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Lattice Input on the Inclusive $\tau$ Decay $V_{us}$ Puzzle

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Recent analyses of flavor-breaking hadronic-$\tau$-decay-based sum rules produce values of $|V_{us}| \sim 3\sigma$ low compared to 3-family unitarity expectations. An unresolved systematic issue is the significant variation in $|V_{us}|$ produced by different prescriptions for treating the slowly converging $D = 2$ OPE series. We investigate the reliability of these prescriptions using lattice data for various flavor-breaking correlators and show the fixed-scale prescription is clearly preferred. Preliminary updates of the conventional $\tau$-based, and related mixed $\tau$-electroproduction-data-based, sum rule analyses incorporating B-factory results for low-multiplicity strange $\tau$ decay mode distributions are then performed. Use of the preferred FOPT $D = 2$ OPE prescription is shown to significantly reduce the discrepancy between 3-family unitarity expectations and the sum rule results.

The conventional inclusive hadronic $\tau$ decay determination of $|V_{us}|$ is obtained by applying the finite energy sum rule (FESR) relation, involving polynomial weight $w(s)$ and kinematic-singularity-free correlator $\Pi(s)$ with spectral function $\rho(s)$,

$$
\int_0^{s_0} w(s)\rho(s) \, ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s)\Pi(s) \, ds,
$$

(1)

to the flavor-breaking (FB) difference $\Delta \Pi_{\tau} \equiv \left[ \Pi_{V+Ag}^{(0+1)} - \Pi_{V+A;us}^{(0+1)} \right]$, where

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$\Pi^{(J)}_{V/A;i,j}(s)$ are the spin $J = 0, 1$ components of the flavor $ij$, vector (V) or axial vector (A) current-current 2-point functions. The spectral functions, $\rho^{(0+1)}_{V/A;i,j}$, hence also $\Delta \rho_{\tau}$, are related to the normalized differential decay distributions, $dR_{V/A;i,j}/ds$, of flavor $ij$ V- or A-current-induced $\tau$ decay widths, $R_{V/A;i,j} \equiv \Gamma[\tau^- \to \nu_\tau \text{hadrons}_{V/A;i,j}(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$, by

$$dR_{V/A;i,j}/ds = 12\pi^2|V_{ij}|^2 S_{\text{EW}} \left[w_\tau(y_\tau)\rho^{(0+1)}_{V/A;i,j}(s) - w_L(y_\tau)\rho^{(0)}(s)\right]/m_\tau^2,$$  \hspace{1cm} (2)

with $y_\tau = s/m_\tau^2$, $V_{ij}$ the ijk CKM matrix element, $w_\tau(y) = (1-y^2)/(1+2y)$, $w_L(y) = y(1-y)^2$, and $S_{\text{EW}}$ a short-distance electroweak correction factor. The $J = 0$ (longitudinal) contributions in (2) are well known phenomenologically and, due to problems with the corresponding $D = 2$ OPE series, usually subtracted from $dR/ds$. \cite{1}

The subtracted result, $dR^{(0+1)}_{V/A;i,j}/ds$, allows the construction of $J = 0 + 1$ reweighted analogues, $R^{w}_{V/A;i,j}(s_0) = \int_{s_0}^{s_\text{max}} ds [w(s)/w_\tau(y_\tau)] dR^{(0+1)}_{V/A;i,j}(s)/ds$, for any $w(s)$ and $s_0 < m_\tau^2$. Defining $\delta R^{w}_{V/A;i,j}(s_0) = [R^{w}_{V/A;ud}(s_0)/|V_{ud}|^2] - [R^{w}_{V/A;us}(s_0)/|V_{us}|^2]$, one has, for $s_0$ large enough to allow use of the OPE on the RHS of (1), \cite{1}

$$|V_{us}|^2 = \sqrt{R^{w}_{V/A;ud}(s_0)/\left[R^{w}_{V/A;ud}(s_0)/|V_{ud}|^2 - \delta R^{w,\text{OPE}}_{V/A}(s_0)\right]}.$$ \hspace{1cm} (3)

This relation has usually been employed in un-reweighted form, with $w = w_\tau$, and the single value $s_0 = m_\tau^2$. \cite{1} This has the advantage that $R^{w}_{V/A;ud,us}(m_\tau^2)$ is determinable from branching fraction information alone, but the disadvantage of precluding tests of the $s_0$- and $w(s)$-independence of the analysis, which could otherwise be used to investigate potential systematic uncertainties (in particular, those associated with the treatment of OPE contributions). Such self-consistency tests were carried out in Refs. \cite{2} \cite{3} \cite{4} and non-trivial $w(s)$- and $s_0$-dependences observed, suggesting shortcomings in the experimental data and/or OPE representation.

