Reconstructing the primary reflections in seismic data by Marchenko redatuming and convolutional interferometry

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SYNTHESISING PRIMARY REFLECTIONS BY MARCHENKO REDATUMING AND
CONVOLUTIONAL INTERFEROMETRY

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Running Head: Synthesising Primary Reflections

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ABSTRACT

State of the art methods to image the Earth’s subsurface using active-source seismic reflection data involve reverse-time migration (RTM). This, and other standard seismic processing methods such as velocity analysis, provide best results only when all waves in the data set are primaries (waves reflected only once). A variety of methods are therefore deployed as pre-processing to predict multiples (waves reflected several times); however, accurate removal of those predicted multiples from recorded data using adaptive subtraction techniques proves challenging, even in cases where they can be predicted with reasonable accuracy. We describe a new, alternative strategy: we construct a parallel data set consisting of only primaries, which is calculated directly from recorded data. This obviates the need for both multiple prediction and removal methods. Primaries are constructed using convolutional interferometry to combine first arriving events of up-going and direct-wave down-going Green’s functions to virtual receivers in the subsurface. The required up-going wavefields to virtual receivers are constructed by Marchenko redatuming. Crucially, this is possible without detailed models of the Earth’s subsurface velocity structure: similarly to most migration techniques, the method only requires surface reflection data and estimates of direct (non-reflected) arrivals between virtual subsurface sources and the acquisition surface. The method is demonstrated on a stratified synclinal model. It is shown both to be particularly robust against errors in the reference velocity model used, and to improve migrated images substantially.

INTRODUCTION

Whereas advanced methods of seismic data processing such as recursive imaging (Malcolm et al., 2009) or full-waveform inversion (Virieux and Operto, 2009) can properly take into account data that includes multiply scattered waves, many current standard processing steps are based on the so-called Born approximation. This approximation assumes that waves have only scattered from heterogeneities in the medium once, thus requiring that data consist only of primaries – singly scattered energy. Such processing steps include normal moveout correction and velocity analysis
and imaging reflectors using standard linear migration (Zhu et al., 1998; Gray et al., 2001). Multiples represent a source of coherent noise for such methods and must be suppressed to avoid artefacts.

Multiples related to reflections from the Earth’s free surface particularly impact on images resulting from seismic marine data, and much effort has been devoted to their removal (see review by Dragoset et al., 2010). By contrast, internal multiples affect both marine and land data, and relatively fewer techniques exist to predict and remove them from reflection data. Berkhout and Verschuur (1997) iteratively extrapolate shot records to successive reflecting boundaries responsible for multiple generation. Jakubowicz (1998) used combinations of three observed reflections to predict and remove multiples, which led to several other variations on that theme (e.g., Behura and Forghani, 2012; Hung and Wang, 2012). However, the above schemes require significant prior information about subsurface reflectors or reflections prior to multiple prediction and removal. Inverse scattering methods for multiple prediction (e.g., Weglein et al., 1997, 2003) do not demand so much information but tend to be computationally expensive (Wang et al., 2012). Meles et al. (2015) proposed an internal multiple prediction method based on a combined use of seismic interferometry and Marchenko redatuming.

Seismic interferometry techniques synthesise Green’s functions between source (or receiver) locations by integrating cross-correlations or convolutions of wavefields recorded by receivers (or emanating from sources) located elsewhere (Campillo and Paul, 2003; Wapenaar, 2004; van Manen et al., 2005; Wapenaar and Fokkema, 2006). With these methods, one of the sources (or receivers) is basically turned into a virtual receiver (or source).

Marchenko redatuming estimates up- and down-going components of Green’s functions between an arbitrary location inside a medium such as the Earth’s subsurface where no sources (or receivers) are placed, and real receivers (or sources) located at the surface (Broggini et al., 2012; Wapenaar et al., 2012, 2014a; da Costa et al., 2014; Wapenaar, 2014). In contrast to interferometry but similarly to standard linear migration methods, Marchenko focusing requires an estimate of
the direct wave from the virtual source (or to the virtual receiver), illumination from only one side of the medium, and no physical sources (or receivers) inside the medium.

In principle redatumed Green’s functions can be used to provide multiple-free images directly (Behura et al., 2014; Broggini et al., 2014; da Costa et al., 2015; van der Neut et al., 2015). However, this approach requires as many virtual sources as there are image points in the subsurface, and many correlation or deconvolution operations. It is thus computationally feasible only if we wish to image a small portion of the subsurface. Marchenko focusing also allows one to perform redatuming of surface reflectivity to a finite number of depth levels, and to apply standard imaging in between those datum levels (Wapenaar et al., 2014b; Ravasi et al., 2015). In that case, however, the redatumed reflectivities include internal multiples reverberating below the redatuming level, which again may diminish the quality of resulting images if they are not removed prior to imaging.

