Adaptive Selection of Antennas for Optimum Transmission in Spatial Modulation

Citation for published version:

Digital Object Identifier (DOI):
10.1109/TWC.2015.2409067

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Publisher's PDF, also known as Version of record

Published In:
IEEE Transactions on Wireless Communications

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Abstract—In this paper, we propose an optimum transmit structure for spatial modulation (SM), a unique single-stream multiple-input multiple-output (MIMO) transmission technique. As a three-dimensional modulation scheme, SM enables a trade-off between the size of the spatial constellation diagram and the size of the signal constellation diagram. Based on this fact, the novel method, named transmission optimized spatial modulation (TOSM), selects the best transmit structure that minimizes the average bit error probability (ABEP). Unlike the traditional antenna selection methods, the proposed method relies on statistical channel state information (CSI) instead of instant CSI, and feedback is only needed for the optimal number of transmit antennas. The overhead for this, however, is negligible. In addition, TOSM has low computational complexity as the optimization problem is solved through a simple closed-form objective function with a single variable. Simulation results show that TOSM significantly improves the performance of SM at various channel correlations. Assuming Rayleigh fading channels, TOSM outperforms the original SM by up to 9 dB. Moreover, we propose a single radio-frequency (RF) chain base station (BS) based on TOSM, which achieves low hardware complexity and high energy efficiency. In comparison with multi-stream MIMO schemes, TOSM offers an energy saving of at least 56% in the continuous transmission mode, and 62% in the discontinuous transmission mode.

Index Terms—Spatial modulation (SM), channel correlation, transmit antenna selection, MIMO.

I. INTRODUCTION

The need to curtail the carbon footprint and the operation cost of wireless networks requires an overall energy reduction of base stations (BSs) in the region of two to three orders of magnitude [3]. At the same time, a significant increase in spectrum efficiency from currently about 1.5 bit/s/Hz to at least 10 bit/s/Hz is required to cope with the exponentially increasing traffic loads [4]. This challenges the design of multiple-input multiple-output (MIMO) systems associated with the BS. A typical long-term evolution (LTE) BS consists of radio-frequency (RF) chains, baseband interfaces, direct current to direct current (DC-DC) converters, cooling fans, etc. Each RF chain contains a power amplifier (PA), and PAs contribute around 65% of the entire energy consumption [5]. The efficiency of state-of-the-art (SOTA) PAs is about 30% only [6], i.e. more than two thirds of the energy is consumed in quiescent power. This drives research on minimizing the overall BS energy consumption instead of the energy required for the RF output stage only. As a result, power optimization of PAs has been studied. In [7], cell discontinuous transmission (DTX) was proposed to enable the BSs to fall into a sleep mode when there is no data to convey, so that the overall energy consumption can be reduced. Based on that concept, another optimization method using on/off PAs was reported in [8], and a similar work was conducted for MIMO orthogonal frequency division multiple access (OFDMA) systems [9]. However, those studies have the following limitations: i) they focus on the operation of RF chains, while modulation schemes are not considered; ii) the optimization is implemented within each individual RF chain; and iii) the benefit is inversely proportional to the traffic load. When the BS has to be operated in the active mode continuously, the above methods would fail to achieve any energy-saving gain. Therefore it is necessary to study energy reduction on a more comprehensive level, including not only hardware operations, but also modulation schemes.

While multi-stream MIMO schemes, such as vertical Bell Labs layered space-time (V-BLAST) and space-time block coding (STBC), offer high spectrum efficiency, unfortunately, they need multiple RF chains that heavily compromise the energy efficiency. Meanwhile, spatial modulation (SM) is a unique single-stream MIMO technique [10]–[12], where the bit stream is divided into blocks and each block is split into two parts: i) the first part activates one antenna from the antenna array while the remaining antennas do not emit a signal; ii) the bits in the second part are modulated by a signal constellation diagram, and sent out through the activated antenna. The use of a single active antenna makes SM a truly energy-efficient MIMO transmission technique, because only one RF chain is required, regardless of the number of transmit antennas used. At the same time, SM ensures spatial multiplexing gains as information is encoded in the antenna index. However, like all other MIMO schemes, SM suffers performance degradation caused by channel correlations [13]. Trying to improve the performance of SM against channel variations, an adaptive method was proposed in [14], where one candidate is selected from several optional SM structures. Although the performance...
of SM can be improved to some extent, this method has the following weaknesses: i) it requires instant channel state information (CSI), and therefore it is not suitable for fast fading channels; ii) the relation between the adaptive selection and the channel correlation has not been exploited; and iii) despite using a simplified modulation order selection criterion, it still requires significant processing power.

In this context, we propose a novel adaptive antenna selection method for optimum transmission in SM. As a three-dimensional modulation scheme, SM enables a trade-off between the size of the spatial constellation diagram and the size of the signal constellation diagram, while achieving the same spectrum efficiency. Based on this unique characteristic, transmission optimized spatial modulation (TOSM) aims to select the best combination of these two constellation sizes, which minimizes the average bit error probability (ABEP). To avoid the prohibitive complexity caused by exhaustive search, a two-stage optimization strategy is proposed. The first step is to determine the optimal number of transmit antennas, and this is performed at the receiver. In the second step, the required number of antennas are selected at the transmitter. In addition to low computational complexity, TOSM needs very limited feedback because of two aspects: i) since it is based on statistical CSI, the frequency of updating is relatively low; and ii) feedback is required only to inform the transmitter of the number of selected antennas, instead of the index of each selected antenna. Therefore, the feedback overhead is negligibly low and not considered in this paper. In addition, assuming the SOTA 2010 power model [15], the overall BS energy consumption is studied for TOSM. The DTX technique is combined with TOSM to further improve the energy efficiency. Compared with our preceding studies in [1] and [2], the contributions in this paper are four-fold: i) a two-stage optimization method is proposed to balance the spatial modulation order with the signal modulation order in SM systems; ii) a complete derivation of a simplified ABEP bound for SM over generalized fading channels is presented; iii) a direct antenna selection method based on circle packing is proposed; and iv) the energy efficiency of TOSM in terms of the BS energy consumption is evaluated for both the continuous mode and the DTX mode.

