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GAVEL: Strategy-Proof Ascending Bid Auction for Dynamic Licensed Shared Access

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ABSTRACT
Licensed Shared Access (LSA) is a new shared spectrum access model that is gaining traction for unlocking incumbent spectrum to mobile network operators in a form similar to licensed spectrum, thus having the potential to alleviate the spectrum crunch below 6 GHz. Short-term spectrum auctions can pave the way for dynamic LSA in the future and to create incentives for incumbents to voluntarily participate in the LSA model, thereby increase spectrum availability. Different from existing auction schemes that are mostly based on the sealed-bid auction format, we consider an ascending bid auction format which is theoretically equivalent to a sealed bid format but comes with better behavioral properties. We develop a novel auction mechanism called GA VEL that follows the ascending bid auction format and is well-suited for the dynamic LSA context. GA VEL, besides being strategy-proof, satisfies the three additional desirable properties of supporting heterogeneous spectrum, fine-grained spectrum sharing and bidder privacy protection. Through simulation-based evaluations, GA VEL is shown to outperform two recently proposed schemes in terms of revenue, social welfare, number of winners and achieving high spectrum utilization while at the same time performing close to the LP based optimal solution.

CCS Concepts
- Networks → Cognitive radios; Network resources allocation;
  Network economics; Theory of computation → Computational pricing and auctions;

Keywords
Network Economics; Spectrum Auction; Ascending Bid Auction;
Licensed Shared Access; Privacy Preserving Auction

1. INTRODUCTION
Mobile data traffic has been experiencing dramatic growth in the past several years and this growth trend is expected to continue for the foreseeable future. Making more spectrum available is an obvious mechanism to cope with this growing demand. However opportunities for clearing spectrum below 6 GHz, where most mobile networks operate currently and will continue to do so in the future, to create new bands for licensed exclusive use by mobile network operators (MNOs) are dwindling. However MNOs prefer licensed spectrum as it offers interference protection and lets them develop services that provide guaranteed quality of service (QoS).

In this context, Licensed Shared Access (LSA) [21] has emerged as a new shared spectrum access model that can unlock substantial amount of licensed spectrum below 6 GHz held by incumbents not concerned with civilian wireless and mobile data communication and enable more efficient use of such spectrum bands. LSA framework allows incumbents to authorize other users (e.g., MNOs) to access all or part of the spectrum licensed to them at designated times and in designated geographical regions as per the sharing rules agreed between them and mediated by the national regulator. From a MNO perspective, LSA model opens up new spectrum bands for use that are qualitatively similar to licensed spectrum to offer guaranteed QoS. While the current LSA use cases reflect relatively longer term authorization of incumbent spectrum to LSA licensees in the order of a few years, it is believed that we will be heading to a future with a dynamic LSA model that features short-term and fine-grained spectrum sharing [24], and potentially involving new operators and business models [3]. Moreover, the current LSA frameworks lack incentives for incumbents to voluntarily open up the spectrum they hold for sharing.

Dynamic spectrum auctions [12] can help address the aforementioned issues with potential LSA spectrum. In general, auctions are an effective market mechanism due to their perceived fairness and efficiency in allocating resources. All the bidders have equal opportunity to win and the resources are sold to bidders who value them the most. In the LSA context, auctions have two main advantages: (i) they allow dynamic allocation of spectrum for short time periods and also even sharing at a fine-grained channel level as opposed to the static allocation of chunks of spectrum over long periods usually spanning years; (ii) they create incentives for the incumbents to participate in the auction, leading to more spectrum availability in the market. When used to coordinate spectrum sharing among LSA licensees, a suitable auction mechanism should be able to support heterogeneous spectrum bands and channels as the LSA spectrum as a whole is expected to be fragmented across different parts of the spectrum with widely different propagation characteristics.

The auction format used can have a significant impact on the effectiveness of the auction scheme [1, 7]. Auctions have been extensively used over the years for dynamic spectrum management
with several different types of strategy-proof auction mechanisms developed to suit different scenarios [32]. However, almost all are based on the sealed-bid auction format, where the auctioneer collects bids from the bidders for all the channels simultaneously to compute winners and market clearing prices. Our key insight is that limitations of existing auction schemes can be attributed to the underlying auction format. So we depart from the convention and consider for the first time the ascending bid auction format, which is theoretically equivalent to a sealed-bid auction, but offers better behavioral properties that help overcome the limitations of sealed-bid auctions (lack of support for heterogeneous channels, privacy protection and false-name bids). We elaborate on this point in Sections 2.2-4.

Our goal in this paper is to design a short-term spectrum auction mechanism based on the ascending bid auction format, that is well suited for the dynamic LSA context. Such a mechanism should handle heterogeneous spectrum and also support fine-grained spectrum sharing, as mentioned above. It should also be strategy-proof to be robust to threats from insincere bidders by guaranteeing that the best strategy for a bidder is to bid truthfully based on its private valuations. However, this does not protect the bidders from the threats of an insincere auctioneer [25]. Bids based on true valuations may reveal the bidders’s utility, which would act as a disincentive for bidders to bid truthfully and may impose the need for bidding strategies. This suggests privacy protection of the bidders as an additional requirement. Privacy protecting auction mechanisms [10,11,19,23,37] that protect the bid privacy mostly order preserving cryptographic tools (such as the Pallimer crypto system) while lacking in other desirable properties like heterogeneous spectrum support.

