Higher-order Representation and Reasoning for Automated Ontology Evolution

HIGHER-ORDER REPRESENTATION AND REASONING FOR AUTOMATED ONTOLOGY EVOLUTION

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Abstract: The GALILEO system aims at realising automated ontology evolution. This is necessary to enable intelligent agents to manipulate their own knowledge autonomously and thus reason and communicate effectively in open, dynamic digital environments characterised by the heterogeneity of data and of representation languages. Our approach is based on patterns of diagnosis of faults detected across multiple ontologies. Such patterns allow to identify the type of repair required when conflicting ontologies yield erroneous inferences. We assume that each ontology is locally consistent, i.e. inconsistency arises only across ontologies when they are merged together. Local consistency avoids the derivation of uninteresting theorems, so the formula for diagnosis can essentially be seen as an open theorem over the ontologies. The system's application domain is physics; we have adopted a modular formalisation of physics, structured by means of locales in Isabelle, to perform modular higher-order reasoning, and visualised by means of development graphs.

1 INTRODUCTION

Artificial intelligence and, more generally, computer science are presently faced with the challenge that autonomous software agents must be able to manipulate their own knowledge. Such knowledge is typically represented in an ontology that conceptualises the entities of the software's application domain and allows the software to reason about such entities at a higher level of abstraction than simply the level of data or information. Just like any abstract model, ontologies are limited representations of the world, which is dynamic and inherently complex. If autonomous systems are to deal with such dynamics, they must be able to autonomously update their own ontologies.

The process of updating an ontology in the face of new information is often called *ontology evolution*. The literature on the subject mostly concentrates on the evolution of ontologies coded in Description Logic for Semantic Web applications [Bundy and Varzinczak, 2009]. The primary accent is on defining logical notions and/or methods to enable a user to maintain the consistency of an ontology either through its lifecycle or in relation to other ontologies. The former case is often related to ontology debugging and yields notions like conservative extensions [Ghilardi et al., 2006], belief revision [Katsuno and Mendelzon, 1991], interactive ontology evolution [Stojanovic et al., 2002], inconsistency repair [Kalyanpur et al., 2006, Lam et al., 2008, Ovchinnikova and Kühnberger, 2007]. The case of multiple ontologies is often related to ontology alignment and yields notions like mapping [Kalfoglou and Schorlemmer, 2003], matching [Doan et al., 2004, Giunchiglia and Shvaiko, 2004], or contextualisation. Despite these valuable research efforts on the dynamics of ontologies, we know of relatively few works that have explicitly considered the problem of applying *automated* mechanisms to repair *locally consistent* but *globally inconsistent* ontologies. In this type of situations belief revision may be insufficient to resolve conflicts between ontologies and the very signature of their representation language may need to be evolved. This opens up many kinds of syntactical manipulations, including splitting a function into parts and changing the arity of a function. An attempt at this kind of automated ontology evolution is described in [McNeill and Bundy, 2007], which investigates an environment in which agents with slightly different ontologies interact with each other. The main goal of the described system, the GALILEO System, is to identify and repair ontological mismatches arising from the heterogeneity in the underlying logical representation, e.g., arity mismatches.

In our view the limited progress in automated ontology evolution described above depends on two circumstances. On the one hand, most of the works address interactive ontology evolution driven by user's instructions. This choice is a pragmatic one, dictated by the need to support ontology developers in their work, rather than by a quest for automation. In many cases, though, that is not enough because what is actually required is ontology evolution *at runtime* performed, for instance, by autonomous agents that communicate with each other in heterogeneous environments, including the Semantic Web. On the other hand, the focus on ontologies coded in Description Logics does not allow for a sufficiently generic analysis and resolution of ontological inconsistencies, even when an approach aims at automating, for instance, the integration of changes in ontologies. As a matter of fact, the limited expressivity of first-order or lesser logics constitutes a limit on the possibility of modelling the ontology evolution process in the same language in which the ontology is coded. Being impossible to quantify over, and thus, to reason about, the predicates, the relations and the functions of the ontology, it is very problematic to formalise and implement a *sufficiently generic* ontology evolution process.

We therefore turned to study automated (as opposed to user-assisted) ontology evolution using higher-order logic (HOL), which provides the benefit of making it possible to express sufficiently generic patterns of evolution. In the framework of the GALILEO system [Bundy and Chan, 2008, Chan and Bundy, 2008], a number of so-called *ontology repair plans* (ORPs) are being developed and implemented in HOL. These mechanisms compile together patterns for diagnosis of conflicts between ontologies and transformation rules for effecting repairs. For both development and testing, we rely on examples of ontology evolution in physics. Many seminal advances in physics are results of ontology evolution, as physicists revise predictive theories when confronted with conflicting experimental evidence. Therefore, in ORPs developed thus far, one of the ontologies represents a predictive theory; a second ontology represents a sensory or experimental set-up for that theory. When the sensory ontology generates a theorem that contradicts a theorem of the theoretical ontology, an ORP is triggered and amends the two ontologies. ORPs may act either as belief revision mechanisms or as signature revision mechanisms or both. Working in HOL provides the additional benefit of formalising concepts and their relationships with a highly expressive representation. We believe this is desirable, because physics concepts are often naturally represented as HOL objects, e.g., the orbit of a star, the rate of change in a quantity, etc.

