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Lattice Input on the Inclusive $\tau$ Decay $V_{us}$ Puzzle

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Recent analyses of flavor-breaking hadronic-τ-decay-based sum rules produce values of $|V_{us}| \sim 3\sigma$ low compared to 3-family unitarity expectations. An unresolved systematic issue is the significant variation in $|V_{us}|$ produced by different prescriptions for treating the slowly converging $D = 2$ OPE series. We investigate the reliability of these prescriptions using lattice data for various flavor-breaking correlators and show the fixed-scale prescription is clearly preferred. Preliminary updates of the conventional τ-based, and related mixed τ-electroproduction-data-based, sum rule analyses incorporating B-factory results for low-multiplicity strange τ decay mode distributions are then performed. Use of the preferred FOPT $D = 2$ OPE prescription is shown to significantly reduce the discrepancy between 3-family unitarity expectations and the sum rule results.

The conventional inclusive hadronic τ decay determination of $|V_{us}|$ is obtained by applying the finite energy sum rule (FESR) relation, involving polynomial weight $w(s)$ and kinematic-singularity-free correlator $\Pi(s)$ with spectral function $\rho(s)$,

$$\int_0^{s_0} w(s)\rho(s) \, ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s)\Pi(s) \, ds,$$

(1)

to the flavor-breaking (FB) difference $\Delta \Pi_{\tau} \equiv \left[ \Pi_{V+A;ud}^{(0+1)} - \Pi_{V+A;us}^{(0+1)} \right]$, where

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$\Pi^{(J)}_{V/A;ij}(s)$ are the spin $J = 0,1$ components of the flavor $ij$, vector (V) or axial vector (A) current-current 2-point functions. The spectral functions, $\rho^{(0+1)}_{V/A;ij}$, hence also $\Delta \rho_{\tau}$, are related to the normalized differential decay distributions, $dR_{V/A;ij}/ds$, of flavor $ij$ V- or A-current-induced $\tau$ decay widths, $R_{V/A;ij} \equiv \Gamma(\tau^- \to \nu_\tau \text{hadrons}_{V/A;ij}(\gamma))/\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma))$, by

$$dR_{V/A;ij}/ds = 12\pi^2|V_{ij}|^2 S_{EW} \left[ w_\tau(y_\tau)\rho^{(0+1)}_{V/A;ij}(s) - w_L(y_\tau)\rho^{(0)}(s) \right]/m_\tau^2,$$  

with $y_\tau = s/m_\tau^2$, $V_{ij}$ the CKM matrix element, $w_\tau(y) = (1-y)^2(1+2y)$, $w_L(y) = y(1-y)^2$, and $S_{EW}$ a short-distance electroweak correction factor. The $J = 0$ (longitudinal) contributions in [2] are well known phenomenologically and, due to problems with the corresponding $D = 2$ OPE series, usually subtracted from $dR/ds$ [2,12].

The subtracted result, $dR^{(0+1)}_{V/A;ij}/ds$, allows the construction of $J = 0 + 1$ reweighted analogues, $R_{V/A;ij}^{w}(s_0) = \int_0^{s_0} ds [w(s)/w_\tau(y_\tau)] dR^{(0+1)}_{V/A;ij}(s)/ds$, for any $w(s)$ and $s_0 < m_\tau^2$. Defining $\delta R_{V/A;ij}^{w}(s_0) = \left[ R_{V/A;ij}^{w}(s_0)/|V_{us}|^2 \right] - \left[ R_{V/A;ij}^{w}(s_0)/|V_{us}|^2 \right]$, one has, for $s_0$ large enough to allow use of the OPE on the RHS of [1,11]

$$|V_{us}| = \sqrt{R_{V/A;ij}^{w}(s_0)}/\left[ R_{V/A;ij}^{w}(s_0)/|V_{us}|^2 - \delta R_{V/A;ij}^{w,OPE}(s_0) \right].$$  

This relation has usually been employed in un-reweighted form, with $w = w_\tau$, and the single value $s_0 = m_\tau^2$. This has the advantage that $R_{V/A;ud,us}(s_0)$ is determinable from branching fraction information alone, but the disadvantage of precluding tests of the $s_0$- and $w(s)$-independence of the analysis, which could otherwise be used to investigate potential systematic uncertainties (in particular, those associated with the treatment of OPE contributions). Such self-consistency tests were carried out in Refs. [2,13,14] and non-trivial $w(s)$- and $s_0$-dependences observed, suggesting shortcomings in the experimental data and/or OPE representation.