2. BACKGROUND AND MOTIVATION

2.1 Licensed Shared Access (LSA) and Desirable Auction Features

Licensed Shared Access (LSA) [21] is a new shared spectrum access framework that allows one or more LSA licensees to access the spectrum that has already been allocated to an incumbent. This framework has been designed to serve the short-term to mid-term needs through a quasi-static allocation of shared spectrum to LSA licensees. Each licensees conform to certain sharing rules included in their rights to use the spectrum. During the period when the license is active, the incumbent forfeits the spectrum access right to the LSA licensees who can guarantee a certain QoS since the resources are now assured.

The LSA framework consists of four main entities, the incumbents, LSA repository, LSA controller and the LSA licensees shown in Fig. 1. The incumbent is the spectrum owner with long term licenses for exclusive access to a spectrum band (e.g., 2.3GHz band in Europe). Incumbents propose sharing agreements that could define temporal, geographical, and power level constraints, so as to protect themselves from interference. The LSA repository is a database which receives from the incumbents the pieces of spectrum in terms of space, time and frequency that are available for sharing along with the conditions they are subject to. The LSA Controller is responsible for managing access to the shared spectrum that has been made available to the LSA licensees based on the sharing rules and incumbent usage provided by the LSA repository. While the LSA framework has a broader scope, where each LSA controller can interface with more than one LSA repository as well as with multiple LSA licensee networks that use different technologies, we focus on a simpler and concrete scenario where there exists one LSA controller per licensee (a MNO). LTE-Advanced (LTE-A) supports carrier aggregation and can aggregate spectrum across different bands. It is conceivable that LTE can use carrier aggregation to leverage the spectrum available under LSA.

In the following, we outline aspects important to consider when designing a short-term auction scheme for the dynamic LSA context.

Spectrum Heterogeneity: Regulatory authorities across different countries are moving towards releasing spectrum under the LSA model. In Europe, the 2.3-2.4 band is being considered for use under LSA [21]. This band is currently used by cordless cameras, telemetry, and defence systems across Europe. Apart from these two, 3.6-4.2 GHz [15] and 4.8-4.9 GHz, respectively used by satellite earth stations and radar systems, are other potential bands that could be made accessible on a shared basis via LSA. It is well known that transmissions across different frequency bands may have different propagation and penetration characteristics. These different characteristics are useful in serving different use cases (e.g., low frequency bands for coverage, high frequency bands for capacity and small cells). The auction mechanism should be able to support bidding for different types of spectrum.

Fine-grained Channels: Sharing spectrum at a fine-grained level in time and space promotes efficient spectrum use. An LSA licensee’s spectrum needs can be bursty. With LSA, the size of the spectrum band being sold in the market may still be too big for users who only need intermittent but guaranteed access to the spectrum. The use of fine-grained channels allow users the ability to acquire the resources it requires but at a lower cost. The availability of fine-grained channels significantly increases the accessibility of such spectrum to more users since the amount of spectrum available in the market is limited. This in turn increases the social welfare in the market as more number of bidders’ demand can now be satisfied. A side effect of this is the increase in competition, resulting in higher revenue for the incumbents, in turn incentivizing the incumbents to release more spectrum in the market.

Privacy Protection: The LSA repository, which may play the role of the auctioneer, could be from a third party vendor in which case the bid information should be protected from it. There are two main threats due to an insincere auctioneer in the LSA context: (i) overcharging winners; (ii) collusion with insincere bidders.

Overcharging: A common pricing scheme is the second pricing, where the auction winner with the highest bid pays the second highest bid (or the highest losing bid). If the bid information of the...
Auctioneer.
SMASHER-GR does not provide privacy protection from a rogue.


tive of the number/types of channels in each bundle. In addition, 
uation for all channel bundles (combinations) he desires irrespec-
uniform valuations where a bidder must have the same uniform val-
trum as well as fine-grained channels. However it is limited due to 
al [33] proposed SMASHER-GR that supports heterogeneous spec-
also do not support fine-grained channel sharing. Lastly, Zheng et 
mechanisms do not protect the bid information from the auctioneer and 
winner and price determination schemes. However these mecha-
nisms do not satisfy all the properties that are of in-

2.2 Limitations of Existing Auction Schemes

Existing schemes do not satisfy all the properties that are of in-
terest to enable spectrum sharing in a dynamic LSA context. Since 
the first truthful short-term spectrum auction [35] was proposed, is-
issues such as heterogeneous channels, privacy protection and false-
name bids have been uncovered. Some of these issues have been 
addressed in subsequent works [5,6,9,33,34,36] but with increas-
ingly complex adaptations to the auction scheme.

Most of the existing auctions for the short-term spectrum mar-
ket are based on the sealed bid auction format. We now discuss 
them with respect to support for the desirable properties outlined 
in the previous subsection: heterogeneous spectrum, fine-grained 
channels and privacy protection.

Heterogeneous Spectrum: When the market has heterogeneous 
spectrum, it leads to additional design challenges in the auction 
mechanism. Bidders would have different valuations for different 
types of spectrum and may need to submit a different bid for these 
different bands of spectrum, hence the need for bid diversity. Ex-
isting auction schemes [8,35,36] mostly treat spectrum as identical 
objects and apply the same conflict graph for different spectrum 
while considering spectrum reuse. Some recent works [5,9,33,34] 
have considered spectrum heterogeneity with increasingly complex 
winner and price determination schemes. However these mech-
nisms do not protect the bid information from the auctioneer and 
also do not support fine-grained channel sharing. Lastly, Zheng et 
al [33] proposed SMASHER-GR that supports heterogeneous spec-
trum as well as fine-grained channels. However it is limited due to 
uniform valuations where a bidder must have the same uniform val-
uation for all channel bundles (combinations) he desires irrespec-
tive of the number/types of channels in each bundle. In addition, 
SMASHER-GR does not provide privacy protection from a rogue 
auctioneer.