Suppose we have an ontology O_t representing the current state of a predictive physics theory and an ontology O_s representing some sensory information arising from an experiment. Suppose these two ontologies disagree over the value of some function *stuff* when it is applied to a vector of arguments $\vec{s}$ of type $\vec{\tau}$. $stuff(\vec{s})$ might, for instance, be the total energy of a ball or the orbit of a planet.

Trigger: If $stuff(\vec{s})$ has two different values in O_t and O_s then the following formula will be triggered, identifying a potential contradiction between theory and experiment.

$$O_t \vdash stuff(\vec{s}) = v_1 \quad (1)$$
$$O_s \vdash stuff(\vec{s}) = v_2 \quad (2)$$
$$O_t \vdash v_1 \neq v_2 \quad (3)$$

where $O \vdash \phi$ means that formula ϕ is a theorem of ontology O.

Figure 1: Trigger of the "Where's My Stuff?" ontology repair plan

In this paper we discuss the *diagnostic mechanism* of the ORP called *Where's my stuff?* (WMS) [Bundy and Chan, 2008]. WMS is triggered when the predicted value returned by a function, which we call *stuff*, conflicts with the observed value of the same function. The trigger formulae of WMS are formalised in Figure 1. The purpose of WMS is to amend the signature of two conflicting ontologies by redefining the function that computes the quantity that is subject to contradiction and that instantiates the higher order variable *stuff*. In practice, WMS deploys an addition-strategy that is quite common in physics. For instance, in order to account for unpredictable yet observed gravitational behaviours in the orbit of a planet or in the stellar orbital velocity in a galaxy, astronomers often postulate the presence of an additional unobserved planet or, resp., of dark matter. Accordingly, WMS redefines the contradictory function (in the examples, the functions orbit, resp., orbital velocity) as the sum of a *visible* part (i.e. the amount calculated by the original function) and an *invisible* part (i.e. the amount that can only indirectly be observed). For WMS's repair operation to be triggered, its diagnostic mechanism must have individuated the function *stuff* and assessed a contradiction between the value of *stuff* in the theoretical and the sensory ontologies.

The workings of such diagnosis allow us to illustrates two points about using a higher-order approach for ontology evolution:

1. The polymorphism of *stuff*, as well as of other symbols like $=$, $\neq$, $<$, $-$, etc. permits the generality of WMS and its applicability over disparate cases.
2. The use of a higher-order theorem prover like Isabelle [Paulson, 1994], allows reasoning over locally consistent but globally inconsistent ontologies that share variables.

More strongly, these two points are important results for both ontology evolution and automated theorem proving, and they represent the main contribution of this paper. They show that current interactive theorem proving technology is capable of inferring the trigger formulae used in ontology repair plans, despite their problematic features described above.

The rest of the paper is structured as follows: §2 gives an overview of two examples of ontology evolution in physics that are used in subsequent sections to evaluate the proposed approach to the representation and reasoning for ontology evolution; §3 describes the structure of the ontological representation and the specific axioms of the theoretical and sensory ontologies; and, §4 highlights the advantages of detecting conflicts between ontologies using HOL. Note that details of ontology repair procedures are not covered in this paper; we refer interested readers to [Bundy and Chan, 2008, Chan and Bundy, 2008] for more complete presentations.

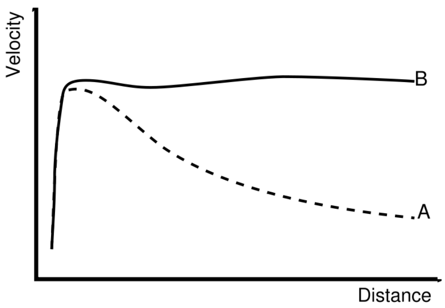
2 TWO EXAMPLES OF ONTOLOGY EVOLUTION IN PHYSICS

In this paper, we base our evaluation of the representation of knowledge and reasoning on two examples of ontology evolution in physics: the *bouncing-ball* paradox and the proposed existence of *dark matter*. Both cases can be emulated by WMS.

The bouncing-ball paradox, as described in [diSessa, 1983], involves dropping a ball from a height above ground and calculating its total energy as the sum of its kinetic energy, which is a function of the ball's velocity, and of its potential energy, which is a function of its height on the ground. The initial amount of total energy of the ball is greater than zero because of its positive amount of potential energy. If the amount of total energy is defined as a summation of kinetic and potential energies, the final amount of total energy of the ball will then be zero Joules because of its zero velocity and zero height at ground level. The paradox is exactly the contradiction between the initial and final amounts of total energy of the ball: the law of conservation of energy requires such amounts to be the same. WMS emulates the usual solution to the paradox and adds to the function that computes the total energy of the ball a third component, for elastic energy. This is the type of energy to which the ball's kinetic energy is transformed at the time of impact with the ground. This solution is equivalent to re-idealising the ball as a spring rather than as a particle.