Using an approach similar to that employed by Meles et al. (2015) to predict multiples, we propose here a new method to estimate primaries directly, based on convolutional interferometry and Marchenko redatuming. This obviates the need to subtract multiples from recorded data, and produces a primaries-only data set that can be inverted without multiple-related artefacts using standard methods of linear migration. We demonstrate the method on a synthetic data set and show that it is particularly robust against errors in initial estimates of the velocity structure.

**METHOD**

Convolutional interferometry uses acoustic reciprocity theorems to express the Green’s function between two locations as (van Manen et al., 2005)

\[
G(x_2, x_1) = \int_S \frac{1}{\rho(x)} \{G(x, x_2) n_t \partial_i G(x, x_1) - n_t \partial_i G(x, x_2) G(x, x_1)\} dS
\]

(1)

Here \(\rho(x)\) denotes density, \(x_1\) and \(x_2\) are two source (or receiver) positions, \(G(x_2, x_1)\) represents the frequency domain Green’s function recorded at \(x_2\) for an impulsive volume injection rate
source at $x_1$, $S$ is an arbitrary boundary of receivers (sources) enclosing either $x_1$ or $x_2$ but not both (Figure 1(a)), and $n_i$ and $\partial_i$ represent the $i^{th}$ Cartesian component of the normal vector to $S$, and of the gradient, respectively. Einstein summation applies over repeated indices, and we have applied source-receiver reciprocity to the expressions in van Manen et al. (2005).

We can approximate equation (1) using a one-way wave propagation formalism and by approximating dipoles (derivatives) as (Wapenaar and Berkhout, 1989; Wapenaar and Fokkema, 2006, Wapenaar et al., 2011)

$$G(x_2, x_1) \approx \int \frac{2/j\omega}{c(x)\rho(x)} \{ G^-(x, x_2)G^+(x, x_1) - G^+(x, x_2)G^-(x, x_1) \} dS,$$

where $c(x)$ indicates wave speed, and $G^{+/−}$ represent down/up-going Green’s function components at the boundary $S$. The main contributions to the evaluation of such interferometric surface integrals come from neighbourhoods of points where the phase of the integrand is stationary (Snieder et al., 2006), and some example stationary points for reflected waves are indicated by circles in Figure 1(a). For the geometries considered here, these stationary points are located inside the medium, and usually the corresponding Green’s functions in the integrand ($G^{+/−}(x, x_1)$ and $G^{+/−}(x, x_2)$) can be neither directly measured nor modelled accurately, as this would require either the presence of receivers in the subsurface or knowledge of the exact subsurface velocity and density distributions. However, Marchenko redatuming provides estimates of all such Green’s functions from surface sources at $x_1$ or $x_2$ to receivers located in the subsurface at points $x$ (Figure 1(a)), given only surface reflection data and an estimate of the direct (non-reflected) wavefield from the surface to $x$ (Broggini et al., 2012; Wapenaar et al., 2012, 2014).

Figure 1(a) illustrates how primary reflections are reconstructed in convolutional interferometry: equations (1) and (2) essentially piece together and integrate wavefields travelling upwards and downwards from around each stationary point, to calculate wavefields that would travel along
each full wave path between $x_1$ and $x_2$. Meles et al. (2015) noted that the number of reflections undergone by an event in $G(x_2,x_1)$ (its scattering order) is equal to the sum of the number of reflections undergone by its constitutive components, $G(x,x_1)$ and $G(x,x_2)$. They concluded that one component of primaries (scattering order = 1) must be a direct wave (scattering order = 0) and one component must be a first-order scattering event (Figure 1). They used that property to synthesize only multiple reflections by convolving components consisting of purely scattered waves (i.e., events with scattering order $\geq 1$); this choice resulted in synthesis of events whose minimum wave path scattering order was 2. The results were subtracted from measured reflection data to reveal the primaries. In this paper we outline a related approach to predict primaries directly.

Primaries are constructed by convolving down-going direct waves and up-going first order scattered waves. Following the standard decomposition of Green’s functions into direct and scattered waves (e.g., $G(x,x') = G_D(x,x') + G_S(x,x')$, where $G_D(x,x')$ represents the component of $G(x,x')$ that does not undergo any reflection), direct waves are uniquely defined for any source-receiver pairs $G_D(x,x_1) \text{ or } G_D(x,x_2)$. By contrast, up-going Green’s functions $G_S^{-}$ comprise many first order scattering events (in addition to multiples). This is illustrated in Figure 1(a), which discriminates the construction of two different primaries. Filled circles indicate points at which direct waves are pieced together with first-arriving events of scattered up-going Green’s functions on surface $S$. The unfilled circle indicates a point where this does not apply: for that point the associated primary reflection $G_S^{-}(x,x_2)$ is not the first scattered arrival. The latter case is more clearly explained in Figure 1(b) where we focus on the construction associated with the unfilled circle in Figure 1(a). Here, the construction associated with a primary in $G(x_2,x_1)$ - the thick solid black ray, is shown to involve the down-going direct wave $G_D^+(x,x_1)$ - dashed black ray, and a singly scattered up-going component of $G^-(x,x_2)$ - thin solid black ray. The latter has a larger travel-time than the first arrival of the up-going Green’s function $G^-(x,x_2)$ - dotted black ray. Thus for arbitrary boundaries $S$, the components associated with primaries do not necessarily
Involves direct waves and first arriving events of up-going Green’s functions (this is especially the case when large off-sets and layers with varying velocity are considered). Finally, note that whenever multiply scattered waves in either $G^-$ or $G^+$ are convolved with any event in $G^+$ or $G^-$, respectively, only multiples are synthetized.

