A study of CP violation in $B^{-/\pm} \rightarrow DK^{\pm/\mp}$ and $B^{+/\pm} \rightarrow D \pi^{+/\mp}$ decays with $D \rightarrow (KSK^{+/\mp})K^{-0} \pi^{+/\mp}$ final states
A study of CP violation in $B^{\pm} \rightarrow DK^{\pm}$ and $B^{\pm} \rightarrow D\pi^{\pm}$ decays with $D \rightarrow K_{S}^{0}K^{\pm}\pi^{\mp}$ final states

LHCb Collaboration

1. Introduction

A precise measurement of the unitarity triangle angle $\gamma = \arg(-V_{ud}V_{ub}^{\ast})$ is one of the most important tests of the Cabibbo Kohayashi Maskawa (CKM) mechanism. This parameter can be accessed through measurements of observables in decays of charged $B$ mesons to a neutral $D$ meson and a kaon or pion, where $D$ labels a $D^{0}$ or $\bar{D}^{0}$ meson decaying to a particular final state accessible to $D^{0}$ and $\bar{D}^{0}$. Such decays are sensitive to $\gamma$ through the interference between $b \rightarrow c\bar{s}s$ and $b \rightarrow u\bar{c}a$ amplitudes. They offer an attractive means to measure $\gamma$ because the effect of physics beyond the Standard Model is expected to be negligible, thus allowing interesting comparisons with other measurements where such effects could be larger.

The determination of $\gamma$ using $B^{\pm} \rightarrow DK^{\pm}$ decays was first proposed for $D$ decays to the CP-eigenstates $K^{+}K^{-}$ and $\pi^{+}\pi^{-}$ (so-called “GLW” analysis) [1,2]. Subsequently, the analysis of the $K^{\pm}\pi^{\mp}$ final state was proposed (named “ADS”) [3,4], where the suppression between the colour favoured $B^{-} \rightarrow D^{0}K^{-}$ and suppressed $B^{-} \rightarrow D^{0}K^{-}$ decays is compensated by the CKM suppression of the $D^{0} \rightarrow K^{+}\pi^{-}$ decay relative to $D^{0} \rightarrow K^{+}\pi^{-}$ resulting in large interference. The LHCb Collaboration has published the two-body ADS and GLW analyses [5], the Dalitz analysis of the decay $B^{\pm} \rightarrow [K_{S}^{0}h^{\pm}h^{\mp}]_{1}K^{\mp}$ ($h = \pi,K$) [6] and the ADS-like analysis of the decay mode $B^{\pm} \rightarrow [K^{\pm}\pi^{\mp}\pi^{\pm}\pi^{0}]_{1}K^{\pm}$ [7], where $[X]_{1}$ indicates a given final state $X$ produced by the decay of the $D$ meson. These measurements have recently been combined to yield the result $\gamma = (72.0^{+14.7}_{-15.6})^{\circ}$ [8], which is in agreement with the results obtained by the BaBar and Belle Collaborations of $\gamma = (69^{+17}_{-10})^{\circ}$ [9] and $\gamma = (68^{+15}_{-14})^{\circ}$ [10], respectively. In analogy to studies in charged $B$ meson decays, sensitivity to $\gamma$ can also be gained from decays of neutral $B$ mesons, as has been demonstrated in the LHCb analysis of $B^{0} \rightarrow [K^{+}K^{-}]_{D}K^{0}$ decays [11].

The inclusion of additional $B^{\pm} \rightarrow DK^{\pm}$ modes can provide further constraints on $\gamma$. In this Letter, an analysis of the $D \rightarrow K_{S}^{0}K^{\mp}\pi^{\mp}$ final states is performed, the first ADS-like analysis to use singly Cabibbo-suppressed (SCS) decays. The two decays, $B^{\pm} \rightarrow [K_{S}^{0}K^{\mp}\pi^{\mp}]_{1}h^{\pm}$ and $B^{\pm} \rightarrow [K_{S}^{0}K^{\mp}\pi^{\mp}]_{1}h^{\mp}$, are distinguished by the charge of the $K^{\mp}$ from the decay of the $D$ meson relative to the charge of the $B$ meson, so that the former is labelled “same sign” (SS) and the latter “opposite sign” (OS).

