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Lattice Input on the Inclusive τ Decay V_{us} Puzzle

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Recent analyses of flavor-breaking hadronic-τ-decay-based sum rules produce values of $|V_{us}| \sim 3\sigma$ low compared to 3-family unitarity expectations. An unresolved systematic issue is the significant variation in $|V_{us}|$ produced by different prescriptions for treating the slowly converging $D = 2$ OPE series. We investigate the reliability of these prescriptions using lattice data for various flavor-breaking correlators and show the fixed-scale prescription is clearly preferred. Preliminary updates of the conventional τ-based, and related mixed τ-electroproduction-data-based, sum rule analyses incorporating B-factory results for low-multiplicity strange τ decay mode distributions are then performed. Use of the preferred FOPT $D = 2$ OPE prescription is shown to significantly reduce the discrepancy between 3-family unitarity expectations and the sum rule results.

The conventional inclusive hadronic τ decay determination of $|V_{us}|$ [1] is obtained by applying the finite energy sum rule (FESR) relation, involving polynomial weight $w(s)$ and kinematic-singularity-free correlator $\Pi(s)$ with spectral function $\rho(s)$,

$$\int_0^{s_0} w(s)\rho(s)\, ds \;=\; -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\Pi(s)\, ds \;, \tag{1}$$

to the flavor-breaking (FB) difference $\Delta\Pi_\tau \;\equiv\; \left[\Pi^{(0+1)}_{V+A;ud} - \Pi^{(0+1)}_{V+A;us}\right]$, where

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$\Pi^{(J)}_{V/A;ij}(s)$ are the spin $J = 0, 1$ components of the flavor ij, vector (V) or axial vector (A) current-current 2-point functions. The spectral functions, $\rho^{(0+1)}_{V/A;ij}$, hence also $\Delta\rho_\tau$, are related to the normalized differential decay distributions, $dR_{V/A;ij}/ds$, of flavor ij V- or A-current-induced τ decay widths, $R_{V/A;ij} \equiv \Gamma[\tau^- \to \nu_\tau \,\text{hadrons}_{V/A;ij}\,(\gamma)]/\Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e(\gamma)]$, by

$$dR_{V/A;ij}/ds \;=\; 12\pi^2 |V_{ij}|^2 S_{EW} \left[w_\tau(y_\tau)\rho^{(0+1)}_{V/A;ij}(s) - w_L(y_\tau)\rho^{(0)}(s) \right] / m_\tau^2 \;, \qquad (2)$$

with $y_\tau = s/m_\tau^2$, V_{ij} the ij CKM matrix element, $w_\tau(y) = (1-y)^2(1+2y)$, $w_L(y) = y(1-y)^2$, and S_{EW} a short-distance electroweak correction factor. The $J = 0$ (longitudinal) contributions in (2) are well known phenomenologically and, due to problems with the corresponding $D = 2$ OPE series, usually subtracted from dR/ds [1,2]. The subtracted result, $dR^{(0+1)}_{V/A;ij}/ds$, allows the construction of $J = 0+1$ reweighted analogues, $R^w_{V+A;ij}(s_0) = \int_0^{s_0} ds\, [w(s)/w_\tau(y_\tau)]\, dR^{(0+1)}_{V+A;ij}(s)/ds$, for any $w(s)$ and $s_0 < m_\tau^2$. Defining $\delta R^w_{V+A}(s_0) = \left[R^w_{V+A;ud}(s_0)/|V_{ud}|^2 \right] - \left[R^w_{V+A;us}(s_0)/|V_{us}|^2 \right]$, one has, for s_0 large enough to allow use of the OPE on the RHS of (1), [1]

$$|V_{us}| \;=\; \sqrt{R^w_{V+A;us}(s_0) / \left[\frac{R^w_{V+A;ud}(s_0)}{|V_{ud}|^2} \;-\; \delta R^{w,OPE}_{V+A}(s_0) \right]} \;. \qquad (3)$$

This relation has usually been employed in un-reweighted form, with $w = w_\tau$, and the single value $s_0 = m_\tau^2$ [1]. This has the advantage that $R^{w_\tau}_{V+A;ud,us}(m_\tau^2)$ is determinable from branching fraction information alone, but the disadvantage of precluding tests of the s_0- and $w(s)$-independence of the analysis, which could otherwise be used to investigate potential systematic uncertainties (in particular, those associated with the treatment of OPE contributions). Such self-consistency tests were carried out in Refs. 2, 3, 4, and non-trivial $w(s)$- and s_0-dependences observed, suggesting shortcomings in the experimental data and/or OPE representation.