With sealed bid auction format, the market clearing price deter-
mination involves identifying the critical neighbor for each channel 
a bidder wins. The critical neighbor is the bidder with the highest 
losing bid. The difficulty in providing support for heterogeneous 
spectrum is primarily due to the additional complexity in identify-
ing the critical neighbor to determine the opportunity cost for each 
channel independently. There are multiple conflict graphs, one per 
channel, and hence each bidder may have a different set of interferr-
ing neighbors for each channel.

Fine-grained Channels: Most existing auction schemes pre-
clude shared use among interfering users and the allocations ensure 
no interference between winners. The auction scheme proposed by 
Kasbekar et al. [16] can be adapted to enable shared use among 
neighbors. However, they do not support heterogeneous channels 
and their approach becomes intractable due to the lack of a bid-
ding language. Kash et al. [17] propose SATYA, a truthful auction 
scheme for spectrum sharing that uses bucketing and ironing of bids 
to maintain monotonicity for truthfulness. While this is the first 
scheme to support channel sharing, it has a few drawbacks. Firstly, 
it does not support bid diversity and heterogeneous spectrum. Sec-
ondly, it has an exponential run time and is only polynomial under 
some restrictions.

As with the heterogeneous spectrum, supporting fine-grained chan-
nels using sealed-bid auctions requires complex price and winner 
determination strategy. This is again due to the additional com-
plexity in identifying a critical neighbor when multiple interfering 
bidders can win the same channel. Satya [17], for example, uses 
a complex scheme involving bucketing and ironing of bids to de-
termine winners which enables fine-grained channels, but lacks in 
support for other desirable properties such as heterogeneous chan-
nels.

Privacy Protection: An insincere auctioneer can leverage the 
bid information to its own advantage [23]. Recently, a few auction 
schemes [10,11,19,28,37] that provide bid privacy have been pro-
posed for dynamic spectrum allocation. Huang et al. [10] propose 
PPS, a strategy proof auction scheme that protects bid informa-
tion from the auctioneer using Paillier’s cryptosystem. However, 
it lacks in several necessary properties such as bid diversity and 
support for heterogeneous channels. Huang et al. [11] proposed 
SPRING, a strategy-proof auction scheme that uses asymmetric 
key encryption to protect the bid information from the auctioneer. 
However, a bidder can bid for only one channel, which severely
limits its use. Ming et al. [19] propose PPER, an auction scheme that guarantees bid privacy and economic-robustness using the reverse simplex method that enables the LP problem to be solved in a distributed fashion. However, the scheme assumes that all channels are homogeneous. Zhu et al. [37] proposed DEAR, which protects the bid privacy with the use of cryptography tools. It is a single price auction where all the winners are expected to pay the same price. In sealed bid auction schemes, the auctioneer receives the bids from all the bidders and is the only entity in the auction to have all the information. This information bias can be avoided by encrypting the bids from the auctioneers. However, encrypting the bid information from the auctioneer also leads to the limitations in these privacy protecting auction schemes as discussed above.

Table 1 qualitatively compares the proposed auction mechanism, GAVEL and its preliminary version [20], with existing schemes in terms of truthfulness and satisfying the above mentioned properties.

2.3 Related Literature

Market mechanisms [14, 31] that have been proposed for resource management in wireless networks can be adopted for winner and price determination of short term spectrum auctions. However, they encounter the same problems as that of a sealed bid auction when applied to short term spectrum access, as a critical neighbor must still be identified for price determination to guarantee truthfulness. As a side note, an auction mechanism can be modeled as a special case of matching with contracts [27] to compute winners and prices. However, existing matching algorithms cannot be applied to dynamic spectrum allocation due to the lack of support for characteristics such as spatial reuse and heterogeneous spectrum.

Several game theory based approaches [13, 29] have been proposed for wireless spectrum management. They can be adopted to determine winners and prices for short term spectrum access. However, they are still based on the sealed bid auction format and create an information bias between the auctioneer and the bidders. Hence these approaches share the same limitations as sealed bid auction schemes.

2.4 Other Key Considerations

**Rich Bidding Language**: The availability of heterogeneous channels in the market, provides the bidders access to a set of substitute and complementary channels with interdependent values. It is essential that the bidders are provided with a rich bidding language to bid for the right set of substitute and complementary channels. For example, in the simplest case, let us assume that a bidder has two substitute channels $C_1$ and $C_2$ available at its location. If the bidder wins $C_1$ channel then its valuation for $C_2$ would drop and it should be able to decrease the bid for $C_2$ reactively. Similarly, if the bidder has two complementary channels $C_3$ and $C_4$ available at its location, and wins channel $C_3$ then the valuation for channel $C_4$ would not change. The bidding language of the auction mechanism should therefore provide bidders the ability to express their preferences on the right combination of substitute and complementary channels.

The fundamental problem is that the existing auction mechanisms support either substitutes or complementary channels in the market but not both. Auction mechanisms for homogeneous spectrum [19, 26, 30, 35] assume all channels in the market are substitutes, while auctions for heterogeneous spectrum [4, 5, 9] assume all channels are complementary. However in a dynamic LSA context, the market for heterogeneous spectrum would consist of both substitutes and complementary channels. For example, channels in the 3.6–4.2 GHz band could be considered substitutes, where as channels in 3.6–4.2GHz are considered complementary to channels in the 2.3–2.4 GHz band.

This problem is common to sealed bid auctions where the bidders express their bids for all the channels simultaneously without being able to express bundle preferences. While this is not a problem for combinatorial auction schemes such as [33, 34], it would require a complex winner and price determination strategy, not to mention the overhead for bidders in computing bids for all such possible combination of channels.