In order to interpret CP-violating effects using these three-body decays it is necessary to account for the variation of the $D$ decay strong phase over its Dalitz plot due to the presence of resonances between the particles in the final state. Instead of employing an amplitude model to describe this phase variation, direct measurements of the phase made by the CLEO Collaboration are used, which are averaged over large regions of the Dalitz plot [12]. The same CLEO study indicates that this averaging can be employed without a large loss of sensitivity. The use of the CLEO results avoids the need to introduce a systematic uncertainty resulting from an amplitude model description.

The analysis uses the full 2011 and 2012 LHCb pp collision data sets, corresponding to integrated luminosities of 1 and 2 fb$^{-1}$ and centre-of-mass energies of $\sqrt{s} = 7$ TeV and 8 TeV, respectively. The results are measurements of CP-violating observables that can be interpreted in terms of $\gamma$ and other hadronic parameters of the $B^{\pm}$ meson decay.
2. Formalism

The decay $B^+ \rightarrow [K^0_S K^+ \pi^-]_{D} K^+$ can proceed via a $D^0$ or $\bar{D}^0$ meson, so that the decay amplitude is the sum of two amplitudes that interfere,

$$A(m_{e2K^+K^-}, m_{e2\pi}^2) = A_{D^0}(m_{e2K^+}, m_{e2\pi}^2) + r_B e^{i(\delta_B + \gamma)} A_{\bar{D}^0}(m_{e2K^+}, m_{e2\pi}^2),$$

(1)

where $A_{D^0}(m_{e2K^+}, m_{e2\pi}^2)$ are the $D^0$ and $\bar{D}^0$ decay amplitudes to a specific point in the $K^0_S K^+\pi^-$ Dalitz plot. The amplitude ratio $r_B$ is $r_B = \frac{|A_{D^0} + A_{\bar{D}^0}|}{|A_{D^0} - A_{\bar{D}^0}|} = 0.089 \pm 0.009$ [8] and $\delta_B$ is the strong phase difference between the $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow \bar{D}^0 K^+$ decays. The calculation of the decay rate in a region of the Dalitz plot requires the evaluation of the integral of the interference term between the two $D$ decay amplitudes over that region. Measurements have been made by the CLEO Collaboration [12], where quantum-correlated $D$ decays are used to determine the integral of the interference term directly in the form of a "coherence factor", $\kappa_{K^0_S K^+\pi^-}$, and an average strong phase difference, $\delta_{K^0_S K^+\pi^-}$, as first proposed in Ref. [13]. The coherence factor can take a value between 0 and 1 and is defined through the expression

$$\kappa_{K^0_S K^+\pi^-} = \frac{s_D}{s_{\bar{D}}} \bigg[ \int A^*_{D}(m_{e2K^+}, m_{e2\pi}^2) A_{\bar{D}}(m_{e2K^+}, m_{e2\pi}^2) \, \delta m_{e2K^+} \, \delta m_{e2\pi} \bigg],$$

(2)

where $A_{\bar{D}} = \int A_{D_{K^+\pi^-}}(m_{e2K^+}, m_{e2\pi}^2) \, \delta m_{e2K^+} \, \delta m_{e2\pi}$.

This avoids the significant modelling uncertainty incurred by the determination of the strong phase difference between the $D^0$ and $\bar{D}^0$ amplitudes at each point in the Dalitz region from an amplitude model. The decay rates, $\Gamma$, to the four final states can therefore be expressed as

$$\Gamma_{SS, DK}^{\pm} = \bigg[ 1 + r_D^2 \bigg] \frac{z}{2} r_B \frac{r_B}{r_D} \kappa_{K^0_S K^+\pi^-} \cos(\delta_B \pm \gamma - \delta_{K^0_S K^+\pi^-})$$