In Figure 2 different partial boundaries (comprising only either horizontal or vertical lines) are used to construct primaries. Filled circles and solid rays indicate points at which direct waves and first arriving events of up-going Green’s functions are pieced together at a stationary point to construct the corresponding primary. Unfilled circles and dashed rays indicate points at which direct waves and later, singly scattered arriving events of up-going Green’s functions are pieced together at a stationary point to construct the corresponding primary.

This shows that for a 1D medium and any $x_1, x_2$ pair, if only first-arriving up-going waves are included then a single horizontal surface $S$ always results in the synthesis of a single primary event (Figure 2(a-c)). By contrast, a vertical portion of $S$ may result in zero, one or several events (Figure 2(d-e)).

Figure 3(a) and (b) show the locations of a single stationary point corresponding to a primary event when different boundaries ($S^u$ and $S^d$) are taken into account. The white and black squares indicate points $x_\alpha, x_\beta$ where either the integrand $G^+(x, x_1)G^-(x, x_2)$ or $G^+(x, x_2)G^-(x, x_1)$ is stationary, respectively. Figures 3(a) and (b) shows that a single stationary point is sufficient to synthesize the sought primary. Up- and down-ward pointing arrows in 3(a) and 3(b) represent normal vectors at stationary points $x_\alpha$ and $x_\beta$; note that for boundaries $S^u$ and $S^d$ these normal vectors are antiparallel at $x_\alpha$ and $x_\beta$. However, if we consider a single boundary containing both $x_\alpha$ and $x_\beta$ (e.g., $S^t$ in Figure 3(c)), we observe that the normals at $x_\alpha$ and $x_\beta$ are parallel. As each single stationary point $x_\alpha$ and $x_\beta$ would construct the sought primary with antiparallel normals (Figure 3(a) and 3(b)), we conclude that their contributions in equation (1) and (2) must sum destructively when parallel normals are used (since changing the normal direction is here
equivalent to multiplying the result by -1): for \( S^t \) in Figure 3(c) the primary is in fact also synthesized by stationary point \( x_γ \).

Keeping in mind the above observations and the limitations concerning performance of the method for different boundaries summarized in Figure 2, if we assume that the first arriving energy of any up-going Green’s function \( G^- (x_1, x) \) is associated with a singly-scattered event, then we can reconstruct primaries by combining such events with direct waves. More precisely, we postulate that primaries, and primaries only, are reconstructed when first-arriving up-going events are convolved with direct down-going Green’s functions. We therefore propose the following approximate representations for primaries:

\[
G_p(x_2, x_1) \approx \sum_i \int_{S_i} \frac{2j\omega}{c(x)\rho(x)} \{G_F^- (x, x_2) G_D^+ (x, x_1)\} dS
\]  

(3a)

\[
G_p(x_2, x_1) \approx \sum_i \int_{S_i} \frac{2j\omega}{c(x)\rho(x)} \{G_D^+ (x, x_2) G_F^- (x, x_1)\} dS
\]  

(3b)

\[
G_p(x_2, x_1) \approx \sum_i \int_{S_i} \frac{j\omega}{c(x)\rho(x)} \{G_F^- (x, x_2) G_D^+ (x, x_1) + G_D^+ (x, x_2) G_F^- (x, x_1)\} dS
\]  

(3c)