The remainder of this paper is organized as follows. Section II describes the system model, including the SM transceiver, the channel model, and the BS power model. In Section III, a two-stage antenna selection method is proposed for optimum transmission in SM. Section IV studies the proposed method in the case of Rayleigh fading channels. Simulation results are presented in Section V to validate the optimization accuracy and the bit error rate (BER) performance of the proposed method. Finally, the paper is concluded in Section VI.

II. SYSTEM MODEL

A. MIMO System and Signal Model

A \( N_t \times N_r \) SM-MIMO system is considered, where \( N_t \) and \( N_r \) are the number of transmit antennas and the number of receive antennas, respectively. Unlike the original SM, only a subset of the transmit antennas is used. The size of the spatial constellation diagram, i.e. the number of utilized transmit antennas, is denoted by \( N \), while the size of the signal constellation diagram is denoted by \( M \). The bit stream is divided into blocks with the length of \( \eta_b \) bits, where \( \eta_b = \log_2(N) + \log_2(M) \) is the number of bits per symbol. Each block is then split into two units of \( \log_2(N) \) and \( \log_2(M) \) bits. The first part activates a single transmit antenna from the spatial constellation diagram, and the currently active antenna is denoted by \( t_{act} \). The second part chooses the corresponding symbol \( \chi_l (1 \leq l \leq M) \) from a specific signal constellation diagram, such as phase shift keying (PSK) or quadrature amplitude modulation (QAM), and sends it out through the activated antenna. The transmitted signal of SM is represented by a vector \( x = [0, \ldots, x_{t_{act}}, \ldots, 0]^T \) of \( N \) elements, where the \( t_{act} \)-th element is \( \chi_l \) and all other elements are zero.

The received signal is given by \( y = HX + w \), where \( H \) stands for the channel matrix and it is described in Section II-B; the vector \( w = [w_1, w_2, \ldots, w_{N_r}]^T \) and \( w_j \), the noise at the \( r \)-th receive antenna, is a sample of complex additive white Gaussian noise with distribution \( CN(0, N_0) \). Across receive antennas, the noise components are statistically independent. The signal-to-noise ratio (SNR) is defined as \( \gamma = E_mL/N_0 \), where \( E_m \) is the average energy per symbol transmission and \( L \) denotes the path loss without shadowing. In addition, the required RF output energy per bit is denoted by \( E_b = E_m/\eta_b \).

The transmitted information bits are decoded by the joint maximum likelihood (ML) detection in [12] as follows:

\[
[\hat{t}_{act}, \hat{l}] = \arg \min_{t,l} \|h_t\chi_l\|^2 - 2\Re\{h^Ht\chi_l\},
\]

where \( h_t \) is the \( t \)-th column of \( H \); \( \Re\{\cdot\} \) is the real part of a complex number; \( \hat{t}_{act} \) and \( \hat{l} \) are the detection results of \( t_{act} \) and \( l \), respectively.

B. Channel Model

1) Channel Distribution: The fading coefficient of the link from the \( t \)-th transmit antenna to the \( r \)-th receive antenna is denoted by \( h_{t,r} = \beta_{t,r} \exp(j\varphi_{t,r}) \), where \( \beta_{t,r} \) and \( \varphi_{t,r} \) are the amplitude and the phase, respectively. The channel fading distribution as well as the CSI is assumed to be known at the receiver. Nakagami-\( m \) fading is considered in this paper, i.e. \( \beta_{t,r} \sim \text{Nakagami}(m_{t,r}, \Omega_{t,r}) \), where \( m_{t,r} \) is the shape parameter (when \( m_{t,r} = 1 \), the channel is Rayleigh fading) and \( \Omega_{t,r} \) is the spread controlling parameter. The phase \( \varphi_{t,r} \) is uniformly distributed between \((-\pi, \pi]\).

2) Channel Correlation: Since we focus on selecting the transmit antennas, the receive antennas are assumed to be independent without loss of the generality. The correlation coefficient between the amplitudes of the two propagation paths from the transmit antennas \( t_i \) and \( t_j \) to the \( r \)-th receive antenna is denoted by \( \rho_{t_i,t_j,r} \). The exponential correlation matrix model in [16] is considered, which is based on the fact that the channel correlation decreases with increasing the distance between antennas. As shown in Fig. 1, the transmit antennas are located in a normalized square area, i.e. the distance between \( t_1 \) and \( t_A \) is unity. The correlation coefficient between \( t_1 \) and \( t_A \) with respect to the \( r \)-th receive antenna is denoted by \( \rho_{s(r)} \).
The number of antennas on each side of the antenna array is formulated as follows:

\[
A = \begin{cases} 
\sqrt{N_t} & \text{if } \log_2 N_t \text{ is even} \\
3 \times \sqrt{\frac{N_t}{2}} & \text{if } \log_2 N_t \text{ is odd}
\end{cases}.
\]  

(2)

When \( \log_2 N_t \) is even, the antennas form a square array with the dimension of \( A \times A \). If \( \log_2 N_t \) is odd, the antennas are placed in the shape shown in Fig. 1(b), where \( A_{\text{inner}} = \sqrt{\frac{N_t}{2}} \).