(3)

where $r_D$ is the amplitude ratio for $D^0 \rightarrow K^0_S K^+\pi^-$ with respect to $D^0 \rightarrow K^0_S K^-\pi^+$ and $z$ is the normalisation factor. Analogous equations can be written for the $B^+ \rightarrow D^+\pi^+$ system, with $r_D^2$ replacing $r_B$ and $\delta_B$ respectively. Less interference is expected in the $B^+ \rightarrow D\pi^+$ system where the value of $r_D^2$ is much lower, approximately 0.015 [8]. These expressions receive small corrections from mixing in the charm system, which, though accounted for in Section 7, are not explicitly written here. At the current level of precision these corrections have a negligible effect on the final results.

Observables constructed using the decay rates in Eq. (3) have a sensitivity to $\gamma$ that depends upon the value of the coherence factor, with a higher coherence corresponding to greater sensitivity. The CLEO Collaboration measured the coherence factor and average strong phase difference in two regions of the Dalitz plot: firstly across the whole Dalitz plot, and secondly within a region $\pm 100$ MeV/c$^2$ around the $K^*(892)^+$ resonance, which decays to $K^0_S \pi^+$, where, though the sample size is diminished, the coherence is higher. The measured values are $\kappa_{K^0_S K^+\pi^-} = 0.73 \pm 0.08$ and $\delta_{K^0_S K^+\pi^-} = 8.3 \pm 15.2^\circ$ for the whole Dalitz plot, and $\kappa_{K^0_S K^+\pi^-} = 1.00 \pm 0.16$ and $\delta_{K^0_S K^+\pi^-} = 26.5 \pm 15.8^\circ$ in the restricted region. The branching fraction ratio of $D^0 \rightarrow K^0_S K^+\pi^-$ to $D^0 \rightarrow K^0_S K^-\pi^+$ decays was found to be $0.592 \pm 0.044$ in the whole Dalitz plot and $0.356 \pm 0.034$ in the restricted region [12].

Eight yields are measured in this analysis, where seven observables are constructed. The charge asymmetry is measured in each of the four decay modes, defined as $A_{SS, DK} = \frac{N_{K^0_S K^+\pi^-} - N_{K^0_S K^-\pi^+}}{N_{K^0_S K^+\pi^-} + N_{K^0_S K^-\pi^+}}$ for the $B^+ \rightarrow [K^0_S K^+\pi^+]_{D} K^+$ mode and analogously for the other modes. The ratios of $B^+ \rightarrow DK^+$ and $B^+ \rightarrow D\pi^+$ yields are determined for the SS and OS decays, $\Gamma_{SS, DK}/\Gamma_{OS, DK}$ and $\Gamma_{SS, DK}/\Gamma_{OS, DK}$ respectively, and the ratio of SS to OS $B^+ \rightarrow D\pi^+$ yields, $\Gamma_{SS}/\Gamma_{OS}$, is measured. The measurements are performed both for the whole Dalitz plot and in the restricted region around the $K^*(892)^+$ resonance.

3. The LHCb detector and data set

The LHCb detector [14] is a single-arm forward spectrometer covering the pseudorapidity range $2 < \eta < 5$, designed for the study of particles containing b or c quarks. The detector includes a high-precision tracking system consisting of a silicon-strip vertex detector surrounding the pp interaction region, a large-area silicon-strip detector located upstream of a dipole magnet with a bending power of about 4 Tm, and three stations of silicon-strip detectors and straw drift tubes placed downstream. The combined tracking system provides a momentum measurement with relative uncertainty that varies from 0.4% at 5 GeV/c to 0.6% at 100 GeV/c, and impact parameter (IP) resolution of 20 μm for tracks with large transverse momentum. Different types of charged hadrons are distinguished by particle identification (PID) information from two ring-imaging Cherenkov (RICH) detectors [15]. Photon, electron and hadron candidates are identified by a calorimeter system consisting of scintillating-pad and preshower detectors, an electromagnetic calorimeter and a hadronic calorimeter. Muons are identified by a system composed of alternating layers of iron and multiwire proporitional chambers.