**Protect from False-name bids**: False-name bids [26] is a cheating technique where a bidder submits multiple bids for the same item by creating multiple fictitious bidders in the market. This can increase the utility for the rogue bidder at the cost of the auctioneer and other bidders. As discussed in [26], this can lead to a severe loss in revenue for the auctioneer when a number of bidders cheat using false-name bids. In this cheating technique, a rogue bidder exploits the auctioneer’s need to identify the critical neighbor for winner and clearing price determination. By creating a fictitious bidder in the neighborhood the bidder can manipulate this critical neighbor determination process and hence this vulnerability is common to all the sealed-bid short-term auctions. To prevent the manipulation of the critical neighbor identification process, ALETHIA uses a modified winner and price determination strategy but that cannot be used for heterogeneous spectrum. Moreover, it does not support bid diversity, privacy protection and fine-grained channels.

2.5 Choice of the Auction Format

The foregoing discussion suggests that limitations of existing auction schemes from the requirements for dynamic LSA context can be linked to the sealed-bid auction format they use. Indeed the choice of the auction format is a key decision in auction design. The sealed bid and ascending bid [22] auctions are two different auction formats that can both identify the minimum Walrasian equilibrium prices and enable truthful bidding. In contrast to the sealed bid auction format (see Fig. 2), in the ascending bid auction framework, shown in Fig.4, the bidders (through a proxy agent) gradually submit their demand set to the auctioneer at increasing prices over multiple rounds until all the demands are met. Hence the outcome is incrementally computed in each round. While they have theoretically equivalent outcomes, their behavioral differences in terms of information bias and critical neighbor identification, as shown in Fig.3, influence the effectiveness of the auction schemes in practice [1, 7].

**Information Bias**: The information exchanged among the bidders and the auctioneer play a critical role in the effectiveness of the auction. In the sealed bid auction format, the bid information is not
3. SYSTEM MODEL

We model the LSA framework as a spectrum market with the aim of enabling dynamic fine-grained sharing of heterogeneous spectrum via short-term auctions. Auctioneers could be the same entity that manages the LSA repository. Each LSA licensee (MNO) interacts with the repository via its LSA controller as shown in Fig. 1. Each MNO could have multiple bidders participating in the auction at any given time with each such bidder representing demand for a certain type of spectrum at a particular location. We consider the spectrum to be heterogeneous and the bidders preference to be diverse. Let $\mathbb{N} = \{1, 2, ..., N\}$ be the set of bidders competing for heterogeneous channels denoted by $C = \{C_1, C_2, ..., C_m\}$ and $|C| = m$.

Conflict graphs are commonly used to represent reuse constraints in spectrum auctions. Conflict graph can be used to represent the constraints posed by the sharing agreement accepted by a bidder of the LSA licensee, including frequency, spatial and temporal conditions. More specifically, each bidder\(^\text{1}\) appears as a node in a conflict graph. Each channel has a conflict graph represented as $G_C = (V_C, E_C)$ where each user $i$ is associated with a vertex $i \in V_C$ and edge $e = (i, j) \in E_C$ exists if users $i$ and $j$ can conflict (interfere) with each other when they use channel $C_k$. Let $N_i \subseteq C$ represent all the neighbors of bidder $i$, i.e., all the vertices $V_C$ in conflict graph $G_C$ such that there exists an edge $E_C$ with bidder $i$.

Each bidder $i \in \mathbb{N}$ has valuations $V^C_i \to \mathbb{R}$ over channels $C_k \in C$, which are considered private values. The consumption set for bidder $i$ is denoted using $X_i$, where bidder $i$’s consumption bundle is given by $x_i = \{x_i^C_1, x_i^C_2, ..., x_i^{C_m}\} \in X_i$, with $0 \leq x_i^C_k \leq 1$ denoting the fraction of channel $C_k$ consumed by bidder $i$. The clearing price of bidder $i$ is denoted by $W_i = \{w_i^C_1, w_i^C_2, ..., w_i^{C_m}\} \in \mathbb{R}$. The utility for bidder $i$ with the consumption bundle $x_i$ can be computed as:

$$\mathcal{U}_i(x_i) = \sum_{C_k \in C} (V^C_i - W^C_i)x_i^C_k$$

The existence vector $d_i^C = \{d_i^C_1, d_i^C_2, ..., d_i^{C_m}\}$ is used to denote whether a channel is available for use for bidder $i$. The allocation vector $a_C = \{a_1^C, a_2^C, ..., a_m^C\}$ where $0 \leq a_i^C_k \leq 1$ is used to denote the proportion of channel $C_k$ that has been allocated to bidder $i$. When bidder $i$ is allocated exclusive use of channel $C_k$, the allocation vector $a_i^C_k = 1$ and when its not allocated $a_i^C_k = 0$.

A Proxy Agent is used to bid on behalf of the bidders in order to control the level of interaction the bidders have with the auctioneer. The bidders express their demand curves (demand as a function of round price) to a proxy bidding agent, who then responds the auctioneer with the appropriate demand values at the round price. The Proxy agent can perform this bidding with the demand curves received during the beginning of the auction and the bidder has the ability to adjust the demand curves in response to higher prices. The frequency of these adjustments determine the level of bidders’ interaction with the auctioneer during the auction process. The demand curves are generated by the bidders during the auction process, using functions that may depend on different parameters such as utility and budget. These functions are independent to the auction mechanism and can be bidder specific.

4. GAVEL

In this section, we develop GAVEL, a strategy-proof auction mechanism that is fundamentally different from the existing auctions schemes. Specifically, it is an ascending bid auction for short-term and fine-grained spectrum sharing, suitable for dynamic LSA context.