The absolute distance between \( t_i \) and \( t_j \) is denoted by \( d_{t_i, t_j} \), and the correlation between those two antennas is given by [16, Eq. (10)]:

\[
\rho_{s(t_i, t_j)} = \rho_{s(r)}, \quad 0 \leq \rho_{s(r)} \leq 1.
\]  

(3)

The average degree of the channel correlations, denoted by \( \rho_{av} \), is calculated by:

\[
\rho_{av} = \frac{1}{N_r} \sum_{r=1}^{N_r} \left( \frac{1}{N_t(N_t-1)} \sum_{t_i=1 \atop t_j \neq t_i=1}^{N_t} \rho_{s(t_i, t_j)} \right).
\]  

(4)

C. Base Station Power Model

In [17], a linear relationship between the RF output power and the overall consumed power of a multi-sector BS was established. The overall BS power consumption is divided into two parts: the load-dependent portion and the constant portion. The former is dependent on the RF output power, while the latter is invariant. In addition, when no data is transmitted, a sleep mode is enabled to reduce the consumption by switching off unneeded components. In this section, a practical BS power model is introduced for the purpose of evaluating the energy efficiency of the proposed method.

1) Power Model: In [15], based on the above literature, a power model named SOTA 2010 was proposed for a one-sector, single-RF-chain BS. Table I specifies the parameters: \( P_{\text{max}} \) is the maximum RF output power; \( P_0 \) and \( P_s \) denote the constant power consumption for the active mode and the sleep mode, respectively; \( \zeta \) stands for the slope that quantifies the load dependence. The instantaneous BS power consumption, denoted by \( P_{in} \), is formulated as a function of the RF output power \( P_{out} \) [15, Eq. (1)]:

\[
P_{in} = \begin{cases} 
P_0 + \zeta P_{out} & \text{if } 0 < P_{out} \leq P_{\text{max}} \\
P_s & \text{if } P_{out} = 0
\end{cases}.
\]  

(5)

Also, the ratio of the time consumed in the active mode and the total period is referred to as the activation ratio \( \mu \).

2) Continuous Mode and DTX Mode: Two modes are considered to operate the RF chains: the continuous mode and the DTX mode [7]. In the continuous mode, the RF chains are always delivering output power of the same level. As a result, \( P_{out} \) is equal to the average RF output power \( \bar{P}_{out} \), and \( \mu = 1 \). Substituting those conditions into (7), the overall BS power consumption in the continuous mode is obtained by:

\[
P_{BS-\text{cont.}} = N_{\text{act}} P_0 + \zeta \bar{P}_{out}.
\]  

(6)

The data rate of the continuous mode is equal to the average data rate, which is denoted by \( R_b = \bar{P}_{out} / E_b \). Conversely, the DTX mode conveys data with full load, and the instantaneous data rate is \( R_{\text{out max}} = P_{\text{max}} / E_b \) that is higher than \( R_b \). Then the system is enabled into sleep mode during the saved time to maintain the same average data rate. The parameter \( \mu \) of the DTX mode is computed by:

\[
\mu = \frac{\bar{R}_b}{R_{\text{out max}}} = \frac{\bar{P}_{out}}{P_{\text{max}}},
\]  

(7)

Substituting (9) and \( P_{out} = P_{\text{max}} \) into (7), the overall BS power consumption in the DTX mode is expressed as:

\[
P_{BS-\text{DTX}} = N_{\text{act}} P_s + \left( \zeta + \frac{N_{\text{act}} (P_0 - P_s)}{P_{\text{max}}} \right) \bar{P}_{out}.
\]  

(8)

III. TOSM OVER GENERALIZED FADE CHANNELS

As mentioned, SM enables to balance the size of the signal constellation diagram with the size of the spatial constellation diagram. To achieve a certain spectrum efficiency \( \eta_s \), there are \( \eta_s + 1 \) possible combinations of \( (M, N) \), based on the requirement of a power of two to provide a full usage in the constellation diagrams. To supply a complete selection range of \( N, N_t \) is set to be equal to \( 2^M \). In this section, an optimization algorithm is proposed to select the best \( (M, N) \) as well as the specific antennas. The context is arranged in four portions: i) an ABEP upper bound is introduced for SM over generalized fading channels; ii) the ABEP upper bound is simplified...
to suit the minimization on the subject of either $M$ or $N$; iii) the optimum $(M, N)$ is obtained by solving the minimization problem; and iv) the required number of transmit antennas are selected from the antenna array.

A. ABEP Upper Bound

The ABEP upper bound of SM with the joint ML detector is given by [18, Eq. (6)]:

$$\text{ABEP} \leq \text{ABEP}_{\text{spatial}} + \text{ABEP}_{\text{signal}} + \text{ABEP}_{\text{joint}},$$

and:

$$\begin{align*}
\text{ABEP}_{\text{spatial}} &= \frac{\log_2(N)}{\eta r} \sum_{t=1}^{M} \text{ABEP}_{\text{SSK}}(l) \\
\text{ABEP}_{\text{signal}} &= \frac{\log_2(M)}{\eta r N} \sum_{l=1}^{N} \text{ABEP}_{\text{MOD}}(t) \\
\text{ABEP}_{\text{joint}} &= \frac{1}{\eta r M N} \sum_{l=1}^{N} \sum_{t=1}^{M} \sum_{j=1}^{N} \text{ABEP}_{\text{MIX}}
\end{align*}$$

with the terms in summations expressed in (13), shown at bottom of the page, where: i) $P_s(l, t)$ is the average symbol error rate (SER) of the $l$-th signal symbol $\chi_l$ emitted from the $t$-th transmit antenna; ii) $N_H(\cdot)$ denotes the Hamming distance between two symbols; iii) the average pair-wise error probability (APEP) is defined as the probability of pair-wise error event which is calculated by (14), shown at bottom of the page. The denotation $E[\cdot]$ is the expectation operator, and $Q(\cdot)$ is the Q-function.