The trigger consists of a hardware stage, based on information from the calorimeter and muon systems, followed by a software stage, which applies a full event reconstruction. The software trigger searches for a track with large $p_T$ and large IP with respect to any pp interaction point, also called a primary vertex (PV). This track is then required to be part of a two-, three- or four-track secondary vertex with a high-p$^2$ sum, significantly displaced from any PV. A multivariate algorithm [16] is used for the identification of secondary vertices consistent with the decay of a b hadron.

Samples of around two million $B^\pm \rightarrow [K^0_S K^+\pi^\pm]_{D} K^\pm$ and two million $B^\pm \rightarrow [K^0_S K^+\pi^\pm]_{D} K^\pm$ decays are simulated to be used in the analysis, along with similarly-sized samples of $B^\pm \rightarrow [K^0_S K^+\pi^-]_{D} \pi^\pm$, $B^\pm \rightarrow [K^0_S K^+K^-]_{D} \pi^\pm$ and $B^\pm \rightarrow [K^0_S K^+\pi^-]_{D} \pi^\pm$ decays that are used to study potential backgrounds. In the simulation, $pp$ collisions are generated using PyTHIA [17] with a specific LHCb configuration [18]. Decays of hadronic particles are described by EvtGen [19], in which final state radiation is generated using Photos [20]. The interaction of the generated particles with the detector and its response are implemented using the GEANT4 toolkit [21] as described in Ref. [22].

4. Candidate selection

Candidate $B^+ \rightarrow [K^0_S K^+\pi^+]_{D} K$ and $B^+ \rightarrow [K^0_S K^+\pi^+]_{D} \pi$ decays are reconstructed in events selected by the trigger and then the candidate momenta are refit, constraining the masses of the neutral $D$ and $K^0_S$ mesons to their known values [23] and the $B^\pm$ meson to originate from the primary vertex [24]. Candidates where the $K^0_S$ decay is reconstructed using "long" pion tracks, which
leave hits in the VELO and downstream tracking stations, are ana-
ysed separately from those reconstructed using “downstream” pion
tracks, which only leave hits in tracking stations beyond the VELO.
The signal candidates in the former category are reconstructed with
a better invariant mass resolution.

The reconstructed masses of the $D$ and $K_S^0$ mesons are re-
quired to be within 25 MeV/$c^2$ and 15 MeV/$c^2$, respectively, of
their known values. Candidate $B^+ \rightarrow D^+ K^\pm$ decays are separated from $B^+ \rightarrow D\pi^\pm$ decays by using PID information from the
RICH detectors. A boosted decision tree (BDT) [25,26] that has been
developed for the analysis of the topologically similar decay mode
$B^+ \rightarrow [K_S^0 h^+ h^-]_0 h^{+\mp}$ is applied to the reconstructed candidates.
The BDT was trained using simulated signal decays, generated uni-
formly over the $D^0$ Dalitz plot, and background candidates taken from
the $B^\pm$ invariant mass region in data between 5700 and
7000 MeV/$c^2$. It exploits the displacement of tracks from the de-
cays of long-lived particles with respect to the PV through the use of $\chi^2$ variables, where $\chi^2$ is defined as the difference in $\chi^2$ of a
given PV fit with and without the considered track. The BDT also
employs the $B^\pm$ and $D$ candidate momenta, an isolation variable
sensitive to the separation of the tracks used to construct the $B^\pm$
candidate from other tracks in the event, and the $\chi^2$ per degree of
freedom of the decay refit. In addition to the requirement placed
on the BDT response variable, each composite candidate is required
to have a vector displacement of production and decay vertices
that aligns closely to its reconstructed momenta. The cosine of
the angle between the displacement and momentum vectors is re-
quired to be less than 0.142 rad for the $K_S^0$ and $D^0$ candidates,
and less than 0.0141 rad (0.0100 rad) for long (downstream) $B^\pm$
candidates.

