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Incremental Graph Computations: Doable and Undoable

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ABSTRACT

The incremental problem for a class $\mathcal{Q}$ of graph queries aims to compute, given a query $Q \in \mathcal{Q}$, graph G, output $Q(G)$ and updates ΔG to G as input, changes ΔO to $Q(G)$ such that $Q(G \oplus \Delta G) = Q(G) \oplus \Delta O$. It is called *bounded* if its cost can be expressed as a polynomial function in the sizes of Q, ΔG and ΔO. It is to reduce computations on possibly big G to small ΔG and ΔO. No matter how desirable, however, our first results are negative: for common graph queries such as graph traversal, connectivity, keyword search and pattern matching, their incremental problems are unbounded.

In light of the negative results, we propose two characterizations for the effectiveness of incremental computation: (a) *localizable*, if its cost is decided by small neighbors of nodes in ΔG instead of the entire G; and (b) *bounded relative to* a batch algorithm $\mathcal{T}$, if the cost is determined by the sizes of ΔG and changes to the affected area that is necessarily checked by $\mathcal{T}$. We show that the incremental computations above are either localizable or relatively bounded, by providing corresponding incremental algorithms. That is, we can either reduce the incremental computations on big graphs to small data, or incrementalize batch algorithms by minimizing unnecessary recomputation. Using real-life graphs, we experimentally verify the effectiveness of our algorithms.

Keywords

incremental computation; graph data management; query optimization

1. INTRODUCTION

For a class $\mathcal{Q}$ of graph queries, *the incremental problem* aims to find an algorithm $\mathcal{T}_\Delta$ that, given a query $Q \in \mathcal{Q}$, a graph G, query answers $Q(G)$ and updates ΔG to G as input, computes changes ΔO to $Q(G)$ such that

$$Q(G \oplus \Delta G) = Q(G) \oplus \Delta O.$$

Here $S \oplus \Delta S$ denotes applying updates ΔS to S, when S is either graph G or query result $Q(G)$. That is, $\mathcal{T}_\Delta$ answers Q in response to ΔG by computing changes to the (old) output $Q(G)$. We refer to $\mathcal{T}_\Delta$ as an *incremental algorithm* for $\mathcal{Q}$, in contrast to *batch algorithms* $\mathcal{T}$ that given Q, G and ΔG, recompute $Q(G \oplus \Delta G)$ starting from scratch.

The need for incremental computations is evident. Real-life graphs G are often big, *e.g.,* the social graph of Facebook has billions of nodes and trillions of edges [23]. Graph queries are expensive, *e.g.,* subgraph isomorphism is NP-complete (cf. [35]). Moreover, real-life graphs are constantly changed. It is often too costly to recompute $Q(G \oplus \Delta G)$ starting from scratch in response to frequent ΔG. These highlight the need for incremental algorithms $\mathcal{T}_\Delta$: we use a batch algorithm $\mathcal{T}$ to compute $Q(G)$ once, and then employ incremental $\mathcal{T}_\Delta$ to compute changes ΔO to $Q(G)$ in response to ΔG. The rationale behind this is that in the real world, changes are typically small, *e.g.,* less than 5% on the entire Web in a week [34]. When ΔG is small, ΔO is often also small, and is much less costly to compute than $Q(G \oplus \Delta G)$, by making use of previous computation $Q(G)$. In addition, incremental computations are crucial to parallel query processing [18, 21] that partitions a big G, partially evaluates queries on the fragments at different processors, treats messages among the processors as *updates*, and conducts iterative computations incrementally to reduce the cost.

When ΔG is small and G is big, can we guarantee that it is more efficient to compute ΔO with $\mathcal{T}_\Delta$ than to recompute $Q(G \oplus \Delta G)$ with $\mathcal{T}$? A traditional characterization is by means of a notion of *boundedness* proposed in [44] and extended to graphs in [17, 38]. It measures the cost of $\mathcal{T}_\Delta$ in $|\mathsf{CHANGED}| = |\Delta G| + |\Delta O|$, the size of the changes in the input and output. We say that $\mathcal{T}_\Delta$ is *bounded* if its cost can be expressed as a polynomial function of $|\mathsf{CHANGED}|$ and $|Q|$. The incremental problem for $\mathcal{Q}$ is *bounded* if there exists a bounded $\mathcal{T}_\Delta$ for $\mathcal{Q}$, and is *unbounded* otherwise.

Bounded $\mathcal{T}_\Delta$ allows us to reduce the incremental computations on big graphs to small graphs. Its cost is determined by $|\mathsf{CHANGED}|$ and query size $|Q|$, rather than by the size $|G|$ of the entire G. In the real world, $|Q|$ is typically small; moreover, $|\mathsf{CHANGED}|$ represents the updating cost that is *inherent to* the incremental problem itself, and is often much smaller than $|G|$. Hence bounded $\mathcal{T}_\Delta$ warrants efficient incremental computation no matter how big G is.

Undoable. No matter how desirable, we show that the incremental problem for $\mathcal{Q}$ is *unbounded* when $\mathcal{Q}$ ranges over graph traversal (RPQ, regular path queries), strongly connected components (SCC) and keyword search (KWS). The negative results hold when ΔG consists of a single edge dele-

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tion or insertion. Add to it the unboundedness of graph pattern matching via subgraph isomorphism (ISO) [17]. For these common queries, a bounded incremental algorithm is beyond reach. That is, by the standard of boundedness, incremental graph algorithms seem not very helpful.

Doable. The situation is not so hopeless. The boundedness of [17,38,44] is often too strong to evaluate incremental algorithms. To characterize the effectiveness of real-life incremental algorithms, we propose two alternative measures.

(1) Localizable computations. We say that the incremental problem for $\mathcal{Q}$ is *localizable* if there exists an incremental algorithm $\mathcal{T}_\Delta$ such that for $Q \in \mathcal{Q}$, G and ΔG, its cost is determined by $|Q|$ and the d_Q-neighbors of nodes in ΔG, where d_Q is decided by $|Q|$ only. In practice, Q is typically small, and so is d_Q. Hence it allows us to reduce the computations on (big) G to small d_Q-neighbors of ΔG.

We show that the incremental problems for KWS and ISO are localizable, although they are unbounded.

(2) Relative boundedness. We often want to incrementalize a batch algorithm $\mathcal{T}$ for $\mathcal{Q}$. For a query $Q \in \mathcal{Q}$ and a graph G, we denote by $G_{(\mathcal{T},Q)}$ the part of data in G inspected by $\mathcal{T}$ when computing $Q(G)$. Given updates ΔG to G, denote by AFF the difference between $(G \oplus \Delta G)_{(\mathcal{T},Q)}$ and $G_{(\mathcal{T},Q)}$.

An incremental algorithm $\mathcal{T}_\Delta$ for $\mathcal{Q}$ is *bounded relative to* $\mathcal{T}$ if its cost is a polynomial in $|\Delta G|$, $|Q|$ and $|\mathsf{AFF}|$. Intuitively, AFF indicates the necessary cost for incrementalizing $\mathcal{T}$, and $\mathcal{T}_\Delta$ incurs this minimum cost, not measured in $|G|$.

We show that RPQ and SCC are relatively bounded, *i.e.,* it is possible to incrementalize their popular batch algorithms $\mathcal{T}$ and minimize unnecessary recomputation of $\mathcal{T}$.

Contributions. The paper studies the effectiveness of incremental graph computations, and provides the following.

(1) Impossibility results. We show that no bounded incremental algorithms exist for RPQ, SCC, and KWS (Section 3). We establish these impossibility results either by elementary proofs or by reductions from incremental graph problems that are already known unbounded. To the best of our knowledge, this work gives the first proofs by reductions for unbounded graph incremental computations.

(2) New characterizations. We characterize localizable incremental computations and relative boundedness in Sections 4 and 5, respectively. We show that the incremental computations above are either localizable (KWS and ISO) or relatively bounded (RPQ and SCC). That is, while these incremental computations are unbounded, they can still be effectively conducted with performance guarantees.

(3) Incremental algorithms. As a proof of concept, we develop localized incremental algorithms for KWS and ISO (Section 4), and bounded incremental algorithms for RPQ and SCC relative to their batch algorithms (Section 5). We also develop optimization techniques for processing batch updates. These extend the small library of existing incremental graph algorithms that have performance guarantees.

(4) Experimental study. We evaluate the algorithms using real-life and synthetic graphs (Section 6). We find that (a) our localizable and relatively bounded incremental algorithms for KWS, RPQ, SCC and ISO are effective. They outperform their batch counterparts even when $|\Delta G|$ is up to 30%, 35%, 25% and 25% of $|G|$, respectively, and are on average 4.9, 6.2, 2.9 and 3.7 times faster when $|\Delta G|$ accounts for 10% of $|G|$. (b) They scale well with $|G|$. For instance, they take 28, 100, 19 and 225 seconds, respectively, on G with 50 million nodes and 100 million edges, under 5% updates, as opposed to 197, 1172, 144 and 2386 seconds by batch algorithms. (c) Our optimization strategies are effective: they improve the performance by 1.6 times on average.

Related work. We categorize the related work as follows.

Bounded incremental algorithms. Proposed in [44], the notion was studied for graph algorithms in [17, 38, 39]. A number of incremental algorithms have been developed for graphs [12, 17, 26, 28, 32, 38–42, 46] (see [16] for a survey). However, their costs are typically studied in terms of *amortized* analysis for averaged operation time of a sequence of unit updates to G, not in the size of changes that is inherent to the incremental problem itself. To the best of our knowledge, bounded algorithms are only in place for the shortest path problems, single-source or all pairs, with positive lengths [38, 39]. It is known that the incremental problem is unbounded for subgraph isomorphism ISO [17], and for single-source reachability to all vertices [38].

As the notion of boundedness is often too strong, a weaker standard was introduced in [17], based on a notion of affected area $\mathsf{AFF}^\forall$. Intuitively, $\mathsf{AFF}^\forall$ covers not only changes ΔO, but also data that is necessarily checked to detect ΔO by *all* incremental algorithms for $\mathcal{Q}$, encoded in auxiliary structures. An incremental algorithm is *semi-bounded* [17] if (a) its cost can be expressed as a polynomial in $|\mathsf{AFF}^\forall|$, $|Q|$ and $|\Delta G|$, and (b) the size of the auxiliary structure is bounded by a polynomial in $|G|$. The incremental problem for graph simulation is shown semi-bounded [17].

This work differs from the prior work in the following. (a) We establish new unboundedness results for RPQ, SCC and KWS, and a new form of reductions as proof techniques. (b) We propose measures for the effectiveness of incremental graph algorithms. In contrast to [17,38,39], localizable algorithms are characterized by d_Q-neighbors of ΔG instead of ΔO or $\mathsf{AFF}^\forall$. Relative boundedness is defined in terms of the affected area AFF relative to a specific algorithm $\mathcal{T}$, as opposed to $\mathsf{AFF}^\forall$ for *all* incremental algorithms for $\mathcal{Q}$ (semi-boundedness). (c) We develop incremental algorithms for RPQ, SCC, KWS and ISO with performance guarantees under the new measures, although they are unbounded.

Locality of graph computations. There have been batch algorithms that capitalize on the data locality of queries, for (parallel) subgraph isomorphism (*e.g.,* [19, 20]). Incoop [9], a generic MapReduce framework for incremental computations, also makes use of the locality of previously computed results in its scheduling algorithm to prevent straggling. To the best of our knowledge, the study of localizable incremental algorithms is the first effort to characterize the effectiveness of incremental algorithms in terms of locality.

Relative boundedness. There has also been work on incrementalizing batch algorithms, notably self-adjusting computations [5,10]. The idea is to track the dependencies between data and function calls as a dynamic dependency graph [6], upon which functions that are affected by the changes in the input can be identified and recomputed. Memorization [36] is used to record and reuse the results of function calls when possible. It is a general-purpose, language-centric technique for programs to automatically respond to modifications to their data. In contrast, relative boundedness is to characterize whether it is *feasible* to incrementalize a given batch algorithm $\mathcal{T}$ with cost measured in the size of affected area AFF inspected by $\mathcal{T}$, not in terms of function calls.

View maintenance. Related is also view maintenance for updating materialized views, which has been studied for relational data [14, 24, 25], object-oriented databases [31], and semi-structured data modeled as graphs [4,46]. Various methods have been proposed, *e.g.,* an algebraic approach of [11] for XML views and the use of key constraints [24] for relations. However, few of them have provable performance guarantees, and fewer can be applied to graph queries. In particular, the techniques of [4, 46] are developed for views specified as selection paths, and do not apply to graph queries studied in this paper. In contrast, we study the boundedness of incremental graph problems and provide algorithms that are localizable or relatively bounded.

2. INCREMENTAL COMPUTATIONS

We first present graph queries studied in this paper, and then formulate their incremental problems.

We start with basic notations.