B. Simplification

Due to the relaxation of linearity requirements, unlike QAM, PSK can work in PA saturation [19]. This makes PSK a more energy efficient modulation scheme. Moreover, results in [18] have shown that, for SM, PSK is not worse than QAM in many cases and in some cases it is even better. Therefore, PSK modulation is applied in the rest of this paper to keep a moderate level of analytical complexity. Every symbol has an equal weight and the average of different signal symbols in (12) can be neglected. In addition, $P_s(t, l)$ reduces to $P_s(t)$.

1) APEP of Spatial Part: At first, we focus on $\text{ABEP}_{\text{spatial}}$ in the union bound. Assuming a high SNR, the APEP of space shift keying (SSK) modulation over correlated Nakagami-$m$ fading channels (with the assumption of $m_{s,t} = m_r$ for $t = 1, 2, \ldots, N_t$) can be obtained based on the moment generate function (MGF) as follows [20, Eq. (15)]:

$$\text{APEP}(t_j \rightarrow t_i) = \gamma^{-N_r} \frac{2^{3N_r-1} \Gamma(N_r + 0.5)}{\sqrt{\pi} \Gamma(N_r + 1)} \prod_{r=1}^{N_r} f(r),$$

where $f(r)$ is computed in (16), shown at bottom of the page. We define:

$$C_{t_i, t_j} = \frac{1}{2} \times 2^{3N_r-1} \Gamma(N_r + 0.5) \prod_{r=1}^{N_r} f(r).$$

Note that $C_{t_i, t_j}$ is constant when $m_r$, $\Omega_{t_r}$, and $\rho_{t_i, t_j, r}$ are given. Consequently, (15) can be rewritten as:

$$\text{APEP}(t_j \rightarrow t_i) = 2C_{t_i, t_j} \gamma^{-N_r}. \tag{18}$$

The $\log_2(N)$ bits provided to the spatial constellation is assumed to be encoded by Gray codes. When a certain antenna is activated, there are $N/2$ other antennas that may cause an error for any particular bit. Therefore the total Hamming distance from one symbol to the other symbols is calculated by:

$$\sum_{t_i \neq t_i = 1}^{N} N_H(t_j \rightarrow t_i) = \frac{N}{2} \log_2(N). \tag{19}$$

Combining (19) and (13a), we have:

$$\text{ABEP}_{\text{SSK}}(l) = \frac{1}{2(N-1)} \sum_{t_i=1}^{N} \sum_{t_j \neq t_i = 1}^{N} \text{APEP}(t_j \rightarrow t_i). \tag{20}$$

\begin{align*}
\text{ABEP}_{\text{SSK}}(l) &= \frac{1}{N \log_2(N)} \sum_{t_i=1}^{N} \sum_{t_j \neq t_i = 1}^{N} N_H(t_j \rightarrow t_i) \text{APEP}((t_j, \chi_l) \rightarrow (t_i, \chi_l)) \\
\text{ABEP}_{\text{MOD}}(t) &= \frac{1}{M \log_2(M)} \sum_{l=1}^{M} P_s(l, t) \\
\text{ABEP}_{\text{MIX}} &= N_H((t_j, \chi_{t_j}) \rightarrow (t_i, \chi_{t_i})) \text{APEP}((t_j, \chi_{t_j}) \rightarrow (t_i, \chi_{t_i}))
\end{align*}

$$\text{APEP}((t_j, \chi_{t_j}) \rightarrow (t_i, \chi_{t_i})) = E \left[ Q \left( \sqrt{\frac{\gamma}{|\alpha_{t_j} \chi_{t_j} - \alpha_{t_i} \chi_{t_i}|^2}}, \frac{1}{4} \right) \right] \tag{14}$$

$$f(r) = \frac{m_r (\Omega_{t_i, r} + \Omega_{t_j, r})^{-2m_r} (1 - \rho_{t_i, t_j, r})^{m_r-1} \sum_{k_r=0}^{k_r} \rho_{t_i, t_j, r} \Gamma(2m_r + 2k_r - 1)}{(\Omega_{t_i, r} \Omega_{t_j, r})^{1-m_r} \Gamma(2m_r)} \left( \frac{\sqrt{\Omega_{t_i, r} \Omega_{t_j, r}}}{\Omega_{t_i, r} + \Omega_{t_j, r}} \right)^{2k_r}$$

$$\Gamma(k_r)$$

$$\Gamma(k_r)$$
Substituting (18) and (20) into (12a), this gives \( \text{ABEP}_{\text{spatial}} \) as a function of \( N \):

\[
\text{ABEP}_{\text{spatial}} = \gamma^{-Nt} \log_2(N) \sum_{t=1}^{N} \sum_{j=1}^{N} C_{ti,j}.
\]  

(21)

2) \text{ABEP of Signal Part}: The average SER of the PSK modulation over Nakagami-\( m \) fading channels is given by (21), Eq. (9.16):

\[
P_s(t) = \frac{1}{\pi} \int_0^{\frac{M-1}{2m-1}} \prod_{r=1}^{Nt} \left( 1 + \frac{\pi t_r \sin^2 \left( \frac{\pi t_r}{M} \right)}{2m r \sin^2 \left( \frac{\gamma}{M} \right)} \right)^{-m_r} d\theta,
\]  

(22)

where \( \pi t_r = \Omega \gamma r \) is the average SNR of the symbol sent from the \( t \)-th transmit antenna at the input of the \( r \)-th receive antenna. The assumption of a high SNR results in \( \gamma \sin^2(\pi/M) \gg 1 \). Hence (22) can be rewritten as:

\[
P_s(t) = \frac{1}{\pi} \sum_{r=1}^{Nt} \prod_{r=1}^{Nt} \left( \frac{2m_r}{\Omega \gamma r \sin^2 \left( \frac{\gamma}{M} \right)} \right)^{m_r}.
\]  