Additional requirements are used to suppress backgrounds from
specific processes. Contamination from $B$ decays that do not con-
tain an intermediate $D$ meson is minimised by placing a minimum threshold of 0.2 ps on the decay time of the $D$ candidate. A po-
tential background could arise from processes where a pion is
misidentified as a kaon or vice versa. One example is the relatively
abundant mode $B^+ \rightarrow [K^0_S \pi^+ \pi^-]_0 h^\pm$, which has a branching frac-
tion around ten times larger than the signal. These are suppressed by
placing requirements on both the $D$ daughter pion and kaon, mak-
ing use of PID information. For $K_S^0$ candidates formed from
long tracks, the flight distance $\chi^2$ of the candidate is used to sup-
press background from $B^+ \rightarrow [K^0_S \pi^+ \pi^-]_0 h^\pm$ decays. Where
multiple candidates are found belonging to the same event, the can-
didate with the lowest value of the refit $\chi^2$ per degree of
freedom is retained and any others are discarded, leading to a re-
duction in the sample size of approximately 0.3%.

The $B^\pm$ invariant mass spectra are shown in Fig. 1 for candi-
dates selected in the whole $D$ Dalitz plot, overlaid with a para-
metric fit described in Section 5. The $D$ Dalitz plots are shown in
Fig. 2 for the $B^+ \rightarrow D K^\pm$ and $B^\pm \rightarrow D\pi^\pm$ candidates that
fall within a nominal $B^\pm$ signal region in $B^0$ invariant mass
(5247–5317 MeV/$c^2$). The dominant $K^*(892)^+$ resonance is clearly
visible within a horizontal band, and the window around this res-
onance used in the analysis is indicated.

5. Invariant mass fit

In order to determine the signal yields in each decay mode, si-
multaneous fits are performed to the $B^\pm$ invariant mass spectra in
the range 5110 MeV/$c^2$ to 5800 MeV/$c^2$ in the different modes,
both for candidates in the whole $D$ Dalitz plot, and for only those
inside the restricted region around the $K^*(892)^+$ resonance. The
data samples are split according to the year in which the data were
taken, the decay mode, the $K_S^0$ type and the charge of the $B$
candidate. The fit is parameterised in terms of the observables described
in Section 2, rather than varying each signal yield in each category
independently.

The probability density function (PDF) used to model the signal
component is a modified Gaussian function with asymmetric tails,
where the unnormalised form is given by

$$ f(m; m_0, \alpha_L, \alpha_R, \sigma) = \begin{cases} \exp[-(m-m_0)^2/(2\alpha_L^2 + \alpha_L(m-m_0)^2)) & \text{for } m < m_0, \\ \exp[-(m-m_0)^2/(2\alpha_R^2 + \alpha_R(m-m_0)^2)) & \text{for } m > m_0. \end{cases} $$

where $m$ is the reconstructed mass, $m_0$ is the mean $B$ mass and $\sigma$
determines the width of the function. The $\alpha_L,R$ parameters govern
the shape of the tail. The mean $B$ mass is shared among all catego-
ries but is allowed to differ according to the year in which the data
were collected. The $\alpha_L$ parameters are fixed to the values de-
termined in the earlier analysis of $B^\pm \rightarrow [K_S^0 \pi^+ \pi^-]_0 h^\pm$ [6]. The $\alpha_R$ parameters are common to the $B^\pm \rightarrow D\pi^\pm$ and $B^+ \rightarrow D^{*+}$,
SS and OS categories, and are allowed to vary in the fit. Only the
width parameters $\alpha_R(B^\pm \rightarrow D\pi^\pm)$ are allowed to vary in the fit. The
ratios $\sigma(B^\pm \rightarrow D\pi^\pm)/\sigma(B^+ \rightarrow D^{*+})$ are fixed accord-
ing to studies of the similar mode $B^\pm \rightarrow [K_S^0 \pi^+ \pi^-]_0 h^\pm$. The fitted values
for $\sigma(B^+ \rightarrow D^{*+})$ vary by less than 10% around 14 MeV/$c^2$.
The total yield of $B^\pm \rightarrow D\pi^\pm$ decays is allowed to vary between
the different $K_S^0$ type and year categories. The yields in the vari-
ous $D$ decay modes and different charges, and all the $B^\pm \rightarrow D^{*+}$
yields, are determined using the observables described in Sec-
tion 2, rather than being fitted directly.