The most obvious potential OPE problem lies in the rather slow convergence of the $D = 2$ OPE series. In terms of the running $\overline{MS}$ quantities $m_s(Q^2)$ and $\bar{a} \equiv \alpha_s(Q^2)/\pi$, the $D = 2$ series, which is known to 4-loops, is given by

$$\left[\Delta\Pi_\tau(Q^2)\right]^{OPE}_{D=2} \;=\; \frac{3}{2\pi^2}\, \frac{m_s^2(Q^2)}{Q^2} \sum_{k=0} c^\tau_k \bar{a}^k \qquad (4)$$

with $c^\tau_k = 1,\ 7/3,\ 19.93,\ 208.75$ for $k = 0 \cdots 3$ [5]. Since $\bar{a}(m_\tau^2) \simeq 0.10$, $c^\tau_3 \bar{a}^3 > c^\tau_2 \bar{a}^2$ at the spacelike point on the contour for all $s_0 \le m_\tau^2$. The problematic convergence complicates the assessment of $D = 2$ truncation errors, and manifests itself, e.g., in the ~ 0.0020 difference in $|V_{us}|$ values obtained using two alternate (CIPT or FOPT) versions of the 4-loop-truncated, w_τ-weighted series.

An alternate determination employs the FB combination $\Delta\Pi_{\tau-EM} \equiv 9\Pi_{EM} - 5\Pi^{(0+1)}_{ud;V} + \Pi^{(0+1)}_{ud;A} - \Pi^{(0+1)}_{us;V+A}$ in place of $\Delta\Pi_\tau$ [6]. Inclusive electroproduction cross-sections fix the electromagnetic (EM) spectral function. By construction, the $\Delta\Pi_{\tau-EM}$ $D = 2$ series is strongly suppressed, having the form (4), with $c^\tau_k \to$

$c_k^{\tau-EM} = 0, -1/3, -4.384, -44.943$ for $k = 0\cdots 3$. The $D = 4$ series is also strongly suppressed. OPE contributions to $\Delta\Pi_{\tau-EM}$ FESRs, hence also estimated OPE errors, are thus very small [6], and the resulting $|V_{us}|$ errors essentially entirely experimental. A check of this predicted suppression is thus of interest.

We investigate the relative merits of the fixed-scale (FOPT-like) and local-scale ($\mu^2 = Q^2$, i.e., CIPT-like) treatments of the $\Delta\Pi_\tau$ $D = 2$ series, and the level of $\Delta\Pi_{\tau-EM}$ suppression, by comparing OPE expectations and lattice data for the two correlator combinations over a range of Euclidean Q^2. Five RBC/UKQCD domain wall fermion ensembles are employed, three, with $m_\pi = 293, 349, 399\ MeV$, having $1/a = 2.31\ GeV$ [8], and two, with $m_\pi = 171, 248\ MeV$, having $1/a = 1.37\ GeV$ [7]. For technical reasons, conserved-local versions of the flavor us 2-point functions are numerically challenging and hence, for $\Delta\Pi_\tau$, local-local versions are used. To check that this does not produce residual lattice artifacts which would impact our conclusions, we have also performed the OPE-lattice comparison, using conserved-local data, for the alternate flavor-diagonal FB combination $\Delta\Pi_{diag} \equiv \Pi_{V;\ell\ell} - \Pi_{V;ss}$, whose $D = 2$ series is very similar to that of $\Delta\Pi_\tau$ ($c_k^\tau \to c_k^{diag} = 1, 8/3, 24.32, 253.69$ for $k = 0\cdots 3$ in (4)). The results confirm those of the local-local study.

Representative OPE-lattice data comparisons for $\Delta\Pi_\tau$ are shown, for the $1/a =$ 2.13 GeV, $m_\pi = 293$ MeV ensemble, in Fig. 1. The left (right) panel comparison employs the fixed-scale (local-scale) prescription for the $D = 2$ OPE series. The fixed-scale versions match much better the Q^2 dependence of the lattice results, with the 3-loop-truncated version thereof best matching the overall normalization.

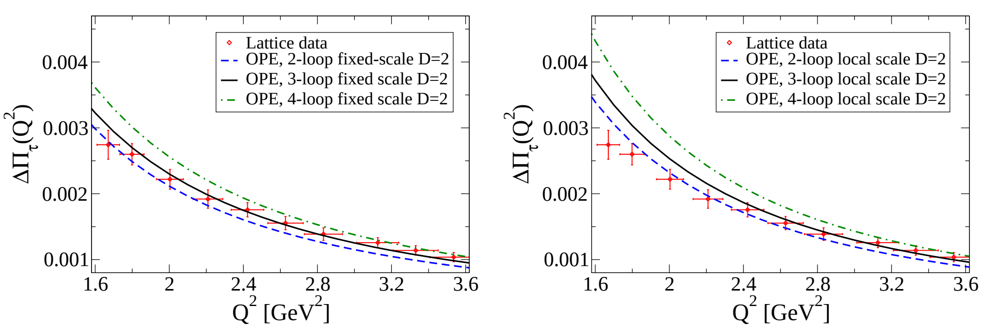

Fig. 1. OPE and lattice $\Delta\Pi_\tau$ data, $1/a = 2.31$ GeV, $m_\pi = 293$ MeV ensemble, $O(\bar{a}^{1,2,3})$ $D = 2$ OPE truncation, fixed-scale (left panel) or local-scale (right panel) $D = 2$ prescription

The comparison of lattice data for $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau-EM}$ confirms the very strong suppression of $\Delta\Pi_{\tau-EM}$ [4] (see Ref. 4 for the relevant figure).