where \( G_p \) stands for the Green’s function’s primary arrivals, \( G_D^+ \) for the direct down-going wave, \( G_F^- \) for the first-arriving events of up-going components of Green’s functions that are created (in our examples) using Marchenko redatuming, and \( S_i \) is a partial boundary \((i=1,2,\ldots)\). Note that equations 3(a), 3(b) and 3(c) correspond to geometries depicted in Figures 3(a), 3(b) and 3(c), respectively. Similarly to the multiple prediction method of Meles et al. (2015), we use multiple horizontal truncated boundaries \( S_i \), similar to that shown by the thick dashed horizontal line in Fig. 3(c). Summing results over multiple horizontal, vertically separated boundaries in equation 3(c) ensures that we can capture all primaries by at least one boundary. While complex subsurface structural geometries may invalidate the above postulate, we expect it to be valid for many realistic media.
Equations 3(a-c) differ crucially from the representation theorems in equations (1) and (2): either 
\[ G_D^+(x, x_1) G_F^-(x, x_2) \text{ or } G_D^+(x, x_2) G_F^-(x, x_1) \] only (equations 3(a) and 3(b)) or their sum (equations 3(c)) are integrated. These equations are consistent with our observation above regarding constructions involving only one stationary point (Figure 3(a) and 3(b)) and destructive summation due to the parallel normal along truncated horizontal boundaries (Figure 3(c)). Due to the latter issue, the second term in the integrand in equation 3(c) has the opposite sign compared to that in equation 1 or 2 which avoids the two terms mutually cancelling out in the case of Figure 3(c). Equations 3(a-c) result in the desired primaries provided that the stationary points of the corresponding integrands are located along the boundaries employed. Note that the integrand in equation 3(c) is stationary whenever either of those in equations 3(a) or 3(b) are. The distribution of stationary points of the integrands in equations 3(a) and 3(b) is a-priori unknown, so boundaries must usually be chosen with a degree of arbitrariness. Such boundaries may therefore contain the stationary points of equation 3(a) but not those of 3(b), or vice versa, and consequently these equations may produce slightly different results on truncated boundaries. To maximize the probability of producing all primaries we need to include as many stationary points as possible (i.e. both those of equations 3(a) and 3(b)). Therefore, herein we apply equation 3(c), even if this may result in incorrect amplitudes (as stationary points of only 3(a) or 3(b), or of both may be located on each employed boundary).

Several first order reflections may be associated with a single more complex reflector (e.g., synclinal geometries as in Figure 4) and Marchenko redatuming will correctly reproduce them all (in the case that sufficient coverage is provided for each event). Equation 3(c) will synthesize the various primaries as long as each constituent ray can be represented as a convolution of direct and first arriving events at different stationary points along the integration boundary (see Figure 4).

The above primary prediction method is based on representation theorems, and in principle could estimate exact phases and amplitudes of primaries. However, inaccuracies in Marchenko Green’s functions or in the implementation of equations (3) may affect the results. For example, equations
(3) require knowledge of velocity \(c(x)\) and density \(\rho(x)\) along integration boundaries \(S_i\). These quantities are rarely known exactly in practical situations, and therefore in our numerical experiments we use the values in the smooth reference model for both \(c(x)\) and \(\rho(x)\) when performing integration along boundaries \(S_i\). However, provided \(c(x)\) and \(\rho(x)\) vary smoothly other than at reflectors, the use of incorrect velocity and density would usually provide accurate kinematics and only affect the amplitudes of predicted primaries. The amplitudes could be corrected by direct comparison with recorded data. Finally, note that each boundary \(S_i\) may generate only one or a subset of all primaries (Figure 2): thus by varying \(S_i\) within equations (3) we also obtain spatial information about which interfaces generate each primary.

We therefore propose the following algorithm to estimate primaries only:

1) Choose a horizontal boundary \(S_i\) in the subsurface. Locate virtual receivers at regularly-sampled locations \(x\) along \(S_i\), and use Marchenko redatuming to compute corresponding up-going Green’s function \(G^-(x,x_p)\), where source locations \(x_p\) span the surface array.

2) Mute events occurring before the direct waves in the up going Green’s functions \(G^-(x,x_p)\) to remove possible Marchenko artefacts (Thorbecke et al., 2013).

3) Pick the first-arriving event in the muted up-going Green’s function \(G_M^-(x,x_p)\) to produce \(G_F^-(x,x_p)\).

4) Apply equation 3(c) to predict primaries \(G_P(x_j,x_k)\) for all \(x_j,x_k\) in the surface array.

5) Repeat steps 1 to 4 using \(S_i\) located at different depths to predict different primaries, then sum the results as specified in equation 3(c).

**NUMERICAL EXAMPLE**

We test the algorithm using a 2-dimensional varying density-velocity synclinal model (Figure 5). We compute synthetic surface seismic data with a finite-difference time domain modelling code and a Ricker source wavelet with central frequency 20 Hz, using absorbing boundaries on all sides.
(thus assuming that surface-related multiples have been removed from recorded data), between 201 colocated sources and receivers equally spaced along the surface of the model shown in Figure 5, with inter-source spacing of 12 m.

Partial boundaries consist of horizontal lines $S_1$ to $S_4$ in Figure 5. Up-going Green’s functions $G^-(x, x_p)$ are estimated at a set of 121 points $x$ along each boundary using Marchenko redatuming. We estimate direct waves $G^+_D(x, x_q)$ using a smooth velocity model (Figure 5(c)). First arriving events of up-going Green’s functions are then picked automatically and windowed.