In addition to the signal PDF, two background PDFs are re-
quired. The first background PDF models candidates formed from
random combinations of tracks and is represented by a linear func-
ction. In the fit within the restricted Dalitz region, where the sam-
ple size is significantly smaller, the slope of the linear function
fitting the $B^\pm \rightarrow D\pi^\pm$ data is fixed to the value determined
in the fit to the whole Dalitz plot. The second background PDF ac-
counts for contamination from partially reconstructed processes.
Given that the contamination is dominated by those processes that
involve a real $D^0 \rightarrow K_S^0 K^{\pm} \pi^{\mp}$ decay, the PDF is fixed to the shape
determined from the more abundant mode $B^+ \rightarrow [K^+ \pi^-]_0 h^\pm$. The
yields of both these background components are free to vary in
each data category.

A further significant background is present in the $B^\pm \rightarrow D^{*+}$
samples due to $\pi \rightarrow K$ misidentification of the much more abund-
ent $B^\pm \rightarrow D\pi^\pm$ mode. This background is modelled in the $B^\pm \rightarrow D^{*+}$
spacings using a Crystal Ball function [27], where the para-
eters of the function are common to all data categories in
the fit and are allowed to vary. The yield of the background in
the $B^\pm \rightarrow D^{*+}$ samples is fixed with respect to the fitted $B^\pm \rightarrow D\pi^\pm$ signal yield using knowledge of the RICH particle identi-
fication efficiencies that is obtained from data using samples of
$D^{*+} \rightarrow [K\pi^0]_0 \pi^\pm$ decays. The efficiency for kaons to be selected is
found to be around 84% and the misidentification rate for pions is
around 4%.

Production and detection asymmetries are accounted for, fol-
lowing the same procedure as in Refs. [5,7]. Values for the $B^\pm$
production and $K$ detection asymmetries are assigned such that the
combination of production and detection asymmetries corresponds
to the raw asymmetry observed in $B^\pm \rightarrow J/\psi K^{\pm}$ decays [28].
The detection asymmetry assigned is $-0.5 \pm 0.7$% for each unit of
strangeness in the final state to account for the differing inter-
actions of $K^+$ and $K^-$ mesons with the detector material. An ana-
logous asymmetry is present for pions, though it is expected to be
much smaller, and the detection asymmetry assigned is $0.0 \pm 0.7$%
Any potential asymmetry arising from a difference between the re-
ponses of the left and right sides of the detector is minimised by
Fig. 1. Distributions of $B^\pm$ invariant mass of the SS and OS samples for the (a, c, e, g) $B^+ \rightarrow DK^-$ and (b, d, f, h) $B^\mp \rightarrow D\pi^\pm$ candidates in the full data sample. The fits are shown for (a, b, e, f) $B^+$ and (c, d, g, h) $B^-$ candidates. Fit PDFs are superimposed.

A further correction is included to account for non-uniformities in the acceptance over the Dalitz plot. This efficiency correction primarily affects the $R_{SS}$ observable, given the difference in the Dalitz distributions for the two $D$ meson decay modes. A correction factor, $\zeta$, is found by combining the LHCb acceptance, extracted from the simulated signal sample, and amplitude models, $A_{SS,OS}(m_{K^0_S K^-}, m_{K^0_S \pi})$, for the Dalitz distributions of the SS or OS decays,

$$
\zeta \equiv \frac{\int D \, dm_{K^0_S K^-}^2 \, dm_{K^0_S \pi}^2 \, |\epsilon(m_{K^0_S K^-}, m_{K^0_S \pi}) \times |A_{OS}(m_{K^0_S K^-}, m_{K^0_S \pi})|^2|}{\int D \, dm_{K^0_S K^-}^2 \, dm_{K^0_S \pi}^2 \, |\epsilon(m_{K^0_S K^-}, m_{K^0_S \pi}) \times |A_{SS}(m_{K^0_S K^-}, m_{K^0_S \pi})|^2|},
$$

