Achieving Arbitrary Signals Transmission Using a Single Radio Frequency Chain

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Abstract—Electrically steerable parasitic array radiator (ESPAR) antenna has been proposed to obtain multiple antenna functionality with only a single radio frequency (RF) chain. However, for some signals, the input resistance of the ESPAR antenna (EA) becomes negative. A negative input resistance would lead to oscillatory/instable behaviour of an EA to transmit signals. In this work, we address this issue of an EA by obtaining approximate signals for transmission, which are close to the ideal signals in the mean squared error (MSE) sense, and satisfy the requirement that the input resistance is positive. Specifically, we formulate and solve an optimization problem to minimize the MSE between the ideal and the approximate signals taking into account the constraint on the input resistance. Closed-form expressions to calculate the approximate signal vector from ideal signal vector are also obtained, resulting in simplified processing at the transmitter. We denote the EA utilizing the novel closed-form expressions as EA with preprocessing (EA-P) and the EA without any preprocessing as the standard EA (EA-S). The performance of EA-P is compared with the performance of EA-S and the standard multiple antenna system, with multiple RF chains, under various communication scenarios. Our simulations show that the performance of EA-S significantly degrades compared with the standard multiple antenna system and an error floor is obtained. However, using the proposed EA-P scheme, the error floor is avoided and nearly the same performance as that of the standard multiple antenna system is achieved.

Index Terms—Reconfigurable antenna, ESPAR, MIMO transmission, a single RF chain, optimization.

I. INTRODUCTION

Multiple-input multiple-output (MIMO) communication has attracted great interest in recent years due to its spatial multiplexing and diversity benefits. These benefits increase as the number of antennas increases. However, each additional antenna element requires an additional radio frequency (RF) chain resulting in increased cost and complexity. In addition, in order to maximize the benefits offered by multiple antennas, the antennas need to be placed sufficiently apart to minimize the coupling. This results in an increase in the size of the device and reduced mobility [1]. Electrically steerable parasitic array radiator (ESPAR), a pattern reconfiguration antenna with only a single active element [2], has been proposed to reduce the cost of multiple antenna devices. Reconfigurable antennas can achieve dynamically changeable antenna properties in frequency, radiation pattern and polarization [3], [4], [5].

The studies in [4], [6], [7], [8] have shown that reconfigurable antennas can improve the performance of MIMO wireless communication systems. For ESPAR antenna (EA), the antenna elements in ESPAR antenna (EA) are placed close together, resulting in strong coupling among antenna elements. This coupling is exploited to control the currents at the antenna elements and the overall radiation pattern is shaped by varying the feeding voltage at the active element and adjusting the impedance of the parasitic elements. Therefore, ESPAR gives multi-antenna functionality utilizing a single RF chain and alleviates the complexity and implementation issues associated with standard multiple antenna systems.

Recently, there has been a lot of research related to different aspects of EA design and to achieve its low-cost implementation in real-world systems. EA based multiple antenna system designs utilizing the beamspace model were proposed in [1], [9], [10], [11]. However, there are several weaknesses in the beamspace MIMO model. In the beamspace model, the receiver cannot guarantee the orthogonality of basic beam patterns and the radiating modes cannot be computed accurately for an arbitrary EA. Moreover, arbitrary channel-dependent precoding cannot be realised for an arbitrary antenna array [12]. Therefore, an alternate model based on the currents at the ports of the transmit antenna was introduced in [13]. The authors in [14] provided the design conditions that an EA has to satisfy in order to support an arbitrary precoding scheme.

The main design condition considered in [14] was that the real part of the input impedance, also labelled...
as input resistance [15], should be positive. Satisfying this condition is essential because a negative input resistance results in a positive reflection coefficient (dB), which implies that the EA is reflecting power back and exhibiting oscillatory/unstable behavior [16]. In a standard multiple antenna system, the input resistance is designed to be positive and therefore, this issue does not arise [15]. However, in the case of the EA system, the value of the input resistance depends on the currents at the antenna elements (which depend on the signal to be transmitted) and the mutual coupling (MC) matrix (which depends on the antenna geometry). If the MC matrix is fixed, which is typically the case for an EA, a change in the antenna signal/current vectors can result in a negative input resistance. This implies that for such signals/current vectors, the EA will reflect back power and exhibit unstable behavior. In order to transmit such signals, a new MC matrix, implying a new EA, needs to be designed. Therefore, one issue with the approach in [14] is that, once an EA is designed, there are still some signals for which the EA will be unstable, as such signals result in a negative input resistance. In order to transmit such signals a new EA needs to be designed.

In this work, a new approach is adopted, i.e., instead of trying to transmit the actual/ideal signals, which give a negative input resistance at the EA; Signals closely approximating the actual/ideal signals which do not lead to a negative input resistance are transmitted. In this way, the EA does not need to be redesigned. To the authors best knowledge, this approach is new and has not been proposed in literature previously. In order to obtain the approximate signal, an optimization problem is proposed to minimize the mean squared error (MSE) between the ideal and approximate signals considering the implementation constraint of the EA. The underlying problem is originally not convex. We recast it to convex form via equivalent reformulation and Lagrange duality. Finally, as practical transmitters require real-time operation, the optimization problem is solved analytically and novel closed-form expressions are derived to easily calculate the approximate current signals for the EA system.

In this paper, we denote the EA utilizing the novel closed-form expressions as EA with preprocessing (EA-P) and the EA without any preprocessing as the standard EA (EA-S). The performance of EA-P is compared with the performance of the EA-S and the standard multiple antenna transmitter in various communication scenarios.

1 Note that recasting a non-convex optimization to a convex optimization problem using Lagrange duality is a well known approach in optimization theory [17]. However, to the best of authors knowledge, none of the previous works related to EA have formulated an optimization problem to obtain the current/signal vectors taking into account the input resistance constraint.

**II. SYSTEM MODEL FOR ESPAR MIMO**

**A. System Model with ESPAR Transmitter**

The circuit diagrams of a $(M+1)$-element EA and a standard multiple antenna are shown in Fig. 1 [18]. The EA consists of a single active element with a RF unit and $M$ parasitic elements without any RF units. On the other hand, the standard multiple antenna consists of $(M+1)$ elements each connected to a RF chain.

As depicted in Fig. 1, $i = [i_0, i_1, \ldots, i_M]^T$ is the vector of currents at the antenna elements, where $T$ denotes the transpose operation. The matrix $Z \in \mathbb{C}^{(M+1) \times (M+1)}$ denotes the MC matrix and it depends on the antenna geometry [19]. $Z_L = \text{diag}(z_0, z_1, z_2, \ldots, z_M)$ is the source impedance matrix where $z_0$ is the source resistance which typically equals $50 \Omega$ [15] and $z_1, z_2, \ldots, z_M$ are the loads at the parasitic elements. For the standard multiple antenna system, the vector of source voltages $v$ is $[v_0, v_1, \ldots, v_M]^T$. The currents are driven by the RF power sources.

**Fig. 1:** Circuit comparison of a) the EA transmitter and b) standard multiple antenna transmitter

Specifically, when the transmitter does not possess the channel state information (CSI), the spatial multiplexing scenario and space-time block precoding are considered. Whereas, when the transmitter possesses the CSI, the performance of EA-P, EA-S and standard multiple antenna with maximum ratio transmission (MRT) are compared. The performance metric compared is the symbol error rate (SER). Our results show that the EA-P performs significantly better compared with the EA-S and gives almost the same performance as the standard multiple antenna transmitter. However, replacing the ideal signal with approximate signal leads to some performance loss. If in addition, when one takes into account the cost and complexity of the EA, the EA-P scheme is well performing and practical.