In the case of the hypothetic existence of dark matter, the evidence for it comes from various sources, for instance, from an anomaly in the orbital velocities of stars in spiral galaxies identified by Rubin in 1975. Given the observed distribution of mass in these galaxies, we can use Newtonian Mechanics to predict that the orbital velocity of each star should be inversely proportional to the square root of its distance from the galactic centre (called its *radius*). However, observation of these stars show their orbital velocities to be roughly constant and independent of their radius. Figure 2 illustrates the predicted and actual graphs. In order to account for this discrepancy it is hypothesed that galaxies contain an invisible halo of, so called, dark matter which does not radiate and can only be measured indirectly. Accordingly WMS adds to the function that computes the stellar orbital velocity a second component that depends on dark matter.

In the next sections we discuss how the physics knowledge underlying these two cases can best be represented in FOL and HOL, respectively, in order for a HOL-theorem prover like Isabelle to diagnose the contradiction between what is expected and what is observed.



This diagram is taken from `http://en.wikipedia.org/wiki/Galaxy_rotation_problem`*. The x-axis is the radii of the stars and the y-axis is their orbital velocities. The dotted line (A) represents the predicted graph and the solid line (B) is the actual graph that is observed.*

Figure 2: Predicted *vs* Observed Stellar Orbital Velocities

3 ONTOLOGICAL REPRESENTATION OF PHYSICS

The language physicists use for expressing relationships between concepts is largely based on mathematics rather than on an expressive logic. It is one of our contributions in this project to provide a logical formalisation of physics formulae and historical examples of ontology evolution. As already mentioned, the need for evolution arises when experimental observations contradict theoretical predictions; thus, to formalise such situations, the predictive theory and sensory data are encapsulated in separate ontologies, which we call O_t and O_s respectively. Such modular representation, though basic, provides a range of benefits, including better control of contradiction, more focused effects of repair, variable certainty and increased reusability. To further modularise the existing knowledge representation, the physics and mathematical theories can be partitioned into small ontologies. Each ontology resembles a small context, e.g., some treatment of physics may depend on separate ontologies for a formal theory and for a naive theory of geometry, which only covers 2-D spaces. Certainty factors can be assigned to an ontology, determining the vulnerability of the theory or experiment to repair. For instance, the ontology of a controversial theory could be valued at a lower confidence than an established one. The structure used for storing and managing the ontologies should therefore be able to accommodate a sufficiently large collection of partitioned ontologies because even fundamental ontologies themselves, e.g., Newtonian Mechanics, arithmetic, etc., may be represented as a collection of smaller ontologies. That said, the structure should also support the management of relations and dependencies between ontologies and their terms.

3.1 Modular Representation as a Development Graph

For the storage and management of the collection of ontologies, we use a formal logical representation called *development graphs* [Autexier et al., 1999], in which nodes and links correspond to ontologies and morphisms, respectively. A logical theory (in our case, an ontology) is characterised by a node, which can be defined to import signatures and axioms from other nodes via *definitional links*. As will be described later, the ontologies are formalised as locales [Ballarin, 2004] in Isabelle, which are mechanisms for performing modular reasoning; each locale corresponds to a node in the development graph. There are also other types of morphisms implemented, the details of which are not covered in this paper. Development graphs are not only useful for formalisation of ontologies, but also for visualisation of the relations between ontologies and the complete structure. Development graphs are already implemented in HETS [Mossakowski et al., 2007], which is a system for the analysis of various specification languages.

3.1.1 Example Representation

To provide a visualisation of the structure of the ontologies for the model of the bouncing-ball paradox, Figure 3 depicts a development graph containing the relevant ontologies (nodes) and definitional links (arcs); it is an illustration of the development graph visualised in HETS. In this representation, the top ontology `BasicPhys` contains the fundamental concepts in physics, e.g., time and events. It extends from `OrderedReals`, which is an internal specification of reals with ordering. The node `ClassicalEnergyConv` contains types specific to energy conversion, e.g., those for various types of energy, including total energy, kinetic energy, and potential energy, but not the theories describing the conversion between types of energy. Note that it extends from `BasicPhys`, so all sorts (types) and operations, if any, are directly imported. Following the path down, the node `OtLaws` contains the theory of energy conversion for particles (without extent) between kinetic and potential energies

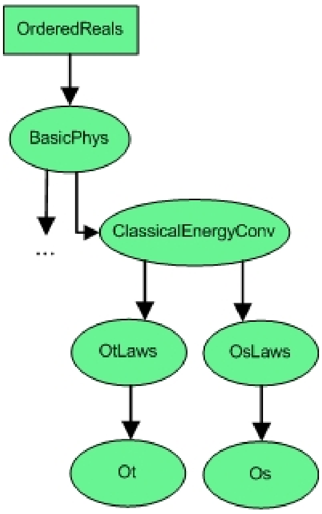


Figure 3: Development graph for the bouncing-ball paradox.