We consider directed *graphs* G represented as (V, E, l), where (1) V is a finite set of nodes; (2) $E \subseteq V \times V$ is a set of edges in which (v, v') denotes an edge from v to v', and (3) each node v in V carries $l(v)$, indicating its label and content, as found in social networks and property graphs.

If (v, w) is an edge in E, we refer to node w as a *successor* of v, and to node v as a *predecessor* of w.

Graph $G_s = (V_s, E_s, l_s)$ is a *subgraph of* G if $V_s \subseteq V$, $E_s \subseteq E$, and for each node $v \in V_s$, $l_s(v) = l(v)$.

Subgraph G_s is *induced by* V_s if E_s consists of all the edges in G such that their endpoints are both in V_s.

2.1 Graph Queries

We study the following four classes of graph queries.

RPQ. Consider directed graphs $G = (V, E, l)$ over a finite alphabet Σ of labels defined on the nodes in V. A *path* ρ from v_0 to v_n in G is a list $(v_0, \ldots, v_n)$, where for $i \in [0, n-1]$, (v_i, v_{i+1}) is an edge in G. The *length* of path ρ is n.

A *regular path query* Q is a regular expression as follows:

$$Q ::= \epsilon \mid \alpha \mid Q \cdot Q \mid Q + Q \mid Q^*.$$

Here (a) ϵ denotes an empty path; (b) α is a label from Σ; (c) $\cdot$ and $+$ are concatenation and union operators, respectively; and (d) Q^* indicates zero or more occurrences of Q.

We use $L(Q)$ to denote the regular language defined by Q, *i.e.,* the set of all strings that can be parsed by Q. For a path $\rho = (v_0, \ldots, v_n)$ in G, we use $l(\rho)$ to denote the labels $l(v_0) \cdots l(v_n)$ of the nodes on ρ. A *match* of Q in G is a pair (v, w) of nodes such that there exists a path ρ from v to w having $l(\rho) \in L(Q)$. RPQ is stated as follows.

- Input: A directed graph G and a regular path query Q.
- Output: The set $Q(G)$ of all matches of Q in G.

It takes $O(|V||E||Q|^2 \log^2 |Q|)$ time to compute $Q(G)$ by using NFA (nondeterministic finite automaton) [29,33], where $|Q|$ is the number of occurrences of labels from Σ in Q [29].

SCC. A subgraph G_s of a directed graph G is a *strongly connected component* of G if it is (a) strongly connected, *i.e.,* for any pair (v, v') of nodes in G_s, there is a path from v to v' and vice versa, and (b) maximum, *i.e.,* adding any node or edge to G_s makes it no longer strongly connected.

symbols	notations
$Q(G)$	the answers to query Q in graph G
ΔG	updates to graph G (edge insertions, deletions)
$G \oplus \Delta G$	the graph obtained by updating G with ΔG
ΔO	updates to old output $Q(G)$ in response to ΔG
$\mathcal{T}$	a batch algorithm for a query class $\mathcal{Q}$
$\mathcal{T}_\Delta$	an incremental algorithm for $\mathcal{Q}$
AFF	changes to the area inspected by a batch algorithm $\mathcal{T}$
$\mathsf{dist}(s, t)$	the shortest distance from node s to t
$G_d(v)$	the d-neighbor of node v in G

Table 1: Notations

We use $\mathsf{SCC}(G)$ to denote the set of all strongly connected components of G. The SCC problem is stated as follows.

- Input: A directed graph G.
- Output: $\mathsf{SCC}(G)$.

It is known that SCC is in $O(|V| + |E|)$ time [43].

KWS. We consider keyword search with distinct roots in the same setting of [37]. A keyword query Q is of the form $(k_1, \ldots, k_m)$, where each k_i is a keyword. Given a directed graph G and a bound b, *a match to Q in G at node r* is a tree $T(r, p_1, \ldots, p_m)$ such that (a) T is a subgraph of G, and r is the root of T, (b) for each $i \in [1, m]$, p_i is a node in T such that $l(p_i) = k_i$, *i.e.,* it matches keyword k_i, (c) $\mathsf{dist}(r, p_i) \leq b$, and (d) the sum $\Sigma_{i \in [1,m]} \mathsf{dist}(r, p_i)$ is the smallest among all such trees. Here for a pair (r, s) of nodes, $\mathsf{dist}(r, s)$ denotes the *shortest distance* from r to s, *i.e.,* the length of a shortest path from r to s. KWS is as follows.

- Input: A directed graph G, a keyword query $Q = (k_1, \ldots, k_m)$, and a positive integer b.
- Output: The set $Q(G)$ of all matches to Q at node r in G within b hops, for r ranging over all nodes in G.

It can be computed in $O(m(|V|\mathsf{log}|V| + |E|))$ time (cf. [45]).

ISO. A *pattern query* Q is a graph (V_Q, E_Q, l_Q), in which V_Q and E_Q are the set of pattern nodes and directed edges, respectively, and each node u in V_Q has a label $l_Q(u)$.

A *match* of Q in G is a subgraph G_s of G that is isomorphic to Q, *i.e.,* there exists a *bijective function* h from V_Q to the set of nodes of G_s such that (a) for each node $u \in V_Q$, $l_Q(u) = l(h(u))$, and (b) (u, u') is an edge in Q iff $(h(u), h(u'))$ is an edge in G_s. The answer $Q(G)$ to Q in G is the set of all matches of Q in G. ISO is stated as follows.

- Input: A directed graph G and a pattern Q.
- Output: The set $Q(G)$ of all matches of Q in G.

It is NP-complete to decide whether $Q(G)$ is empty (cf. [35]).

2.2 Incremental Query Answering

We next formalize incremental computation problems.

Updates. We consider *w.l.o.g.* the following *unit updates*:

- edge insertion: ($\mathsf{insert}\ e$), possibly with new nodes, and
- edge deletion: ($\mathsf{delete}\ e$).

A *batch update* ΔG to graph G is a sequence of unit updates.

Incremental problem. For a class $\mathcal{Q}$ of graph queries, the incremental problem is stated as follows.

- Input: Graph G, query $Q \in \mathcal{Q}$, old output $Q(G)$, and updates ΔG to the input graph G.
- Output: Updates ΔO to the output such that

$$Q(G \oplus \Delta G) = Q(G) \oplus \Delta O.$$

We study the problem for RPQ, SCC, KWS and ISO.

The notations of this paper are summarized in Table 1.

3. BOUNDED PROBLEMS: UNDOABLE

This section shows the following impossibility results.

Theorem 1: *The incremental problem is unbounded for*
- ◦ *regular path queries (*RPQ*),*
- ◦ *strongly connected components (*SCC*), and*
- ◦ *keyword search (*KWS*),*

even under a unit edge deletion and a unit edge deletion. □

Together with the unboundedness of ISO [17], Theorem 1 tells us that it is impossible to find bounded incremental algorithms for all the graph query classes presented in Section 2. The negative results are rather robust: the incremental problems are already unbounded under unit updates.

Before we give a proof, we first review the notion of boundedness of [17,38], and introduce a form of Δ-reductions.

Boundedness. An incremental algorithm $\mathcal{T}_\Delta$ for a graph query class $\mathcal{Q}$ is *bounded* if its cost can be expressed as a polynomial of $|\mathsf{CHANGED}|$ and $|Q|$, where $|\mathsf{CHANGED}| = |\Delta G| + |\Delta O|$. Following [17,38], we require $\mathcal{T}_\Delta$ to be *locally persistent*. Such $\mathcal{T}_\Delta$ may use (a) auxiliary structures associated with each node v of G, to keep track of intermediate results at v, and (b) pointers to its successors and predecessors. However, no global auxiliary information is allowed, such as pointers to nodes other than its neighbors; similarly for edges. The algorithm starts an update from the nodes or edges involved in ΔG, and traverses G following the edges of G. The choice of which edge to follow depends only on the information accumulated in the current processing of G since global information from prior passes is not maintained.

Reductions. We now introduce Δ-*reduction*. Consider two classes of graph queries $\mathcal{Q}_1$ and $\mathcal{Q}_2$. For $i \in [1, 2]$, we represent an instance of (the computational problem for) $\mathcal{Q}_i$ as $I_i = (Q_i, G_i)$, where $Q_i \in \mathcal{Q}_i$ and G_i is a graph.

A Δ-*reduction* from $\mathcal{Q}_1$ to $\mathcal{Q}_2$ is a triple (f, f_i, f_o) of functions such that for each instance $I_1 = (Q_1, G_1)$ of $\mathcal{Q}_1$,

(1) $f(I_1)$ is an instance $I_2 = (Q_2, G_2)$ of $\mathcal{Q}_2$; and

(2) for all updates ΔG_1 to G_1,

  (a) $f_i(\Delta G_1)$ computes updates ΔG_2 to G_2; and

  (b) $f_o(\Delta O_2)$ computes ΔO_1, where ΔO_i denotes updates to $Q_i(G_i)$ in response to ΔG_i for $i \in [1, 2]$,

in polynomial-time (PTIME) in $|\Delta G_1| + |\Delta O_1|$ and $|Q_1|$.

Intuitively, f maps the instances of $\mathcal{Q}_1$ to $\mathcal{Q}_2$; f_i maps input updates ΔG_1 to ΔG_2, and f_o maps output updates O_2 back to O_1, both in PTIME in the size of Q_1 and changes in the input and output of instance (Q_1, G_1), where (Q_2, G_2) corresponds to (Q_1, G_1) via function f. Hence if $\mathcal{Q}_2$ has a bounded incremental algorithm, then so does $\mathcal{Q}_1$. Equivalently, if $\mathcal{Q}_1$ is unbounded, neither is $\mathcal{Q}_2$. That is, Δ-reduction preserves boundedness (see Appendix for a proof).

Lemma 2: *If there exists a Δ-reduction from $\mathcal{Q}_1$ to $\mathcal{Q}_2$ and if the incremental problem for $\mathcal{Q}_2$ is bounded, then the incremental problem for $\mathcal{Q}_1$ is also bounded.* □

Proof of Theorem 1. Based on Δ-reduction, we outline a proof, which reveals the challenges to the development of incremental algorithms. The proofs for RPQ, SCC and KWS are nontrivial (see Appendix and [2]). For each query class, we need to give two proofs: one under a unit edge deletion, and the other under a unit insertion. Indeed, a problem may be unbounded under deletions (resp. insertions) but be bounded under insertions (resp. deletions). An example is SSRP, the single-source reachability problem to all vertices. It is to decide, given a graph G and a node v_s in G, whether there exists a path from v_s to v_t for all nodes v_t in G. It is known that SSRP is unbounded under unit edge deletions but bounded under unit edge insertions [38].

RPQ. We show that the incremental problem for RPQ is unbounded under a unit edge deletion by Δ-reduction from SSRP, whose incremental problem is unbounded under unit deletions. We show the unboundedness under unit edge insertions by giving an elementary proof. We construct an instance (Q, G) of RPQ, and show by contradiction that there exists no bounded incremental algorithm that can correctly compute $Q(G \oplus \Delta G)$ in response to updates ΔG to G.

SCC. We prove the unboundedness of the case under a unit edge deletion also by Δ-reduction from SSRP. The case under unit edge insertions is verified by contradiction.

KWS. We show that the incremental problem for KWS is unbounded under unit edge insertions by Δ-reduction from subgraph isomorphism ISO, whose incremental problem is unbounded under edge insertions when Q is a tree [17]. The case under unit edge deletions is proved by contradiction. □

4. LOCALIZABLE COMPUTATIONS

Not all is lost. Despite Theorem 1, there exist efficient incremental algorithms for RPQ, SCC, KWS and ISO with performance guarantees under new characterizations for the effectiveness of incremental algorithms. In this section we introduce one of the standards, namely, localizable incremental computations. We first present the notion (Section 4.1). We then show that the incremental problems for KWS and ISO are localizable (Section 4.2 and Appendix, respectively).

4.1 Locality of Incremental Computations

We start with a few notations. (a) In a graph G, we say that a node v' is *within d hops* of v if $\mathsf{dist}(v, v') \leq d$ by taking G as an undirected graph. (b) We denote by $V_d(v)$ the set of all nodes in G that are within d hops of v. (c) The *d-neighbor* $G_d(v)$ of v is the subgraph of G induced by $V_d(v)$, in which the set of edges is denoted by $E_d(v)$.

Consider a graph query class $\mathcal{Q}$. An incremental algorithm $\mathcal{T}_\Delta$ for $\mathcal{Q}$ is *localizable* if its cost is determined only by $|Q|$ and the sizes of the d_Q-neighbors of those nodes on the edges of ΔG, where d_Q is determined by the query size $|Q|$.

The incremental problem for $\mathcal{Q}$ is called *localizable* if there exists a localizable incremental algorithm for $\mathcal{Q}$.