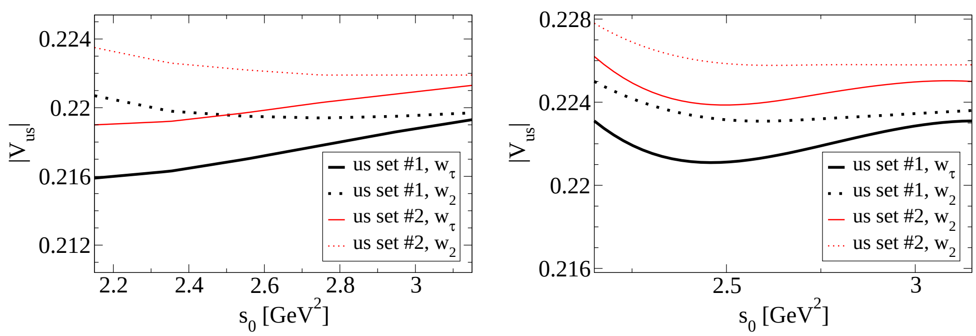

Fig. 2. $|V_{us}|$ from preliminary updates of the FB $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau-EM}$ FESRs

We turn to preliminary updates of the $|V_{us}|$ analyses. For the $D = 2$ OPE series, we employ the 3-loop-truncated FOPT prescription favored by lattice data, and for the ud spectral integrals, OPAL data [9], as updated in Ref. 10. For the us spectral integrals, recent B-factory results are used for the $K\pi$ [11], $K^-\pi^-\pi^+$ [12] and $K_s\pi^-\pi^0$ [13] exclusive mode distributions, and ALEPH results [14], updated for current branching fractions (BFs), for all other modes. Contributions from the latter lie higher in the spectrum, and have much larger errors. The B-factory distributions are unit normalized, and also require current BFs for their overall scales. We work with BFs obtained in a $\pi_{\mu 2}$, $K_{\mu 2}$-constrained HFAG fit, supplemented by the update to $B[\tau^- \to K_s^0\pi^-\pi^0\nu_\tau]$ produced by the recent Belle result [13]. Other non-trivial shifts in the us BFs also remain possible. To illustrate the changes to $|V_{us}|$ that could result, we consider also an alternate set of us BFs with the recent larger, but not yet finalized, BaBar results [15] for $B[\tau^- \to K^-\, n\,\pi^0\nu_\tau]$, $n \leq 3$, used in place of those of the HFAG fit. The first set of us BFs is labelled "us BF set #1" below, the second, alternate set "us BF set #2". Changes to the us BFs alter the inclusive us spectral distribution, and hence can affect both the magnitude of $|V_{us}|$ and the s_0-dependence of the results. The significantly larger preliminary BaBar $K^-\pi^0$ BF is particularly relevant for the FB FESRs considered here, which weight more strongly the low-s part of the spectrum. We consider FESRs employing the weights w_τ and $w_2(y) = (1-y)^2$. w_2 weights less strongly the higher-s, large-error region of the us spectral distribution. Differences between results obtained using the two different weights can thus point to issues with the us spectral distribution.

$|V_{us}|$ results obtained from the w_τ and w_2 versions of the $\Delta\Pi_\tau$ FESR are shown, as a function of s_0, and also the choice of the input us BF set, in the left panel of Fig. 2. Similar results for the $\Delta\Pi_{\tau-EM}$ FESR are shown in the right panel. w_2 results, which are less sensitive to the large-error high-s region, show better s_0-stability in both cases. For w_τ, s_0-stability is also better for the $\Delta\Pi_{\tau-EM}$ case, where OPE contributions are suppressed. The convergence of w_τ results to the more stable w_2 ones as $s_0 \to m_\tau^2$, seen for both the $\Delta\Pi_\tau$ and $\Delta\Pi_{\tau-EM}$ FESRs, suggests the possibility of residual OPE problems in the w_τ case, where cancellations on the contour play a larger role. Finally we note that results obtained using the FOPT prescription preferred by the lattice data agree better with 3-family unitarity expectations than do those (not shown here) obtained using CIPT, as do those obtained using us BF set #2 in place of us BF set #1. More details of these analyses will be presented elsewhere.

We close by stressing the preference for FOPT over CIPT for the $D = 2$ OPE series. The prescription which underlies CIPT (of summing logarithmic terms to all orders while truncating the series of non-logarithmic terms), though plausible, is motivated by heuristic arguments not generally valid for divergent series [16], and performs poorly when tested against lattice data for the FB correlators.

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