for *all* objects and time moments, i.e. $\forall o{:}Obj,t{:}Time.\ TE(o,t) = KE(o,t) + PE(o,t)$; that is, it predicts that potential energy can be converted to only kinetic energy because elastic and other types of energy are neglected. Moreover, it contains definitions as well, e.g., $\forall o.Obj,t.Time.\ KE(o,t) = \frac{1}{2}Mass(o,t).Vel(o,t)^2$. The theoretical ontology `Ot` extends from `OtLaws`, which imports the same predictive theory as that in `OtLaws`. In addition, it contains axioms specifying that the initial velocity is zero and the height is greater than zero. In contrast, the sensory ontology `Os` extends from `OsLaws`, which itself extends from `ClassicalEnergyConv`. `OsLaws` contains similar axioms as `OtLaws`, but at a lower level of generality. The axioms of `OsLaws` cover only the specific entities and events involved in the experiment, e.g., $TE(ball,End(drop)) = KE(ball,End(drop)) + PE(ball,End(drop))$ which restricts the definition to the particular ball being dropped and the particular dropping event involved. `Os`, unlike `Ot`, contains axioms specifying that the final height and velocity are both zero.

The development graph of the dark matter example exhibits a similar structure, so it is omitted to avoid repetition.

3.2 Axiomatisations of Theoretical and Sensory Ontologies

Representational choices need to be made regarding the axiomatisations of O_t and O_s. The ontological representation of the predictive theory as O_t is relatively straightforward as O_t requires access to the same physics laws as those needed in the case study, which are encoded as axioms; the axioms are contained in O_tLaw, but are exported to O_t. As briefly described already, O_s has access to the same axiomatised laws in O_t, but with lesser generality; these are contained in O_sLaw, but are exported to O_s. Instead of expressing the laws over the entire relevant domain, the domain of quantification in O_s is specific to the entities involved in the experiment. Therefore, O_s makes a lesser commitment than O_t because it commits itself only to the entities of the experiment. For the bouncing-ball paradox, the axioms of the O_t and O_s are:

$$
\begin{aligned}
&Ax(O_t) ::= \{\\
&\forall p{:}Part,t_i,t_j{:}Mom.TE(p,t_i) = TE(p,t_j),\\
&\forall p{:}Part,t{:}Mom.\ TE(p,t) = KE(p,t) + PE(p,t),\\
&\forall p{:}Part,t{:}Mom.\ KE(p,t) ::= \frac{1}{2}Mass(p,t).Vel(p,t)^2,\\
&\forall p{:}Part,t{:}Mom.\ PE(p,t) ::= Mass(p,t).G.\\
&Height(p,t)\\
&\}
\end{aligned}
$$

$$
\begin{aligned}
&Ax(O_s) ::= \{\\
&TE(ball,End(drop)) = KE(ball,End(drop)) +\\
&PE(ball,End(drop)),\\
&KE(ball,End(drop)) ::= \frac{1}{2}Mass(ball,End(drop)).\\
&Vel(ball,End(drop))^2,\\
&PE(ball,End(drop)) ::= Mass(ball,End(drop)).G.\\
&Height(ball,End(drop))\\
&\}
\end{aligned}
$$

where $TE(p,t)$, $KE(p,t)$, and $PE(p,t)$ respectively denote the amount of total energy, kinetic energy, and potential energy of an object p at a time moment t; $Mass(p,t)$ and $Vel(p,t)$ respectively denote the mass and velocity of p at t; G is the gravitational constant; $Start(drop)$ and $End(drop)$ respectively denote the start and end of the dropping of the ball.

For the dark matter case study, the axioms of O_t and O_s are:

$$
\begin{aligned}
&Ax(O_t) ::= \{\\
&\forall o{:}Obj, g{:}Galaxy.\ AngVel(o,g) = \frac{OrbVel(o,g)}{Rad(o,g)}, && (4)\\
&\forall o{:}Obj, g{:}Galaxy.\ Rad(o,g) > 0, && (5)\\
&\forall o{:}Obj.\ GraphA(o) = (Rad(o,MWay), && (6)\\
&AngVel(o,MWay).Rad(o,MWay))\\
&\}
\end{aligned}
$$

$$Ax(O_s) ::= \{$$

$$\forall o{:}Obj.\ AngVel(o, MWay) = \frac{OrbVel(o, MWay)}{Rad(o, MWay)}, \tag{7}$$

$$\forall o{:}Obj.\ Rad(o, MWay) > 0, \tag{8}$$

$$\forall o{:}Obj.\ GraphB(o) = (Rad(o, MWay), OrbVel(o, MWay)) \tag{9}$$

$$\}$$

where $AngVel(o,g)$ and $OrbVel(o,g)$ respectively denote the angular and orbital velocities of an object o in the galaxy g; $Rad(o,g)$ returns radius between o and the centre of the galaxy g; and $MWay$ is our galaxy, Milky Way.