The rest of the paper is organized as follows. The system model along with the working of an EA is explained in section II. The processing required for the EA-P is derived by solving an optimization problem in Section III. The performance of the EA-P is compared with the performance of the EA-S and standard multiple antenna transmitters, in different communication scenarios, in section IV. Finally, the main results are summarized in the concluding Section V.
voltage supply of each antenna element through fixed impedances \( z_0, z_1, z_2, \ldots, z_M \) [20], [21].

In the case of the EA system, \( v \) is \( \left[ v_0, 0, \ldots, 0 \right]^T \). The currents at the elements are varied by varying the input voltage \( v_0 \) at the active element and the loads \( z_1, z_2, \ldots, z_M \) at the parasitic elements. When feeding the active element, the currents are induced on the parasitic elements due to the mutual coupling between antenna elements. The MC matrix \( Z \) depends on the array’s geometry and is given as

\[
Z = \begin{bmatrix}
Z_{00} & Z_{01} & \ldots & Z_{0M} \\
Z_{10} & Z_{11} & \ldots & Z_{1M} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{M0} & Z_{M2} & \ldots & Z_{MM}
\end{bmatrix},
\]

(1)

According to Ohm’s law, the port current vectors for both circuits can be expressed as

\[
i = (Z + Z_L)^{-1} v.
\]

(2)

If current \( i \), which is the function of \( Z \) and \( Z_L \) in (2), flows through the antenna elements, then the input impedance, \( Z_{in} \), is given by [14]

\[
Z_{in} = Z_{00} + \sum_{m=1}^{M} Z_{0m}i_m.
\]

(3)

Note that \( Z_{in} \) is a function of \( Z \) and current vector \( i \) as shown in (3) and \( i \) varies with voltage feeding and loads value \( Z_L \) in (2). Therefore, \( Z_{in} \) is depends on the MC matrix \( Z \) as well as the loads \( Z_L \). After transmission from an EA, the received signal at the receiver, with \( n_r \) antennas, is given as [13], [22], [23]

\[
y = H i + n,
\]

where \( H \in \mathbb{C}^{n_r \times (M+1)} \) is channel matrix and \( n \in \mathbb{C}^{n_r \times 1} \) denotes the noise vector at the receiver. The receiver is assumed to be a standard multiple antenna receiver. The noise model is assumed to be additive white Gaussian noise (AWGN) with zero mean and unit variance.

B. Signal Transmission Using an EA Transmitter

In order to radiate the signal, the currents at the antenna element need to be varied based on the transmission signals. In the case of the standard multiple antenna system, the currents are varied by varying the input voltage at each antenna element. However, in the case of an EA, the loads at the parasitic elements and the voltage feeding at the active element needs to be varied. As discussed previously, for a certain transmission signal, it is possible that the voltage and loadings are such that they lead to a negative input resistance and result in an unstable EA. Let \( \hat{i} = [i_0, i_1, \ldots, i_M]^T \) denote desired current vector corresponding to ideal signals that is required to be transmitted by the EA. Utilizing (4), the input impedance, \( Z_{in} \), can be calculated for \( \hat{i} \) as

\[
Z_{in} = Z_{00} + \sum_{m=1}^{M} \frac{Z_{0m} \hat{i}_m}{\hat{i}_0}.
\]

(5)

If for some signals, \( \hat{i} \), the input resistance becomes negative, we propose that instead of transmitting the ideal signal, a signal closely approximating the ideal signal, which does not lead to a negative input resistance, is transmitted. In this way, the EA system does not need to be redesigned based on the transmission signal. As mentioned previously, we denote this EA with preprocessing as EA-P.

III. SIGNAL PROCESSING FOR AN EA-P

In this section, we obtain the current vectors approximating the ideal current vectors that leads to a positive input resistance. One can easily obtain the corresponding values of the voltage and the loads for an EA-P by using the approximate currents.

A. Problem Formulation

The problem to obtain the configuration for the EA-P, i.e. obtaining the values of the voltage and the loads, for each transmission signal can be formulated as an optimization problem to minimize the MSE between the currents corresponding to the ideal and approximate transmission signals, under the practical consideration of positive input resistance, and is given as

\[
\min_{v_0, z_1, z_2, \ldots, z_M} \left\| i - \hat{i} \right\|_2^2
\]

\[
\text{st. } \Re \left\{ Z_{00} + \sum_{m=1}^{M} \frac{Z_{0m} \hat{i}_m}{\hat{i}_0} \right\} \geq \xi,
\]

\[
i = (Z + Z_L)^{-1} v,
\]

(6a)

(6b)

(6c)

where \( \hat{i} \) denotes the approximate current vectors. The objective of this optimization problem is to find the voltage feeding \( v_0 \) and the loads \( z_1, z_2, \ldots, z_M \) to minimize MSE between the ideal and approximate current vectors. As input resistance is required to be positive, a positive variable, \( \xi \), is introduced in constraint (7b) to make sure that the input resistance is positive. Choosing \( \xi > 0 \) ensures stability of the EA. The impact of value of \( \xi \) on SER performance will be discussed in Subsection IV-C. A constraint on the real and imaginary part of impedance values can be considered. However, running extensive simulations, we found that the range of load values lies between \([-500, 500]\). The load values in this range are acceptable for antenna design [9] and therefore this constraint is omitted in this work. As the
configuration variables $Z_{L}$ and $v$ only appear in the (6c), the optimization problem in (6) can be solved in two steps. The first step is to obtain the approximate currents, $\hat{i}$, which can be obtained by solving the optimization problem with only constraint (6b). Once $\hat{i}$ is obtained, the loads and the voltage feeding can be calculated using (6c).

If for a certain ideal current vectors $\hat{i}$, the corresponding input resistance is greater than $\xi$, the EA can radiate the corresponding signal without any issue and in this case $i = \hat{i}$ and no optimization is required. The set for current vectors which yield input resistance greater than $\xi$ can be represented as

$$\mathbb{I}_{P_{\xi}} = \left\{ i \in \mathbb{R} \left( Z_{00} + \sum_{m=1}^{M} \frac{Z_{0m}i_{m}}{i_{0}} \right) \geq \xi \right\}. \quad (7)$$

Similarly, the unsupported set $\mathbb{I}_{N_{\xi}}$ is the complement of $\mathbb{I}_{P_{\xi}}$.

**B. Alternate representation for the input resistance constraint**

In order to simplify the constraint (6b), let $R_{m}$ and $X_{m}$ denote the real part and the imaginary part of $Z_{0m}$, respectively. $w_{2m+1}$ and $\hat{w}_{2m+2}$ denote the real part and the imaginary part of $i_{m}$, respectively. Similarly, $w_{2m+1}$ and $\hat{w}_{2m+2}$ denote the real part and the imaginary part of $i_{m}$, respectively. As all these variables are complex, we reformulate them from the complex domain into the real domain by separating their real and imaginary parts. Then the ideal and approximate current vectors can be written as new vectors $w = [\hat{w}_{1}, w_{2}, \hat{w}_{3}, w_{4}, \ldots, w_{2M+1}, \hat{w}_{2M+2}]^{T}$ and $\hat{w} = [w_{1}, w_{2}, w_{3}, w_{4}, \ldots, w_{2M+1}, w_{2M+2}]^{T}$, respectively.