(23)

When \( M \geq 4 \), we have: i) \( \int_0^{\frac{M-1}{2m-1}} \prod_{r=1}^{Nt} 2m_r d\theta \approx \prod_{r=1}^{Nt} 2m_r d\theta \), which is independent of the signal constellation size \( M \); ii) \( \sin(\pi/M) \approx \pi/M \). The average shape parameter of the fading distributions across all receive antennas is denoted by \( m_r = 1/Nr \sum_{r=1}^{Nt} m_r \). Then the average SER of PSK can be formulated as follows:

\[
P_s(t) = \frac{M^{2m_r}}{m_r \gamma^{Nt}} \prod_{r=1}^{Nt} \left( \frac{2m_r}{\Omega \gamma r \sin^2 \left( \frac{\gamma}{M} \right)} \right)^{m_r}.
\]  

(24)

Substituting (24) into (12b), a simplified \( \text{ABEP}_{\text{signal}} \) is obtained by:

\[
\text{ABEP}_{\text{signal}} = \frac{\mathcal{B}_N}{\eta_s} M^{2m_r} \gamma^{-Nt} \gamma^{m_r}.
\]  

(25)

with

\[
\mathcal{B}_N = \frac{1}{\pi^{Nt}} \prod_{r=1}^{Nt} \left( \frac{m_r}{\Omega \gamma r \sin^2 \left( \frac{\gamma}{M} \right)} \right)^{m_r}.
\]  

(26)

Note that similar to \( \text{ABEP}_{\text{spatial}} \) in (21), \( \text{ABEP}_{\text{signal}} \) is also a function of \( N \) after replacing \( M \) by \( 2^N / N \).

3) \text{ABEP of Joint Part}: The symbols of the PSK modulation are expressed as \( \chi_j = \exp(j\varphi_j) \), where \( \varphi_l = 2\pi(l-1)/M \). Thus, (14) can be rewritten as (27), shown at bottom of the page. Substituting (18) and (28) into (12c), \( \text{ABEP}_{\text{joint}} \) is derived as follows:

\[
\text{ABEP}_{\text{joint}} = \frac{\eta_s 2^N - M \log_2 M - N \log_2(N)}{\gamma^{Nt}} \left( \frac{1}{N(N-1)} \sum_{l=1}^{Nt} \sum_{j=1}^{Nt} C_{ti,j} \right) \gamma^{-Nt}.
\]  

(29)

\[
N_{\text{H}} \left( (t_j, \chi_{t_j}) \rightarrow (t_i, \chi_{t_i}) \right) = \frac{(M-1)N}{2} \log_2(N) + \frac{(N-1)M}{2} \log_2(M).
\]  

(28)
of \((M, N)\) that minimizes the ABEP of SM; and ii) select the specific antennas from the antenna array.

C. Optimal Selection of the Number of Transmit Antennas

In this step, the minimization of the simplified ABEP with respect to \(N\) (or \(M\)) is implemented for a given scenario, which is comprised of the spectrum efficiency, the number of receive antennas, the SNR, the fading distribution, and the correlation coefficient. The term \(1/\eta_s\) in (30) is a positive constant, hence it can be removed without affecting the optimization result. In addition, the difference between antennas is not considered in this step. As a result, \(\overline{B}_N\) and \(\overline{C}_N\) are replaced by \(\overline{B}_{N_t}\) and \(\overline{C}_{N_t}\) to avoid the dependence on the antenna dissimilarity. The optimization problem can thus be formulated as (32), shown at the bottom of the page.

Note that both \(\overline{C}_{N_t}^{-1}N_t\) and \(\overline{B}_{N_t}^{-1}N_t\) are constant for a certain scenario. In other words, they are independent of the variable \(M\). The optimization problem in (32) is a non-linear programming problem and can be solved numerically [22]. The optimization result of \(\log_2(M)\) could be, and most likely is, a non-integer. However, without considering special encoding methods such as fractional bit encoding [23], both \(M\) and \(N\) must be a power of two to supply a full usage in the constellation diagram. This can readily be achieved by comparing the ABEP values of the two nearest integers around the optimal \(M\). Afterwards, the best combination of \((M, N)\) is obtained and denoted by \((M_{\text{opt}}, N_{\text{opt}})\).

D. Direct Antenna Selection

The second step is to select a subarray of \(N_{\text{opt}}\) antennas from the size-\(N\) antenna array. The chosen subset should achieve the minimum ABEP of all subarrays with the same size. Since \(\overline{B}_N\) in (30) is irrelevant to the channel correlations, the problem is equivalent to finding the subarray with a minimum \(\overline{C}_N\). Like the traditional transmit antenna selection (TAS) methods, this issue can be solved by an exhaustive search. However, this results in an unaffordable complexity for a large \(\eta_s\). Taking \(\eta_s = 6\) and \(N_{\text{opt}} = 16\) as an example, the full search space is about \(5 \times 10^{14}\), which is prohibitive for practical implementations. Here we propose a novel TAS method based on circle packing, which can directly determine the selection.