The representation of O_t (and O_sLaws) adopted may appear *ad-hoc* as it requires O_s to have access to the same laws as O_t but expressed at the lowest level of generality. However, this reflects a situation where the sensory data in O_s is interpreted under the context of the theory O_t, so the data would be interpreted using the laws available in O_t. The minimal exportation of the theoretical laws, therefore, involves the same laws but at the lowest level of generality. This interpretation is analogous to a physicist's making sense of new sensory data in accordance to his/her current physics theory and understanding of the initial experiment setting. Of course, there are other alternative representations that are worthy of further work, each implying a different philosophy of the content of O_s and of the set of deducible theorems:

(a) O_s has access to the same axiomatised laws as those imported by O_t and have the same domain of quantification, so O_s has access to the same physics as O_t. One shortcoming of this representation is the loss of distinction between predictive theory and mere experimental evidence. Adopting such representation is equivalent to modelling the evolution of a physicist's ontological understanding of physics when confronted by *two* conflicting sets of data.

(b) O_s shares only the language of O_t and does not import any physics laws and definitions, so O_s is a knowledgebase. In order to derive a contradiction between the two ontologies without losing the distinction between them, the reasoning mechanism must then be able to access both O_t's and O_s's axioms. This is more general than those described above because, in an environment in which there are multiple theoretical ontologies confronting the same O_s, the deducible theorems then depend on the axioms of the particular theoretical ontology.

(c) O_s is not committed to share the language or the axioms of O_t, so this is the most general representation. The terms in O_t and O_s still need to be related in order to perform reasoning. One approach is to introduce metal-level relations between terms across the two ontologies, perceiving each as a different *context* [McCarthy and Buvac, 1998]. The reasoning mechanism needs to account for both the meta-level relations between terms and object-level formulation, which significantly increases the complexity of the task.

Undoubtedly, representation (c) is the most interesting avenue given the potentially high level of extensibility and generality. It is similar to the current representation in the way that both address the need for interpretation of the data in O_s in the context of O_t. However, the required axioms for contextual reasoning in the current representation are explicitly encoded in *OsLaws* at the object-level, which can handle only O_t, whereas in (c) these may be represented at the meta-level, which can handle multiple, arbitrary contexts. That said, we believe the representation we have currently adopted strikes a reasonable balance between expressivity and complexity.

4 REASONING FOR AUTOMATED ONTOLOGY EVOLUTION

The initial reasoning step for ontology evolution is to determine whether a conflict exists between the ontologies and which ontology repair plan should be triggered. Without a robust and correct reasoning mechanism, a repair plan could be triggered to modify an already correct ontology or might not be triggered when it is supposed to be: giving rise to false-positives and false-negatives, respectively. Using the representation described previously in §3, the diagnosis can take place by translating nodes into logical theories in the logic of a particular theorem prover and then attempting to deduce trigger formulae in those theories.

4.1 Detection of Conflicts between Ontologies

If we reason with an inconsistent ontology, then every formula is deducible in multiple ways, leading to an explosion of provable theorems. Therefore, when we prove whether a trigger formula is a theorem of some given ontologies to detect a conflict between them, the ontologies cannot be first merged.

For reasoning in both first-order and higher-order logics, we use the interactive theorem prover Isabelle because its emphasis on proving higher-order theorems. Isabelle, although powerful, does not offer tools tailored for reasoning with modular ontologies. Fortunately, there are at least two workarounds for reasoning modularly without reverting to merging the axioms of the ontologies and giving rise to an inconsistent set of axioms. One is to specify the ontologies as separate Isabelle theories and another is to specify them as separate locales [Ballarin, 2004], which are mechanisms for defining local scopes in a proof. The locales approach is more attractive because each of our ontologies can be viewed as an individual context and theorems can be proved in the context of a specified locale. That said, each locale corresponds to a node in a development graph as each node represents an ontology, which is analogous to a context. In this section, we present the procedure for the diagnosis of conflict in the bouncing-ball and dark matter case studies. The type of conflict under scrutiny is that formalised in the WMS ontology repair plan [Bundy and Chan, 2008], of which the trigger is specified in Figure 1. Note that some parts of the proofs are been omitted due to space limitations.

4.1.1 By First-Order Proof Calculus

To trigger WMS in the bouncing-ball paradox presented earlier, we need to show that (1), (2), and (3) are deducible. The trigger formulae can be instantiated with the following substitution:

$$\{TE/stuff,\ \langle ball, End(drop)\rangle/\vec{s},\ TE(ball, Start(drop))/v_1, 0/v_2\}$$

With this substitution, the instantiated form of (1), (2), and (3) is:

$$O_t \vdash TE(ball, End(drop)) = TE(ball, Start(drop)) \quad (10)$$

$$O_s \vdash TE(ball, End(drop)) = 0 \quad (11)$$

$$O_t \vdash TE(ball, Start(drop)) \neq 0 \quad (12)$$

where, in physics terms, (12) comes from the initial condition that the ball is suspended from a positive height, (10) comes from the law of conservation of energy so that the total amount of energy at the start and at the end of the drop should be the same, and (11) is deduced from the observation that at the end of the drop both the velocity and the height of the ball are zero. The representation of the paradox does not require quantification over functions, so first-order logic is sufficient for the representation. To prove the trigger formulae in this example, HOL is therefore used to reason over FOL, as demonstrated in the Isabelle proof below:

```
typedecl Obj
typedecl Event
types Energy = real   Time = real
```