**Proposition 1**

The input resistance can be expressed as

$$\Re \left\{ \sum_{m=1}^{M} \frac{Z_{0m}i_{m}}{i_{0}} \right\} = \frac{w^{T}A\hat{w}}{w_{1}^{2} + w_{2}^{2}}, \quad (8)$$

where the matrix $A$ is

$$A = \begin{bmatrix}
R_{0} & 0 & \frac{R_{1}}{2} & -\frac{X_{1}}{2} & \ldots & \frac{R_{M}}{2} & -\frac{X_{M}}{2} \\
0 & R_{0} & \frac{R_{1}}{2} & \frac{X_{1}}{2} & \ldots & \frac{R_{M}}{2} & \frac{X_{M}}{2} \\
-\frac{X_{1}}{2} & \frac{R_{1}}{2} & 0 & 0 & \ldots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
\frac{R_{M}}{2} & -\frac{X_{M}}{2} & 0 & 0 & \ldots & 0 & 0 \\
\frac{X_{M}}{2} & \frac{R_{M}}{2} & 0 & 0 & \ldots & 0 & 0
\end{bmatrix}. \quad (9)$$

**Proof:** The proof is provided in Appendix A. \hfill \blacksquare

As $\xi$ is positive, by substituting (8) into (6b), we get $w^{T}A\hat{w} \geq \xi (w_{1}^{2} + w_{2}^{2}) = \xi (w^{T}Gw)$, where $G = \text{diag}(1, 1, 0, \ldots, 0)$. The constraint (6b) can be expressed as $w^{T}B\hat{w} \geq 0$, where $B = A - \xi G$. The optimization problem to obtain the approximate current vectors can be then reformulated as

$$\begin{align}
\min_{w} & \quad \| \hat{w} - w \|_{2}^{2}, \\
\text{st.} & \quad w^{T}B\hat{w} \geq 0. \quad (10a)
\end{align}$$

The convexity of (10) depends on the matrix $B$ as the objective function (10a) is convex.

**Proposition 2**

The optimization problem in (10) is non-convex as $B$ is an indefinite matrix. The eigenvalues of $B$ are given by (11).

**Proof:** The proof is provided in Appendix B. \hfill \blacksquare

**C. Reformulation of the Nonconvex-Constrained Optimization Problem**

For the problem in (10), the objective function is quadratic convex and the constraint is a quadratic non-convex inequality. As strong duality holds for (10) [17], its solution can be obtained by solving its dual problem. The Lagrangian of this optimization problem is

$$L (w, \delta) = \| \hat{w} - w \|_{2}^{2} + \delta ( -w^{T}B\hat{w} ) = w^{T} ( I - \delta B ) w - 2w^{T}\hat{w} + \hat{w}^{T}\hat{w}, \quad (12)$$

where $\delta$ is Lagrange multiplier, and $I$ is an identity matrix of size $(M + 1)$. The stationarity condition is

$$\nabla_{w} L (w, \delta) = 2 ( I - \delta B ) w - 2\hat{w} = 0. \quad (13)$$

As $B$ is a square matrix, the optimal value, $w (\delta)$, which satisfies (13) is

$$w (\delta) = ( I - \delta B )^{-1} \hat{w}. \quad (14)$$

The dual function of (10) can be expressed using the Lagrangian as

$$g (\delta) = \inf_{w} \left( w^{T} ( I - \delta B ) w - 2w^{T}\hat{w} + \hat{w}^{T}\hat{w} \right). \quad (15)$$

After substituting (14) into (15), it can be noticed that $g (\delta) = -\infty$ if matrix $( I - \delta B )$ is not a positive semidefinite matrix or $w^{T}$ is not in the range of $( I - \delta B )$. Therefore, the dual problem can be expressed as

$$\begin{align}
\max_{\delta} & \quad -w^{T} ( I - \delta B )^{-1} \hat{w} + \hat{w}^{T}\hat{w}, \\
\text{st.} & \quad I - \delta B \in S_{+}^{2(M+1)}, \quad (16a) \\
& \quad w \in \mathbb{R}^{(M+1)}, \quad (16b) \\
& \quad \delta \geq 0, \quad (16c) 
\end{align}$$

where $S_{+}^{n}$ denotes the set of symmetric positive semidefinite $n \times n$ matrices, and $\mathbb{R} ( I - \delta B )$ denotes the range of $( I - \delta B )$. This can be further reformulated to obtain the closed-form expression of the optimal value of $\delta$. 
where \( \sigma \) is obtained by separating the real and imaginary parts of ideal current vectors \( \hat{i} = (\hat{i}_0, \hat{i}_1, \ldots, \hat{i}_M) \), and its \( k \)-th entry is

\[
\hat{w}_k = \left\{ \begin{array}{ll}
\Re\left(\hat{i}_{\hat{k}_1}\right) & k = 1, 3, \ldots, 2M + 1 \\
\Im\left(\hat{i}_{\hat{k}_2}\right) & k = 2, 4, \ldots, 2M + 2
\end{array} \right.
\]

(23)

where \( \Im(\hat{i}_{\hat{k}_2}) \) denotes the imaginary part of \( \hat{i}_{\hat{k}_2} \).

Theorem 1 gives the expression to calculate the approximate current vectors. When the input resistance from the ideal current vectors is no less than \( \xi \), the ideal current vectors can be realized by the EA-P without any optimization operation. Otherwise, the EA-P transmits the approximate current vectors which are obtained from Theorem 1.

The loads and feeding, for the EA-P, can be calculated based on the approximate current vectors as [14],

\[
v_0 = z_0 i_0 + \sum_{j=0}^{M} Z_{0j} i_j,
\]

(24a)

\[
z_m = -\sum_{j=0}^{M} Z_{mj} i_j, \quad m = 1, 2, \ldots, M.
\]

(24b)

From these expressions, it can be noted that the load values depend on the normalized current vectors and MC matrix, and the input power to the active element is controlled by the feeding. When the normalized current vectors are fixed, the load values are independent of the feeding circuit.

The steps to obtain the configuration for the EA-P for every transmit signal are summarized in Algorithm 1.

E. Discussion

1) Complexity: For the computation of the approximate currents, the EA-P algorithm involves only matrix operations and no iteration is required. The eigenvalues and corresponding eigenvectors of the MC matrix need to be computed only once. In other words, steps 1 to 3 in Algorithm 1 need to be calculated only once and can be used for the subsequent signal transmissions. Thus, the complexity from step 1 to 3 is negligible. Steps 4

...
After solving the optimization problem, the loads $g$ are independent of the feeding circuit and the matching circuit from Theorem 1 and (24b) as mentioned in [14]. A dynamic impedance matching is necessary. The generator/source impedance of the EA. In [14], a dynamic impedance matching circuit is proposed to compensate the impedance mismatch between the source and input impedance of the EA.

When considering dynamic matching circuit, the optimization problem will not be changed. The load values are independent of the feeding circuit and the matching circuit from Theorem 1 and (24b) as mentioned in the Subsection III-D. Therefore, after matching the impedance, one only needs to calculate the new feeding voltage [14]. As shown in Fig. 2, the input voltage to ESPAR is $V_{in} = \sum_{j=0}^{M} Z_{0j} i_j$. As the matching impedance is $Z_{dm} = \Re \{Z_{in}\} - \Im \{Z_{in}\}$, the new feeding voltage can be calculated as

$$V_{0} = \frac{V_{in}}{Z_{in}} (Z_{dm} + Z_{in})$$

$$= \sum_{j=0}^{M} Z_{0j} i_j \left( \frac{2\Re \{Z m\}}{\sum_{m=1}^{M} Z_{m0} i_m} + \frac{\sum_{m=1}^{M} Z_{0m} i_m}{i_0} \right)$$

$$Z_{00} + \frac{\sum_{m=1}^{M} Z_{0m} i_m}{i_0}$$

(25)

3) Stability and implementation of Parasitic loads:

After solving the optimization problem, the loads $z_1$ to $z_M$ might lead to a negative resistance value. A stable circuit for generating parasitic loads with a negative real part has been proposed in [26], [27]. The parasitic elements might require some gain to produce the required currents. However, this is not the aim of this work and it is assumed that the parasitic elements are efficiently implemented and stable. As in [16], a system structure is unstable if the input port impedance has a negative real part. Therefore, provided that the parasitic loads with a negative real part are stable, the resulting EA structure will be stable if the input resistance is positive. Thus, it is necessary to satisfy this condition of stability for an EA.