Intuitively, if $\mathcal{T}_\Delta$ is localizable, it can compute ΔO by inspecting only $G_{d_Q}(v)$, *i.e.,* nodes within d_Q hops of nodes v in ΔG. In practice, $G_{d_Q}(v)$ is often small. Indeed, (a) Q is typically small; *e.g.,* 98% of real-life pattern queries have radius 1, and 1.8% have radius 2 [22]; hence so is d_Q; and (b) real-life graphs are often sparse; for instance, the average node degree is 14.3 in social graphs [13]. Hence, $\mathcal{T}_\Delta$ can reduce the computations on possibly big G to small $G_{d_Q}(v)$.

The main results of this section are as follows.

Theorem 3: *The incremental problem is localizable for* KWS *and* ISO *under batch updates.* □

That is, while the incremental problems for KWS and ISO are unbounded, we can still effectively conduct their incremental computations by making big graphs "small".

As a constructive proof of Theorem 3, we next develop localizable incremental algorithms for KWS. The incremental algorithms for ISO are similar and are outlined in Appendix.

Algorithm: IncKWS^+

Input: A graph G with $\mathsf{kdist}(\cdot)$, keyword query Q and bound b, matches $Q(G)$, and an edge (v, w) to be inserted.
Output: The updated matches $Q(G \oplus \Delta G)$ and kdist lists.

1. **for each** k_i in Q with $\mathsf{kdist}(w)[k_i].\mathsf{dist} < \min(\mathsf{kdist}(v)[k_i].\mathsf{dist} - 1, b)$ **do**
2. $\quad$ $\mathsf{kdist}(v)[k_i].\mathsf{dist} := \mathsf{kdist}(w)[k_i].\mathsf{dist} + 1$;
3. $\quad$ $\mathsf{kdist}(v)[k_i].\mathsf{next} := w$; queue $q_i := nil$; $q_i.\mathsf{enqueue}(v)$;
4. $\quad$ **while** q_i is not empty **do**
5. $\quad\quad$ node $u := q_i.\mathsf{dequeue}()$;
6. $\quad\quad$ **for each** predecessor u' of u such that $\mathsf{kdist}(u)[k_i].\mathsf{dist} < \min(\mathsf{kdist}(u')[k_i].\mathsf{dist} - 1, b)$ **do**
7. $\quad\quad\quad$ $\mathsf{kdist}(u')[k_i].\mathsf{dist} := \mathsf{kdist}(u)[k_i].\mathsf{dist} + 1$;
8. $\quad\quad\quad$ $\mathsf{kdist}(u')[k_i].\mathsf{next} := u$; $q_i.\mathsf{enqueue}(u')$;
9. **for each** u_1'' and u_2'' involved in a changed $\mathsf{kdist}(u)[k_i].\mathsf{next}$ **do**
10. $\quad$ replace (u, u_1'') with (u, u_2'') in all the matches of $Q(G)$ or add matches to $Q(G \oplus \Delta G)$ by including (u, u_2'');
11. **return** $Q(G \oplus \Delta G)$ (including revised $Q(G)$) and $\mathsf{kdist}(\cdot)$;

Figure 1: Algorithm IncKWS^+

4.2 Localizable Algorithms for KWS

We first provide localizable algorithms for KWS under unit edge insertions and deletions. We then develop a localizable incremental algorithm for KWS to process batch updates.

Data structures. We start with an auxiliary structure. Recall that a KWS query consists of a list Q of keywords and an integer bound b. For each node v in graph G, we maintain a *keyword-distance* list $\mathsf{kdist}(v)$. Its entries are of the form $(\mathsf{keyword}, \mathsf{dist}, \mathsf{next})$, where dist is the shortest distance from v to a node labeled $\mathsf{keyword}$ in Q, and next indicates the node on this shortest path next to v. A single shortest path is selected with a predefined order in case of a tie. Hence each root uniquely determines a match if it exists. Such keyword-distance lists are obtained after the execution of a batch algorithm. Indeed, existing batch approaches [8,27,30] for KWS traverse G to find shortest paths from nodes to others matching keywords in Q. While they vary in search and indexing strategies, they all maintain something like $\mathsf{kdist}(\cdot)$.

(1) Unit insertions. Inserting an edge to graph G may shorten the shortest distances from nodes to those matching keywords in Q, which is reflected as changes to dist and next in the keyword-distance lists on G. Based on this, we present an incremental algorithm, referred to as IncKWS^+ and shown in Fig. 1, to process unit edge insertions.

Given ΔG consisting of $\mathsf{insert}(v, w)$, IncKWS^+ inspects whether it inflicts any change to shortest paths of existing matches; if so, it propagates the changes, revises $\mathsf{kdist}(v)$ entries for affected nodes v and updates the matches accordingly. It proceeds until no more revision is needed. The search is confined in the b-neighbors of nodes in ΔG, and hence localizable, where b is the bound in the KWS query.

More specifically, IncKWS^+ first checks whether (v, w) is on a shorter path within the bound b from v to nodes labeled k_i in Q. If so, $\mathsf{kdist}(v)$ is adjusted by updating dist and next (lines 2-3). IncKWS^+ then propagates the change to the ancestors of v if their kdist entries are no longer valid (lines 4-8). An FIFO (first-in-first-out) queue q_i is used to control the propagation, following BFS (breadth-first-search). Each time when a node u is dequeued from q_i, the predecessors of u are inspected to check whether u triggers updated shortest path from them within bound b, followed by updating their kdist entries when needed (lines 6-8). These predecessors may be inserted into queue q_i for further checking (line 8).

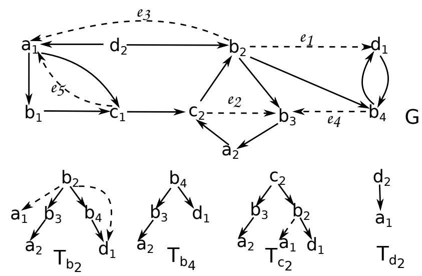


Figure 2: Example graph and matches of KWS

After revising the data structures, IncKWS^+ computes $Q(G \oplus \Delta G)$ based on the changes to next in $\mathsf{kdist}(\cdot)$'s (lines 9-10), either by replacing some edges in existing matches, or by including new matches not in $Q(G)$. Note that all such affected edges are inside the $2b$-neighbors of ΔG.

Example 1: Figure 2 gives a graph G (with all solid edges and dotted e_2, e_5). Consider $Q = (a, d)$ and bound 2. Two trees T_{b_2} and T_{d_2} in $Q(G)$ are shown in Fig. 2 (solid edges).

When edge e_1 is added to G, denote by G_1 the graph after the insertion. IncKWS^+ finds that the shortest distance from b_2 to nodes matching d in G_1 is reduced to 1 from 2. Thus it updates the entries in $\mathsf{kdist}(b_2)[d]$ and propagates the change to b_2's predecessors. The propagation stops at c_2 since the shortest distance from it to d nodes reaches bound 2. The values of $\langle \mathsf{dist}, \mathsf{next} \rangle$ in kdist lists on G are updated as follows.

IncKWS^+	before insertion	after insertion
$\mathsf{kdist}(b_2)[d]$	$\langle 2, b_4 \rangle$	$\langle 1, d_1 \rangle$
$\mathsf{kdist}(c_2)[d]$	$\langle \perp, nil \rangle$	$\langle 2, b_2 \rangle$

Then IncKWS^+ revises T_{b_2} by replacing the path starting with edge (b_2, b_4) by (b_2, d_1) to get T'_{b_2} in $Q(G_1)$, and a new match T_{c_2} (solid edges in Fig. 2) is added to $Q(G_1)$. $\square$

Correctness & complexity. IncKWS^+ updates $\mathsf{kdist}(\cdot)$'s correctly: it revises only entries in which dist values are decreased, and checks all affected entries by propagating the changes. From this the correctness of IncKWS^+ follows.

IncKWS^+ is in $O(m(|V_b(w)| + |E_b(w)|) + |V_b(w)||E_{2b}(w)|)$ time. Updating $\mathsf{kdist}(\cdot)$'s takes $O(m(|V_b(w)|+|E_b(w)|))$ time in total (lines 1-8), where m is the number of keywords in Q. Observe the following: (a) each node with updated kdist is verified at most m times to check all the keywords in Q; and (b) only the data in $G_b(w)$ is inspected since change propagation stops as soon as the shortest distance exceeds b, *i.e.,* $\mathsf{kdist}(\cdot)$'s are partially updated for matches within bound b. Updating $Q(G)$ (lines 9-10) takes $O(|V_b(w)||E_{2b}(w)|)$ time since the roots of the affected matches are within b hops of w, and their edges to be adjusted are at most $2b$ hops away from w. Therefore, algorithm IncKWS^+ is localizable.

(2) Unit deletions. The incremental algorithm for processing unit $\mathsf{delete}(v, w)$ is shown in Fig. 3, denoted by IncKWS^-. In contrast to edge insertions, some shortest distances in kdist lists may be increased by $\mathsf{delete}(v, w)$. The main idea of IncKWS^- is to identify those entries in $\mathsf{kdist}(\cdot)$'s that are *affected* by ΔG, and compute changes to dist and next. Similar to IncKWS^+, updating $\mathsf{kdist}(\cdot)$'s is confined within the b-neighbors of ΔG by inspecting only those distances no longer than bound b. The identification and computation are separated into two phases in IncKWS^-.

After consulting whether (v, w) is on a shortest path from v to some node labeled keyword k_i within bound b, IncKWS^- propagates the change to v's predecessors if needed with the help of a stack a_i, and each predecessor that may have

Algorithm: IncKWS^-

Input: G with $\mathsf{kdist}(\cdot)$, Q, b, $Q(G)$ as in IncKWS^+, and $\mathsf{delete}(v, w)$.
Output: The updated matches $Q(G \oplus \Delta G)$ and kdist lists.

1. **for each** k_i in Q with $w = \mathsf{kdist}(v)[k_i].\mathsf{next}$ **and** $\mathsf{kdist}(w)[k_i] < b$ **do**
2. queue $q_i := nil$; stack $a_i := nil$; $a_i.\mathsf{push}(v)$; mark v *affected*;
3. **while** a_i is not empty **do**
4. node $u := a_i.\mathsf{pop}()$;
5. **for each** predecessor u' of u that $u = \mathsf{kdist}(u')[k_i].\mathsf{next}$ **and** $\mathsf{kdist}(u')[k_i] \leq b$ **do**
6. $a_i.\mathsf{push}(u')$; mark u' *affected*;
7. **for each** *affected* node u **do**
8. compute dist and next for $\mathsf{kdist}(u)[k_i]$ based on those u's successors that are *not affected*;
9. $q_i.\mathsf{insert}(u, \mathsf{kdist}(u)[k_i].\mathsf{dist})$;
10. **while** q_i is not empty **do**
11. $(u, d) := q_i.\mathsf{pull\_min}()$;
12. **for each** predecessor u' of u with $d < \min(\mathsf{kdist}(u')[k_i].\mathsf{dist} - 1, b)$ **do**
13. $\mathsf{kdist}(u')[k_i].\mathsf{dist} := d + 1$; $\mathsf{kdist}(u')[k_i].\mathsf{next} := u$;
14. $q_i.\mathsf{decrease}(u', \mathsf{kdist}(u')[k_i].\mathsf{dist})$;
15. **for each** u_1'' and u_2'' involved in a changed $\mathsf{kdist}(u)[k_i].\mathsf{next}$ **do**
16. replace (u, u_1'') with (u, u_2'') in all the matches of $Q(G)$ or remove matches from $Q(G)$ by excluding (u, u_1'');
17. **return** $Q(G \oplus \Delta G)$ (updated $Q(G)$ above) and $\mathsf{kdist}(\cdot)$;

Figure 3: Algorithm IncKWS^-

an updated shortest path to nodes matching k_i is marked *affected w.r.t.* k_i (lines 1-6). The propagation is similar to that of IncKWS^+, by inspecting next values, and is conducted in the b-neighbors of v. Then the *potential* kdist entries for those affected nodes are computed based on their successors that are *not affected w.r.t.* k_i (line 8), and affected nodes with their potential dist values (as keys) are inserted into priority queue q_i (line 9) to compute exact dist values later. Indeed, the exact values of dist and next may depend on the affected successors, whose values also need to be determined.

The exact values of dist and next are computed in the second phase (lines 10-14). For node u with minimum dist that is removed from q_i, IncKWS^- checks whether it leads to a new shortest path within bound b originated from predecessor u' of u (lines 11-12). If so, values in $\mathsf{kdist}(u')[k_i]$ are updated, and the key of u' in q_i is decreased (lines 13-14).

The process continues until q_i becomes empty. Matches in $Q(G)$ are updated using the latest kdist lists (lines 15-16).

