lower frequencies, the tests corroborate more changes in correlations between stable and shock periods. Moreover, in oil shock periods we find that correlations among stock markets tend to increase despite they are already high.

Analyzing the effect of stock market crashes, during recent 2008 and 2011 episodes, there are statistically significant changes in correlation of stock markets and oil, supporting contagion. The results suggest that the U.S. stock market is in the front line regarding contagion with oil market both for oil and stock market shocks. At lower frequencies, we find evidence of statistical increases in correlation for several financial crisis. These changes in return co-movements are in line with reassessments of fundamentals that originate and propagate crisis. For instance, oil peaks due to wars in the Middle East reflect changes in equilibrium supply and demand relationships and financial crisis often occur due to currency devaluations, liquidity constraints or change in macroeconomic fundamentals.

The analysis conducted has a number of implications of interest to policy makers, but also to the construction of optimal portfolio diversification strategies as changes in correlation impacts portfolio weights. The issue whether correlations will move to pre-crisis levels is an important issue for diversification strategies. In our sample period, oil and stock markets shocks shift correlations, but on the post-crisis correlations tend to go back to pre-crisis values. However, we do not observe this after 2008.
References


MATLAB (2010). *version 7.10.0 (R2010a)*. Natick, Massachusetts: The MathWorks Inc.


Figures and Tables

Figure 1. Prices of stock indexes and oil.
Table I
Summary statistics

This table presents the summary statistics of the daily returns of stock market indexes and oil. **Oil prices (Oil)** are settlement prices of the continuous oil futures contract of West Texas Intermediate oil traded on the New York Mercantile Exchange. **stock market indexes of Germany, Japan, the United Kingdom (U.K.) and the United States (U.S.) are from Datastream.** The returns are the first differences of the logarithm of prices. By column, we report the annualized mean, the annualized standard deviation (sd), the skewness, the kurtosis, the p-values of the Jarque-Bera test statistics, the autocorrelations of orders one and fifteen of returns ($\rho(1)$ and $\rho(15)$, respectively) and the p-values of the Ljung-Box test statistics. The sample period ranges from February 27, 1990 to November 22, 2011.

<table>
<thead>
<tr>
<th></th>
<th>Oil</th>
<th>Germany</th>
<th>Japan</th>
<th>U.K.</th>
<th>U.S.</th>
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</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.076</td>
<td>0.025</td>
<td>-0.025</td>
<td>0.050</td>
<td>0.076</td>
</tr>
<tr>
<td>sd</td>
<td>0.389</td>
<td>0.214</td>
<td>0.230</td>
<td>0.198</td>
<td>0.184</td>
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<tr>
<td>skewness</td>
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<td>-0.055</td>
<td>0.053</td>
<td>-0.172</td>
<td>-0.274</td>
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<tr>
<td>kurtosis</td>
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<td>12.056</td>
<td>7.003</td>
<td>12.091</td>
<td>11.667</td>
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<td>p-value-JB</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>$\rho(1)$</td>
<td>-0.001</td>
<td>0.030</td>
<td>0.021</td>
<td>0.013</td>
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<tr>
<td>$\rho(15)$</td>
<td>0.025</td>
<td>0.018</td>
<td>-0.015</td>
<td>0.002</td>
<td>-0.023</td>
</tr>
<tr>
<td>p-value-Q(15)</td>
<td>0.000</td>
<td>0.000</td>
<td>0.002</td>
<td>0.000</td>
<td>0.000</td>
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</table>
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Figure 3: Evolution of oil prices together with the oil events. Source: www.wtrg.com/prices.htm.
Figure 4. Confidence intervals for wavelet correlations between stock markets and oil, given oil shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 5. Confidence intervals for wavelet correlations between stock markets and oil, given oil shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
Figure 6. Confidence intervals for wavelet correlations between stock markets, given oil shocks. Detail level 3 (D3).
Figure 7. Confidence intervals for wavelet correlations between stocks markets, given oil shocks. Detail level 6 (D6).
Figure 8. Confidence intervals for wavelet correlations between stock markets and oil, given financial shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 9. Confidence intervals for wavelet correlations between stock markets and oil, given financial shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
Supplementary appendix for “Correlations between oil and stock markets: A wavelet-based approach”

Belén Martín-Barragán  Sofia Ramos  Helena Veiga

This appendix has been provided by the authors to clarify some details and report robust tests to the study presented in the original article.

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A. Tables: Dates of shocks

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Starting and ending dates of oil shocks

<table>
<thead>
<tr>
<th>Shock</th>
<th>starting date</th>
<th>ending date</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRAQ war</td>
<td>20-Mar-2003</td>
<td>30-Jan-2004</td>
</tr>
<tr>
<td>oil PEAK</td>
<td>01-Jul-2008</td>
<td>31-Oct-2008</td>
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</table>

Table II
Dates of financial shocks

<table>
<thead>
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<th>Shock</th>
<th>Date</th>
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</thead>
<tbody>
<tr>
<td>Asian Financial crisis</td>
<td>22/10/1997</td>
</tr>
<tr>
<td>Long Term Capital Management crisis</td>
<td>03/09/1998</td>
</tr>
<tr>
<td>Dot com bubble crisis</td>
<td>10/03/2000</td>
</tr>
<tr>
<td>September 11</td>
<td>11/09/2001</td>
</tr>
<tr>
<td>Lehman Brothers failure</td>
<td>09/09/2008</td>
</tr>
<tr>
<td>Succession of falls in US stock market</td>
<td>28/07/2011</td>
</tr>
</tbody>
</table>
B. Analysis of change of correlation between stock markets and oil (Brent futures price)