As the correlation coefficient \(\rho_{t_i, t_j}\) is inversely proportional to the distance \(d_{t_i, t_j}\), a rational solution is to maximize the minimum geometric distance between any pair of the chosen antennas. This is equivalent to the circle packing problem in mathematics which can be worked out numerically [24]. Fig. 2 shows the circle packing solutions for various numbers of antennas, where the antennas are located at the circle centers. In the original problem, each circle must fit inside the square boundary. The problem at hand is slightly different where the circle centers are restricted to be inside the boundary, and in Fig. 2 this is shown by dashed lines. It is worth noting that this solution requires fully flexible positions. Thus, we refer to it as ideal circle packing (ICP). However, the antenna positions are fixed in practice, and the subarray cannot be perfectly allocated by ICP. Instead, a realistic circle packing (RCP) is developed by selecting those antennas closest to the ideal positions. In Fig. 3, an RCP solution is demonstrated for the case of \(N_t = 32\) and \(N = 8\). As can be observed, the selection presents a similarity to the solution for \(N = 8\) in Fig. 2. With an increase of \(N_t\), the RCP solution becomes closer to ICP as the antenna array supplies a larger flexibility in positions.

IV. TOSM OVER C.I.D. RAYLEIGH FADING CHANNELS

In this section, we study TOSM in the special case of channel fading, Rayleigh fading. We present that TOSM is independent

\[
M_{\text{opt}} = \arg \min_M \quad \frac{\overline{B}_{N_t}}{\gamma^{N_t}N_t} M^{2\pi r N_t} + \frac{\overline{C}_{N_t}}{\gamma^N N_t} (2^{\eta_s N_t} - M \log_2(M)) , \quad \text{subject to:} \quad 1 \leq M \leq 2^{\eta_s} \tag{32}
\]
of SNR in this particular scenario, and a look-up table can be built to quickly determine the best choice of \((M, N)\). In addition, given a target bit error rate (BER) and transmission rate, a closed-form expression of the BS energy consumption is derived for TOSM. This allows us to evaluate the proposed scheme analytically.

A. ABEP of TOSM4

For correlated and identically distributed (c.i.d.) Rayleigh fading channels, we have \(m_r = 1\) and \(\Omega_{r, r} = \Omega\) for all \(t\) and \(r\). As a result, \(\mathcal{T}_N\) in (26) and \(\mathcal{C}_N\) in (31) reduce to:

\[
B = \left(\frac{2}{\Omega}\right)^{N_r} \frac{1}{\pi^{2N_r+1}} \int_0^\pi (\sin \theta)^{2N_r} d\theta, \tag{33}
\]

and

\[
C = \frac{4^{N_r-1} \Gamma(N_r + 0.5)}{\Omega^{2N_r} \sqrt{\pi} \Gamma(N_r + 1)} \left( \sum_{k=0}^{\infty} \frac{\Gamma(2k + 1) \rho_k}{4^k k! \Gamma(k + 1)} \right)^{N_r}. \tag{34}
\]

Correspondingly, the ABEP expression in (30) reduces to:

\[
\text{ABEP} = \frac{B(M)^{2N_r} + C(2^{N_r} \eta_s - M \log_2(M))}{\eta_s^{N_r}}. \tag{35}
\]

The term \(\eta_s \gamma^{N_r}\) is a positive constant, and therefore can be removed without influencing the optimization result. After further removing the constant term \(C^2 \eta_s\), the optimization problem is reduced to:

\[
M_{\text{opt}} = \arg \min_M \quad B(M)^{2N_r} - CM \log_2(M)
\]

subject to: \(1 \leq M \leq 2^{\eta_s}\). \tag{36}

It is not difficult to find that the optimal solution of TOSM over c.i.d. Rayleigh fading is independent of SNR. Also, \(M_{\text{opt}}\) is independent of \(\eta_s\) when \(\eta_s\) is large enough. In other words, for a certain \(N_r\) and a large enough \(\eta_s\), the optimal solution is only dependent on \(\rho_{\text{rev}}\). As shown in Table II, this allows us to build a look-up table to quickly decide the best SM deployment against different channel correlation degrees.

B. Base Station Energy Consumption based on TOSM

Substituting \(\gamma = E_m L/N_0\) into (35), the required \(E_m\) using TOSM is computed by:

\[
E_m = \frac{N_0}{L} \left( \frac{F(M_{\text{opt}})}{\eta_s R_e} \right)^{\frac{1}{\eta_s}}, \tag{37}
\]

where \(F(M) = B(M)^{2N_r} + C(2^{N_r} \eta_s - M \log_2(M))\) and \(R_e\) denotes the value of the target BER. Substituting (37) into \(P_m = E_m R_0/\eta_s\), the required RF output power is obtained by:

\[
P_m = \frac{R_0 N_0}{\eta_s L} \left( \frac{F(M_{\text{opt}})}{\eta_s R_e} \right)^{\frac{1}{\eta_s}}. \tag{38}
\]

The BS energy consumption per bit \(E_{\text{BS}}\) is given by:

\[
E_{\text{BS}} = \frac{P_{\text{BS}}}{R_0}. \tag{39}
\]

1) Continuous Mode: In the continuous mode, the energy consumption per bit of a BS based on TOSM is obtained by substituting (8) and (38) into (39) with \(N_{\text{act}} = 1\):

\[
E_{\text{BS}} = \frac{P_0}{R_b} \frac{t}{\eta_s L} \left( \frac{F(M_{\text{opt}})}{\eta_s R_e} \right)^{\frac{1}{\eta_s}}. \tag{40}
\]