4.1 Overview

The basis for GAVEL is the ascending-bid auction mechanism proposed by Ausubel [2] that is shown to be efficient and also replicates the outcome of a Vickrey-Clarke-Groves (VCG) auction. But Ausubel’s mechanism is not intended for dynamic spectrum shar-
Auction Phase

Figure 4: Schematic of the system architecture with GAVEL.

Figure 5: Auction Example: Graph showing conflict between five bidders with the channels available for use (Ai) and private values of those channels (Vi).

Fig. 4 shows the overall architecture of the system based on GAVEL. The timeline for the system in operation is seen as a sequence of epochs, each consisting of a short Auction Phase followed by a much longer Spectrum Use Phase. The duration of an epoch could be a few minutes to several hours depending on the use case. Each Auction Phase consists of one or more rounds involving interaction between the auctioneer (LSA Repository) and bidders (could be multiple per LSA Licensee) as part of the auction to meet the spectrum demand of the bidders subject to their valuations, spectrum availability and mutual conflicts. The bidders interaction with the auctioneer can be managed using a proxy agent as described in Section 3. At the beginning of Auction Phase, the auctioneer announces the initial price, i.e., the reserve price. It then waits for a bidding period to receive demands from the bidders at that price. Depending on the demands, the auctioneer may allocate one or more channels to some of the bidders at the current round price. The auction then may proceed to another round by increasing the reserve price to bring down excess demand. This process may continue over several rounds until there is no more demand to be fulfilled. At the end of the auction phase, bidders whose bids are successful, proceed to use the spectrum they won in the following Spectrum Use Phase subject to the sharing conditions, until the end of that epoch. This process repeats in the next epoch and so on.

4.2 Detailed Description

We now describe GAVEL in more detail. Referring to Fig. 4, at the beginning of an epoch (round \(t = 1\)), the auctioneer announces a reserve price vector \(p_1 = (p_1^1, p_1^2, ..., p_1^n)\) for the channels, and the bidders respond with the demand vector \(D_t(1) = (d_{C_i}^1(1), d_{C_j}^2(1), ..., d_{C_k}^m(1))\) where \(0 < d_{C_i}^1(1) \leq c_i^1\) is the portion of channel \(C_k\) that bidder \(i\) desires at price \(p_1^1\) from what is available for use (\(c_i^1\)). If a bidder desires exclusive use of a channel then its demand would be 1. The round price controls the demand from each bidder in the sense that the decision to bid for a channel is determined by the number of channels within its private valuations. Only the channels that have higher valuations than the current round price would be in demand from the bidder. This channel has to be available for the bidder to use which is determined by the LSA Controller with information from the LSA Repository. The demands are assumed to be weakly decreasing with increasing price:

\[
\forall i \in \mathbb{N}, \forall C_k \in \mathbb{C}, \forall t \geq 1, d_{C_i}^1(t) \in \{0, d_{C_i}^1(1)\}
\]

At each round \(t\) with price vector \(p_t\), for channel \(C_k \in \mathbb{C}\), the auctioneer determines if for any bidder \(i\) the aggregate demand of \(i\) and its neighbors in the conflict graph \(G_{C_k}\) is low enough to satisfy \(i\)'s demand. If so, \(d_{C_i}^1(t)\), the portion of channel \(C_k\) demanded by bidder \(i\), is credited to the bidder.

\[
d_{C_i}^t = \begin{cases} 
\frac{d_{C_i}^1}{\sum_{j \in N_i \cap A_{C_k}} d_{C_j}^1(t)} & \text{if } \sum_{j \in N_i \cap A_{C_k}} d_{C_j}^1(t) \leq 1, \\
0 & \text{otherwise}.
\end{cases}
\]

 Appropriately, the current round price \(p_t^C\) for channel \(C_k\) is added to \(W_t^C\), the total price to be paid by the bidder at the end of the auction.

The above process repeats with increasing round prices until there is no demand from the bidders. The channels won by the bidders are now assigned to them and removed from their list of available spectrum. In order to prevent the neighbors from rebidding for the channels that they cannot use (because one of its neighbors already won the channel), we remove these channels from the neighbors availability list as well. All the bidders that won channel \(C_k\) are no longer treated as active players in the auction scheme for channel \(C_k\). The round price is now reset to the spectrum reserve price and the multi-round auction process is repeated until there is no demand for channels at the reserve price.

Note that to allow for spatial reuse, we view only the neighbors of a node in the conflict graph as its competing bidders. For example in the conflict graph shown in Fig. 5 A competes with B and C in the auction; C competes with A, B and D; and E competes only with D. This is unlike the classical VCG auction or Ausubel’s mechanism where all bidders compete with each other.

4.3 Example

We now illustrate the working of the GAVEL auction mechanism for the example in Fig. 5. For simplicity we assume the bidders demand for channels are independent, i.e. their demand for one channel does not influence their demand for another. To observe the progress of the auction, we set the proxy agent to interact with the bidder in each round to provide updated demand curves. Note that in Fig. 5(a), the set of channels available at each bidder with “A:”. As a specific example, bidder A has three channels available (1, 2 and 3). A fraction of the channel is assigned to a bidder only if the LSA controller identifies if its available for use to the bidder.
In this particular scenario, the demand is high with all the bidders desiring for (a fraction of) every channel available to them. The price vector announced by the auctioneer at different rounds and demand vector of bidders at different rounds in the auction scheme are shown in Table 2. The demand vector of bidders shown at round 1, are the fractions of channels required by the bidders.