For subsurface points $x_A$ and $x_B$ in Figure 5(a) and $x'$ spanning the surface sources, we discuss the picking process in more detail. The presence of refractions in the data and the inaccuracy of the modelled direct waves that are used in Marchenko redatuming result in artefacts (indicated by arrows in Figure 6) contaminating the up-going components of the estimated Green’s functions $G^-(x_{A,B}, x')$ (Thorbecke et al., 2013). To simplify the picking procedure and avoid artefacts in the estimation of primaries, we mute up-going Green’s functions so that we do not consider any energy occurring before the corresponding direct waves $G^+_D(x_{A,B}, x')$ (Figures 6(b) and 6(e)), whose kinematics are indicated by solid black lines in Figures 6(a) and 6(d). We then pick first-arriving events $G^+_F(x_{A,B}, x')$ in the muted gathers $G^+_M(x_{A,B}, x')$ by simple windowing (Figure 6(c) and 6(f)). Despite inaccuracies in these wavefields and the consequent errors in picking, primaries were relatively well reconstructed through application of equation 3(c), with only small, low amplitude artefacts (Figure 7). Note that the triplication of the primary associated with the synclinal interface, indicated by red arrows in Figure 7, is synthesized correctly (see Figure 4).

We then apply reverse time migration (RTM) to both the observed data and the estimated primaries using the smoothed reference velocity model (Figure 5(c)). Resulting images are shown in Figure 8. Linear migration of internal multiples results in many multiple-related artefacts contaminating the conventional image (as indicated by red arrows in Figure 8(a)). RTM of primaries only provides a much cleaner image. Minor artefacts below the first reflector (as indicated by the red arrow) are due to imperfect picking of first-arriving events in $G^-$. Amplitude reduction in lateral
portions of the image (left and right sides in Figure 8(b)) are due to inaccuracies in the Marchenko estimates caused by poor illumination of virtual receivers located in the leftmost and rightmost portions of the model.

**DISCUSSION**

We have presented a new method to synthesize primaries based on Marchenko redatuming and convolutional interferometry. The method requires reflection data for a set of collocated surface sources and receivers, and estimates of the direct wave from the surface to each virtual receiver in the subsurface, which may be challenging for complex models. Nevertheless, we showed that primaries are predicted reasonably well despite using a highly smoothed velocity model for direct wave estimation (Figure 5(c)). This is because Marchenko redatuming is a time domain method, and small errors in timing of direct waves result in time shifts of opposite sign in up- and down-going components. Since these components are convolved in our method (equations 3) and hence their phases (travel times) are added, such errors cancel and have little effect on results (see cartoon in Figure 9). This is better illustrated in Figure 10, where Marchenko gathers obtained with different velocity models are compared. Gathers (a) and (c) show down- and up-going Green’s functions retrieved using the correct velocity model for point $x_A$ ($x = 1480$ m, $z = 530$ m) and $x'$ spanning the surface sources in Figure 5(a). Gathers (b) and (d) show down- and up-going Green’s functions that were obtained using a velocity model perturbed by 10%. For the zero-offset trace collected above $x_A$, the down- and up-going components at the stationary point in the convolutional gathers are indicated by arrows in Figure 10. Note that when using the wrong velocity model, exactly opposite shifts in time affect down- and up-going gathers. Their convolution is therefore almost unaffected by the relative errors and the primary is synthesized correctly. This is shown in Figure 11, where the primary associated with the synclinal interface is properly reconstructed despite the use of the wrong velocity model in the Marchenko redatuming step.
The need to convolve first arriving events of up-going Green’s functions and down-going direct waves requires the use of Marchenko redatuming to estimate these components over standard wavefield extrapolation methods as are used in RTM for example. The latter methods usually produce up-going gathers contaminated by non-physical, coherent events associated with internal multiples. Some of these artefacts may occur before the first-arriving event of the true up-going Green’s function (see Figure 12). They would then be picked and jeopardize the performance of the primary method, as convolving these events with direct waves would reproduce internal multiples.

Automating the picking process required by our method can be challenging as small artefacts in the up-going Green’s function gathers might be picked erroneously (Thorbecke et al., 2013). While the demultiple method of Meles et al. (2015) which is also based on Marchenko redatuming and convolutional interferometry does not require any picking, its performance is limited by errors in (adaptive) multiple subtraction procedures, similarly to all other multiple removal algorithms published to-date (Guitton and Verschuur, 2004). This limitation is entirely removed by the primaries synthesis algorithm proposed here.

The choice of the integration boundaries $S_i$ plays a key role in our algorithm. To reproduce all of the primaries with correct relative amplitudes, each $S_i$ should lie above each reflector. If several boundaries are chosen within a homogeneous stratum, the same primary will be predicted multiple times. Stacking over different boundaries (as prescribed by equation 3(c)) would then result in errors in the relative amplitudes of the different primaries, similarly to multiples in the method of Meles et al. (2015).