Example 2: Recall Q, G_1 and $Q(G_1)$ from Example 1. Suppose that e_2 is now removed from G_1. This makes the shortest path from c_2 to a_2 in T_{c_2} split, and IncKWS^- marks node c_2 affected with keyword a. Since the shortest distance from successor b_2 of c_2 to nodes matching a equals the bound 2, IncKWS^- concludes that node c_2 cannot be the root of a match, and removes T_{c_2} of Example 1 from $Q(G_1)$. □

<u>*Correctness & complexity*.</u> The correctness of IncKWS^- is verified just like for IncKWS^+, except that the exact values of $\mathsf{kdist}(v)$ may depend on multiple affected successors of v.

IncKWS^- runs in $O(m(|V_b(w)|\log|V_b(w)| + |E_b(w)|) + |V_b(w)||E_{2b}(w)|)$ time, including $O(|V_b(w)||E_{2b}(w)|)$ for updating matches in addition to the cost for computing changes to $\mathsf{kdist}(\cdot)$'s. Its first phase (lines 1-9) takes $O(m(|V_b(w)| + |E_b(w)|))$ time since only the affected shortest paths of length bounded by b are identified. The second phase (lines 10-14) takes $O(m(|V_b(w)|\log|V_b(w)| + |E_b(w)|))$ time, the same as computing b-bounded shortest path from affected nodes to m sinks, *i.e.,* nodes labeled a keyword from Q.

(3) Batch updates. We next give an incremental algorithm, denoted by IncKWS (not shown), to process batch updates $\Delta G = (\Delta G^+, \Delta G^-)$, where ΔG^+ and ΔG^- denote edge insertions and deletions, respectively. We assume *w.l.o.g.* that there exist no delete e in ΔG^- and insert e in ΔG^+ for the same edge e, which can be easily detected.

Given batch updates ΔG, IncKWS inspects whether each unit edge deletion and insertion causes any change to existing matches, *i.e.,* whether some of existing shortest paths become invalid and new shortest paths have to be generated; if so, it propagates the changes and updates the affected keyword-distance lists. The algorithm updates the same entry at most once even if it is affected by multiple updates in ΔG, by *interleaving* different change propagation with a global data structure to accommodate the effects of different unit updates. It works in three phases, as outlined below.

(a) IncKWS first identifies the affected nodes *w.r.t.* each keyword k_i in Q due to ΔG^- within the b-neighbors of ΔG^-, and computes their potential dist and next values, using the same strategy of IncKWS^-. Here all the affected nodes *w.r.t.* k_i and their potential dist values are inserted into a *single* priority queue q_i to further compute exact values.

(b) The algorithm then checks whether each $\mathsf{insert}(v, w)$ leads to the creation of a shorter path within bound b when neither v nor w is affected *w.r.t.* k_i by ΔG^-. Insertions with affected nodes are not considered since dist value at w may no longer be correct due to ΔG^-, or this edge has already been inspected to compute potential dist value for node v. If so, dist and next values are updated for $\mathsf{kdist}(v)$. Unlike IncKWS^+ that propagates this change to ancestors of v directly, it inserts node v and the updated dist value into queue q_i to interleave $\mathsf{insert}(v, w)$ with other updates in ΔG.

(c) After these, IncKWS computes exact next and dist values of $\mathsf{kdist}(\cdot)$'s, in the same way as we do in IncKWS^- by making use of queue q_i. Note that all potential changes to $\mathsf{kdist}(\cdot)$'s caused by ΔG, including both deletions and insertions, are collected into the same q_i; in this way the algorithm guarantees that the exact value, *i.e.,* shortest distance, is decided at most once for each entry affected. Matches in $Q(G)$ are updated accordingly within the $2b$-neighbors of ΔG at last.

Example 3: Consider Q and G of Example 1, and batch updates ΔG that insert edges e_1, e_3, e_4 and delete e_2 and e_5.

Given these, IncKWS first identifies the affected nodes c_1 and c_2 *w.r.t.* a, and finds that the potential value of the corresponding dist exceeds the bound 2. Then it processes insertions; *e.g.,* the insertion of e_3 leads to decreased shortest distance from b_2 to a nodes, and the change is propagated to c_2 for computing the exact value of $\mathsf{kdist}(c_2)[a]$, *i.e.,* IncKWS interleaves insert e_3 and delete e_2 to decide the exact shortest distance from c_2 to a nodes. The other updates are handled similarly. Based on these, it replaces the two branches of T_{b_2} with (b_2, a_1) and (b_2, d_1), respectively, and adds match T_{b_4} in Fig. 2. A new match T'_{c_2} is also generated, where path (c_2, b_3, a_2) in T_{c_2} of Example 1 is replaced by (c_2, b_2, a_1). □

<u>*Correctness & complexity.*</u> For the correctness of IncKWS, observe the following. (a) Each node that is affected *w.r.t.* keyword k_i by any unit update in ΔG is inspected. (b) The dist values for these nodes are monotonically increasing and correctly computed, similar to its counterpart in IncKWS^-.

IncKWS is in $O(m(|V_b(\Delta G)|\log|V_b(\Delta G)| + |E_b(\Delta G)|) + |V_b(\Delta G)||E_{2b}(\Delta G)|)$ time, where $V_b(\Delta G)$ (resp. $E_b(\Delta G)$)

denote the nodes (resp. edges) of the union of b-neighbors of nodes in ΔG. Note that the final kdist value of each affected node *w.r.t.* any keyword k_i is determined once by using the global priority queue q_i. The complexity analysis is similar to that of IncKWS^-, except that here the $2b$-neighbors of all the nodes involved in ΔG are possibly accessed.

Since the costs of IncKWS^+, IncKWS^- and IncKWS are determined by m and the size of $2b$-neighbors of nodes involved in ΔG for a given bound b, they are all localizable.

Remark. Although the incremental algorithms for KWS are developed for a constant b, they can be readily extended to cope with b that varies. More specifically, when change propagation stops at node v due to bound b, we can annotate v as a “breakpoint” *w.r.t.* b, and the set of all such breakpoints is stored as a “snapshot” of graph G *w.r.t.* b. When given a larger b', the snapshot is firstly restored and each breakpoint is regarded as a unit update to G, *i.e.,* as input to the incremental algorithm with b' in addition to ΔG, from where the change propagation continues. In this way, KWS queries with different b values can be answered using the same data structure, *i.e.,* keyword-distance list that is consistently updated. Indeed, we only need to store the snapshot of G *w.r.t.* the maximum b that is encountered.

5. RELATIVE BOUNDEDNESS

We next introduce relative boundedness, another alternative characterization for the effectiveness of incremental computations. We first formalize the notion in Section 5.1. We then develop relatively bounded incremental algorithms for RPQ and SCC in Sections 5.2 and 5.3, respectively.

5.1 Relative Boundedness

Consider a batch algorithm $\mathcal{T}$ for a query class $\mathcal{Q}$ that is proven effective and being widely used in practice. For a query $Q \in \mathcal{Q}$ and a graph G, we denote by $G_{(\mathcal{T},Q)}$ the data inspected by $\mathcal{T}$ when computing $Q(G)$, including data in G and possibly auxiliary structures used by $\mathcal{T}$. For updates ΔG to G, we denote by AFF the difference between $(G \oplus \Delta G)_{(\mathcal{T},Q)}$ and $G_{(\mathcal{T},Q)}$, *i.e.,* the difference in the data inspected by $\mathcal{T}$ for computing $Q(G \oplus \Delta G)$ and for $Q(G)$.

An incremental algorithm $\mathcal{T}_\Delta$ for $\mathcal{Q}$ is *bounded relative to* $\mathcal{T}$ if its cost can be expressed as a polynomial function in $|\Delta G|$, $|Q|$ and $|\mathsf{AFF}|$ for $Q \in \mathcal{Q}$, graph G and updates ΔG. Note that the changes ΔO to $Q(G)$ are included in AFF.

Intuitively, we only incrementalize batch algorithms $\mathcal{T}$'s that have been verified effective. As batch algorithms have been studied for decades for graphs, a number of such algorithms are in place. When incrementalizing such algorithms, relative boundedness is to characterize the effectiveness of the incrementalization, *i.e.,* whether it minimizes unnecessary recomputation in response to updates ΔG. It suffices to develop $\mathcal{T}_\Delta$ bounded relatively to one of such $\mathcal{T}$'s.

Note that for a class $\mathcal{Q}$ of graph queries, one can find localizable incremental algorithms only if $\mathcal{Q}$ has the data locality, *i.e.,* to decide whether v is in the answer $Q(G)$ to a query Q, it suffices to inspect the d_Q-neighbor of v. However, many graph queries do not have the data locality, *e.g.,* RPQ and SCC. For such queries, we can explore relatively bounded incremental algorithms. Moreover, even when $\mathcal{Q}$ has the data locality, we want to find incremental algorithms that are both localizable and bounded relative to a practical batch algorithm of $\mathcal{Q}$. Such algorithms are particularly needed for large queries Q (*i.e.,* when diameter d_Q of Q is large).

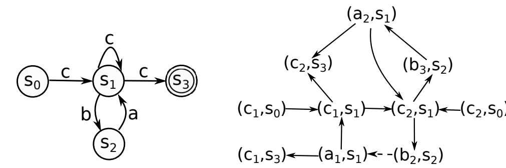


Figure 4: NFA M_Q and intersection graph of M_Q, G

We should remark that there are other alternative effectiveness characterizations for incremental graph algorithms, *e.g.,* a classification in terms of incremental complexity. We focus on localizability and relative boundedness in this paper since they are easy to verify and use in practice.

The main results of this section are as follows.

Theorem 4: *There are bounded incremental algorithms for* RPQ *and* SCC *relative to their batch counterparts.* □

As a proof, we present relatively bounded algorithms for RPQ and SCC. As will be seen in Section 6, these algorithms are effective although none of the query classes is bounded.

5.2 Incrementalization for RPQ

We start with RPQ. Given a regular path query Q and a graph G, it is to compute the set $Q(G)$ of matches of Q in G, *i.e.,* pairs (v, w) of nodes in G such that v can reach w by following a path in the regular language defined by Q.

We incrementalize a batch algorithm $\mathsf{RPQ_{NFA}}$ [29, 33] for RPQ. We first review $\mathsf{RPQ_{NFA}}$ and identify its AFF. We then give a bounded incremental algorithm relative to $\mathsf{RPQ_{NFA}}$.

Batch algorithm. Algorithm $\mathsf{RPQ_{NFA}}$ consists of two phases. Given Q and G, it first translates Q into an NFA M_Q [29], and then computes $Q(G)$ by traversing G guided by M_Q [33]. Its time complexity is $O(|V||E||Q|^2 \log^2 |Q|)$.

More specifically, $M_Q = (S, \Sigma, \delta, s_0, F)$, where S is a finite set of *states*, Σ is the *alphabet*, δ is the *transition function* that maps $S \times \Sigma$ to the set of subsets of S, $s_0 \in S$ is the *initial state*, and $F \subseteq S$ is the set of *accepting states*. There are other methods for constructing NFA, *e.g.,* the one based on partial derivatives [7]. We adopt the algorithm of [29] since it constructs smaller NFA than [7] and takes less time.

After M_Q is in place, the second phase starts, traversing the *intersection graph* $G_I = (V_I, E_I, l_I)$ of G and M_Q [33]. Here $V_I = V \times S$, $l_I(v, s) = l(v)$, $E_I \subseteq V_I \times V_I$ and $((v, s), (v', s'))$ is in E_I if and only if $(v, v') \in E$ and $s' \in \delta(s, l(v'))$. Each node v in G is *marked* with a set $v.\mathsf{pmark}(\cdot)$ of *markings*, where $v.\mathsf{pmark}(u)$ is a set of states s in S, indicating that there exists a path ρ from u to v in G such that (u, s_0) reaches (v, s) following the corresponding path ρ_I of ρ in G_I. When node v is visited in state s, only the successor v' of v with $\delta(s, l(v')) \neq \emptyset$ are inspected. The markings prevent a node from being visited more than once in the same state. It includes (u, v) in $Q(G)$ if $v.\mathsf{pmark}(u) \cap F \neq \emptyset$, *i.e.,* there exist state $s \in v.\mathsf{pmark}(u)$ and a path ρ_I from (u, s_0) to (v, s) such that $l_I(\rho_I) \in L(Q)$.

Example 4: Consider an RPQ query $Q = c \cdot (b \cdot a + c)^* \cdot c$ over the graph G of Fig. 2. Its NFA M_Q and a fragment of the intersection graph G_I of G and M_Q are shown in Fig. 4 (excluding dotted edge $((b_2, s_2), (a_1, s_1))$).

$\mathsf{RPQ_{NFA}}$ traverses G_I and marks the nodes in G with states of M_Q. Note that there exist paths from (c_1, s_0) to (c_2, s_3) and from (c_2, s_0) to (c_2, s_3) in G_I; thus the accepting state

Algorithm: IncRPQ

Input: A graph G with $\mathsf{pmark_e}(\cdot)$, regular path query Q and NFA M_Q, matches $Q(G)$, and batch updates $(\Delta G^+, \Delta G^-)$.
Output: The updated matches $Q(G \oplus \Delta G)$ and markings $\mathsf{pmark_e}(\cdot)$.