In the first round of the auction with price \( p_1 = (1,1,1) \), the auctioneer sets a reserve price of 1 for channels 1, 2, and 3. The five bidders A, B, C, D, and E bid for fractions of channels based on their demand (as they have higher valuations than the current round price). For instance, bidder A has a demand of 60% of channel C1, exclusive use of channel C2 and 50% of channel C3. Note that at bidder A, \( \sum_{j \in N \cup \{A\}} d_j^C(p_1) = d_A^C + d_B^C + d_C^C = 0.7 < 1 \), so bidder A is credited with channel C3. Since there is excess demand, the auction proceeds to subsequent rounds with the price getting incremented at each round. At price vector \( p = (4,6,3) \), at bidder E, \( \sum_{j \in N \cup \{E\}} d_j^C(p_2) = d_E^C + d_D^C = 0.7 < 1 \).

Since the cumulative demand from E’s neighbors show no competition, i.e., at this point in the auction, E is guaranteed to win channel 3. Similarly, bidder C does not have demand for channel C1 anymore, so bidders A and B win channel 1 to be shared between them at 60% and 30% respectively. At price vector \( p = (4,8,8) \), bidder A loses interest in channel C2, and bidder C has no more demand for C3. So, B and C are credited channel 2 at price 8. Finally, at \( p = (4,8,10) \), the market clears with bidder A winning channels C1 and C3, bidder B winning C1 and C2, bidders C, D, and E winning channels C1, C2 and C3 respectively.

It can be clearly seen from the above example that the result of the auction is efficient: the auction has allocated the channels to the bidders who value them the most. The formal proof is provided in section 4.4. It can also be seen that the resultant pricing for channels won is equivalent to that of a VCG auction. For example, bidder A wins channel C1 at the reserve price and channel C1 at the opportunity cost (\( p^C = 4 \)) of bidder C which is the highest losing bid. Similarly, bidder B wins C2 channel at the highest losing bid (\( p = 8 \)) amongst its neighbors and so on.

**Complexity.** GAVEL runs in \( O(NM) \) where, N, M and R are the number of bidders, number of channels, and the number of rounds respectively. In one auction round, the neighborhood demand is checked for each bidder and for each channel to enable the channel credit process (NM). With R rounds, GAVEL has a computational complexity of \( O(NM) \) and hence runs in polynomial time. In practice, the values of both M and R are also quite small, further supporting the applicability of GAVEL in an online scenario.

### 4.4 GAVEL Properties

**Theorem 4.1.** GAVEL is individually rational.

**Proof.** If bidder i wins channel \( C_k \) at round \( t \) with price \( p_i^C \), then it means it has positive demand at round \( t \), i.e., \( d_i^C(t) = 0 \). Therefore, GAVEL is individually rational for all bidders.

**Theorem 4.2.** GAVEL is truthful. The dominant strategy for a bidder is to bid truthfully.

**Proof.** In order to prove that an auction mechanism is truthful, we need to show: (i) the pricing function does not depend on the bid of the winning bidder; and (ii) it is monotonic, i.e., if bidder i wins a channel at bid \( p \), then his winning bid at any bid \( p' > p \).

It is indeed the case that pricing function in GAVEL does not depend on the bid of the winning bidder. In any given round, the price that a bidder needs to pay for the channels it is credited in round \( t \) is the round price \( p_t \), which does not have any relation with the bidder’s bid.

Now to the monotonicity. Assume bidder i wins a channel at price \( p_t \), round \( t \) and at any of its subsequent rounds \( t^* > t \), the cu-

---

**Table 2: GAVEL Illustration: Price and Demand Vector over different rounds**

<table>
<thead>
<tr>
<th>Round</th>
<th>Price Vector</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(1,1,1)</td>
<td>(0.6,1,0.5)</td>
<td>(0.3,0.4,0)</td>
<td>(1,0.4,0,2)</td>
<td>(0,0.2,1)</td>
<td>(0,1,0.5)</td>
<td>Ch 3 credited to A</td>
</tr>
<tr>
<td>2</td>
<td>(4,6,3)</td>
<td>(0.6,1,0.5)</td>
<td>(0.3,0.4,0)</td>
<td>(0,0.4,0,2)</td>
<td>(0,0,1)</td>
<td>(0,1,0.5)</td>
<td>Ch 1 to A and B, Ch 2 to E</td>
</tr>
<tr>
<td>3</td>
<td>(4,8,8)</td>
<td>(0,0,0)</td>
<td>(0.3,0.4,0)</td>
<td>(0,0,4)</td>
<td>(0,1,0,1)</td>
<td>(0,1,0.5)</td>
<td>Ch 2 credited to bidders B and C</td>
</tr>
<tr>
<td>4</td>
<td>(4,8,10)</td>
<td>(0,0,0)</td>
<td>(0.3,0.4,0)</td>
<td>(0,0,4)</td>
<td>(0,1,0,1)</td>
<td>(0,1,0)</td>
<td>Ch 3 credited to bidder D</td>
</tr>
</tbody>
</table>
mulative demand of $i$’s neighbors $\sum_{j \in N_{i'}} d_i^j(t) \geq \sum_{j \in N_{i'}} d_i^j(t^*)$. The only way bidder $i$ cannot win the channel at higher price $p^*$ is if the aggregate demand of $i$’s neighbors increases with the bid $p^*$. This is not possible since we have assumed that the demand vectors are weakly decreasing with higher round prices, which results in a monotonically non-increasing demand for each channel with each new round. Thus $i$ will always win the channel at any bid $p^* > p$.

Therefore, GAVEL is a truthful mechanism.

**Remark.** With the properties of individual rationality and truthfulness, and given GAVEL is a multi-unit single auction scheme implies that it is also economically robust.

**Theorem 4.3.** GAVEL protects against the frauds of an insincere auctioneer.

**Proof.** The auction should be able to protect the bidder from the auctioneer overcharging winners and colluding with other greedy bidders.