The most obvious potential OPE problem lies in the rather slow convergence of the $D = 2$ OPE series. In terms of the running $\overline{MS}$ quantities $m_s(Q^2)$ and $\bar{a} \equiv \alpha_s(Q^2)/\pi$, the $D = 2$ series, which is known to 4-loops, is given by

$$[\Delta \Pi_{\tau}(Q^2)]^{(OPE)}_{D=2} = \frac{3}{2\pi^2} \frac{m_s^2(Q^2)}{Q^2} \sum_{k=0}^{4} c_k^\tau \bar{a}_k^{k}$$  

with $c_k^\tau = 1, 7/3, 19.93, 208.75$ for $k = 0 \cdots 3$ [5]. Since $\bar{a}(m_s^2) \simeq 0.10$, $c_k^\tau \bar{a}_k > c_3^\tau \bar{a}_3$ at the spacelike point on the contour for all $s_0 \leq m_\tau^2$. The problematic convergence complicates the assessment of $D = 2$ truncation errors, and manifests itself, e.g., in the $\sim 0.0020$ difference in $|V_{us}|$ values obtained using two alternate (CIPT or FOPT) versions of the 4-loop-truncated, $w_\tau$-weighted series.

An alternate determination employs the FB combination $\Delta \Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi_{ud,us}(0+1) + \Pi^{(0+1)}_{ud,us} - \Pi_{us,V+A}$ in place of $\Delta \Pi_{\tau}$ [11]. Inclusive electroproduction cross-sections fix the electromagnetic (EM) spectral function. By construction, the $\Delta \Pi_{\tau-EM} D = 2$ series is strongly suppressed, having the form [11], with $c_k^\tau \rightarrow$
\(c_k^{\tau-EM} = 0, -1/3, -4.384, -44.943\) for \(k = 0 \cdots 3\). The \(D = 4\) series is also strongly suppressed. OPE contributions to \(\Delta \Pi_{\tau-EM}\) FESRs, hence also estimated OPE errors, are thus very small and the resulting \(|V_{us}|\) errors essentially entirely experimental. A check of this predicted suppression is thus of interest.

We investigate the relative merits of the fixed-scale (FOPT-like) and local-scale (\(\mu^2 = Q^2\), i.e., CIPT-like) treatments of the \(\Delta \Pi_\tau\) \(D = 2\) series, and the level of \(\Delta \Pi_{\tau-EM}\) suppression, by comparing OPE expectations and lattice data for the two correlator combinations over a range of Euclidean \(Q^2\). Five RBC/UKQCD domain wall fermion ensembles are employed, three, with \(m_\pi = 293, 349, 399\) MeV, having \(1/a = 2.31\) GeV, and two, with \(m_\pi = 171, 248\) MeV, having \(1/a = 1.37\) GeV. For technical reasons, conserved-local versions of the flavor \(us\) 2-point functions are numerically challenging and hence, for \(\Delta \Pi_\tau\), local-local versions are used. To check that this does not produce residual lattice artifacts which would impact our conclusions, we have also performed the OPE-lattice comparison, using conserved-local data, for the alternate flavor-diagonal FB combination \(\Delta \Pi_{diag} \equiv \Pi_{V;\ell\ell} - \Pi_{V;ss}\), whose \(D = 2\) series is very similar to that of \(\Delta \Pi_\tau\) (\(c_k^{\tau} \rightarrow c_k^{diag} = 1, 8/3, 24.32, 253.69\) for \(k = 0 \cdots 3\) in (4)). The results confirm those of the local-local study.

Representative OPE-lattice data comparisons for \(\Delta \Pi_\tau\) are shown, for the \(1/a = 2.31\) GeV, \(m_\pi = 293\) MeV ensemble, in Fig. 1. The left (right) panel comparison employs the fixed-scale (local-scale) prescription for the \(D = 2\) OPE series. The fixed-scale versions match much better the \(Q^2\) dependence of the lattice results, with the 3-loop-truncated version thereof best matching the overall normalization.