A constant for representing the ball[1]:

```
consts ball :: Obj
```

Each of the ontologies outlined in Figure 3 is specified as a locale. Locale `BasicPhys` corresponds to the node `BasicPhys` in Figure 3, which represents an ontology specifying the language of O_t and O_s:

```
locale BasicPhys =
  fixes Vel :: "Obj ⇒ Time ⇒ real"
  and Height :: "Obj ⇒ Time ⇒ real"
  and Start :: "Event ⇒ Time ⇒ real"
...
```

The locale `ClassicalEnergyConv` corresponds to the node `ClassicalEnergyConv` in the development graph, which imports the signature from `BasicPhys` and extends it with a language for representing various types of energies:

```
locale ClassicalEnergyConv =
  fixes TE :: "Obj ⇒ Time ⇒ real"
  and KE :: "Obj ⇒ Time ⇒ real"
  and PE :: "Event ⇒ Time ⇒ Energy"
...
```

Locale `OtLaws` corresponds to the node `OtLaws` in the development graph and contains the axioms constituting the definitions of total energy (without elasticity), potential energy, kinetic energy, the law of conservation of energy, and the gravitational constant[2]:

```
locale OtLaws = BasicPhys +
  assumes te: "TE p t = PE p t + KE p t"
  and pe: "PE p t = Mass p×G×Height p
t"
  and ke: "KE p t = 0.5×Mass p×Vel p
t×Vel p t"
...
```

Locale `Ot` corresponds to `Ot` and asserts the values of the initial velocity, initial height, and mass of the ball:

```
locale Ot = OtLaws +
  assumes vinit: "Vel ball (Start drop)
= 0"
  and hinit: "Height ball (Start drop)
> 0"
  and mass: "Mass ball > 0"
```

[1] `a::τ` denotes that `a` is of type τ.

[2] If `F` is a function, `F x` denotes the application of `F` to `x`.

Locale OsLaws corresponds to the node OsLaws and represents an ontology containing axioms based on the laws over a specific domain:

```
locale OsLaws = BasicPhys +
  assumes te: "TE p t = PE p t + KE p t"
  and pe: "PE p t = Mass p×G×Height p
t"
  and ke: "KE p t = 0.5×Mass p×Vel p
t×Vel p t"
...
```

The locale Os corresponds to Os and asserts the values obtained from observation, i.e. the final values of the velocity, height, and mass of the ball:

```
locale Os = OsLaws +
  assumes vfin: "Vel ball (End drop) =
0"
  and hfin: "Height ball (End drop) =
0"
  and mass: "Mass ball > 0"
```

To prove the trigger formulae in Figure 1, we need to be able to deduce the total amount of energy from basic quantities, e.g., velocity, height and mass. Isabelle has a rich library of mathematical theorems, which significantly helps the rewriting of equations and the substitution of values. Three theorems are needed to be proven in the ontologies: one proposes that there exists a moment when the final amount of total energy is equal to the initial amount of total energy, the final amount of total energy is zero, and the initial amount of total energy is not zero. Hence, the first proof goal to be discharged is (1), which can be proved using the instantiation (10):

```
lemma (in Ot) lem1: "TE ball (End drop)
= TE ball (Start drop)"
  using cons   by auto

theorem (in Ot) OtWMS1: "EX (stuff::?'a
⇒ ?'b) s v1. stuff s = v1"
proof (intro exI) qed (rule lem1)
```

The second proof goal to be discharged is (2), which can be proved using the instantiation (11):

```
lemma (in Os) lem2: "TE ball (End drop)
= 0"
  using mass vfin hfin te ke pe g
  by auto

theorem (in Os) OsWMS1: "EX
(stuff::?'a=>?'b) s v2. stuff s = v2"
proof (intro exI) qed (rule lem2)
```

The final proof goal to be discharged is (3), which can be proved using the instantiation (12):

```
theorem (in Ot) OtWMS2: "TE ball (Start
drop) ≠ 0"
proof ... qed
```

4.1.2 By Higher-Order Proof Calculus

The proposed existence of dark matter is a case study which requires a higher-order representation. As described earlier, since the two graphs in Figure 2 are compared, function objects can be used to represent graphs in the formulation. Formulae (1), (2), and (3) can be instantiated with the following substitution:

$$\{\lambda s \in g.\ \langle Rad(s), OrbVel(s)\rangle / stuff,\ \langle MWay\rangle/\vec{s},\ Graph_p/v_1,\ Graph_a/v_2\}$$

which gives the instantiated form of the trigger formulae as follows:

$$O_t \vdash \lambda s \in MWay.\ \langle Rad(s), OrbVel(s)\rangle = Graph_p \quad (13)$$

$$O_s \vdash \lambda s \in MWay.\ \langle Rad(s), OrbVel(s)\rangle = Graph_a \quad (14)$$

$$O_t \vdash Graph_p \neq Graph_a \quad (15)$$

where $OrbVel(s)$ is the orbital velocity of star s, $Rad(s)$ is the radius of s from the centre of the Milky Way, and $MWay$ is our own galaxy, represented as the set of stars it contains. Formula (14) shows the predicted graph, $Graph_p$: the orbital velocity decreases roughly inversely with the square root of the radius (see Figure 2). This graph is deduced by Newtonian Mechanics from the observed distribution of the visible stars in the Milky Way. Formula (15) shows the actual observed orbital velocity graph, $Graph_a$: it is almost a constant function over most of the values of s (see Figure 2). Note the use of λ-abstraction to create graph objects as unary functions. These two graphs are unequal (15), within the range of legitimate experimental variation.