The most obvious potential OPE problem lies in the rather slow convergence of the $D = 2$ OPE series. In terms of the running $\overline{\text{MS}}$ quantities $m_\alpha(Q^2)$ and $a \equiv \alpha_s(Q^2)/\pi$, the $D = 2$ series, which is known to 4-loops, is given by

$$[\Delta \Pi_\tau(Q^2)]^{\text{OPE}}_{D=2} = \frac{3}{2\pi^2} \frac{m_\alpha^2(Q^2)}{Q^2} \sum_{k=0} \frac{c_k a^k}{k^2},$$ \hspace{1cm} (4)

with $c_k = 1, 7/3, 19.93, 208.75$ for $k = 0 \cdots 3$. Since $a(m_\tau^2) \simeq 0.10, c_k a^3 > c_k a^2$ at the spacelike point on the contour for all $s_0 \leq m_\tau^2$. The problematic convergence complicates the assessment of $D = 2$ truncation errors, and manifests itself, e.g., in the $\sim 0.0020$ difference in $|V_{us}|$ values obtained using two alternate (CIPT or FOPT) versions of the 4-loop-truncated, $w_\tau$-weighted series.

An alternate determination employs the FB combination $\Delta \Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi^{(0+1)}_{ud,us} + \Pi^{(0+1)}_{ud,A} - \Pi^{(0+1)}_{us;V,A}$ in place of $\Delta \Pi_\tau$. \cite{2} Inclusive electroproduction cross-sections fix the electromagnetic (EM) spectral function. By construction, the $\Delta \Pi_{\tau-EM} D = 2$ series is strongly suppressed, having the form \cite{1}, with $c_k \to$
\( c_k^{\tau-EM} = 0, -1/3, -4.384, -44.943 \) for \( k = 0 \cdots 3 \). The \( D = 4 \) series is also strongly suppressed. OPE contributions to \( \Delta \Pi_{\tau-EM} \) FESRs, hence also estimated OPE errors, are thus very small \( 6 \), and the resulting \( |V_{us}| \) errors essentially entirely experimental. A check of this predicted suppression is thus of interest.

We investigate the relative merits of the fixed-scale (FOPT-like) and local-scale (\( \mu^2 = Q^2 \)), i.e., CIPT-like) treatments of the \( \Delta \Pi_{\tau} \) \( D = 2 \) series, and the level of \( \Delta \Pi_{\tau-EM} \) suppression, by comparing OPE expectations and lattice data for the two correlator combinations over a range of Euclidean \( Q^2 \). Five RBC/UKQCD domain wall fermion ensembles are employed, three, with \( m_{\pi} = 293, 349, 399 \, \text{MeV} \), having \( 1/a = 2.31 \, \text{GeV} \), and two, with \( m_{\pi} = 171, 248 \, \text{MeV} \), having \( 1/a = 1.37 \, \text{GeV} \). For technical reasons, conserved-local versions of the flavor us 2-point functions are numerically challenging and hence, for \( \Delta \Pi_{\tau} \), local-local versions are used. To check that this does not produce residual lattice artifacts which would impact our conclusions, we have also performed the OPE-lattice comparison, using conserved-local data, for the alternate flavor-diagonal FB combination \( \Delta \Pi_{\text{diag}} \equiv \Pi_{V;\ell\ell} - \Pi_{V;ss} \), whose \( D = 2 \) series is very similar to that of \( \Delta \Pi_{\tau} \) (\( c_k^{\text{diag}} \rightarrow c_k^{\text{diag}} = 1, 8/3, 24.32, 253.69 \) for \( k = 0 \cdots 3 \) in \( 4 \)). The results confirm those of the local-local study.

Representative OPE-lattice data comparisons for \( \Delta \Pi_{\tau} \) are shown, for the \( 1/a = 2.13 \, \text{GeV}, m_{\pi} = 293 \, \text{MeV} \) ensemble, in Fig. 1. The left (right) panel comparison employs the fixed-scale (local-scale) prescription for the \( D = 2 \) OPE series. The fixed-scale versions match much better the \( Q^2 \) dependence of the lattice results, with the 3-loop-truncated version thereof best matching the overall normalization.