2) DTX Mode: Similarly, based on (10), we can compute \(E_{\text{BS}}\) in the DTX mode as follows:

\[
E_{\text{BS}} = \frac{P_s}{R_b} + \frac{N_0}{\eta_s L} \left( \frac{P_0 - P_s}{P_{\text{max}}} \right) \frac{F(M_{\text{opt}})}{\eta_s R_e}^{\frac{1}{\eta_s}}. \tag{41}
\]
In this section, Monte Carlo results are presented to validate the performance of our scheme. Two cases are studied: i) c.i.d. Nakagami-\(m\) fading; and ii) c.i.d. Rayleigh fading. In the first case, we focus on verifying the accuracy of the simplified ABEP bound. The reason for choosing the second case is three-fold: i) multipath fading is typically modeled by a Rayleigh distribution; ii) by fixing the parameter \(m\), the main trends of TOSM in relation to the channel correlation can be studied; and iii) a look-up table is formed within this scenario to exhibit the optimum transmit structure. In addition, the performance of TOSM is compared with some other schemes including the optimum transmit structure. In Fig. 4, the simplified ABEP expression of SM in (32) is verified against simulation results. To present an extensive comparison, several scenarios are considered by varying the shape factor, the number of receive antennas, the spectrum efficiency and \(E_b/N_0\). A unit spread controlling factor is assumed. For a certain scenario, the BER curve of SM is shown as a function of the average correlation degree. As can be seen, in general, the theoretical curves match the simulation results well. Since the simplified ABEP is an approximation of the ABEP upper bound in [18], we expect some deviations especially at high channel correlations. Despite this, the simplified ABEP is still very close to the simulation results.

A. Accuracy of the Simplified ABEP

Fig. 4. The simplified ABEP bound of SM versus simulations.

V. SIMULATION RESULTS

The BER performance of the proposed RCP approach is evaluated against two baseline schemes: i) the exhaustive search (ES); and ii) the worst case where the neighbouring antennas are selected. We refer to this scheme as worst selection (WS) in the sequel.

Fig. 6 and Fig. 7 present the BER performance of RCP for \(\eta_s = 4\) and \(5\), respectively. Due to the intractable complexity of ES, the results when \(\eta_s > 5\) are not presented. In addition, the antenna area is assumed to be the same to ensure a fair comparison for different \(\eta_s\). Therefore, \(\rho_s\) is used instead of \(\rho_{av}\). As shown, the RCP scheme achieves almost the same performance as ES with a gap of less than 0.3 dB. Furthermore, the negligible difference between RCP and ES is barely affected by the channel correlations, whereas the performance of WS becomes much worse as the correlation increases. To achieve the same BER value of \(1 \times 10^{-4}\) in the case of selecting 8 out

B. Optimality of the Look-up Table

The deployment of \((M, N)\) for TOSM over c.i.d. Rayleigh fading channels is shown in Table II. We use gray colour to highlight the situations in which the optimal number of transmit antennas is one, i.e. SM is not applied. The following outcomes are observed: i) given a certain \(\eta_s\) and \(N_r\), the optimal \(N\) decreases as \(\rho_{av}\) increases. In other words, it is better to use SM with fewer transmit antennas at high channel correlations; ii) if \(N_r = 1\), the best choice regresses to a single transmit-antenna scheme for an extremely high \(\rho_{av}\); iii) when \(N_r\) is increased, SM is suitable for more cases of \((\eta_s, \rho_{av})\), and the optimal number of transmit antennas becomes larger; and iv) for a certain \(N_r\) and \(\rho_{av}\), the best selection of \(M\) maintains a constant when \(\eta_s\) is large enough.

The look-up table can be extended to any needed situation and easily used to configure the deployment of the TOSM system. In practice, \(N_r\) and \(\eta_s\) are usually fixed for a BS. As a result, the only parameter that needs to be determined is the correlation coefficient, which can be obtained through the structured correlation estimator [25]. Using \(\eta_s = 6\) and \(N_r = 2\), for example, Fig. 5 shows the simulation results of various fixed-SM schemes at \(E_b/N_0 = 25\) dB. As shown, SM using \(N = 16\) outperforms \(N = 8\) when \(\rho_{av}\) is below 0.5. However, the opposite result happens for \(0.6 \leq \rho_{av} \leq 0.9\). At an extremely high correlation of \(\rho_{av} = 1\), two transmit-antenna SM achieves the lowest BER. It can be observed that Table II fits the simulation results.

C. BER Performance of Direct Antenna Selection

The BER performance of the proposed RCP approach is evaluated against two baseline schemes: i) the exhaustive search (ES); and ii) the worst case where the neighbouring antennas are selected. We refer to this scheme as worst selection (WS) in the sequel.

Fig. 5. BER performance of fixed-SM schemes over c.i.d. Rayleigh fading.
of 32 antennas, in comparison with WS, RCP obtains an energy saving of 1.1 dB and 2.0 dB at $\rho_s = 0.1$ and 0.9, respectively.

**D. BER Performance of TOSM**

The complexity of the MIMO system depends on the number of RF chains rather than the total number of transmit antennas. Despite the requirement of large antennas at the transmitter, TOSM needs only one RF chain. For this reason, it is reasonable to compare our approach to fixed-SM schemes with the same $\eta_s$. Based on the obtained optimal $N$, we evaluate the BER performance of TOSM. Assuming $N_r = 2$ and $E_b/N_0 = 25$ dB, Figs. 8–10 show the BER results against the channel correlation for $\eta_s = 4$, 5 and 6, respectively. The case of $N = 1$ is referred to as single-input multiple-output (SIMO).