1. set $\mathsf{aff_s} := \mathsf{identAff}(G, \mathsf{pmark_e}(\cdot), \Delta G^-)$; queue $q := nil$;
2. **for each** (v, u, s) in $\mathsf{aff_s}$ **do**
3.   update dist, mpre for $v.\mathsf{pmark_e}(u)[s]$ based on its cpre;
4. $q.\mathsf{insert}((v, u, s), v.\mathsf{pmark_e}(u)[s].\mathsf{dist})$;
5. **for each** edge insertion of (v, w) in ΔG^+ **do**
6.   **if** edge (v, w) leads to a smaller $w.\mathsf{pmark_e}(u)[s].\mathsf{dist}$ for node u and state s **and** (v, u, s) is not in $\mathsf{aff_s}$ **then**
7.   update dist, mpre, cpre for $w.\mathsf{pmark_e}(u)[s]$;
8. $q.\mathsf{insert}((w, u, s), w.\mathsf{pmark_e}(u)[s].\mathsf{dist})$;
9. update $\mathsf{pmark_e}(\cdot)$ based on queue q and NFA M_Q;
10. update $Q(G)$ to get $Q(G \oplus \Delta G)$;
11. **return** $Q(G \oplus \Delta G)$ and $\mathsf{pmark_e}(\cdot)$;

Figure 5: Algorithm IncRPQ

s_3 is included in markings $c_2.\mathsf{pmark}(c_1)$ and $c_2.\mathsf{pmark}(c_2)$. Therefore, (c_1, c_2) and (c_2, c_2) are returned by $\mathsf{RPQ_{NFA}}$. □

Auxiliary structures. The marking $v.\mathsf{pmark_e}(u)$ is of the form $\langle \mathsf{state}, \mathsf{dist}, \mathsf{cpre}, \mathsf{mpre} \rangle$, where (a) dist is the shortest distance from (u, s_0) to (v, state) in G_I, (b) (v', s') is contained in $v.\mathsf{pmark_e}(u)[s].\mathsf{cpre}$ if there exists an entry in $v'.\mathsf{pmark_e}(u)$ for state s' such that $s \in \delta(s', l(v))$ and (v', v) is in G, *i.e.,* $v.\mathsf{pmark_e}(u)[s].\mathsf{cpre}$ stores predecessors of node (v, s) in G_I that are on a path starting from (u, s_0); and (c) (v', s') is in $v.\mathsf{pmark_e}(u)[s].\mathsf{mpre}$ if $v'.\mathsf{pmark_e}(u)[s'].\mathsf{dist} + 1 = v.\mathsf{pmark_e}[s].\mathsf{dist}$, *i.e.,* mpre keeps track of those predecessors on shortest paths. The auxiliary information is computed by $\mathsf{RPQ_{NFA}}$ without increasing its complexity.

Characterization of AFF. We identify AFF, *i.e.,* the difference between $G_{(\mathsf{RPQ_{NFA}}, Q)}$ and $(G \oplus \Delta G)_{(\mathsf{RPQ_{NFA}}, Q)}$, as changes to the markings. Indeed, the markings are the data that $\mathsf{RPQ_{NFA}}$ necessarily inspects, since updates to markings trigger different behaviors of $\mathsf{RPQ_{NFA}}$ when computing $Q(G \oplus \Delta G)$ and $Q(G)$. For instance, a change to dist in $v.\mathsf{pmark_e}(u)[s]$ indicates that (v, s) is reached in BFS through a different path from (u, s_0) and state s is included in $v.\mathsf{pmark}(u)$ in $\mathsf{RPQ_{NFA}}$ at a different level of the BFS tree.

Incremental algorithm. Based on markings, we develop incremental algorithms that are bounded relative to $\mathsf{RPQ_{NFA}}$. The boundedness is accomplished by updating markings only when there exists corresponding difference between the data inspected by $\mathsf{RPQ_{NFA}}$. For unit edge deletions and insertions, the algorithms are similar to their counterparts for KWS (Section 4.2), guided by changes to dist. Below we just present an algorithm for processing batch updates.

The algorithm is denoted as IncRPQ and shown in Fig. 5. It first invokes procedure identAff (not shown) to identify a set $\mathsf{aff_s}$ of (v, u, s) triples, where $v.\mathsf{pmark_e}(u)[s].\mathsf{dist}$ is no longer valid due to edge deletions (line 1). Similar to how IncKWS^- identifies affected entries of keyword-distance lists (Section 4.2), identAff checks the values of mpre and cpre in markings. For example, if $v.\mathsf{pmark_e}(u)[s].\mathsf{mpre}$ becomes empty, it checks whether (v, s) is in $v'.\mathsf{pmark_e}(u)[s'].\mathsf{mpre}$ for each successor v' of v and $s' \in \delta(s, l(v'))$. If so, (v, s) is removed, and identAff continues to check the successors of v'. IncRPQ then updates the corresponding (potential) dist values of triples in $\mathsf{aff_s}$ based on the current cpre, *i.e.,* the remaining candidate predecessors after removing affected entries. These triples with dist values are inserted into priority queue q (lines 2-4) for deciding exact markings later on.

Thereafter, IncRPQ processes insertions in ΔG^+ by checking whether they yield smaller dist values in some markings (lines 5-6), and update them accordingly (line 7). Again, the updated triples are added to queue q (line 8). IncRPQ determines exact markings based on queue q (line 9) following a monotonically increasing order of updated dist, similar to IncKWS, while NFA M_Q is used to guide the propagation. By grouping updated triples in queue q, the algorithm reduces redundant computations when processing ΔG.

Finally, given the updated markings, $Q(G \oplus \Delta G)$ is computed by taking new pairs of nodes marked with accepting states in F and removing invalid ones from $Q(G)$ (line 10).

Example 5: Recall batch updates ΔG to G from Example 3. These inflict the deletion of $((c_2, s_1), (b_3, s_2))$ and insertion of $((b_2, s_2), (a_1, s_1))$ to the intersection graph G_I of Example 4. IncRPQ first finds that triple (b_3, c_2, s_2) is affected by the deletion. The change is propagated to the decedents of (b_3, s_2) in G_I, and potential values of $\langle \mathsf{dist}, \mathsf{mpre} \rangle$ for affected entries are computed. After these, it decides exact values after processing insertions; some are shown below.

IncRPQ	before updates	after updates
$b_3.\mathsf{pmark_e}(c_2)[s_2]$	$\langle 2, \{(c_2, s_1)\} \rangle$	$\langle \bot, nil \rangle$
$a_2.\mathsf{pmark_e}(c_2)[s_1]$	$\langle 3, \{(b_3, s_2)\} \rangle$	$\langle \bot, nil \rangle$
$c_2.\mathsf{pmark_e}(c_2)[s_3]$	$\langle 4, \{(a_2, s_1)\} \rangle$	$\langle 5, \{(c_1, s_1)\} \rangle$
$c_1.\mathsf{pmark_e}(c_2)[s_3]$	$\langle \bot, nil \rangle$	$\langle 4, \{(a_1, s_1)\} \rangle$
$c_1.\mathsf{pmark_e}(c_1)[s_3]$	$\langle \bot, nil \rangle$	$\langle 5, \{(a_1, s_1)\} \rangle$

Note that although the previous path from (c_2, s_0) to (c_2, s_3) is split, accepting state s_3 remains in marking $c_2.\mathsf{pmark_e}(c_2)$ since another path connecting these two nodes in G_I is formed as a result of insertions. Indeed, IncRPQ combines the processes for $\mathsf{delete}(c_2, b_3)$ and $\mathsf{insert}(b_2, a_1)$ to compute exact value of $c_2.\mathsf{pmark_e}(c_2)[s_3]$. Based on these, it adds (c_2, c_1) and (c_1, c_1) to obtain $Q(G \oplus \Delta G)$, as accepting state s_3 is included in the corresponding markings. □

Correctness & complexity. One can verify that IncRPQ correctly updates markings by induction on the number of changed entries. IncRPQ is in $O(|\mathsf{AFF}| \log |\mathsf{AFF}|)$ time. Indeed, (a) affected triples are added to set $\mathsf{aff_s}$ and queue q at most once by BFS traversal; (b) each of procedure identAff (line 1), computing potential values (lines 2-4) and processing edge insertions (lines 5-8) takes $O(|\mathsf{AFF}|)$ time by using M_Q and cpre, where to compute potential values, $O(|\mathsf{AFF}|)$ predecessors are processed directly via cpre, without inspecting the entire neighbors; and (c) computing the latest values of markings (line 9) needs $O(|\mathsf{AFF}| \log |\mathsf{AFF}|)$ time by using heaps for queue q, just like fixing dist values for affected nodes in IncKWS (Section 4.2). Note that $|Q|$ is counted in $|\mathsf{AFF}|$. All these steps have costs bounded by a function of $|\mathsf{AFF}|$. Hence IncRPQ is bounded relative to $\mathsf{RPQ_{NFA}}$.

5.3 Incrementalization for SCC

We next investigate the incremental problem for SCC. Given a graph G, it is to compute $\mathsf{SCC}(G)$, *i.e.,* the set of all strongly connected components in G. In the sequel we abbreviate a strongly connected component as an scc.

We incrementalize Tarjan's algorithm [43] for SCC. We refer to the batch algorithm as Tarjan. Below we first review the basic idea of Tarjan, and identify its AFF.

Batch algorithm. Tarjan traverses a directed graph G via repeated DFS (depth-first search) to generate a spanning forest $\mathcal{F}$, such that each scc corresponds to a subtree of a

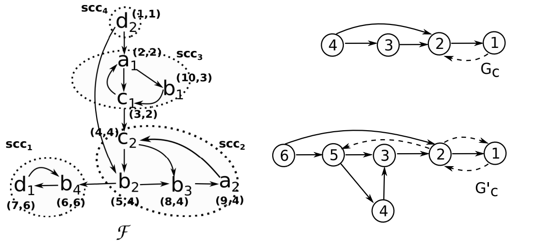


Figure 6: DFS forest of G and contracted graphs

Algorithm: IncSCC$^+$

Input: A graph G with $\mathsf{num}(\cdot)$, $\mathsf{lowlink}(\cdot)$, contracted graph G_c, $\mathsf{SCC}(G)$ and an edge (v, w) to be inserted.
Output: $\mathsf{SCC}(G \oplus \Delta G)$ and updated $\mathsf{num}(\cdot)$, $\mathsf{lowlink}(\cdot)$ and G_c.

1. **if** v and w are within the same scc (tree) T **then**
2. $\quad T := T \oplus \Delta G$; update $\mathsf{num}(\cdot)$, $\mathsf{lowlink}(\cdot)$ for T;
3. **if** $r(\mathsf{scc}(v)) > r(\mathsf{scc}(w))$ **then** update G_c;
4. **if** $r(\mathsf{scc}(v)) < r(\mathsf{scc}(w))$ **then**
5. $\quad \mathsf{aff_r} := \mathsf{DFS_f}(G_c, w, r(\mathsf{scc}(v)))$; $\mathsf{aff_l} := \mathsf{DFS_b}(G_c, v, r(\mathsf{scc}(w)))$;
6. $\quad$ **if** $\mathsf{Tarjan}(\mathsf{aff_l} \cup \mathsf{aff_r}, v)$ has *non-singleton* cycle C **then**
7. $\quad\quad$ merge the corresponding components of nodes in C;
8. $\quad\quad$ update $\mathsf{num}(\cdot)$, $\mathsf{lowlink}(\cdot)$ for the new component;
9. $\quad$ **else** $\mathsf{reallocRank}(\mathsf{aff_l}, \mathsf{aff_r})$;
10. **return** $\mathsf{SCC}(G \oplus \Delta G)$ and updated $\mathsf{num}(\cdot)$, $\mathsf{lowlink}(\cdot)$, and G_c;

Figure 7: Algorithm IncSCC$^+$

tree T in $\mathcal{F}$ with a designated *root*. It reduces SCC to finding the roots of corresponding subtrees in $\mathcal{F}$.

More specifically, each node v in G is assigned a unique integer $v.\mathsf{num}$, denoting the order of v visited in the traversal. The edges of G fall into four classes by DFS: (a) *tree arcs* that lead to nodes not yet discovered during the traversal; (b) *fronds* that run from descendants to their ancestors in a tree; (c) *reverse fronds* that are from ancestors to descendants in a tree; and (d) *cross-links* that run from one subtree to another. In addition, $v.\mathsf{lowlink}$ is maintained, representing the *smallest* num of the node that is in the *same* scc as v and is reachable by traversing zero or more tree arcs followed by *at most one* frond or cross-link. It determines whether v is the root of the subtree corresponding to an scc by checking whether $v.\mathsf{lowlink} = v.\mathsf{num}$, and if so, generates the scc accordingly. It uses a stack to store nodes that have been reached during DFS but have not been placed in an scc. A node remains on the stack if and only if there exists a path in G from it to some node earlier on the stack.