If this is not guaranteed, the EA will be unstable and oscillation is possible [16]. From the expression (24), the loads $z_1$ to $z_M$ might lead to a negative resistance value. If there exists a load value $z_m$ at the $m$-th parasitic element, its real part is negative, $\Re (z_m) < 0$. From (24b), we have

$$\Re (z_m) = \Re \left( -\sum_{j=0}^{M} \frac{Z_{mj} i_j}{i_m} \right) = -\Re \left( \sum_{j=0}^{M} \frac{Z_{mj} i_j}{i_m} \right) < 0,$$

(26)

As this is a active load, the input impedance at the $m$-th parasitic element is $Z_{in}^{(m)} = \frac{V_{in}^{(m)}}{i_m^{(m)}} = \sum_{j=1}^{M} Z_{mj} i_j \frac{i_j}{i_m}$, and its real part is $\Re \left( Z_{in}^{(m)} \right) = \Re \left( \sum_{j=1}^{M} Z_{mj} i_j \frac{i_j}{i_m} \right)$

From (26), we have $\Re \left( Z_{in}^{(m)} \right) > 0$. Thus, this guarantees the stability of the system while $z_1$ to $z_M$ might have negative real parts.

IV. APPLICATION OF THE ALGORITHM IN VARIOUS COMMUNICATION SCENARIOS

In this section, we analyse and compare the performance of the EA-P, EA-S, and the standard multiple
antenna transmitters in various communication scenarios.

A. Each Antenna Element Transmits the Same Signal

First, we consider the simplest scenario in which all the antenna elements transmit the same signal \( s \), i.e. \( \hat{s} = [s, s, \ldots, s]^T \). In this case, substituting \( \hat{s} \) in (3), yields the input resistance, \( R_{in} \), as

\[
R_{in} = \Re \left( \frac{Z_{00} + \sum_{m=1}^{M} Z_{0m} s}{s} \right) = R_0 + \sum_{m=1}^{M} R_m. \tag{27}
\]

From (27), it can be noted that the input resistance is independent of the transmitted signal and only depends on the MC matrix. Therefore, any EA characterized by a MC matrix which satisfies the condition, \( R_0 + \sum_{m=1}^{M} R_m > 0 \), can be employed to transmit the same signal from every antenna element. If this condition is generally satisfied in a practical EA system, radiating the same signal is generally feasible. Thus, in this scenario, the EA-P gives the same performance as EA-S and the standard multiple antenna transmitter.

B. Spatial Multiplexing

Next we consider the scenario in which each antenna element, transmits a different signal. In this case, \( \hat{s} = [s_0, s_1, \ldots, s_M]^T \) where \( s_m \) denotes the signal at the \( (m+1) \)-th antenna element. The input resistance can be expressed as

\[
R_{in} = \Re \left( \frac{Z_{00} + \sum_{m=1}^{M} Z_{0m} s_m}{s_0} \right). \tag{28}
\]

From (28), it can be noted that unlike the previous scenario, whether a signal can be transmitted not only depends on the MC matrix but also on the transmission signal. The transmission signal depends on the modulation scheme employed. As an example, assume that the modulation scheme is \( N \)-PSK. The signal at the \( (m+1) \)-th element can be denoted as \( s_m = \exp(j \phi_m) \), where \( \phi_m = \frac{2 \pi m n_m}{N}, n_m \in \{0, 1, \ldots, (N-1)\} \) is the phase of the signal at the \( (m+1) \)-th antenna element and \( N \) is the order of the PSK signal constellation [9]. In this case, the input resistance can be expressed as (29).

Therefore, in order to transmit different \( N \)-PSK signals for each antenna element, the MC matrix of the EA is required to satisfy

\[
R_{in} > 0, \text{ for } n_m, n_0 \in \{0, 1, \ldots, N-1\}. \tag{30}
\]

In the special case of BPSK modulation, when the transmit signal set is \( \{\exp(j0), \exp(j\pi)\} \), the input resistance can be simply expressed as \( R_{in} = R_0 + \sum_{m=1}^{M} a_m R_m \), where, \( a_m = \begin{cases} 1 & \text{for } \phi_m = \phi_0 \\ -1 & \text{for } \phi_m \neq \phi_0 \end{cases} \). Thus, the BPSK signal can be transmitted by the EA when it satisfies the condition \( R_{in} = R_0 + \sum_{m=1}^{M} a_m R_m > 0 \), for all \( a_m \in \{1, -1\} \). These conditions, for BPSK and \( N \)-PSK modulation, might not be satisfied for some EAs, and therefore in such cases, EA-P can be employed to achieve transmission of those signals.

Numerical Simulation: As an example, in Fig. 3, we plot the input resistance against the phase of the transmission signal for an EA with two antenna elements. In the simulations, only the transmitter is assumed to be equipped with an EA and a standard multiple antenna receiver is considered. Without loss of generality, the number of antennas at the receiver, \( n_r \), is assumed to be one and the channel is Rayleigh faded. The modulation scheme is 16-PSK. In Fig. 3, the green colored region denotes \( R_{in} > 0 \) and the red region denotes \( R_{in} \leq 0 \). The black points are all possible phase combinations of the two 16-PSK signals.

Fig. 3(a) is for the EA with MC matrix given by \( Z^{(1)} \), and Fig. 3(b) is for the EA with MC matrix given by \( Z^{(2)} \) and \( Z^{(3)} \). These three MC matrices for 2-element ESPAR in the IE3D antenna software are

\[
Z^{(1)} = \begin{bmatrix} 45.12 - j16.60 & 42.39 - j29.51 \\ 42.39 - j29.51 & 21.12 - j29.64 \end{bmatrix},
\]

\[
Z^{(2)} = \begin{bmatrix} 52.81 - j11.09 & 40.27 - j20.75 \\ 40.27 - j20.75 & 52.81 - j11.09 \end{bmatrix},
\]

\[
Z^{(3)} = \begin{bmatrix} 465.4 - j659.5 & -24.06 + j34.93 \\ -24.06 + j34.93 & 21.12 - j157.2 \end{bmatrix}.
\]

These have been designed in [14], [28].

It can be observed from the figure that some combinations of the 16-PSK signals cannot be supported using \( Z^{(1)} \). For this EA, by employing EA-P, the transmission of these signal combinations is possible. Whereas, for the EA with \( Z^{(2)} \) and \( Z^{(3)} \), all possible signal combinations can be supported. Comparing \( Z^{(1)} \), \( Z^{(2)} \), and \( Z^{(3)} \), it can be noted that the EA, whose MC matrix has higher self impedance at the feeding element can support more signal combinations. In the Fig. 3(a) and Fig. 3(b), the MC matrix \( Z^{(1)} \), \( Z^{(2)} \), and \( Z^{(3)} \) is obtained using the IE3D antenna design software. Furthermore, the feasible regions for linear EAs with different antenna spacing are shown in the Fig. 3(c) and Fig. 3(d), respectively. The MC matrices, in this case, are obtained using the induced electromotive force method (IEFM) for different antenna spacing [29]. The feasible region for two different antenna spacing \( d = \frac{\Delta}{10} \) and \( d = \frac{\Delta}{4} \) for linear EAs are shown in the Fig. 3(c) and Fig. 3(d), respectively. It can
$$R_{in} = \Re \left( Z_{00} + \sum_{m=1}^{M} Z_{0m} \exp \left( j (\phi_m - \phi_0) \right) \right) = R_0 + \sum_{m=1}^{M} \left( R_m \cos \frac{2\pi (n_m - n_0)}{N} - X_m \sin \frac{2\pi (n_m - n_0)}{N} \right).$$

(29)

Fig. 3: The feasible region for different EAs when 16-PSK is transmitted.