**Overcharging Winners:** In order to overcharge the winners the auctioneer must have the knowledge of the winning bidder’s valuation. In GAVEL, bidders never have to reveal its demand curve beyond the winning price: A bidder $i$ who wins channel $C_k$ with valuation $V_i^{C_k}$ at price $w_i^{C_k} < V_i^{C_k}$ stops bidding at $w_i^{C_k}$ and hence the valuation $V_i^{C_k}$ is never revealed to the auctioneer.

**Collusion with greedy bidders:** For this trick to work, the greedy bidder must bid above its own valuation but less than the winning bidder’s valuation in order to make a profit. In GAVEL, it is impossible for the greedy bidder to do this without risking winning the channel at a price higher than its valuation, as the winning bidder’s valuation is never revealed. In which case the greedy bidder will suffer a negative utility and the auctioneer will lose revenue from the sale of that channel, as the bidder is now unable to pay. Therefore GAVEL protects against the frauds of an insincere auctioneer.

**Theorem 4.4.** GAVEL protects against false-name bids.

**Proof.** To show that GAVEL protects against false-name bids, we need to show that a bidder cannot increase his utility with false-name bids, given that the other bidders and their bids remain the same.

This is a proof by contradiction. Suppose bidder $i$ wins $x$ fraction of channel $C_k$ at price $p_i$ and with a fictitious bidder $i'$ it wins $x'$ fraction of channel $C_k$ where $x \neq x'$. In GAVEL, $x$ fraction of channel $C_k$ can be won by bidder $i$ at price $p_i$ if and only if at round $t$, $\sum_{j \in N_i} d_i^j(t) \leq 1 - x$. With a fictitious bidder, if it wins $x'$ fraction of channel $C_k$ then it means $\sum_{j \in N_i} d_i^j(t) \leq 1 - x'$. Since $i$ and $i'$ has the same set of neighbors ($N_i = N_i$), this can only happen if the demand from the neighbors change, which is a contradiction. Therefore GAVEL protects against false-name bids.

5. EVALUATION

For our evaluation, we follow the auctioning based LSA framework described in section 1 with the LSA controller as the auctioneer and the LSA licensees with non-zero demand as bidders. The bidders act as their own proxy agent with access to channel valuations. We compare the results obtained with different auction mechanisms. Specifically, we compare GAVEL against two recently proposed truthful combinatorial auction schemes, AEGIS-MP [34] and SMASHER-GR [33], which supports heterogeneous spectrum. AEGIS-MP is based on the English Clock auction format in which losing bidders are allowed to increase their bids or shrink their bundles until the auction ends. It uses a greedy mechanism for channel allocation and identifies critical neighbors to enable truthful bidding. However, it does not support fine-grained channels. SMASHER-GR is a sealed bid auction that uses the notion of virtual channels to supports fine-grained channels. The bidders are expected to have uniform valuations for any channel bundles they are interested in. Note that both these scheme do not provide bid privacy protection.

To model an urban environment, we consider an area with a realistic distribution of about 2000 houses per square kilometer. We set the number of channels in the market to 20 and to simulate heterogeneous spectrum in the market, we divide these channels into three bands, 700 MHz, 2.4 GHz and 3.5 GHz with 80m, 30m, and 10m interference range respectively. For each of these three bands, the average number of neighbors within the interference range is about 20, 12 and 6 respectively.

**Methodology:** The auction schemes take as input the (i) conflict graph for each channel in the market (ii) the valuation of channels for all bidders and (iii) the price vector. For AEGIS-MP the demand from bidders are only for exclusive channel use, since they do not support fine-grained channels. We adapt demand in the market by changing the bidders’ channel valuations. For SMASHER-GR we use uniform valuations for all channel bundles. We rerun these auction schemes 10 times for increasing demand in the market. The demand in the market signifies the fraction of channels the bidders have positive valuations for. For example, at 20% demand, on an average the bidders have positive valuations for about 20% of the channels available for use at their location. We evaluate the performance of GAVEL on the following metrics while varying demand in the market: (i) Revenue; (ii) Social Welfare; (iii) Spectrum Utilization and (iv) Percentage of Winners. To benchmark auction mechanisms with the optimum, we use the following LP formulations that maximize each of these four metrics individually.

![Figure 6: Performance of various metrics with increasing demand](image-url)
Social Welfare: \[
\max_a \sum_{i \in N} \sum_{C \in \mathbb{C}} \psi_i^C a_i^C \quad (3)
\]

Revenue: \[
\max_a \sum_{i \in N} \sum_{C \in \mathbb{C}} \psi_i^C a_i^C \quad (4)
\]

Number of Winners: \[
\max_a \sum_{i \in S} Y_i \quad (5)
\]

Spectrum Utilization: \[
\max_a \sum_{i \in N} \sum_{C \in \mathbb{C}} \delta_i^C a_i^C \quad (6)
\]

Subject to,
\[
\sum_{j \in N(i) \cup \{a\}} a_j^C \leq 1
\]

\[
\delta_i^C a_i^C = a_i^C, \quad \forall i \in \mathbb{N}, \forall k \in \mathbb{C}
\]

\[
0 \leq a_i^C \leq 1
\]

where,

\[
Y_i = 0 \text{ if } \sum_{C \in \mathbb{C}} a_i^C = 0
\]

\[
Y_i = 1 \text{ if } \sum_{C \in \mathbb{C}} a_i^C > 0
\]

Results shown for optimal solution are obtained by solving the LP using the GUROBI solver.