where $\epsilon(m_{K^0_S K^-}, m_{K^0_S \pi})$ is the efficiency at a point in the Dalitz plot. The typical deviation of $\zeta$ from unity is found to be around 5%. The acceptance is illustrated in Fig. 3, where bins of variable size are used to ensure that statistical fluctuations due to the finite size of the simulated sample are negligible. The Dalitz distributions are
The signal purity in a process is around 44% [12]. This accounts for the higher value of the signal yield derived from the fit to data, and are typically corrected accordingly. The biases are determined by generating and fitting a large number of simulated samples using input values obtained from the fit to data, and are typically found to be around 2%. The fit results are corrected accordingly.

Due to the restricted sample size under study, small biases exist in the determination of the observables. The biases are determined by generating and fitting a large number of simulated samples using input values obtained from the fit to data, and are typically found to be around 2%. The fit results are corrected accordingly.

The fit projections, with long and downstream $K_S^0$-type categories merged and 2011 and 2012 data combined, are given for the fit to the whole Dalitz plot in Fig. 1. The signal purity in a nominal mass range from 5247 MeV/c² to 5317 MeV/c² is around 85% for the $B^+ \to D \pi^+$ decays and around 96% for the $B^0 \to D \pi^+$ decays. The signal yields derived from the fits to both the whole Dalitz distribution and the restricted region around the $K^*(892)^0$ resonance are given by considering the relevant Dalitz divisions in which the acceptance is determined, and the limited size of the simulated sample available to determine the acceptance. The correlations between $R_{SS/OS}$ and the ratios $R_{DK/DT,SS}$ and $R_{DK/DT,OS}$ are -16% (-13%) and +16% (+16%), respectively, for the fit to the whole Dalitz plot ($K^*(892)^0$ region).

The correlation between the $R_{DK/DT,SS}$ and $R_{DK/DT,OS}$ ratios is +11% (+15%). Correlations between the asymmetry observables are all less than 1% and are neglected.

### Table 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>Whole Dalitz plot</th>
<th>$K^*(892)^0$ region</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS</td>
<td>145 ± 15</td>
<td>1841 ± 47</td>
</tr>
<tr>
<td>OS</td>
<td>71 ± 10</td>
<td>1267 ± 37</td>
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### Table 2

<table>
<thead>
<tr>
<th>Observable</th>
<th>Whole Dalitz plot</th>
<th>$K^*(892)^0$ region</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{SS,OS}$</td>
<td>1.528 ± 0.058 ± 0.025</td>
<td>2.57 ± 0.13 ± 0.06</td>
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<tr>
<td>$R_{DK,SS}$</td>
<td>0.092 ± 0.009 ± 0.004</td>
<td>0.084 ± 0.011 ± 0.003</td>
</tr>
<tr>
<td>$R_{DK,OS}$</td>
<td>0.066 ± 0.009 ± 0.002</td>
<td>0.056 ± 0.013 ± 0.002</td>
</tr>
<tr>
<td>$A_{SS,DK}$</td>
<td>0.040 ± 0.091 ± 0.018</td>
<td>0.026 ± 0.109 ± 0.029</td>
</tr>
<tr>
<td>$A_{OS,DK}$</td>
<td>0.233 ± 0.129 ± 0.024</td>
<td>0.336 ± 0.208 ± 0.026</td>
</tr>
<tr>
<td>$A_{SS,DT}$</td>
<td>-0.025 ± 0.024 ± 0.010</td>
<td>-0.012 ± 0.028 ± 0.010</td>
</tr>
<tr>
<td>$A_{OS,DT}$</td>
<td>-0.052 ± 0.029 ± 0.017</td>
<td>-0.054 ± 0.043 ± 0.017</td>
</tr>
</tbody>
</table>

### 6. Systematic uncertainties

The largest single source of systematic uncertainty is the knowledge of the efficiency correction factor that multiplies the $R_{SS/OS}$ observable. This uncertainty has three sources: the uncertainties on the CLEO amplitude models, the granularity of the Dalitz divisions in which the acceptance is determined, and the limited size of the simulated sample available to determine the LHCb acceptance. Of these, it is the modelling uncertainty that is dominant. In addition, an uncertainty is assigned to account for the fact that interference is neglected in the computation of the efficiency correction factor, which is shared between the $D \pi^+$ and $D K^+$ systems.