In our synthetic test for simplicity we used horizontal integration boundaries, but this is not a requirement of the method (arbitrarily shaped boundaries can be used). We also considered the ideal case of single integration boundaries between each pair of reflectors. This choice not only provides good relative amplitudes of primaries, but also allows easier implementation of the method, as first arriving up-going events are all associated with the same interface. However, in
practical applications, erroneous or approximate prior estimates of the subsurface structure may result in integration boundaries that intersect interfaces (Meles et al., 2015). In Figure 13 we consider a similar situation by employing integration boundaries $I_1$ to $I_3$ (Figure 5) and source number 101. When intersecting boundaries are used, primaries (black curves) are only partially properly reconstructed (Figures 13(a), (b) and (c)). Note that in Figures 13(a-c) two primaries are partially synthesized for complementary receiver subsets, and white arrows indicate artefacts in the primary gathers. These artefacts are due to non-stationary contributions of the integrand at points where boundaries intersect an interface. This is better illustrated in Figure 14, where with the aid of a cartoon we investigate in more detail the artefacts labelled A and B in Figure 13(b), corresponding to source 101 and receiver 93 (dashed line in Figure 13(b)). These artefacts are not stationary with respect to the location of the boundary lines, and hence are eventually attenuated by summing over multiple boundaries (black arrows in Figure 13(d)).

Complex subsurface structures may require many integration boundaries to avoid the problems discussed above. In standard applications, a fairly dense grid of redatuming points throughout the medium (similar to that provided locally by $I_1$ to $I_3$ in Figure 5) could be used. As a consequence, the proposed algorithm would become rather expensive, and its total costs might approach that of redatuming via MDD to all subsurface points. However, in contrast to MDD our method does not require any inversion, and furthermore produces multiple-free data at the ground acquisition surface. This is an important difference with respect to MDD redatuming, as our method can be used not only to perform RTM but also to provide primaries-only data for other linearized processing steps that require linearized data (e.g., velocity analysis).

Note that there are other subsurface structural geometries that may invalidate our postulate that primaries are synthesized by the convolution of a direct wave with the first arriving event of the up-going wavefield. In Figure 15 we show two further challenging scenarios where the convolution of first arriving up-going fields and direct waves (a) does not construct a primary event, or (b) produces a refracted wave. Clearly when these types of refracted waves occur, the method is not expected to give perfect results. Note also that Marchenko redatuming is based on
up/down decomposition, and the presence of horizontally propagating waves may affect its performances.

The cost of one iteration of Marchenko redatuming, which we may consider as the unit cost \((U)\) for this methodology, is equivalent to that of a multi-dimensional convolution for each source-receiver pair at the acquisition level. A total number of \(n_r \cdot n_s\) convolutions is then required, where \(n_r\) and \(n_s\) are the number of receivers and sources, respectively. As in standard Marchenko applications source and receivers are collocated, the unit cost involves \(n_r^2\) convolutions. Note that the number of sources and receivers can increase dramatically when moving from 2D to 3D applications, and so does the Unit cost.

Several iterations \((n_i)\) are required before convergence is achieved (in our synthetic tests we used \(n_i = 6\) iterations). In 2D and 3D integration boundaries are lines and surfaces, respectively. Each integration boundary may comprise of up to hundreds (in 2D) or thousands (in 3D) of virtual receiver locations \((n_v)\), depending on the size of the model, in order to conform to the Nyquist criterion. Finally, several or many boundaries \((n_b)\) may be needed for complex subsurface geometries. In total, the overall cost of the method is given by: \(n_i \cdot n_v \cdot n_b \cdot U\). Optimization of the computational cost could be achieved by minimizing/optimizing the number of boundaries to be used and by involving other primary events in the Marchenko redatumed up-going Green’s functions in addition to first arriving waves. A more detailed discussion about the computation costs in Marchenko redatuming can be found in Behura et al. (2014).

Finally, note that Marchenko redatuming was used in this manuscript as it provides separated wavefields at subsurface receiver locations. However, any method and acquisition configuration that allow up/down-going wavefields to be estimated in the subsurface could be used within the algorithm discussed here. Any improvement in either quality or efficiency of those methods compared to Marchenko redatuming would then be expected to be inherited by our method.
CONCLUSIONS

We presented a new method to predict primary reflections based on Marchenko redatuming and convolutional interferometry. The method was demonstrated on acoustic data and proved to be stable with respect to inaccuracies in the redatumed Green’s functions. The synthesized primaries were used to produce images free of multiple-related artefacts via linear reverse-time migration. For simplicity, the method has been tested on a surface-related multiples waves free dataset recorded for collocated sources and receivers. Extension to datasets collected in standard acquisition setups and including ghosts and surface related multiples will be the topic of future research. Applications connected to other methods such as full-waveform inversion and velocity analysis will also be investigated.