![Fig. 1](image-url) OPE and lattice \( \Delta \Pi_{\tau} \) data, \( 1/a = 2.31 \, \text{GeV}, m_{\pi} = 293 \, \text{MeV} \) ensemble, \( O(\bar{a}^{1,2,3}) \) \( D = 2 \) OPE truncation, fixed-scale (left panel) or local-scale (right panel) \( D = 2 \) prescription.

The comparison of lattice data for \( \Delta \Pi_{\tau} \) and \( \Delta \Pi_{\tau-EM} \) confirms the very strong suppression of \( \Delta \Pi_{\tau-EM} \) (see Ref. \( 4 \) for the relevant figure).
We turn to preliminary updates of the $|V_{us}|$ analyses. For the $D = 2$ OPE series, we employ the 3-loop-truncated FOPT prescription favored by lattice data, and for the $ud$ spectral integrals, OPAL data $[9]$ as updated in Ref. $[10]$. For the $us$ spectral integrals, recent B-factory results are used for the $K\pi$ $[11]$, $K^-\pi^+\pi^0$ $[12]$ and $K_s\pi^-\pi^0$ $[13]$ exclusive mode distributions, and ALEPH results $[14]$, updated for current branching fractions (BFs), for all other modes. Contributions from the latter lie higher in the spectrum, and have much larger errors. The B-factory distributions are unit normalized, and also require current BFs for their overall scales. We work with BFs obtained in a $\pi\mu^2$, $K\mu^2$-constrained HFAG fit, supplemented by the update to $B[\tau^- \rightarrow K^0\pi^-\pi^0\nu_\tau]$ produced by the recent Belle result $[13]$. Other non-trivial shifts in the $us$ BFs also remain possible. To illustrate the changes to $|V_{us}|$ that could result, we consider also an alternate set of $us$ BFs with the recent larger, but not yet finalized, BaBar results $[15]$ for $B[\tau^- \rightarrow K^- n \pi^0\nu_\tau]$, $n \leq 3$, used in place of those of the HFAG fit. The first set of $us$ BFs is labelled “$us$ BF set #1” below, the second, alternate set “$us$ BF set #2”. Changes to the $us$ BFs alter the inclusive $us$ spectral distribution, and hence can affect both the magnitude of $|V_{us}|$ and the $s_0$-dependence of the results. The significantly larger preliminary BaBar $K^-\pi^0$ BF is particularly relevant for the FB FESRs considered here, which weight more strongly the low-$s$ part of the spectrum. We consider FESRs employing the weights $w_\tau$ and $w_2(y) = (1 - y)^2$. $w_2$ weights less strongly the higher-$s$, large-error region of the $us$ spectral distribution. Differences between results obtained using the two different weights can thus point to issues with the $us$ spectral distribution.

$|V_{us}|$ results obtained from the $w_\tau$ and $w_2$ versions of the $\Delta\Pi_\tau$ FESR are shown, as a function of $s_0$, and also the choice of the input $us$ BF set, in the left panel of Fig. 2. Similar results for the $\Delta\Pi_{\tau-EM}$ FESR are shown in the right panel. $w_2$ results, which are less sensitive to the large-error high-$s$ region, show better $s_0$-stability in both cases. For $w_\tau$, $s_0$-stability is also better for the $\Delta\Pi_{\tau-EM}$ case,
where OPE contributions are suppressed. The convergence of \( w_\tau \) results to the more stable \( w_2 \) ones as \( s_0 \to m_{\tau}^2 \), seen for both the \( \Delta\Pi_\tau \) and \( \Delta\Pi_\tau-EM \) FESRs, suggests the possibility of residual OPE problems in the \( w_\tau \) case, where cancellations on the contour play a larger role. Finally we note that results obtained using the FOPT prescription preferred by the lattice data agree better with 3-family unitarity expectations than do those (not shown here) obtained using CIPT, as do those obtained using \( us \) BF set \#2 in place of \( us \) BF set \#1. More details of these analyses will be presented elsewhere.

We close by stressing the preference for FOPT over CIPT for the \( D=2 \) OPE series. The prescription which underlies CIPT (of summing logarithmic terms to all orders while truncating the series of non-logarithmic terms), though plausible, is motivated by heuristic arguments not generally valid for divergent series\(^\text{16}\), and performs poorly when tested against lattice data for the FB correlators.

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