Example 6: Figure 6 depicts the DFS forest $\mathcal{F}$ obtained by applying Tarjan on graph G of Fig. 2. Each node is annotated with its (num, lowlink). There are four scc's. The corresponding contracted graph G_c (see below) is also shown in Fig. 6 (solid edges), where node i refers to scc_i in G. □

Auxiliary structures. To incrementalize Tarjan, we maintain the values of num and lowlink after traversing G, and annotate the edges with the type that they fall into. Besides, a *contracted graph* G_c is constructed by contracting each scc into a single node. The graph G_c maintains a counter for the number of cross-links from one node to another. Each node v in G_c has a *topological rank* $r(v)$, initially the order of the scc to which v corresponds in the output sequence of Tarjan. Indeed, the topological sorting of scc's is a byproduct of Tarjan as nodes of each scc are popped from the stack recursively. These can be obtained by slightly revising Tarjan without increasing its complexity or changing its logic.

It is shown that $r(v) > r(v')$ if (v, v') is a cross-link in G_c [43], an invariant property on which we will capitalize.

Characterization of AFF. The affected area AFF includes the following: (a) changes to lowlink and num of nodes when computing $\mathsf{SCC}(G \oplus \Delta G)$, since accurate lowlink and num values determine the correctness of Tarjan; (b) v's successors for each node v whose $v.\mathsf{lowlink}$ changes, since the lowlink value of v is determined by comparing with lowlink or num of its successors; and (c) the neighbors of v for each node v whose $v.\mathsf{num}$ changes, since these neighbors are affected in this case and are necessarily checked by Tarjan.

We next give bounded incremental algorithms relative to Tarjan, under unit insertions, deletions, and batch updates.

(1) Unit insertions. Inserting an edge may result in combining two or more scc's into a single one. This happens if and only if a cycle is formed with the corresponding nodes of these scc's in the contracted graph after the insertion.

Employing the contracted graph G_c, we propose incremental algorithm IncSCC$^+$, shown in Fig. 7, to process unit insertion of edge (v, w). Intuitively, IncSCC$^+$ checks whether (v, w) inflicts a cycle in G_c, and combines some of the scc's in $\mathsf{SCC}(G)$ when necessary to get $\mathsf{SCC}(G \oplus \Delta G)$. It separates different types of (v, w), and makes use of topological ranks based on the invariant property mentioned above. Relatively boundedness is guaranteed since every change to the rank of an scc inspected by algorithm IncSCC$^+$ corresponds to a change of lowlink or num, and thus is in AFF.

More specifically, if v and w are within the same scc T, then nothing changes for the other scc's. In this case, IncSCC$^+$ only applies ΔG to T and computes the changes to num and lowlink, by applying Tarjan on the changed parts (lines 1-2). Otherwise consider the topological ranks of $\mathsf{scc}(v)$ and $\mathsf{scc}(w)$ in G_c, where $\mathsf{scc}(v)$ (resp. $\mathsf{scc}(w)$) refers to the corresponding scc node to which v (resp. w) belongs.

(a) If $r(\mathsf{scc}(v)) > r(\mathsf{scc}(w))$, then no new scc is generated, and IncSCC$^+$ only updates the graph G_c by inserting edge $(\mathsf{scc}(v), \mathsf{scc}(w))$ or increasing the counter of edges connecting their corresponding scc's (line 3). As the order of topological ranks in G_c is not affected in this case, it concludes that graph G_c is still acyclic and $\mathsf{SCC}(G \oplus \Delta G) = \mathsf{SCC}(G)$.

(b) If $r(\mathsf{scc}(w)) > r(\mathsf{scc}(v))$, *i.e.,* if the order of these two ranks becomes "incorrect", IncSCC$^+$ identifies the *affected area* $\mathsf{aff_l}$ and $\mathsf{aff_r}$, two subgraphs of G_c induced by nodes whose ranks are no longer valid, through a bi-directional search. It invokes procedure $\mathsf{DFS_f}$ to conduct a forward DFS traversal from w to find nodes with topological ranks greater than that of v, followed by a backward traversal $\mathsf{DFS_b}$ from v to find nodes having ranks less than that of w (lines 4-5). If a cycle C is formed in the affected area, the corresponding scc's of the nodes in C are merged into one to obtain $\mathsf{SCC}(G \oplus \Delta G)$; this is followed by updating num and lowlink values in the new scc (lines 6-8). Otherwise, although the output is unaffected, it reallocates the topological ranks of nodes in the affected area such that $r(v) > r(v')$ when (v, v') is in G_c, using procedure reallocRank (not shown) (line 9), *i.e.,* the relationship of topological ranks still holds. Procedure reallocRank sorts the previous ranks of those nodes in $\mathsf{aff_l}$ and $\mathsf{aff_r}$, and reassigns them in an ascending order, first $\mathsf{aff_r}$ and then $\mathsf{aff_l}$. Indeed, nodes in $\mathsf{aff_r}$ should have lower ranks than those in $\mathsf{aff_l}$ due to the edge insertion.

Example 7: Continuing with Example 6, consider insertion of edge $e_4 = (b_4, b_3)$ into G. Observe that the topological ranks $r(\mathsf{scc}(b_4)) < r(\mathsf{scc}(b_3))$ in G_c; thus IncSCC^+ identifies the affected area that consists of nodes 1 and 2 and forms a cycle. Then scc_1 and scc_2 are merged to get the output. □

Correctness & complexity. The correctness of IncSCC^+ is warranted by the following properties: (a) scc's are merged in response to an edge insertion if and only if they form a cycle in the contracted graph; and (b) the topological ranks of the nodes on any path in G_c decrease monotonically.

IncSCC^+ is in $O(|\mathsf{AFF}| \log |\mathsf{AFF}|)$ time. The cost for updating lowlink and num by Tarjan on the affected parts is $O(|\mathsf{AFF}|)$. Besides this, it only visits those nodes in the contracted graph with updated ranks, and their neighbors. The number of nodes visited does not exceed $|\mathsf{AFF}|$ since there must be changes to num and lowlink in the scc's that they refer to. Cycle detection is done in $O(|\mathsf{AFF}|)$ time and rank reallocation takes $O(|\mathsf{AFF}| \log |\mathsf{AFF}|)$ time via sorting by using heaps. Hence IncSCC^+ is bounded relative to Tarjan.

(2) Unit deletions. When edge (v, w) is deleted from G, an scc may be split into multiple ones. However, the output is unchanged if v still reaches w after deletion. We give an incremental algorithm for SCC under unit deletions, denoted by IncSCC^-. Intuitively, it examines the reachability from v to w by using num and lowlink maintained, and computes new scc's in $\mathsf{SCC}(G \oplus \Delta G)$ when v no longer reaches w in the same scc. The reachability checking is done as a byproduct of change propagation to num and lowlink, from which relatively boundedness is obtained. For the lack of space, we defer the details of IncSCC^- to [2].

(3) Batch updates. We now present algorithm IncSCC to process $\Delta G = (\Delta G^+, \Delta G^-)$, provided in [2]. It handles multiple updates in groups instead of one by one, to reduce redundant cost. IncSCC consists of two steps.

(a) IncSCC first processes *intra-component* updates, where the endpoints of an updated edge are in the same scc. All updates to the same scc are grouped and processed together. It starts with edge insertions, and adjusts values of num and lowlink following IncSCC^+. Inserted edges are processed following a descending order determined by the num values of their source nodes. Then, following the same processing order, IncSCC^- is invoked to handle deletions grouped together, to reduce redundant updates to num and lowlink values. After these, Tarjan is called on the affected scc's at most once to generate new scc's in $\mathsf{SCC}(G \oplus \Delta G)$.

(b) IncSCC then handles *inter-component* updates, for edge updates in which the endpoints fall in different scc's. After updating G_c with deletions, forward and backward traversals are performed to find the affected areas for all inter-component insertions, similar to IncSCC^+. However, IncSCC stores these areas in a global structure aff, and checks the existence of cycles formed by nodes from this global affected area, instead of processing unit updates one by one. Components are merged, and $\mathsf{num}(\cdot)$ and $\mathsf{lowlink}(\cdot)$ are revised, along the same lines as IncSCC^+ to get $\mathsf{SCC}(G \oplus \Delta G)$.

Finally, topological ranks are reallocated if needed, and $\mathsf{SCC}(G \oplus \Delta G)$ is returned (see [2] for details).

Example 8: Consider batch updates ΔG of Example 3. The intra-component deletions of e_2 and e_5 are firstly handled. Since $e_2 = (c_2, b_3)$ is a reverse frond in scc_2, IncSCC just deletes it from scc_2. Deletion of e_5 is processed as described in Example 9 (Appendix). Thereafter, the remaining three inter-component insertions in ΔG are handled by retrieving the affected area on contracted graph G'_c. Note that nodes 1 to 5 are covered by affected area aff that constitutes an scc in G'_c, hence all the previous scc's in $\mathsf{SCC}(G)$ except scc_4 (d_2) are merged to obtain $\mathsf{SCC}(G \oplus \Delta G)$ in IncSCC. □

Correctness & complexity. The correctness of IncSCC follows from the correctness of IncSCC^+ and IncSCC^-. IncSCC takes $O(|\mathsf{AFF}|(|\Delta G| + \log |\mathsf{AFF}|))$ time. Indeed, processing intra-component updates needs $O(|\Delta G||\mathsf{AFF}|)$ time since each update to the auxiliary structures in AFF is checked at most $|\Delta G|$ times; and handling inter-component updates takes $O(|\Delta G||\mathsf{AFF}| + |\mathsf{AFF}| \log |\mathsf{AFF}|)$ time, where each node with updated ranks in G_c is accessed by at most $|\Delta G|$ different bi-directional searches; the time for final rank reallocation is in $O(|\mathsf{AFF}| \log |\mathsf{AFF}|)$ as all such nodes are collected in aff. Thus IncSCC is bounded relative to Tarjan.

6. EXPERIMENTAL EVALUATION

Using real-life and synthetic data, we conducted three sets of experiments to evaluate the impacts of (1) the size $|\Delta G|$ of batch updates; (2) the complexity of queries Q for KWS, RPQ and ISO (see below); and (3) the size $|G|$ of graphs on our incremental algorithms, compared with their batch counterparts and some existing dynamic algorithms.

Experimental setting. We used the following datasets.

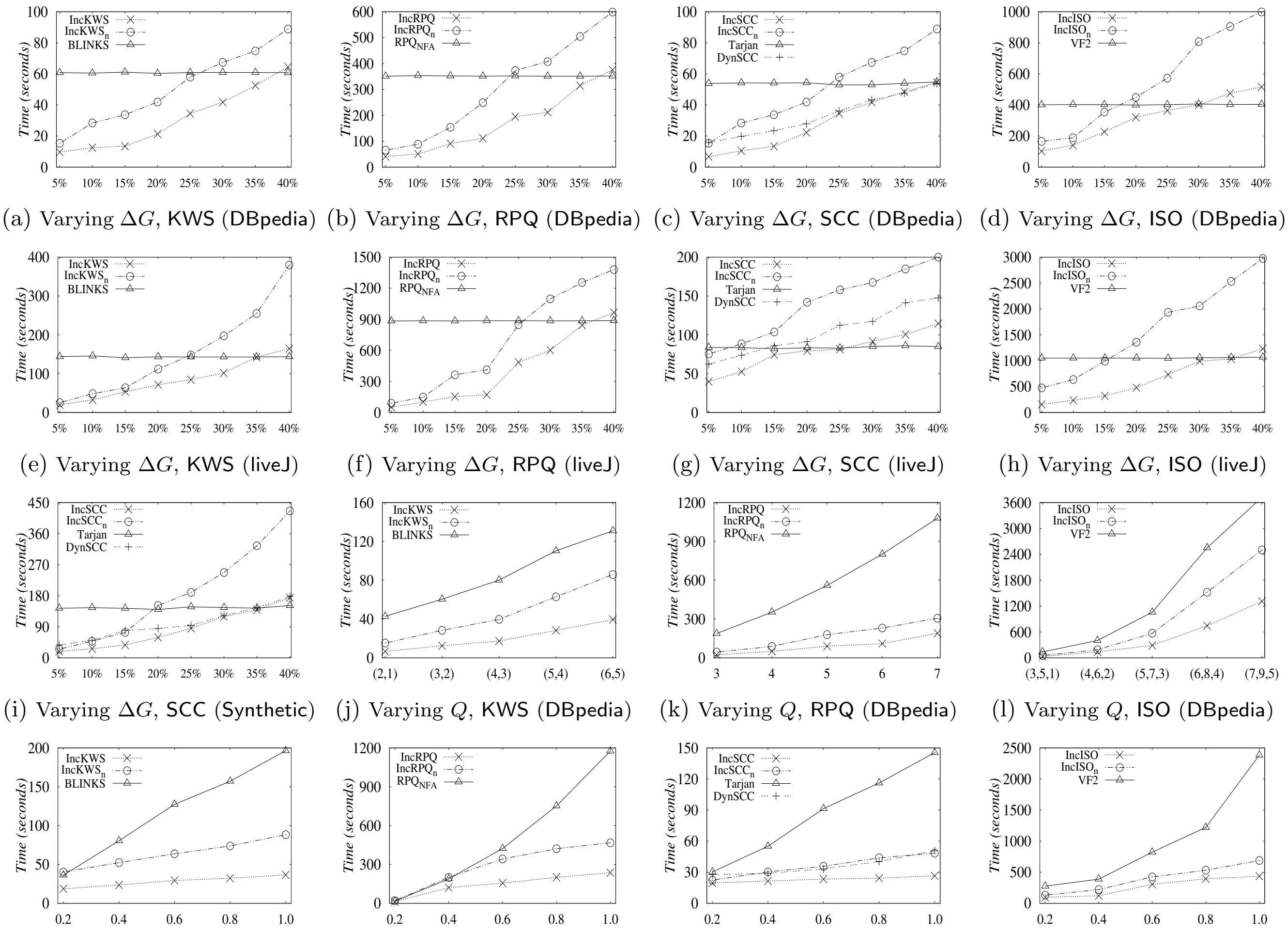
Graphs. We used two real-life graphs: (a) DBpedia, a knowledge graph [1] with 4.3 million nodes, 40.3 million edges and 495 labels; and (b) *LiveJournal* (liveJ in short), a social network [3] with 4.9 million nodes, 68.5 million edges and 100 labels. We also designed a generator to produce synthetic graphs G, controlled by the number of nodes $|V|$ (up to 50 million) and number of edges $|E|$ (up to 100 million), with labels drawn from an alphabet Σ of 100 symbols.