The following proof illustrates the power of using Isabelle's higher-order proof calculus to detect a conflict between the two HOL ontologies, which exhibits a similar structure to the previous proof:

```
typedecl Obj
types Spiral = "Obj set"  Time = real
```

A constant for representing the Milky Way:

```
consts MWay :: Spiral
```

Locale BasicPhys corresponds to BasicPhys in the development graph, which represents the ontology containing only the language of locales O_t and O_s in this particular case study:

```
locale BasicPhys =
  fixes OrbVel :: "Obj => Obj set ⇒
real"
  and GrphP :: "Obj ⇒ real×real"
...
```

```
  assumes cab: "∃P. P ≠ {} ∧ (∀x. x∈P
⟶ CurveA x ≠ CurveB x)"
  and gcp: "GrphP = CurveP"
  and gca: "GrphA = CurveA"

locale OtLaws = BasicPhys +
  assumes radgtzero: "Rad p g > 0"
  and ovabsov: "OrbVel p g = abs
(OrbVel p g)"
```

Locale Ot contains the defintion of angular velocity in terms of orbital velocity. it also asserts that its graph is a plot of the product of angular velocity by radius:

```
locale Ot = OtLaws +
  assumes avel: "AngVel p g = OrbVel p g
/ Rad p g"
  and ga: "GrphP p = (Rad p MWay,
AngVel p MWay × Rad p MWay)"

locale OsLaws = BasicPhys +
  assumes radgtzero: "∀p. p∈MWay ⟶ Rad
p MWay > 0"
  and ovabsov: "∀p. p∈MWay ⟶ OrbVel p
MWay = abs (OrbVel p MWay)"
```

Locale Os explicitly asserts that its graph is a plot of the orbital velocity of stars in the Milky Way.

```
locale Os = OsLaws +
  assumes gb: "GrphA p = (Rad p MWay,
OrbVel p MWay)"
```

Similar to the previous proof, the first proof goal to be discharged here is (1), which can be proved using the instantiation (14):

```
lemma (in Ot) lem1: "(λ g s. (Rad s g,
OrbVel s g)) MWay = GrphP"
  apply (simp add: expand_fun_eq)
  ...

theorem (in Ot) OtWMS1: "∃(stuff::?'a ⇒
?'b) s v1. stuff s = v1"
proof (intro exI) qed (rule lem1)
```

The second proof goal to be discharged is (2), which can be proved using the instantiation (15):

```
lemma (in Os) lem2: "(λ g s. (Rad s g,
OrbVel s g)) MWay = GrphA"
  using gb
  by (simp add: expand_fun_eq)

theorem (in Os) OsWMS1: "∃(stuff::?'a ⇒
?'b) s v2. stuff s = v2"
proof (intro exI) qed (rule lem2)
```

The final proof goal to be discharged is (3), which can be proved using the instantiation (15):

```
theorem (in Ot) OtWMS2: "GrphA ≠ GrphP"
  using cab gca gcp by auto
```

5 DISCUSSION

The two case studies presented have shown the benefits of representing the predictive theory and the sensory data as separate ontologies. By encoding each ontology as an individual locale that is locally consistent, each of the three parts of the WMS trigger formulae is simply an open theorem of the relevant ontology. If the two were merged, then there would be an explosion of uninteresting theorems. Moreover, the case studies have demonstrated the need for higher-order logic and the power of using a higher-order theorem prover such as Isabelle for aiding automated ontology evolution. For example, Isabelle's polymorphic meta-logic is particularly useful for the detection of the trigger formula because *stuff* has a polymorphic type $a \Rightarrow b$ and, before diagnosis, how it is to be instantiated is not known. An obvious advantage is that the type of *stuff* is a variable, which provides a sufficiently high level of generality in the trigger formula. As shown in the proof for the bouncing-ball paradox, *stuff* is instantiated by TE with type $Obj \Rightarrow Time \Rightarrow real$, whereas in that for the existence of dark matter, *stuff* is instantiated by $\lambda s\, t.\langle Radius\ s\ t, Orb\ s\ t\rangle$ with type $Obj \Rightarrow real \times real$. Moreover, the proof of the trigger formula (2) requires the comparision using the equality and the inequality operators, which are polymorphic as well. For example, real numbers and functions are compared in the bouncing-ball and the dark matter case studies respectively, so the operators have defined meanings on reals in one scenario and on functions in another.

On the representational aspect, if a less expressive logic, e.g., DL or FOL, was adopted, it would be impossible to reason over function objects. Significant changes to the representation would be required in order to perform the described kind of reasoning. For example, in the dark matter case study, the representation of the function of the orbit of a star could no longer be a functional object, but a (possibly infinite) set of positional points in a 3-D space, which we believe is unnatural.