From Fig. 3(c) and Fig. 3(d), the proportion of supported signal combinations decreases as the antenna spacing is reduced. Therefore, in this scenario, for signal transmission, the EA-P algorithm becomes indispensable.

The SER performance of the system where the transmitter transmits different data streams from each antenna is shown in Fig. 4 and Fig. 5. For simplicity, we denote the results from standard multiple antenna as SMA in the simulation figures. The SER is plotted against signal-to-noise ratio (SNR). The modulation scheme considered in these figures is 16-PSK and 16-QAM, respectively.

As any positive $\xi$ can guarantee signal transmission using an EA, without loss of generality, $\xi$ is set equal to one. Three EAs with MC matrices, $Z^{(1)}$, $Z^{(2)}$, and $Z^{(3)}$ respectively, are considered and the SER performances of EA-P, EA-S, and the standard multiple antenna transmitters are compared. In the case of the standard multiple antenna transmitter, $\hat{i}$ is transmitted exactly. For the EA-P transmitter, the transmission policy is given in Theorem 1. For a fair comparison of SER with respect to transmit SNR, the transmitted signal $i$ is normalized by $\hat{P}$, where $P$ and $P$ are power based on ideal signals and practical signals, respectively, and the normalization factor is controlled by varying the voltage feeding. In the case of EA-S, no signal is stably transmitted when input resistance is negative and transmitted otherwise. It can be observed in Fig. 4, that as the EA with MC matrix, $Z^{(2)}$, and $Z^{(3)}$, can support all signal combinations for 16-PSK modulation, and the performances of the EA-P, EA-S and standard multiple antenna transmitters are similar. However, for the EA with MC matrix, $Z^{(1)}$, all possible signal combinations cannot be supported using the EA-S. Therefore, in this case, the SER performance of EA-S degrades significantly compared with the standard multiple antenna transmitter. However, it can be observed that by employing the proposed EA-P, performance similar to that of the standard multiple antenna transmitter can be achieved. In Fig. 5, the results are shown for 16-QAM modulation scheme. It can be observed that in this case, the EAs with both $Z^{(1)}$ and $Z^{(2)}$, do not support all possible signal combinations and the performance of EA-S transmitter is significantly degraded. However, nearly the same SER performance can be achieved as in the case of the standard multiple antenna transmitter by using the proposed EA-P.

This also highlights the usefulness of our scheme. All signal combinations could be transmitted by using the EA with MC matrix, $Z^{(2)}$, for 16-PSK. But once the modulation scheme changed to 16-QAM, some signal combinations became infeasible. But the same EA can be used to transmit signals with any modulation scheme by using the proposed EA-P.
Spatial Multiplexing with EA and SMA under 16-QAM

Alamouti code with EA and SMA under 16-QAM

\[ s_1 = \text{exactly transmitted according to Alamouti code} \]

depends on the MC matrix and also the transmission

C. Transmit Diversity

One of the main advantages of multiple antennas is that they provide diversity gain which results in significant performance improvement. There are different approaches to achieve transmit diversity gain depending on the CSI availability at the transmitter.

1) Transmitter without CSI: When the transmitter does not possess CSI, the diversity gain can be achieved by using space-time block codes (STBC). One of the simplest STBC that gives diversity gain is the Alamouti code. In the case of a two elements EA, given a signal vector \( [s_0, s_1]^T \), the ideal port current vectors is

\[ \hat{i} = \begin{bmatrix} s_0 & -s_1 \\ s_1 & s_0 \end{bmatrix} \]

The corresponding input impedance for these two symbol periods are

\[ R_{in}^{(1)} = \Re \left( Z_{00} + \frac{Z_{01}s_1}{s_0} \right) = R_0 + \Re \left( \frac{Z_{01}s_1}{s_0} \right), \]

\[ R_{in}^{(2)} = \Re \left( Z_{00} + \frac{Z_{01}s_0}{s_1} \right) = R_0 + \Re \left( \frac{Z_{01}s_0}{s_1} \right). \] (31)

The supported signal vector set \( [s_0, s_1]^T \), which can be exactly transmitted according to Alamouti code by an EA, is

\[ \mathcal{S}_{PK} = \left\{ [s_0, s_1] \mid R_{in}^{(1)} > 0, R_{in}^{(2)} > 0 \right\}. \] (32)

Again, in this case, whether a signal can be supported depends on the MC matrix and also the transmission signals. In the case of N-PSK modulation, where the transmission symbols \( s_0 \) and \( s_1 \) have phases \( \phi_0 \) and \( \phi_1 \), respectively. The input resistances are

\[ R_{in}^{(1)} = R_0 + R_1 \cos (\phi_1 - \phi_0) - X_1 \sin (\phi_1 - \phi_0), \]

\[ R_{in}^{(2)} = R_0 - R_1 \cos (\phi_1 - \phi_0) + X_1 \sin (\phi_1 - \phi_0). \] (33)

Numerical Simulation: The SER performance of the system where the transmitter employs Alamouti code is shown in Fig. 6. The modulation scheme is 16-QAM. Three EAs with MC matrices, \( Z^{(1)}, Z^{(2)}, \) and \( Z^{(3)} \), respectively, are considered and the SER performance for EA-P, EA-S and the standard multiple antenna transmitters are compared. It can be observed that the performance for EA-S system with \( Z^{(1)} \) and \( Z^{(2)} \) degrades significantly as the EAs with MC matrices, \( Z^{(1)} \) and \( Z^{(2)} \) do not support all possible signal combinations. However, again by using EA-P, the SER is improved and nearly the same performance as that of the standard multiple antenna transmitter can be achieved.

2) Transmitter with CSI: When the transmitter has CSI, transmit diversity can be achieved by precoding using the CSI. One such precoding scheme, called MRT, is considered here. Assuming that symbol \( s \) is to be transmitted, the ideal port current vectors for the transmitter is \( \hat{i} = \frac{h}{\|h\|_2} s \), where \( h = [h_0, h_1, \ldots, h_M]^T \in \mathbb{C}^{M+1} \) and \( h_m \) denotes the channel from the \( (m+1) \)-th element of EA transmitter to the signal antenna receiver [30].

The corresponding input impedance can be obtained as

\[ Z_{in} = Z_{00} + \sum_{m=1}^{M} \frac{Z_{0m} h_m}{i_0} = Z_{00} + \sum_{m=1}^{M} \frac{Z_{0m} h_j}{i_0}. \] (34)

From (34), it can be noted that the input impedance is the function of channel and the MC matrix only, and that it is independent of the data symbols. This implies that, for the EA-P, in the case of a block fading channel, in which the channel is constant over a block
We denote the real and imaginary part of \( Z \) antenna array. As an example, a circular antenna array. For a Rayleigh faded channel, the measurement of suitability of an EA can be the probability that \( R_{in} > 0 \), i.e. \( \mathcal{P} = \Pr(R_{in} > 0) \). An EA can be selected which has the highest probability \( \mathcal{P} \). Note that \( \mathcal{P} = \Pr(R_{in} > 0) = 1 - F_{R_{in}}(0) \), where \( F_{R_{in}} \) denotes the cumulative distribution function (CDF) of \( R_{in} \).