Updates ΔG are randomly generated for real-life and synthetic data, controlled by size $|\Delta G|$ and a ratio ρ of edge insertions to deletions. We use $\rho = 1$ unless stated otherwise, *i.e.,* the size of the data graphs G remain stable.

Query generators. We randomly generated 30 queries of KWS, RPQ and ISO with labels drawn from the graphs. More specifically, (1) KWS queries are controlled by the number m of keywords and bound b; (2) RPQ queries are controlled by the size (recall size $|Q|$ of a regular path query from Section 2.1) and the numbers of occurrences of $\cdot$, $+$ and Kleene $*$; and (3) ISO queries are controlled by the number of nodes $|V_Q|$, the number of edges $|E_Q|$ and the diameter d_Q, *i.e.,* the length of longest shortest path between any two nodes in Q when taken as an undirected graph.

Algorithms. We implemented the following algorithms, all in Java. (1) Incremental algorithms (a) IncKWS (Section 4.2), IncRPQ (Section 5.2), IncSCC (Section 5.3) and IncISO (see Appendix); (b) $\mathsf{IncKWS_n}$, $\mathsf{IncRPQ_n}$, $\mathsf{IncSCC_n}$ and $\mathsf{IncISO_n}$, which process unit updates in batch ΔG one by one by calling their algorithms for unit updates developed in this work; (c) DynSCC that combines the incremental algorithm in [26] to process insertions and decremental algorithm in [32] for deletions. (2) Batch algorithms BLINKS [27] for KWS, $\mathsf{RPQ_{NFA}}$ for RPQ, Tarjan for SCC, and VF2 [15] for ISO.

We did the experiments on an Amazon EC2 r3.4xlarge instance, powered by Intel Xeon processor with 2.3GHz, with



(a) Varying ΔG, KWS (DBpedia) (b) Varying ΔG, RPQ (DBpedia) (c) Varying ΔG, SCC (DBpedia) (d) Varying ΔG, ISO (DBpedia)

(e) Varying ΔG, KWS (liveJ) (f) Varying ΔG, RPQ (liveJ) (g) Varying ΔG, SCC (liveJ) (h) Varying ΔG, ISO (liveJ)

(i) Varying ΔG, SCC (Synthetic) (j) Varying Q, KWS (DBpedia) (k) Varying Q, RPQ (DBpedia) (l) Varying Q, ISO (DBpedia)

(m) Varying G, KWS (Synthetic) (n) Varying G, RPQ (Synthetic) (o) Varying G, SCC (Synthetic) (p) Varying G, ISO (Synthetic)

Figure 8: Performance evaluation

122 GB memory and 320GB SSD storage. Each experiment was run 5 times and the average is reported here.

Experimental results. We next report our findings.

Exp-1: Impact of $|\Delta G|$. We first evaluated the impact of $|\Delta G|$ on the performance of IncKWS, IncRPQ, IncSCC and IncISO, compared with (a) their batch counterparts, and (b) incremental IncKWS$_n$, IncRPQ$_n$, IncSCC$_n$ and IncISO$_n$, and DynSCC for SCC. We conducted the experiments (a) on real-life graphs by varying $|\Delta G|$ from $2.2M$ to $17.6M$ in 2.2M increments over DBpedia and from $3.7M$ to $29.6M$ in 4M increments over liveJ, which account for 5% to 40% of each graph; and (b) synthetic G with $|G| = (50M, 100M)$ by varying $|\Delta G|$ from $7.5M$ to $60M$ in 7.5M increments, *i.e.,* 5% to 40% of $|G|$, for SCC; the results for KWS, RPQ and ISO on synthetic graphs are consistent with their counterparts on real-life graphs, and hence are not reported here.

(1) KWS. Fixing $m = 3$ and $b = 2$, we report the performance of IncKWS on DBpedia and liveJ in Figures 8(a) and 8(e), respectively. We find the following. (a) IncKWS outperforms BLINKS from 6.3 times to 2.8 times over DBpedia, and from 7.3 times to 2 times over liveJ, when $|\Delta G|$ varies from 5% to 20% of $|G|$. In fact, IncKWS does better than BLINKS when $|\Delta G|$ is up to 35% and 30% of $|G|$, respectively. These verify the effectiveness of localizable incremental algorithm IncKWS. (b) IncKWS is from 1.6 to 2 and 1.3 to 1.7 times faster than IncKWS$_n$ in the same setting. This validates the effectiveness of our optimization strategies on batch updates. (c) The larger $|\Delta G|$ is, the slower IncKWS and IncKWS$_n$ are, as expected. However, when $|\Delta G|$ increases, the gap between the performance of IncKWS and IncKWS$_n$ gets larger, which is more evident on liveJ. That is, IncKWS scales better with $|\Delta G|$. In contrast, BLINKS is indifferent to $|\Delta G|$. (d) IncKWS is efficient: it takes 12 and 32 seconds over DBpedia and liveJ, respectively, when $|\Delta G|$ is 10% of $|G|$, as opposed to 61 and 146 seconds by BLINKS. (e) The ratio ρ of insertions to deletions in ΔG has no impact on the performance of IncKWS, by varying ρ while keeping $|\Delta G|$ unchanged (not shown).

(2) RPQ. We then evaluated the relatively bounded algorithm IncRPQ. Fixing $|Q| = 4$, Figures 8(b) and 8(f) show that (a) IncRPQ is from 8.6 to 3.2 times faster than RPQ$_{NFA}$ on DBpedia, and from 12.7 to 4.1 times faster on liveJ, when $|\Delta G|$ varies from 5% to 20% of $|G|$. (b) IncRPQ consistently does better than IncRPQ$_n$. The improvement is on average 2.3 times when $|\Delta G|$ is about 15% of $|G|$. (c) IncRPQ scales better with $|\Delta G|$ than IncRPQ$_n$, especially when $|\Delta G|$ is large. (d) IncRPQ is insensitive to ρ.

(3) SCC. Figures 8(c), 8(g) and 8(i) report the performance for SCC over DBpedia, liveJ and synthetic graphs, respec-

tively. We find the following. (a) IncSCC is from 8 to 1.5, 2.3 to 1.2, and 7.7 to 1.7 times faster than Tarjan over DBpedia, liveJ and synthetic graphs, respectively, when $|\Delta G|$ varies from 5% to 25% of $|G|$. These verify the effectiveness of incrementalizing batch algorithm Tarjan. It is from 1.7 to 2.6, 1.9 to 2.1, and 1.4 to 2.2 times faster than $\mathsf{IncSCC_n}$ in the same setting. (b) IncSCC performs better than DynSCC. For instance, IncSCC is on average 2.1 times faster than DynSCC when $|\Delta G|$ varies from 5% to 15% of $|G|$ over synthetic graphs. In particular, DynSCC does not do well with small $|\Delta G|$ due to its additional cost for maintaining dynamic data structures even when the output remains stable. (c) IncSCC works better on DBpedia than on liveJ since there are large scc's in liveJ, which take up to 77% of $|G|$, and need to be split in response to ΔG. (d) IncSCC is insensitive to ρ, similar to IncKWS and IncRPQ.

(4) ISO. Fixing $|Q| = (4, 6, 2)$, *i.e.,* pattern queries with 4 nodes, 6 edges and diameter 2, we evaluated localizable IncISO. As shown in Figures 8(d) and 8(h) on DBpedia and liveJ, respectively, (a) IncISO behaves better than VF2 and $\mathsf{IncISO_n}$ when $|\Delta G|$ is no more than 25% of $|G|$; it is from 5.6 to 1.8 times faster than VF2 and from 2.4 to 2.6 times faster than $\mathsf{IncISO_n}$, respectively, for $|\Delta G|$ from 5% to 25% of $|G|$. (b) The gap between the performance of IncISO and $\mathsf{IncISO_n}$ gets larger when $|\Delta G|$ grows. (c) IncISO and $\mathsf{IncISO_n}$ take longer to process edge insertions than deletions for the same $|\Delta G|$. This is because matches to be removed can be directly identified and hence, IncISO is faster for deletions. We also find that IncISO is insensitive to ρ.

(5) Unit updates. Using the same set of queries, we also evaluated the performance of these algorithms on processing unit updates, which consist of either a unit insertion or a unit edge deletion. As expected, the improvements of incremental algorithms are substantial. More specifically, IncKWS, IncRPQ, IncSCC and IncISO outperform their batch counterparts by 89 times, 221 times, 37 times, and 393 times on average, respectively (not shown). Moreover, IncSCC is 5.7 times faster than DynSCC on average.

Exp-2: Query complexity. We next evaluated the impact of queries Q, by varying different parameters of Q. We focused on KWS, RPQ and ISO, as SCC has a constant query. We fixed $|\Delta G| = 4.4M$, *i.e.,* 10% of $|G|$, and used DBpedia.

(1) KWS. We varied (m, b) from $(2, 1)$ to $(6, 5)$ for KWS queries. As shown in Figure 8(j), (a) the larger (m, b) is, the longer time is taken by all the algorithms, as expected. (b) IncKWS performs well on real-life queries. For queries with 4 keywords and bound 3, it takes 17 seconds over DBpedia, as opposed to 44 seconds by BLINKS. It works better on sparse DBpedia than on liveJ (not shown). (c) IncKWS outperforms the other algorithms, consistent with Fig. 8(a).

(2) RPQ. Varying $|Q|$ from 3 to 7, the results in Fig. 8(k) tell us the following. (a) IncRPQ is efficient: it returns answers within 190 seconds for all the queries, as opposed to 1080 seconds by $\mathsf{RPQ_{NFA}}$ and 326 seconds by $\mathsf{IncRPQ_n}$. (b) The occurrences of Kleene $*$ have little impact on all the algorithms, as the size of NFA M_Q only depends on the number of node labels in Q. (c) IncRPQ outperforms $\mathsf{RPQ_{NFA}}$ and $\mathsf{IncRPQ_n}$ on all the queries; this is consistent with Fig. 8(b).

(3) ISO. Varying $|Q| = (|V_Q|, |E_Q|, d_Q)$ from $(3, 5, 1)$ to $(7, 9, 5)$, we evaluated the impact of pattern queries. Figure 8(l) shows that all algorithms take longer over larger $|Q|$, as expected. However, (a) IncISO outperforms VF2 and $\mathsf{IncISO_n}$ in all the cases, for the same reasons given above. (b) IncISO does well: it takes 290 seconds when $|Q| = (5, 7, 3)$, but VF2 and $\mathsf{IncISO_n}$ take 1160 and 570 seconds, respectively.

Exp-3: Impact of $|G|$. We finally evaluated the impact of $|G|$ using synthetic graphs. Fixing $|\Delta G| = 15M$ and using the same set of queries tested in Exp-1, we varied $|G|$ with scale factors from 0.2 to 1. Figures 8(m), 8(n), 8(o) and 8(p) report the performance for KWS, RPQ, SCC and ISO, respectively. Observe the following. (a) All the incremental algorithms are less sensitive to $|G|$ compared with their batch counterparts. (b) Incremental algorithms scales well with $|G|$ and are feasible on large graphs.