6 CONCLUSION

Further progress in handling automated ontology evolution is now urgent, due to the demand created by multi-agent systems. We have outlined two main challenges to the development of mechanisms supporting automated ontology evolution, i.e. designing a modular representation and performing reasoning across modular ontologies. The latter imposes a relatively greater challenge in our domain as it demands an unusual use of higher-order theorem proving with interactive provers. As described, a formal logical structure is adopted to store and manage ontologies and ontologies themselves are treated as expressive logical theories. Evident by the two described examples from physics, our work is showing the advantages of the unusual use of Isabelle for higher-order reasoning with modular ontologies and the visualisation of the structure as a development graph.

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REFERENCES

[Autexier et al., 1999] Autexier, S., Hutter, D., Mantel, H., and Schairer, A. (1999). Towards an Evolutionary Formal Software-Development Using CASL. *Lecture Notes In Computer Science; Vol. 1827*, pages 73–88.

[Ballarin, 2004] Ballarin, C. (2004). Locales and locale expressions in Isabelle/Isar. *Lecture Notes in Computer Science*, 3085:34–50.

[Bundy and Chan, 2008] Bundy, A. and Chan, M. (2008). Towards ontology evolution in physics. In Hodges, W., editor, *Procs. Wollic 2008*. LNCS, Springer-Verlag.

[Bundy and Varzinczak, 2009] Bundy, A., L. J. Q. G. and Varzinczak, I. (2009). In *Notes of the IJCAI-09 Workshop on Automated Reasoning about Context and Ontology Evolution*. AAAI Press.

[Chan and Bundy, 2008] Chan, M. and Bundy, A. (2008). Inconstancy: An ontology repair plan for adding hidden variables. In Bringsjord, S. and Shilliday, A., editors, *Symposium on Automated Scientific Discovery*, number FS-08-03 in Technical Report, pages 10–17. AAAI Press. ISBN 978-1-57735-395-9.

[diSessa, 1983] diSessa, A. (1983). Phenomenology and the evolution of intuition. In Stevens, A. and Gentner, D., editors, *Mental Models*, pages 15–33. Erlbaum.

[Doan et al., 2004] Doan, A., Madhavan, J., Domingos, P., and Halevy, A. (2004). Ontology matching: A machine learning approach. *Handbook on Ontologies in Information Systems*, pages 397–416.

[Ghilardi et al., 2006] Ghilardi, S., Lutz, C., and Wolter, F. (2006). Did I damage my ontology? A case for conservative extensions in description logic. *Proceedings of the Tenth International Conference on Principles of Knowledge Representation and Reasoning (KR-2006)*, pages 187–197.

[Giunchiglia and Shvaiko, 2004] Giunchiglia, F. and Shvaiko, P. (2004). Semantic matching. *The Knowledge Engineering Review*, 18(03):265–280.

[Kalfoglou and Schorlemmer, 2003] Kalfoglou, Y. and Schorlemmer, M. (2003). Ontology mapping: the state of the art. *The Knowledge Engineering Review*, 18(1):1–31.

[Kalyanpur et al., 2006] Kalyanpur, A., Parsia, B., Sirin, E., and Grau, B. (2006). Repairing unsatisfiable concepts in OWL ontologies. In *ESWC*, pages 170–184. Springer.

[Katsuno and Mendelzon, 1991] Katsuno, H. and Mendelzon, A. O. (1991). On the difference between updating a knowledge base and revising it. In *KR*, pages 387–394.

[Lam et al., 2008] Lam, J., Sleeman, D., Pan, J., and Vasconcelos, W. (2008). A fine-grained approach to resolving unsatisfiable ontologies. *Lecture Notes in Computer Science*, 4900:62.

[McCarthy and Buvac, 1998] McCarthy, J. and Buvac, S. (1998). Formalizing context (expanded notes). *Computing natural language*, 81:13–50.

[McNeill and Bundy, 2007] McNeill, F. and Bundy, A. (2007). Dynamic, automatic, first-order ontology repair by diagnosis of failed plan execution. *International Journal On Semantic Web and Information Systems*, 3(3):1–35. Special issue on ontology matching.

[Mossakowski et al., 2007] Mossakowski, T., Maeder, C., and Lüttich, K. (2007). The Heterogeneous Tool Set. In Grumberg, O. and Huth, M., editors, *TACAS 2007*, volume 4424 of *Lecture Notes in Computer Science*, pages 519–522. Springer-Verlag Heidelberg.

[Ovchinnikova and Kühnberger, 2007] Ovchinnikova, E. and Kühnberger, K. (2007). Automatic Ontology Extension: Resolving Inconsistencies. *OTT'06*, 1:93.

[Paulson, 1994] Paulson, L. C. (1994). *Isabelle: A generic theorem prover*. Springer-Verlag.

[Stojanovic et al., 2002] Stojanovic, L., Maedche, A., Motik, B., and Stojanovic, N. (2002). User-Driven Ontology Evolution Management. In *Proceedings of the 13th International Conference on Knowledge Engineering and Knowledge Management. Ontologies and the Semantic Web*, pages 285–300. Springer-Verlag London, UK.