The following trends are observed: i) fixed-SM with more antennas is not always better than those using fewer antennas. This signifies that the benefit does not simply come from employing more transmit antennas; ii) TOSM always performs better than or equal to fixed-SM schemes, which validates the optimization results; and iii) when $\eta_s$ increases, TOSM employs more transmit antennas and performs much better than the fixed-SM with a small $N$. Specifically, TOSM slightly outperforms fixed-SM with $N = 2$ at both low and high correlations for $\eta_s = 4$. However, for $\eta_s = 5$ and 6, TOSM can always achieve a significant gain except when the channel correlation is extremely high. Similar, but less pronounced trends are
noticed at lower SNRs. In Fig. 11, the BER performance of TOSM is shown as a function of $E_b/N_0$ for $\eta_s = 6$. As can be seen, TOSM significantly outperforms the other schemes for all presented SNRs and various channel correlation degrees. When the channels are independent, i.e., $\rho_s = 0$, TOSM saves energy in the regions of 0.8 dB, 8.7 dB, and 15.1 dB relative to SSK, fixed-SM with $N = 2$, and SIMO, respectively. As $\rho_s$ increases, TOSM outperforms SSK more significantly. Conversely, fixed-SM with $N = 2$ is only slightly affected by the channel correlation, and the advantage of TOSM is diminishing with an increase of $\rho_s$. However, the gain of TOSM over fixed-SM with $N = 2$ still exceeds 4 dB at $\rho_s = 0.8$.

E. Energy Consumption

The overall BS energy consumption of TOSM is studied in contrast to V-BLAST and STBC as well as fixed-SM. The simulations are restricted in two aspects. On the one hand, the number of transmit antennas for V-BLAST is limited due to the constraint $N_t \geq N_r$. On the other hand, when using more transmit antennas, SM can save more energy in RF chains. The motivation here is to validate the energy efficiency of TOSM. To carry out a relatively fair comparison, the case of $\eta_s = 4$ and $N_r = 4$ is chosen, where TOSM employs four transmit antennas for the channel correlation varied from 0 to 0.8. The same $4 \times 4$ MIMO structure is arranged for both V-BLAST in [26] and rate 3/4 STBC in [27]. In addition, the path loss is assumed to be 100 dB without considering the shadowing.

Fig. 12 gives the transmit energy consumption results for a target BER value of $1 \times 10^{-4}$. It is noticed that, TOSM provides a remarkable and stable gain of around 5 dB in comparison with STBC and more with V-BLAST. The overall BS energy consumptions in both the continuous mode and the DTX mode are shown in Fig. 13. To maintain a certain $E_b$, $P_{max}$ leads to a ceiling of the transmission rate. For this reason, a bit rate of 30 Mbit/s is chosen to ensure the BS works physically, and we compare TOSM with STBC to show the trends. The following outcomes are observed: i) using the DTX mode for TOSM provides a gain of around 1.4 dB over the continuous mode; ii) TOSM significantly outperforms STBC for both the continuous mode and the DTX mode with energy-saving gains of at least 6.3 μJoule/bit (56%) and 5.8 μJoule/bit (62%), respectively; iii) compared with multi-stream MIMO schemes, TOSM requests much less energy because of the single RF chain requirement; iv) in both modes, the gain obtained by TOSM increases as $\rho_s$ increases; and v) TOSM saves more energy in the DTX mode than the continuous mode, especially at high channel correlations. When $\rho_s = 0.8$, for example, TOSM outperforms STBC by 7.7 μJoule/bit (57%) and 8.0 μJoule/bit (63%) in the continuous mode and the DTX mode, respectively.

F. Maximum Transmission Rate

In contrast to multi-stream MIMO schemes, SM requires only one RF chain, and therefore less energy is requested to drive it. However, the maximum RF output power of SM is $1/N_{act}$ of the MIMO scheme with $N_{act}$ active antennas, and this restricts the maximum transmission rate of SM. Fig. 14 shows $R_b$ as a function of the channel correlation in the same scenario of the previous subsection. As can be seen, the maximum transmission rate of fixed-SM outperforms V-BLAST, but
the required number of transmit antennas from an antenna array. In addition, a look-up table was built in the case of c.i.d. Rayleigh fading, which can readily be used to determine the optimum transmit structure. Results show that TOSM improves the BER performance of the original SM significantly, and outperforms V-BLAST and STBC greatly in terms of the overall BS energy consumption. A further study shows that TOSM is more energy efficient when combined with the DTX mode than the continuous mode. Furthermore, the issue with respect to the maximum transmission rate in the SM systems has been addressed, which is caused by the limited output power of a single RF chain. It was shown that TOSM uplifts the maximum transmission rate of the original SM greatly, and diminishes the gap between SM and STBC significantly. All these merits make TOSM a highly energy-efficient, low-complexity scheme to satisfy the requirement of high data rate transmission, and an ideal candidate for massive MIMO [28]. Further research will extend the optimum transmit structure to generalized SM [29], where several antennas are activated simultaneously.

VI. CONCLUSION

In this paper, we proposed an optimum transmit structure for SM, which balances the size of the spatial constellation diagram and the size of the signal constellation diagram. Instead of using exhaustive search, a novel two-stage TAS method has been proposed for reducing the computational complexity, where the optimal number of transmit antennas and the specific antenna positions are determined separately. The first step is to obtain the optimal number of transmit antennas by minimizing a simplified ABEP bound for SM. In the second step, a direct antenna selection method, named RCP, was developed to select the required number of transmit antennas from a antenna array. In addition, a look-up table was built in the case of c.i.d. Rayleigh fading, which can readily be used to determine the optimum transmit structure. Results show that TOSM improves the BER performance of the original SM significantly, and outperforms V-BLAST and STBC greatly in terms of the overall BS energy consumption. A further study shows that TOSM is more energy efficient when combined with the DTX mode than the continuous mode. Furthermore, the issue with respect to the maximum transmission rate in the SM systems has been addressed, which is caused by the limited output power of a single RF chain. It was shown that TOSM uplifts the maximum transmission rate of the original SM greatly, and diminishes the gap between SM and STBC significantly. All these merits make TOSM a highly energy-efficient, low-complexity scheme to satisfy the requirement of high data rate transmission, and an ideal candidate for massive MIMO [28]. Further research will extend the optimum transmit structure to generalized SM [29], where several antennas are activated simultaneously.

REFERENCES


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