As an example, the CDF of \( R_{in} \) is derived for a circular antenna array. As an example, a 5-element ESPAR is shown in Fig. 7. In the case of a circular antenna array, \( Z_{0m} = Z_{01} \), for \( m = 2,3,\ldots,M \). We denote the real and imaginary part of \( Z_{01} \) by \( R_1 \) and \( X_1 \), respectively. Substituting \( R_1 \) and \( X_1 \) into (34), yields the input resistance as

\[
R_{in} = \Re \left( Z_{00} + Z_{01} \sum_{m=1}^{M} \frac{h_j}{h_0} \right)
= R_0 + R_1 \Re \left( \sum_{m=1}^{M} \frac{h_j}{h_0} \right) - X_1 \Im \left( \sum_{m=1}^{M} \frac{h_j}{h_0} \right).
\]  

(35)

Let \( \kappa = \sum_{m=1}^{M} h_m \). Denoting the real and imaginary parts of \( \kappa \) by \( r_\kappa \) and \( x_\kappa \), respectively, the input resistance can be expressed as \( R_{in} = R_0 + R_1 r_\kappa - X_1 x_\kappa \). As \( r_\kappa \) and \( x_\kappa \) are the random variables, in order to obtain the CDF of \( R_{in} \), the distributions of \( r_\kappa \) and \( x_\kappa \) are required. For a Rayleigh faded channel, \( h_j \sim \mathbb{CN}(0,\sigma^2I) \), for all \( j \in \{0,1,\ldots,M\} \), where \( \mathbb{CN} \) denotes a complex Gaussian distribution. As \( h_m \) is a complex Gaussian, the sum of \( h_m \) also follows complex Gaussian distribution, \( \sum_{m=1}^{M} h_j \sim \mathbb{CN}(0,M\sigma^2I) \). This implies that the random variable \( \kappa \) is a ratio of two independent complex Gaussian variables, and the joint PDF of the \( r_\kappa \) and \( x_\kappa \) can be given as [32]

\[
f(r_\kappa, x_\kappa) = \frac{M}{\pi} \left( r_\kappa^2 + x_\kappa^2 + M \right)^{-2}.
\]  

(36)

The CDF for \( R_{in} \) can be obtained by solving

\[
F_{R_{in}}(r_{in}) = \int \int_{r_{in} \leq r} f(r_\kappa, x_\kappa) \, dr_\kappa \, dx_\kappa.
\]  

(37)

Substituting \( R_{in} \) and (36) into (37), we have

\[
F_{R_{in}}(r_{in}) = \int \int_{R_0 + R_1 r_\kappa - X_1 x_\kappa \leq r_{in}} \frac{M}{\pi} \left( r_\kappa^2 + x_\kappa^2 + M \right)^{-2} \, dr_\kappa \, dx_\kappa.
\]  

(38)

After solving the integrals in (38), the CDF of \( R_{in} \) can be obtained and is given in the proposition below.

**Proposition 5** The CDF of input resistance for a circular EA is given as

\[
F_{R_{in}}(r_{in}) = \frac{1}{2} + \frac{r_{in} - R_0}{2 \sqrt{M (X_1^2 + R_1^2) + (r_{in} - R_0)^2}}.
\]  

(39)

The PDF of input resistance is given as

\[
f_{R_{in}}(r_{in}) = \frac{1}{2 \sqrt{M (X_1^2 + R_1^2) + R_0^2}} - \frac{(r_{in} - R_0)^2}{2 \left( M (X_1^2 + R_1^2) + (r_{in} - R_0)^2 \right)^{3/2}}.
\]  

(40)

The probability that a circular EA can transmit an exact MRT precoded signal is

\[
\mathcal{P} = \Pr(R_{in} > 0) = 1 - F_{R_{in}}(0) = \frac{1}{2} + \frac{R_0}{2 \sqrt{M (X_1^2 + R_1^2) + R_0^2}}.
\]  

(41)

**Proof:** The proof is provided in Appendix E. □

From this proposition, it can be noted that the probability that a circular EA can support an exact MRT precoded signal, increases as the self-impedance of the active element increases. Furthermore, this probability decreases as the number of elements increases or the mutual coupling impedance between the active element and parasitic elements increases. These factors show that the probability increases as the mutual coupling is reduced. However, the basic work principle of an EA is the mutual coupling that exists due to the small distance between the elements, therefore, a relatively large mutual coupling is desired for EA operation. Thus, in this case, the probability \( \mathcal{P} \) will decrease and our proposed EA-P will become crucial to achieve signal transmission using the EA.

**Numerical Simulation:** In Fig. 8 and Fig. 9, the CDF of \( R_{in} \), plotted using (39), is compared with the
CDF obtained via Monte Carlo simulation for different EAs. It can be observed that the analytical results match well with the simulation results for MC matrix $Z^{(1)}$, $Z^{(2)}$, and $Z^{(3)}$ in Fig. 8. The probability on $R_{in} < 0$ decreases as $R_0$ increases. The CDFs are compared under different element numbers and antenna spacing for circular linear EAs in Fig. 9, it can be noted that reducing the number of the elements or increasing antenna spacing reduces the probability on $R_{in} < 0$.

The SER performances, in the case of MRT, are compared for circular EAs with different number of elements and different spacings in Fig. 10. It can be observed from Fig. 10 that the SER performance varies with the number of antenna elements and the antenna spacing. Increasing the number of the elements or reducing antenna spacing reduces the SER. It can be observed for the EA-S that the SER performance are very poor and an error floor is obtained. In addition, the error floor can be avoided using EA-P and especially at low SNRs, the SER performance of the EA-P transmitter is similar to that of the standard multiple antenna transmitter. Thus, the SER performance can be improved significantly using EA-P. However, at high SNRs there is a slight loss in performance as the number of elements is increased. This happens because the proportion of unsupported signals increases with increasing antenna elements.

In Fig. 11, the SER performances of the EA-P is shown with varying $\xi^4$. It can be observed that the SER increases with the increasing $\xi$. This happens due to a stronger requirement on the input resistance and results in an approximate signal that is more different from the ideal signal.

V. CONCLUSION

ESPAR is a reconfigurable antenna in which the parasitic loads are varied to achieve arbitrary signal transmission. For some signals, varying the parasitic loads results in a negative input resistance. A negative input resistance implies that the antenna structure is unstable to transmit these signals. In this work, we showed that instead of those actual signals, one can transmit approximate signals, which satisfy the input resistance constraint, without much loss in performance. We obtained a simple closed-form expression for calculating the approximate signals which can be easily implemented at the transmitter. Our simulation results, conducted for various communication scenarios, show

4The value of $\xi$ affects the input power from the active element, and therefore, it will also affect the efficiency of an EA. However, as this work focus on the stability and feasibility of an EA, the efficiency of an EA will be discussed in future work.

---

**Fig. 8:** The CDF of input resistance

**Fig. 9:** The CDF of input resistance with respect to different antenna spacings

**Fig. 10:** SER performance comparison of the EA transmitter and standard multiple antenna transmitter in the case of maximum ratio transmission under 16-QAM.
that the EA-P transmitter gives significant improvement over the performance of the EA-S transmitter and performs nearly the same as the standard multi-antenna system.