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FIGURE CAPTIONS

**Figure 1** Geometrical configurations that construct primaries from convolutional interferometry. Stars are sources at $x_1$ and $x_2$, dashed lines indicate ideal receiver boundaries $S$. (a) Circles indicate stationary points associated with primary reflections between $x_1$ and $x_2$. At each such points convolutional interferometry connects direct and first-order scattering events to create primary waves between $x_1$ and $x_2$. Filled circles indicate stationary points $x$ connecting direct waves $G_D^+(x, x_1)$ and the *first arriving reflection* in $G^{-}(x, x_2)$, or $G_D^+(x, x_2)$ and the *first arriving reflection* in $G^{-}(x, x_1)$. The unfilled circle indicates a stationary point $x$ not connecting $G_D^+(x, x_1)$ and a first arriving reflection in $G^{-}(x, x_2)$. This is better illustrated in (b): the primary reflection event $G(x_2, x_1)$ indicated by the thick ray is constructed by joining an up-going scattered event in $G^{-}(x, x_2)$ and a direct wave $G_D^+(x, x_1)$ (thin solid and dashed rays, respectively). In this case, the first arriving event of $G^{-}(x, x_2)$ is a reflection coming from a different layer (thick dotted ray). The desired primary scattered component (thin solid ray) is the *second* arrival in $G^{-}(x, x_2)$.

**Figure 2** Similarly to Figure 1 but for limited portions of boundary $S$ and different positions $x_2$. Solid rays indicate events involving direct waves and first arriving events of $G^{-}(x, x_1)$ which are therefore reconstructed by equation (3). Dashed rays indicate constructions that do not involve first arriving events of $G^{-}(x, x_1)$ and which therefore are not reconstructed by equation (3).

**Figure 3** Black squares (a and b) indicate points $x_\alpha$ and $x_\beta$ associated with an identical primary event when different boundaries ($S^u$ and $S^d$) are considered. Arrows represent antiparallel normals at $x_\alpha$ and $x_\beta$. Black squares indicate points where either the integrand $G^+(x, x_1)G^{-}(x, x_2)$ or $G^+(x, x_2)G^{-}(x, x_1)$ is stationary at $x_\alpha$ and $x_\beta$, respectively. (c) Mutual cancellation of stationary point contributions occurs in this case due to parallel normals at $x_\alpha$ and $x_\beta$. The gray
square indicates an additional stationary point at $x_\gamma$ whose contribution results in the depicted primary.

**Figure 4** Complex reflector producing several primary reflections (thick and thin rays). The proposed method will synthesize each event that can be obtained as a convolution of direct and first-arriving events of the up-going Green’s function at a stationary point along the integration boundary. Both events here are reconstructed by different stationary points (circles) on the same boundary $\mathcal{S}$.

**Figure 5** (a) Velocity and (b) density models used to compute reflection data. (c) Smooth velocity model used to compute direct waves. $S_i$ to $S_4$ and $I_1$ to $I_3$ represent surfaces used for integration in equation 3(c). Locations of sources 61, 101, 123 and 141 are shown in (a).

**Figure 6** (a, d) Up-going Green’s functions at virtual receivers $x_A$ and $x_B$ in Figure 5(a) respectively, provided by Marchenko redatuming. Black solid lines indicate the kinematics of associated direct waves, while black arrows indicate artefacts. (b, e) gathers muted before the direct wave showing reduced artefacts. (c, f) first-arriving scattered events picked (windowed) from panels (b,e).

**Figure 7** (a) Observed reflection data for source 61 and (b) estimated primaries. The red arrows indicate two correctly synthesized events associated with a triplication from the same reflector (see Figure 3). Each predicted primary corresponds to an actual primary reflection (indicated by blue curves in (a) and (b)). Low amplitude artefacts contaminating gathers b, d and f (black arrows) are due to inaccuracies in the picking process (see Figure 6(b-c)). (c,d) and (e,f): as for (a,b) but for source 101 and 141, respectively. Source locations are shown in Figure 5(a).
**Figure 8** (a) RTM Image obtained by migrating the recorded data (primaries and internal multiples). Blue and red arrows indicate true reflectors and internal-multiple related artefacts, respectively. (b) RTM image obtained by migrating the primaries predicted by equation 3(c). Blue and red arrows indicate true reflectors and picking-related artefacts, respectively. Note that both images have saturated gray-scales at 25% of their maximum amplitude in order to highlight weaker, multiple-related artefacts.

**Figure 9** (a) The square indicates the virtual receiver location when the exact velocity model is used for Marchenko redatuming. Solid and dashed rays represent up-going and down-going Green’s function components, respectively. (b) Same as (a), but using an erroneous velocity model effectively shifts the position of the virtual receiver created by Marchenko redatuming.

**Figure 10** Down-going ((a) and (b)) and up-going ((c) and (d)) Green’s functions estimates as provided by Marchenko redatuming using exact ((a) and (c)) and erroneous ((b) and (d)) velocity models. Opposite time shifts apply to down- and up-going gathers when erroneous velocity models are used (white arrows).

**Figure 11** (a) Observed reflection data for source 123. (b) Estimated primaries using a single convolutional line $S_3$ and a velocity model perturbed by 10%. The predicted primary corresponds to an actual primary reflection (indicated by white curves in (a) and (b)).