Summary. From the experiments we find the following. (1) Incremental algorithms, either localizable or relatively bounded, are more effective than their batch counterparts in response to updates. When $|\Delta G|$ varies from 5% to 20% of $|G|$ for the three full-size graphs G, IncKWS, IncRPQ, IncSCC and IncISO outperform BLINKS, $\mathsf{RPQ_{NFA}}$, Tarjan and VF2 from 6.9 to 2.4 times, 11.6 to 2.8 times, 3.4 to 1.7 times, and 7.9 to 2 times on average, respectively. They outperform the batch algorithms even when $|\Delta G|$ is up to 30%, 35%, 25% and 25% of $|G|$, respectively. (2) Incremental algorithms scale well with $|G|$ and are feasible on real-life graphs when ΔG is small, as commonly found in practice. For instance, IncKWS, IncRPQ, IncSCC and IncISO take 9, 42, 7 and 113 seconds, respectively, when updates account for 5% of DBpedia, as opposed to 62, 355, 54 and 427 seconds by their batch counterparts. (3) Our optimization strategies for batch updates effectively improve the performance by 1.6 times on average.

7. CONCLUSION

We have established undoable and doable results for incremental graph computations. We have shown that the incremental problems for RPQ, SCC and KWS are unbounded under unit updates. However, we have proposed alternative characterizations for the effectiveness of incremental graph computations, and shown that RPQ, SCC, KWS and ISO are either localizable or bounded relative to their batch counterparts, by providing incremental algorithms with corresponding performance guarantees. Our experimental results have verified that the incremental algorithms substantially outperform their batch counterparts and scale well with large graphs, justifying the effectiveness of the new standards.

One topic for future work is to classify graph queries commonly used in practice, characterize their incremental computations, and identify performance guarantees for their incremental algorithms when possible. Another topic is to identify practical conditions under which unbounded incremental problems become bounded or relatively bounded.

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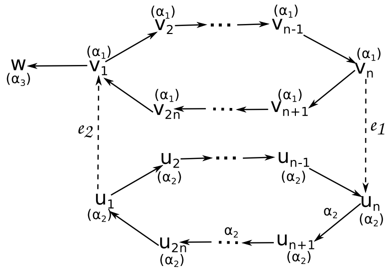

Figure 9: Unboundedness for RPQ

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Appendix: Proofs and Algorithms

Proof of Lemma 2

Assume that there exists a bounded incremental algorithm $\mathcal{T}_\Delta$ for $\mathcal{Q}_2$. We show that a bounded incremental algorithm $\mathcal{T}'_\Delta$ for $\mathcal{Q}_1$ can be built from $\mathcal{T}_\Delta$ and the Δ-reduction (f, f_i, f_o) from $\mathcal{Q}_1$ to $\mathcal{Q}_2$. Given an instance $I_1 = (Q_1, G_1)$ of $\mathcal{Q}_1$, we first compute a corresponding instance $f(I_1) = (Q_2, G_2)$ of $\mathcal{Q}_2$. Then for each update ΔG_1 to G_1, $\mathcal{T}'_\Delta$ transforms it to $f_i(\Delta G_1)$ and invokes the bounded incremental algorithm $\mathcal{T}_\Delta$ on G_2, Q_2, $Q_2(G_2)$ and $f_i(\Delta G_1)$ to obtain ΔO_2, *i.e.,* the corresponding changes to $Q_2(G_2)$. Thereafter, it transforms the updates ΔO_2 back to ΔO_1 leveraging function f_o. As (f, f_i, f_o) is a Δ-reduction, it concludes that $f_o(\Delta O_2) = \Delta O_1$, where ΔO_1 denotes the updates to $Q_1(G_1)$ in response to ΔG_1, and $\mathcal{T}'_\Delta$ takes PTIME in $|\Delta G_1| + |\Delta O_1|$ and $|Q_1|$ to compute ΔO_1, *i.e.,* $\mathcal{T}'_\Delta$ is a bounded incremental algorithm for $\mathcal{Q}_1$. From this Lemma 2 follows. □

Proof of Theorem 1

We give a proof for RPQ, and defer the proofs for SCC and KWS to [2] due to the lack of space.

RPQ. We consider first updates consisting of a unit edge deletion, and then the case of a unit edge insertion.

(1) Deletions. We prove the unboundedness of the incremental problem for RPQ under a unit edge deletion by Δ-reduction from the single source reachability problem to all vertices (SSRP). Given a graph $G = (V, E, l)$ and a node $v_s \in V$, SSRP is to decide whether node v_i is reachable from v_s for all $v_i \in V$. The answer is expressed as Boolean value $r(v_i)$ associated with v_i. The incremental problem for SSRP is unbounded under a unit edge deletion [38].

Given an instance I_1 of SSRP, *i.e.,* a graph $G_1 = (V_1, E_1, l_1)$ and a distinguished node v_s in G_1, we construct an instance I_2 of RPQ, *i.e.,* a graph $G_2 = (V_2, E_2, l_2)$ and a regular path query Q_2, by using function f such that the reachability $r(v_i)$ from v_s to v_i in G_1 changes in response to ΔG_1 iff (if and only if) there exists a corresponding change in the output of Q_2 on G_2 in response to ΔG_2, where input and output updates of the two instances are mapped by functions f_i and f_o, respectively (see Section 3).

More specifically, G_2 is constructed from G_1 with each node v_i replaced by v'_i. All the edges in G_1 remain unchanged, *i.e.,* $(v'_i, v'_j) \in E_2$ iff $(v_i, v_j) \in E_1$. Furthermore, $l_2(v'_i) = \alpha_1$ when $v'_i = v'_s$, and $l_2(v'_i) = \alpha_2$ otherwise, where v'_s corresponds to source node v_s in G_1. Query Q_2 is defined as $\alpha_1 \cdot (\alpha_2)^*$. Then one can verify that v_i is reachable from v_s in G_1 iff the node pair (v'_s, v'_i) is a match of Q_2 in G_2. Indeed, the source node of each match in $Q_2(G_2)$ must be v'_s since all paths having label α_1 originate from v'_s.

Given $\mathsf{delete}(v_i, v_j)$ in ΔG_1, function f_i returns corresponding (v'_i, v'_j) to be deleted from G_2, *i.e.,* $\Delta G_2 = f_i(\Delta G_1)$. Then the changes ΔO_2 to $Q_2(G_2)$ consist of node pairs (v'_s, v'_i) removed. Clearly, v'_i is no longer reachable from v'_s in G_2 and v_i is not reachable from v_s in G_1; hence ΔO_1 is the set of such $r(v_i)$ changed from true to false, which can be computed by $f_o(\Delta O_2)$ directly. Thus, a one-to-one mapping between the changes of I_1 and I_2 is obtained via linear-time functions f_i and f_o.

Putting these together, (f, f_i, f_o) is a Δ-reduction and RPQ is unbounded under a unit edge deletion by Lemma 2.

(2) Insertions. We next show that RPQ is unbounded under a unit edge insertion by contradiction. Consider graph G shown in Fig. 9 (excluding dotted edges), which consists of two cycles (v_1, v_2), $\ldots$, (v_{2n-1}, v_{2n}), (v_{2n}, v_1) and (u_1, u_2), $\ldots$, (u_{2n-1}, u_{2n}), (u_{2n}, u_1), and an edge (v_1, w). Each node v_i in G has label α_1 for $i \in [1, 2n]$, while u_i is labeled α_2. Node w is labeled α_3 that is distinct from α_1 and α_2. Query Q is defined as $\alpha_1 \cdot (\alpha_1)^* \cdot \alpha_2 \cdot (\alpha_2)^* \cdot \alpha_3$. Denote by Δ_1 the insertion of $e_1 = (v_n, u_n)$, and by Δ_2 the insertion of $e_2 = (u_1, v_1)$. Let graph $G_1 = G \oplus \Delta_1$, $G_2 = G \oplus \Delta_2$ and $G_3 = G_1 \oplus \Delta_2$. One can verify that $Q(G) = Q(G_1) = Q(G_2) = \emptyset$, while $Q(G_3) = \{(v_i, w) \mid i \in [1, 2n]\}$.

Assume by contradiction that there exists a bounded incremental algorithm $\mathcal{T}_\Delta$ for RPQ under a unit edge insertion. Then $\mathcal{T}_\Delta(G, Q, Q(G), \Delta_1)$ and $\mathcal{T}_\Delta(G, Q, Q(G), \Delta_2)$ are both in $O(1)$ time since only a unit update is applied to G and none of the outputs is affected for the fixed query Q. We next show that this leads to contradiction.

Let $T_s(G, \Delta G)$ denote the sequence of nodes visited in executing $\mathcal{T}_\Delta(G, Q, Q(G), \Delta G)$, referred to as its *trace*. Observe that $\mathcal{T}_\Delta(G, Q, Q(G), \Delta_2)$ and $\mathcal{T}_\Delta(G_1, Q, Q(G_1), \Delta_2)$ must behave differently as the outputs of these two are different, in which $\mathcal{T}_\Delta(G_1, Q, Q(G_1), \Delta_2)$ computes $Q(G_3)$ exactly. This can happen only if $T_s(G, \Delta_2)$ and $T_s(G_1, \Delta_2)$ contain some node associated with different information in G and G_1 as $\mathcal{T}_\Delta$ traverses the graph from the nodes involved in Δ_2, *i.e.,* u_1 or v_1. Since G_1 is obtained by applying Δ_1 to G, these nodes must be included in $T_s(G, \Delta_1)$ with information updated. Observe that if a node v in G is visited during the execution of a locally persistent algorithm $\mathcal{T}_\Delta$ to process ΔG, then each node on some undirected path from the position of ΔG to v is also inspected by $\mathcal{T}_\Delta$. Denote by v_d the the first node having different information in $T_s(G, \Delta_2)$ and $T_s(G_1, \Delta_2)$. Then $T_s(G, \Delta_1)$ and $T_s(G, \Delta_2)$ include all the nodes on an undirected path from the position of Δ_1 to that of Δ_2 through v_d. However, the length of this path is $O(n)$, which contradicts the assumption that $\mathcal{T}_\Delta(G, Q, Q(G), \Delta_1)$ and $\mathcal{T}_\Delta(G, Q, Q(G), \Delta_2)$ both take constant time. □

Localizable Algorithm for ISO (Section 4)

Recall that given a pattern query Q and a graph G, ISO is to compute the set $Q(G)$ of all matches of Q in G, *i.e.,* all subgraphs of G that are isomorphic to Q. Observe that the deletion of an edge e may cause the removal of matches that include e from $Q(G)$. Conversely, insertion of $e = (v, w)$ may add new matches to $Q(G)$ and all these matches are within $G_{d_Q}(v)$ and $G_{d_Q}(w)$, where d_Q is the length of the longest shortest path between any two nodes in Q when taken as undirected graph, *i.e.,* the *diameter* of Q.

Based on this, we outline a localizable incremental algorithm, denoted by IncISO, for ISO under batch updates (not shown). It works as follows. (1) Collect the set ΔG^- of all edge deletions in ΔG. For each edge deletion of e, remove those matches including e from $Q(G)$, by inspecting the d_Q-neighbors of the two nodes on e, where d_Q is the diameter of Q. (2) For the rest of updates in ΔG, *i.e.,* edge insertions ΔG^+, extract the union of d_Q-neighbors of the nodes involved in these edge insertions, denoted by $G_{d_Q}(\Delta G^+)$. (3) Invoke an existing batch algorithm (*e.g.,*$\mathsf{VF2}$ [15]) for ISO to compute $Q(G_{d_Q}(\Delta G^+))$ all together rather than one by one, and add those matches to $Q(G)$ that are not in $Q(G)$.

Obviously, the cost of IncISO can be expressed as a function of $|Q|$ and $|G_{d_Q}(\Delta G)|$, instead of the size $|G|$ of the entire graph G. In other words, IncISO is localizable, and hence so is ISO. Note that $G_{d_Q}(\Delta G)$ also includes the d_Q-neighbors of nodes involved in edge deletions.

Putting this together with the algorithms presented in Sections 4.2, we complete the proof of Theorem 3.

In our experimental study, we compare IncISO with another algorithm $\mathsf{IncISO_n}$, which applies the batch algorithm on d_Q-neighbor of each update one by one.

Incrementalization for SCC (Section 5.3)

Example 9: Consider deleting edge $e_5 = (c_1, a_1)$ from G of Fig. 2, which is a frond in scc_3 (see Example 6). Since the $\mathsf{lowlink}$ value of c_1 increases to 3 and equals its num after deletion, procedure $\mathsf{chkReach}$ concludes that c_1 no longer reaches root a_1 of scc_3. In light of this, IncSCC^- computes new scc's on affected scc_3 to update the output, *i.e.,* scc_3 is split into three components. The contracted graph G'_c after the deletion is also shown in Fig. 6 (solid edges). □