**APPENDIX A**

**PROOF OF PROPOSITION 1**

*Proof:* From (5), the input resistance is (42).

Eq. (8) follows as a simple consequence of rewriting the quadratic form in (42) in the form of an appropriate matrix expansion.

**APPENDIX B**

**PROOF OF PROPOSITION 2**

*Proof:* As $B$ is a real symmetric matrix and it can be diagonalized as $B = QAQ^T$, where $A$ is a real diagonal matrix and its elements are eigenvalues of $B$, the columns of the real orthogonal matrix $Q$ are corresponding eigenvectors of $B$. For the sake of simplification, the eigenvalues $A$ are set in ascending order. Let $\zeta$ denote the eigenvalues of $B$, the matrix $B - \zeta I$ can be written as

$$
B - \zeta I = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix},
$$

where

$$
B_{11} = \begin{bmatrix}
R_0 - \zeta & 0 \\
0 & R_0 - \zeta
\end{bmatrix},
$$

$$
B_{12} = \frac{R_0 - \zeta}{2} \begin{bmatrix}
-\frac{\zeta}{2} & \cdots & -\frac{\zeta}{2} \\
\frac{\zeta}{2} & \cdots & \frac{\zeta}{2}
\end{bmatrix},
$$

$$
B_{21} = B_{12}^T,
$$

$$
B_{22} = \text{diag}(-\zeta, -\zeta, \ldots, -\zeta).
$$

As $B_{22}$ is invertible, the determinant of the matrix $B - \zeta I$ can be rewritten as (45).

Setting $\det (B - \zeta I) = 0$, the eigenvalues of $B$ are (11). We can find that $\zeta_1$ and $\zeta_2$ are negative whereas $\zeta_3$ and $\zeta_4$ are positive eigenvalues, thus $A$ is $\text{diag}(\zeta_1, \zeta_2, 0, \ldots, 0, \zeta_3, \zeta_4)$. The matrix $B$ is not a positive or a negative semidefinite matrix. The constraint (10b) is nonconvex and the optimization problem (10) is a nonconvex problem.

**APPENDIX C**

**PROOF OF PROPOSITION 3**

*Proof:* From proposition 2, $B$ is a real symmetric matrix and it can be eigenvalue decomposed as $B = QAQ^T$. Here, $A = \text{diag}(\zeta_1, \zeta_2, 0, \ldots, 0, \zeta_3, \zeta_4)$ is a diagonal matrix whose diagonal elements are the eigenvalues of $B$ in ascending order. $Q$ is an orthogonal matrix whose columns are the corresponding eigenvectors. Then $I - \delta B = Q(I - \delta A)Q^T$. The eigenvalues of $(I - \delta B)$ are $1 - \delta \zeta_1, 1 - \delta \zeta_2, 1, \ldots, 1, 1 - \delta \zeta_3, 1 - \delta \zeta_4$ and the columns of $Q$ are corresponding eigenvectors of $(I - \delta B)$. Therefore, the feasible set of the optimization problem (16) is

$$
C = \{ \delta | 0 \leq \delta \leq 1/\zeta_3 \},
$$

and it can be rewritten as the optimization conditions as (17b).

For the objective function (16a), $\hat{w}^T \hat{w}$ can be simply removed form the objective function as it is constant. Similarly, the objective function is reformulated as

$$
\hat{w}^T(I - \delta B)^{-1} \hat{w} = (Q^T \hat{w})^T(I - \delta A)^{-1}(Q^T \hat{w})^T,
$$

Setting $g = [g_1, g_2, \ldots, g_{2M+1}, g_{2M+2}]^T = Q^T \hat{w} \in \mathbb{R}^{2(M+1)}$, (47) can be written as

$$
\hat{w}^T(I - \delta B)^{-1} \hat{w} = \frac{g_1^2 + g_2^2}{1 - \delta \zeta_1} + \frac{g_{2M+1}^2 + g_{2M+2}^2}{1 - \delta \zeta_3} + \sum_{k=3}^{2M} g_k.
$$

From (48), $\sum_{k=3}^{2M} g_k$ depends on the ideal currents $\hat{w}$ and matrix $B$, it is therefore independent of the optimal value $\delta$. It can be removed from the objective function. Setting $r_1 = g_1^2 + g_2^2$, $r_2 = g_{2M+1}^2 + g_{2M+2}^2$, the objective function (16a) can be further reformulated as

$$
\frac{r_1}{1 - \delta \zeta_1} + \frac{r_2}{1 - \delta \zeta_3}.
$$

The optimization problem can be reformulated as (17) from (46) and (49).
which means

\[ R \left\{ Z_{00} + \frac{\sum_{m=1}^{M} Z_{0m} i_m}{i_0} \right\} = R_0 + \frac{\sum_{m=1}^{M} \left[ \left( R_m w_{2m+1} - X_m w_{2m+2} \right) w_1 + \left( R_m w_{2m+2} + X_1 w_{2m+1} \right) w_2 \right]}{w_1^2 + w_2^2} \]

\[ = \frac{1}{w_1^2 + w_2^2} \left[ R_0 w_1^2 + R_0 w_2^2 + \sum_{k=1}^{M} \left( R_m w_{2m+1} w_1 - X_m w_{2m+2} w_1 + R_m w_{2m+2} w_2 + X_1 w_{2m+1} w_2 \right) \right]. \]

\[ \det (B - \zeta I) = \det (B_{22}) \det (B_{11} - B_{12} B_{22}^{-1} B_{21}) = (-\zeta)^{2M-2} \left( \zeta^2 - 2 (R_0 - \xi) \zeta - \sum_{m=1}^{M} \left( R_m^2 + X_m^2 \right) \right)^2. \]

**APPENDIX D**

**PROOF OF PROPOSITION 4**

*Proof:* As when \( \delta = \frac{1}{\delta_3^2} \) or \( \delta = \frac{1}{\delta_1^2} \), the objective function of (17a) cannot obtain minimal value, there are two possible cases from (20).

A. *Case 1:* \( \sigma_2 = 0, \delta = 0, \) and \( \sigma_1 \geq 0 \)

In this case, we can rewrite (19) as \( \nabla_\delta L_\delta (\delta, \sigma_1, \sigma_2) = \zeta_1 r_1 + \zeta_3 r_2 - \sigma_1 = 0. \) Thus, we have \( \sigma_1 = \zeta_1 r_1 + \zeta_3 r_2 \), which means \( \zeta_1 r_1 + \zeta_3 r_2 \geq 0. \)

B. *Case 2:* \( \sigma_2 = 0, \sigma_1 = 0, \) and \( \delta \geq 0 \)

The stationarity condition of the KKT conditions (20) can be written as \( \nabla_\delta L_\delta (\delta, \sigma_1, \sigma_2) = \frac{\zeta_1 r_1}{(1-\delta_3)} - \frac{\zeta_3 r_2}{(1-\delta_3)} = 0. \)

As \( \zeta_1 < 0, \zeta_3 > 0, r_1, r_2 \geq 0, \) (20) can be further simplified as \( \frac{\zeta_1 r_1}{(1-\delta_3)} = \frac{\zeta_3 r_2}{(1-\delta_3)}. \) Then the value of \( \delta \) is \( \delta = \frac{\zeta_1 r_1}{\sqrt{\zeta_1 r_1 + \zeta_3 r_2}}. \)

In this case, we have \( \frac{\zeta_1 r_1 + \zeta_3 r_2}{(1-\delta_3)} < \frac{\zeta_1 r_1}{(1-\delta_3)} + \frac{\zeta_3 r_2}{(1-\delta_3)} = 0. \) Then, we have \( \zeta_1 r_1 + \zeta_3 r_2 < 0. \)