**Figure 12** (a) $\mathbf{G}^- (\mathbf{x}_C, \mathbf{x}')$ estimate as provided by Marchenko redatuming. Black arrow indicates the first-arriving event. (b) Standard extrapolation gather at $\mathbf{x}_C$ derived by RTM. White arrows indicate internal-multiple related artefacts occurring before the first arriving physical energy (black arrow).
Figure 13 (a) Estimated primaries corresponding to source 101 and integration boundary $I_1$. Exact primaries are indicated by black curves. White arrows indicate artefacts. (b) and (c): as for (a) but when boundaries $I_2$ and $I_3$ are employed, respectively. (d) Summation of gathers (a) to (c). Black arrows indicate residual low amplitude artefacts. In (b) the dashed line indicates receiver 93, labels A and B denote two distinct artefacts, and C corresponds to the true primary associated with the syncline.

Figure 14 (a) The white circle indicates a non-stationary point of $G_D^+(x, x_2)G_F^-(x, x_1)$ with $G_F^-(x, x_1)$ being generated by reflector $R_B$. As $x$ passes through the interface $R_A$ from the left of the figure, $G_F^-(x, x_1)$ is generated by reflector $R_A$ as shown in panel (b), and cancellation of the non-stationary contribution in (a) does not take place (and for different geometries similar arguments would apply also to $G_D^+(x, x_1)G_F^-(x, x_2)$). (c) Integrand of equation 3(c) for source 101, receiver 93 and integration line $I_2$. Artefacts A, B in Figure 13(b) are due to non-stationary events (A and B in Figure 14(b)) associated with discontinuities in the integrand. For simple models we expect $G_D^+(x, x_{1,2})$ to be continuous functions of $x$. Therefore, discontinuities of the integrand depend on discontinuities in $G_F^-(x, x_{1,2})$. Finally, C indicates the stationary contribution of the integrand that actually produces the true primary event in 13(b).

Figure 15 Complex geometries and refractions that invalidate our assumption concerning the synthesis of primary reflections via convolutional interferometry of first arriving up-going fields and direct waves. In (a), a primary event associated with a diffractor (gray dot) is seen to involve the convolution of horizontally propagating direct and scattered fields at the stationary point (white circle) on $S_1$ due bending of rays. Note that the assumption would be valid if a different boundary like $S_2$ had been employed (black circles are the corresponding stationary points). (b) A refracted event is constructed by convolving first arriving up-going Green’s functions $G^-(x, x_1)$, solid
line) and down-going direct wave \( (G_D(x, x_2), \text{solid line}) \). In this case, the reflected wave (dashed line) may have larger traveltime to the stationary point (black circle) than the refracted arrival.
**FIGURES**

**Figure 1** Geometrical configurations that construct primaries from convolutional interferometry. Stars are sources at \( x_1 \) and \( x_2 \), dashed lines indicate ideal receiver boundaries \( S \). (a) Circles indicate stationary points associated with primary reflections between \( x_1 \) and \( x_2 \). At each such points convolutional interferometry connects direct and first-order scattering events to create primary waves between \( x_1 \) and \( x_2 \). Filled circles indicate stationary points \( x \) connecting direct waves \( G_D^+(x, x_1) \) and the *first arriving reflection* in \( G^- (x, x_2) \), or \( G_D^-(x, x_2) \) and the *first arriving reflection* in \( G^- (x, x_1) \). The unfilled circle indicates a stationary point \( x \) not connecting \( G_D^+(x, x_1) \) and a first arriving reflection in \( G^- (x_2, x) \). This is better illustrated in (b): the primary reflection event \( G(x_2, x_1) \) indicated by the thick ray is constructed by joining an up-going scattered event in \( G^- (x, x_2) \) and a direct wave \( G_D^+(x, x_1) \) (thin solid and dashed rays, respectively). In this case, the first arriving event of \( G^- (x, x_2) \) is a reflection coming from a different layer (thick dotted ray). The desired primary scattered component (thin solid ray) is the *second arrival* in \( G^- (x, x_2) \).
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Figure 11 (a) Observed reflection data for source 123. (b) Estimated primaries using a single convolutional line $S_3$ and a velocity model perturbed by 10%. The predicted primary corresponds to an actual primary reflection (indicated by white curves in (a) and (b)).
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Figure 25 Complex geometries and refractions that invalidate our assumption concerning the synthesis of primary reflections via convolutional interferometry of first arriving up-going fields and direct waves. In (a), a primary event associated with a diffractor (gray dot) is seen to involve the convolution of horizontally propagating direct and scattered fields at the stationary point (white circle) on $S_1$ due bending of rays. Note that the assumption would be valid if a different boundary like $S_2$ had been employed (black circles are the corresponding stationary points). (b) A refracted event is constructed by convolving first arriving up-going Green’s functions ($G^-(x, x_1)$, solid line) and down-going direct wave ($G^+_D(x, x_2)$, solid line). In this case, the reflected wave (dashed line) may have larger traveltime to the stationary point (black circle) than the refracted arrival.