Figure 1. Confidence intervals for wavelet correlations between stock markets and oil (Brent futures price), given oil shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 2. Confidence intervals for wavelet correlations between stock markets and oil (Brent futures price), given oil shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
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C. Analysis of change of correlation between stock markets and oil (WTI spot price)

Figure 5. Confidence intervals for wavelet correlations between stock markets and oil, given oil shocks (WTI spot price). Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 6. Confidence intervals for wavelet correlations between stock markets and oil, given oil shocks (WTI spot price). Detail levels 4, 5 and 6 (D4, D5 and D6).
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D. Analysis of change of correlation between stock markets of Gulf Cooperation Council (GCC) countries and oil

D.1. WTI futures price

Figure 9. Confidence intervals for wavelet correlations between GCC index and oil (WTI futures price), given oil shocks. Detail levels 1 to 6 (D1 to D6).
Figure 10. Confidence intervals for wavelet correlations between GCC index and oil (WTI futures price), given financial shocks. Detail levels 1 to 6 (D1 to D6).
D.2. Brent futures price

Figure 11. Confidence intervals for wavelet correlations between GCC index and oil (Brent futures price), given oil shocks. Detail levels 1 to 6 (D1 to D6).

Figure 12. Confidence intervals for wavelet correlations between GCC index and oil (Brent futures price), given financial shocks. Detail levels 1 to 6 (D1 to D6).
D.3. WTI spot price

Figure 13. Confidence intervals for wavelet correlations between GCC index and oil (WTI spot price), given oil shocks. Detail levels 1 to 6 (D1 to D6).

Figure 14. Confidence intervals for wavelet correlations between GCC index and oil (WTI spot price), given financial shocks. Detail levels 1 to 6 (D1 to D6).
E. Analysis of change of correlation between stock markets (for other countries in G7)

E.1. Oil shocks

Figure 15. Confidence intervals for wavelet correlations between stock markets, given oil shocks. Detail level 1 (D1).
Figure 16. Confidence intervals for wavelet correlations between stocks markets, given oil shocks. Detail level 2 (D2).
Figure 17. Confidence intervals for wavelet correlations between stock markets, given oil shocks. Detail level 3 (D3).
Figure 18. Confidence intervals for wavelet correlations between stock markets, given oil shocks. Detail level 4 (D4).
Figure 19. Confidence intervals for wavelet correlations between stock markets, given oil shocks. Detail level 5 (D5).
Figure 20. Confidence intervals for wavelet correlations between stock markets, given oil shocks. Detail level 6 (D6).
E.2. Financial shocks

Figure 21. Confidence intervals for wavelet correlations between stock markets, given financial shocks. Detail level 1 (D1).
Figure 22. Confidence intervals for wavelet correlations between stocks markets, given financial shocks. Detail level 2 (D2).
Figure 23. Confidence intervals for wavelet correlations between stocks markets, given financial shocks. Detail level 3 (D3).
Figure 24. Confidence intervals for wavelet correlations between stocks markets, given financial shocks. Detail level 4 (D4).
Figure 25. Confidence intervals for wavelet correlations between stocks markets, given financial shocks. Detail level 5 (D5).
Figure 26. Confidence intervals for wavelet correlations between stocks markets, given financial shocks. Detail level 6 (D6).
F. Analysis of change of correlation between stock markets and oil
(for other countries in G7, WTI futures)

F.1. Oil shocks

![Graphs showing wavelet correlations between stock markets and oil, given oil shocks for Canada, France, and Italy at different detail levels (D1, D2, D3).]

**Figure 27.** Confidence intervals for wavelet correlations between stock markets and oil, given oil shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 28. Confidence intervals for wavelet correlations between stock markets and oil, given oil shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
F.2. Financial shocks

Figure 29. Confidence intervals for wavelet correlations between stock markets and oil, given financial shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 30. Confidence intervals for wavelet correlations between stock markets and oil, given financial shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
G. Analysis of change of correlation between stock markets and oil
(for other countries in G7, WTI spot)

G.1. Oil shocks

Figure 31. Confidence intervals for wavelet correlations between stock markets and oil (WTI spot), given oil shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 32. Confidence intervals for wavelet correlations between stock markets and oil (WTI spot), given oil shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
G.2. Financial shocks

Figure 33. Confidence intervals for wavelet correlations between stock markets and oil (WTI spot), given financial shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 34. Confidence intervals for wavelet correlations between stock markets and oil (WTI spot), given financial shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
H. Analysis of change of correlation between stock markets and oil 
(for other countries in G7, Brent futures)

H.1. Oil shocks

Figure 35. Confidence intervals for wavelet correlations between stock markets and oil (Brent futures), given oil shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 36. Confidence intervals for wavelet correlations between stock markets and oil (Brent futures), given oil shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).
H.2. Financial shocks

Figure 37. Confidence intervals for wavelet correlations between stock markets and oil (Brent futures), given financial shocks. Detail levels 1, 2 and 3 (D1, D2 and D3).
Figure 38. Confidence intervals for wavelet correlations between stock markets and oil (Brent futures), given financial shocks. Detail levels 4, 5 and 6 (D4, D5 and D6).