Social Welfare: The social welfare achieved in the market for different auction schemes are compared against the optimal solution in Fig 6(a). As the demand in the market increases, the social welfare in the market also increases, as more bidders’ demands get satisfied. The superior performance of GAVEL observed with respect to AEGIS-MP and SMASHER-GR can be attributed to two main factors. First, GAVEL does not limit any channel assignment opportunities in order to retain the strategy-proof nature of the scheme. Second, the use of fine-grained channels enables GAVEL to satisfy a significantly larger number of bidders’ demand when compared to AEGIS-MP. When compared to the optimal solution, the social welfare achieved in GAVEL is lower primarily due to the lack of complete information of channel demands in the market at each bidder.

It is worth noting that maximization of overall social welfare is not an objective in GAVEL. The consideration of a local neighborhood in the conflict graph for a channel is necessary to enable truthful bidding. However this does not always maximize social welfare. To illustrate this, consider the conflict graph shown in Fig. 5. A channel exclusively assigned to bidder C would be unavailable to bidders A, B, and D. In GAVEL, bidder C will be assigned the channel if it is the highest bidder, even though it may lower the overall social welfare. Despite this, GAVEL achieves social welfare closer to the optimal when compared to AEGIS-MP and SMASHER-GR for reasons discussed above.

Revenue: The revenue generated in the market with varying demand is shown in Fig 6(b). It can be seen GAVEL achieves higher revenue than the other two schemes because of two reasons. First, the ability to allocate fine-grained channels increases competition in the network, thereby increasing the total revenue generated. While SMASHER-GR also supports fine-grained channels it only allows uniform valuation for channel bundles which results in significant loss of revenue. Second, unlike the other auction schemes, GAVEL does not group bidders into groups to identify and charge a critical price from the winning bidders. Instead, each winning bidder pays the exact opportunity cost in the network for his access to the channel.

Spectrum Utilization: The percentage of spectrum utilized with varying demand in the market is shown in Fig. 6(c). The optimal solution shows the amount of possible spectrum utilization in the scenario. As demand in the market increases, spectrum utilization also increases. When there is about 60% demand in the network, the spectrum utilization achieved is more than 90% for all the auction mechanisms. This is due to the limited number of channels in the market. Also considering that it over allocates channels to bidders who may not end up using these channels, the effective spectrum utilization should be even lower.

Percentage of Winners: Fig. 6(d), shows the percentage of winners in the auction. The optimal scheme shows the maximum number of winners possible in this scenario. It can be seen that for the auction mechanisms as the demand in the market increases, there is a lower number of winners, due to the increase in competition. Unlike the auction schemes, the optimal solution is able to generate more winners with increasing demand as it increases the opportunity for a bidder to win. GAVEL has a significant increase in the percentage of winners (20% better) when compared to AEGIS-MP due to the use of fine-grained channels. Without the ability to allocate fine-grained channels, only one bidder can win the channel in a neighborhood. On the other hand, SMASHER-GR has a high percentage of winners since it uses a greedy scheme to allocate channels and support fine-grained channels. However even with a high number of winners SMASHER-GR achieves a lower social welfare and revenue due to the simplicity of the supported valuations, where all the channel bundles a bidder desires has the same valuation.

Figure 7: Revenue and Number of Rounds vs Price Increment Step Size

Round Price Increment: Unlike sealed bid auction schemes, GAVEL is an iterative mechanism that spans several rounds. The number of rounds is determined by the step size of the price increment in each round. To analyze the effect of this price increment, we fix the demand in the market to 60% to compute revenue and number of rounds for different price increment values shown in Fig. 7. The highest revenue is obtained with the smallest step size while the auction takes most number of rounds to complete. Increasing the step size decreases the revenue because a higher step size drives out demand quickly and gets the auction to a point when effectively all the bidders have zero demand even though there are still unallocated channels, explaining the drop in revenue. Clearly, a higher step size leads to auction completing in fewer rounds. Interesting point to note is about how reduction in revenue with increasing step size relates to the reduction in number of rounds. It can be seen from Fig. 7 that increasing the step size from 1 to 5 results in about 15% loss of revenue but reduces the number of rounds by about 70%, suggesting a value for step size that keeps the duration and overhead of auction minimal without hurting the revenue much.

To summarize, GAVEL generates higher revenues for the incum-
bents by enabling fine-grained channels while efficiently using opportunities for channel reuse. This is an incentive for the incumbents to share more spectrum in the market, which can in turn benefit bidders. GAVEL achieves higher social welfare which means a higher number of bidders are satisfied, while protecting the bid privacy from the auctioneer. This is an incentive for the bidders to participate in the auction leading to more competition and efficient use of the wireless spectrum.

6. CONCLUSION

In this paper, we have considered use of short-term spectrum auctions for the emerging dynamic LSA regulatory framework. From this perspective, we have examined the limitations of existing auction schemes and observe that they can be attributed to the behavior of the underlying auction format used (the sealed-bid auction) in almost all existing schemes. Our key insight is that the alternative ascending bid auction format, while theoretically equivalent to a sealed-bid auction, has different behavioral properties that naturally offer freedom from the limitations of existing sealed bid based auction schemes.

Leveraging the aforementioned insight, we have proposed GAVEL, a short-term spectrum auction scheme based on the ascending bid auction format. GAVEL is well suited for the dynamic LSA context in that it supports heterogeneous spectrum, fine-grained channels and has a rich bidding language that enables bidders to obtain the right combination of substitute and complementary channels. We prove that GAVEL is strategy-proof, economically robust, privacy preserving and protects against false-name bids. Via extensive simulations, GAVEL is shown to achieve higher social welfare and revenue than two recently proposed auction schemes while being close to the optimal solution (obtained from a LP formulation) in terms of social welfare, revenue, number of winning bidders and spectrum utilization.

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8. REFERENCES