C. *Discussion*

In this subsection, we apply logic to prove that the case 1 and the case 2 are logically equivalent to the statement that the ideal signal can be transmitted by ES2R antenna. Then we have \( D_1 \Leftrightarrow D_3 \) and \( D_2 \Leftrightarrow D_4. \)

Now, we can see the practical meaning of these two cases. Substituting ideal currents into constraint (10b), we have \( \hat{w}_T^T B w = (Q^T \hat{w})^T A Q \hat{w} = g^T A g = \zeta_1 r_1 + \zeta_3 r_2. \) Thus, we have \( D_1 \Rightarrow D_3 \) and \( D_2 \Rightarrow D_4. \)

In the case 1, as \( \sigma_1 \geq 0, \zeta_1 r_1 + \zeta_3 r_2 \geq 0. \) Thus, \( C_1 \Rightarrow D_3 \) and then \( C_1 \Rightarrow D_4. \) If \( D_P \) is true, then we have \( \zeta_1 r_1 + \zeta_3 r_2 \geq 0 \) as \( D_P \Rightarrow D_1 \Rightarrow D_3. \) Since there are only two cases of solutions, we have \( C_1 \) is true, which means \( D_P \Rightarrow C_1. \) Thus, we have \( C_1 \Leftrightarrow D_P, \) which implies that case 1 is equivalent that the ideal currents \( \hat{i} \) can be supported by EA. Similarly, we have \( C_2 \Leftrightarrow D_N, \) the ideal currents cannot be implemented by ES2R antenna.

**APPENDIX E**

**PROOF OF PROPOSITION 5**

*Proof:* From (38), the proposition can be proved in these two cases.

A. *Case 1, \( R_1 > 0 \)*

In this case, by setting \( l_1 = \frac{\zeta_1}{\zeta_1^2}, l_2 = \frac{\zeta_3 - R_0}{\zeta_1^2}, \) the CDF of input resistance is

\[ F_{R_1}(r_m) = \int_{r_m}^{\infty} \int_{r_m \zeta_1 x_1 + \zeta_1 x_2}^{\infty} f(r, x) dr dx \]

\[ = \frac{M}{\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left( r_1^2 + x_1^2 + M \right)^{-2} dr dx. \]

The indefinite integral with respect to \( r_1 \) can be calculated by using \([33, \text{eq. (2.110.2)}]\) and \([33, \text{eq.} \]

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and it is
\[
\int \left( r_\kappa^2 + x_\kappa^2 + M \right)^{-2} dr_\kappa = \frac{r_\kappa}{2(M + x_\kappa^2)((M + x_\kappa^2 + r_\kappa^2)} + \frac{\arctan \left( \frac{r_\kappa}{\sqrt{M + x_\kappa^2}} \right)}{2(M + x_\kappa^2)^{3/2}}.
\]

Applying Newton-Leibniz formula, the definite integral with respect to \( r_\kappa \) is (53).

Substituting (53) into (51), the double integral can be separated into the sum of three integrals. The indefinite integral of the first part, \( F_1(x_\kappa) \), can be calculated by using [33, eq. (2.01.15)] and [33, eq. (2.01.2)] and doing some algebraic manipulations, and it is (54).

Similarly, the indefinite integral of the second part, \( F_2(x_\kappa) \), can be calculated from [33, eq. (2.64.5)] and it is
\[
F_2(x_\kappa) = \int \frac{\pi/2}{2(M + x_\kappa^2)^{3/2}} dx_\kappa = \frac{\pi}{4} \frac{x_\kappa}{M \sqrt{M + x_\kappa^2}}.
\]

The definite integral of third part, \( F_3(x_\kappa) \), can be firstly simplified by applying the theorem of integration by parts, and then calculated by using [33, eq. (2.01.15)] and [33, eq. (2.01.2)]. After some algebraic manipulations, we have (56).

According to Newton-Leibniz formula and after some algebraic manipulations, the probability when the real part of the input impedance is less than \( r_{in} \) under the condition \( R_1 \geq 0 \) is
\[
F_{R_{1in}}(r_{in}) = \frac{1}{2} + \frac{l_2}{2 \sqrt{M + l_1^2} + l_2^2}.
\]

As \( l_1 = \frac{x_1}{R_1} \) and \( l_2 = \frac{x_2 - R_0}{R_1} \), putting them into (57), we have (39). The PDF of \( R_{1in} \) can be obtained from its CDF as (40). The probability of the real part of the input impedance is bigger than 0 under the condition \( R_1 > 0 \) is (41).

### B. Case 2: \( R_1 < 0 \)

From (37), in this case, the probability is
\[
F_{R_{1in}}(r_{in}) = \int_{r_\kappa = \frac{x_1}{R_1}}^{+\infty} f(r_\kappa, x_\kappa) dr_\kappa dx_\kappa = \frac{M}{\pi} \int_{-\infty}^{+\infty} \int_{l_1, x_\kappa + l_2}^{+\infty} \left( r_\kappa^2 + x_\kappa^2 + M \right)^{-2} dr_\kappa dx_\kappa.
\]

Applying Newton-Leibniz formula based on (52), we have
\[
\int_{l_1, x_\kappa + l_2}^{+\infty} \left( r_\kappa^2 + x_\kappa^2 + M \right)^{-2} dr_\kappa = \frac{\pi/2}{2(M + x_\kappa^2)^{3/2}} - \frac{\arctan \left( \frac{l_1 x_\kappa + l_2}{\sqrt{M + x_\kappa^2}} \right)}{2(M + x_\kappa^2)^{3/2}}.
\]

Substituting (59) into (58) and after some algebraic manipulations, the CDF and PDF when \( R_1 < 0 \) is (39), (40), respectively. Similarly, the probability of the real part of the input impedance is bigger than 0 under the condition \( R_1 < 0 \) is (41).

### References


\[ \int_{-\infty}^{l_1 x_\kappa + l_2} \left( \frac{x^2 + x_{\kappa}^2 + M}{2} \right)^{-2} dx_\kappa = \frac{l_1 x_\kappa + l_2}{2(M + x_{\kappa}^2) \left( M + x_{\kappa}^2 + (l_1 x_\kappa + l_2)^2 \right)} + \frac{\pi/2}{2(M + x_{\kappa}^2)^{3/2}} + \frac{\arctan \left( \frac{l_1 x_\kappa + l_2}{\sqrt{M + x_{\kappa}^2}} \right)}{2(M + x_{\kappa}^2)^{3/2}} \]

(53)

\[ F_1(x_\kappa) = \int \frac{l_1 x_\kappa + l_2}{2(M + x_{\kappa}^2) \left( M + x_{\kappa}^2 + (l_1 x_\kappa + l_2)^2 \right)} dx_\kappa = \frac{2l_2 \arctan \left( \frac{x_{\kappa}}{\sqrt{M + x_{\kappa}^2}} \right)}{4 \left( l_1^2 M + l_2^2 \right)^{1/2} \sqrt{M}} \]

(54)

\[ F_2(x_\kappa) = \int \frac{\arctan \left( \frac{l_1 x_\kappa + l_2}{\sqrt{M + x_{\kappa}^2}} \right)}{2(M + x_{\kappa}^2)^{1/2}} dx_\kappa = \frac{1}{2} \int \arctan \left( \frac{l_1 x_\kappa + l_2}{\sqrt{M + x_{\kappa}^2}} \right) d \frac{x_{\kappa}}{M + x_{\kappa}^2} \]

(56)
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