![Fig. 1. OPE and lattice \(\Delta \Pi_\tau\) data, \(1/a = 2.31\) GeV, \(m_\pi = 293\) MeV ensemble, \(O(a^{1,2,3})\) \(D = 2\) OPE truncation, fixed-scale (left panel) or local-scale (right panel) \(D = 2\) prescription](image)

The comparison of lattice data for \(\Delta \Pi_\tau\) and \(\Delta \Pi_{\tau-EM}\) confirms the very strong suppression of \(\Delta \Pi_{\tau-EM}\) (see Ref. for the relevant figure).
We turn to preliminary updates of the $|V_{us}|$ analyses. For the $D = 2$ OPE series, we employ the 3-loop-truncated FOPT prescription favored by lattice data, and for the $ud$ spectral integrals, OPAL data $^9$ as updated in Ref. $^{10}$. For the $us$ spectral integrals, recent B-factory results are used for the $K\pi$ $^{11}$, $K^-\pi^-\pi^+$ $^{12}$ and $K_s\pi^-\pi^0$ $^{13}$ exclusive mode distributions, and ALEPH results $^{14}$, updated for current branching fractions (BFs), for all other modes. Contributions from the latter lie higher in the spectrum, and have much larger errors. The B-factory distributions are unit normalized, and also require current BFs for their overall scales. We work with BFs obtained in a $\pi_\mu^2$, $K_\mu^2$-constrained HFAG fit, supplemented by the update to $B[\tau^- \to K^0\pi^-\pi^0\nu_\tau]$ produced by the recent Belle result $^{13}$. Other non-trivial shifts in the $us$ BFAs also remain possible. To illustrate the changes to $|V_{us}|$ that could result, we consider also an alternate set of $us$ BFs with the recent larger, but not yet finalized, BaBar results $^{15}$ for $B[\tau^- \to K^-n\pi^0\nu_\tau]$, $n \leq 3$, used in place of those of the HFAG fit. The first set of $us$ BFs is labelled “$us$ BF set #1” below, the second, alternate set “$us$ BF set #2”. Changes to the $us$ BFs alter the inclusive $us$ spectral distribution, and hence can affect both the magnitude of $|V_{us}|$ and the $s_0$-dependence of the results. The significantly larger preliminary BaBar $K^-\pi^0$ BF is particularly relevant for the FB FESRs considered here, which weight more strongly the low-$s$ part of the spectrum. We consider FESRs employing the weights $w_\tau$ and $w_2(y) = (1-y)^2$. $w_2$ weights less strongly the higher-$s$, large-error region of the $us$ spectral distribution. Differences between results obtained using the two different weights can thus point to issues with the $us$ spectral distribution.

$|V_{us}|$ results obtained from the $w_\tau$ and $w_2$ versions of the $\Delta \Pi_\tau$ FESR are shown, as a function of $s_0$, and also the choice of the input $us$ BF set, in the left panel of Fig. 2. Similar results for the $\Delta \Pi_{\tau-EM}$ FESR are shown in the right panel. $w_2$ results, which are less sensitive to the large-error high-$s$ region, show better $s_0$-stability in both cases. For $w_\tau$, $s_0$-stability is also better for the $\Delta \Pi_{\tau-EM}$ case,
where OPE contributions are suppressed. The convergence of $w_\tau$ results to the more stable $w_2$ ones as $s_0 \to m_\tau^2$, seen for both the $\Delta \Pi_{+}$ and $\Delta \Pi_{-EM}$ FESRs, suggests the possibility of residual OPE problems in the $w_\tau$ case, where cancellations on the contour play a larger role. Finally we note that results obtained using the FOPT prescription preferred by the lattice data agree better with 3-family unitarity expectations than do those (not shown here) obtained using CIPT, as do those obtained using $us$ BF set #2 in place of $us$ BF set #1. More details of these analyses will be presented elsewhere.

We close by stressing the preference for FOPT over CIPT for the $D = 2$ OPE series. The prescription which underlies CIPT (of summing logarithmic terms to all orders while truncating the series of non-logarithmic terms), though plausible, is motivated by heuristic arguments not generally valid for divergent series and performs poorly when tested against lattice data for the FB correlators.

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