Uncertainties on the parameters that are fixed in the PDF are propagated to the observables by repeating the fit to data whilst...
varying each fixed parameter according to its uncertainty. An additional systematic uncertainty is calculated for the fit to the restricted $K^*(892)^\pm$ region, where the $D\pi^\pm$ combinatorial background slopes are fixed to the values determined in the fit to the whole Dalitz plot.

Uncertainties are assigned to account for the errors on the $B^\pm$ production asymmetry and the $K^\pm$ and $\pi^\pm$ detection asymmetries. The effect of the detection asymmetry depends on the pion and kaon content of the final state, and the resulting systematic uncertainty is largest for the $A_{SS,DK}$ and $A_{OS,DS}$ observables.

The absolute uncertainties on the particle identification efficiencies are small, typically around 0.3% for kaon efficiencies and 0.03% for pion efficiencies. Of the four main sources of systematic error, these result in the smallest uncertainties on the experimental observables.

In Table 3, the sources of systematic uncertainty are given for each observable in the fit to the whole Dalitz plot. Similarly those for the fit in the restricted region are given in Table 4.

### 7. Interpretation and conclusions

The sensitivity of this result to the CKM angle $\gamma$ is investigated by employing a frequentist method to scan the $\gamma - r_B$ parameter space and calculate the $\chi^2$ probability at each point, given the measurements of the observables with their statistical and systematic uncertainties combined in quadrature, accounting for correlations between the statistical uncertainties. The effects of charm mixing are accounted for, but $CP$ violation in the decays of $D$ mesons is neglected. Regions of $1\sigma$, $2\sigma$ and $3\sigma$ compatibility with the measurements made are indicated by the dark, medium and light blue regions, respectively, in Fig. 4. The small sample size in the current data set results in a bound on $\gamma$ that is only closed for the $1\sigma$ contour.

Although it is not possible to measure $\gamma$ directly using these results alone, it is of interest to consider how this result relates to the previous LHCb $\gamma$ determination, obtained from other $B^\pm \to D K^{\pm}$ modes [8], since it will be included in future combinations. In order to aid this comparison, the scans of the $\gamma - r_B$ space plots are shown in Fig. 4(a) for the measurement made using the whole $D \to K^{0}_S K\pi$ Dalitz plot and in Fig. 4(b) for that made in the restricted region. The current LHCb average, extracted from a combination of $B^{\pm} \to D K^{\pm}$ analyses [8], is shown as a point with error bars at $\gamma = 72.0^\circ$ and $r_B = 0.089$. The LHCb average lies within the $2\sigma$ region allowed by the measurements presented in this Letter. It is interesting to note that the bound determined in the restricted region space indicates a more stringent constraint when using the restricted region, where the coherence is higher. This, and the fact that the measurements in this Letter are limited by their statistical precision, motivates the use of this region in future analyses of these decays in a larger data sample. Combination with analyses in other, more abundant channels with sensitivity to the same parameters will yield more stringent constraints upon $\gamma$.

In summary, for the first time a measurement of charge asymmetries with associated observables is presented in the decay modes $B^{\pm} \to [K^{0}_S K^{\mp} \pi^{\pm}]_{dh}^{\pm}$ and $B^{\pm} \to [K^{0}_S K^{\mp} \pi^{\pm}]_{dh}^{\mp}$, and no significant $CP$ violation is observed. The results of the analysis are consistent with other measurements of observables in related $B^{\pm} \to D K^{\pm}$ modes, and will be valuable in future global fits of the CKM parameter $\gamma$.

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