Elasticity of phase-Pi (Al₃Si₂O₇(OH)₃) - A hydrous aluminosilicate phase

Citation for published version:

Digital Object Identifier (DOI):
10.1016/j.pepi.2017.05.016

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
Physics of the Earth and Planetary Interiors

General rights
Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact openaccess@ed.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.
Accepted Manuscript

Elasticity of phase-Pi (Al$_3$Si$_2$O$_7$(OH)$_3$)- a hydrous aluminosilicate phase

Ye Peng, Mainak Mookherjee, Andreas Hermann, Suraj Bajgain, Songlin Liu, Bernd Wunder

PII: S0031-9201(17)30027-4
DOI: http://dx.doi.org/10.1016/j.pepi.2017.05.016
Reference: PEPI 6044

To appear in: Physics of the Earth and Planetary Interiors

Received Date: 27 January 2017
Revised Date: 11 May 2017
Accepted Date: 22 May 2017

Please cite this article as: Peng, Y., Mookherjee, M., Hermann, A., Bajgain, S., Liu, S., Wunder, B., Elasticity of phase-Pi (Al$_3$Si$_2$O$_7$(OH)$_3$)- a hydrous aluminosilicate phase, Physics of the Earth and Planetary Interiors (2017), doi: http://dx.doi.org/10.1016/j.pepi.2017.05.016

This is a PDF file of an unedited manuscript that has been accepted for publication. As a service to our customers we are providing this early version of the manuscript. The manuscript will undergo copyediting, typesetting, and review of the resulting proof before it is published in its final form. Please note that during the production process errors may be discovered which could affect the content, and all legal disclaimers that apply to the journal pertain.
Elasticity of phase-Pi (Al$_3$Si$_2$O$_7$(OH)$_3$)- a hydrous aluminosilicate phase

Ye Peng$^1$, Mainak Mookherjee$^1$, Andreas Hermann$^2$, Suraj Bajgain$^1$, Songlin Liu$^{1,3}$, and Bernd Wunder$^4$

$^1$Earth Materials Laboratory, Department of Earth, Ocean and Atmospheric Sciences, Florida State University, Tallahassee, FL 32306, USA

$^2$School of Physics and Astronomy, James Clerk Maxwell Building, The University of Edinburgh, Edinburgh, EH9 3FD, U.K.

$^3$CAS Key Laboratory of Crust-Mantle Materials and Environments, School of Earth and Space Sciences, University of Science and Technology of China, Hefei, China

$^4$GFZ, German Research Center for Geoscience, 14473 Potsdam, Germany

*corresponding author’s email: mmookherjee@fsu.edu

Abstract: Phase-Pi (Al$_3$Si$_2$O$_7$(OH)$_3$) is an aluminosilicate hydrous mineral and is likely to be stable in hydrated sedimentary layers of subducting slabs. Phase-Pi is likely to be stable between the depths of 60 and 200 km and is likely to transport water into the Earth’s interior. Here, we use first principles simulations based on density functional theory to explore the crystal structure at high-pressure, equation of state, and full elastic stiffness tensor as a function of pressure. We find that the pressure volume results could be described by a finite strain fit with $V_0$, $K_0$, and $K_0'$ being 310.3 Å$^3$, 133 GPa, and 3.6 respectively. At zero pressure, the full elastic stiffness tensor shows significant anisotropy with the diagonal principal components $C_{11}$, $C_{22}$, and $C_{33}$ being 235, 292, 266 GPa respectively, the diagonal shear $C_{44}$, $C_{55}$, and $C_{66}$ being 86, 92, and 87 GPa
respectively, and the off-diagonal stiffness $C_{12}, C_{13}, C_{14}, C_{15}, C_{16}, C_{23}, C_{24}, C_{25}, C_{26}, C_{34}, C_{35}, C_{36}, C_{45}, C_{46}$, and $C_{56}$ being $73, 78, 6, -30, 15, 61, 17, 2, 1, -13, -15, 6, 3, 1, and 3 GPa respectively. The zero pressure, shear modulus, $G_0$ and its pressure derivative, $G'_0$ are 90 GPa and 1.9 respectively. Upon compression, hydrogen bonding in phase-Pi shows distinct behavior, with some hydrogen bonds weakening and others strengthening. The latter eventually undergo symmetrization, at pressure greater (> 40 GPa) than the thermodynamic stability of phase-Pi. Full elastic constant tensors indicate that phase-Pi is very anisotropic with $AV_p \sim 22.4\%$ and $AV_s \sim 23.7\%$ at 0 GPa. Our results also indicate that the bulk sound velocity of phase-Pi is slower than that of the high-pressure hydrous aluminosilicate phase, topaz-OH.

**Key-words:** High-Pressure, phase-Pi, equation of state, elasticity, anisotropy, subduction zone

**1. Introduction**

Water plays an important role in the long-term sustenance of solid Earth activities. For instance, water lowers the melting temperatures of silicate rocks and could lead to melting in the deep Earth (Hirschmann, 2006). Water, present in trace quantities within nominally anhydrous minerals, affects the transport properties including rheology (e.g., Mei and Kohlstedt, 2000), viscosity (e.g., Ichikawa et al., 2015) and electrical conductivity (e.g., Wang et al., 2006). Water is transported into the deep Earth via subduction of hydrated lithosphere containing hydrous mineral phases (Kawamoto, 2006). These minerals are distinct from the nominally anhydrous phases and contain
structurally bound hydroxyl groups i.e., OH\(^{-}\) groups occur in well-defined crystallographic sites (Smyth, 2006).

It is well known that sedimentary layers are typically less dense compared to the subducting oceanic crust and lithospheric mantle and hence may not readily subduct. However, based on geochemical evidences i.e., \(^{10}\)Be signature in island arc volcanism, it is well known that sedimentary layers do undergo subduction and are recycled back at arc (Tera et al., 1986, Morris and Tera, 1989, White and Dupre, 1986). Estimates based on deep sea drilling suggest that the amount of sediment subduction is of the order of ~1-2 x10\(^{15}\) g/yr (Plank and Langmuir, 1993; Rea and Ruff, 1996). Among the sediments, contribution from terrigenous sediments dominates. Contribution from carbonate sediment is minor (Rea and Ruff, 1996). The sedimentary layer also helps to subduct water on the order of 1 x10\(^{15}\) g/year (Rea and Ruff, 1996). Part of the subducted water is recycled back through island arc volcanism, however a part of the water is carried into the deep Earth. The water helps in hydrating the subducting lithosphere and the overlying mantle. While the net water contribution from oceanic crust is likely to be greater than the sediments (Peacock, 1990), owing to the greater thermal stability of the minerals stabilized in sedimentary layer, they may be effective in transporting water into the deep Earth in warmer subduction zones (Ono, 1998).

The mineralogy of subducted sediments can be understood within a simplified ternary system, where the end member chemical components are represented by- Al\(_2\)O\(_3\)-SiO\(_2\)-H\(_2\)O (ASH) (Peacock, 1990; Ono, 1999; Schmidt et al., 1998; Schreyer et al., 1995; Wunder et al., 1993a,b). Several hydrous mineral phases are stable in the ASH ternary system. These include gibbsite (Al(OH)\(_3\)), diaspore (AlOOH), kaolinite (Al\(_2\)Si\(_2\)O\(_5\)(OH)\(_4\)),
topaz-OH (Al₂SiO₄(OH)₂), phase-Pi (Al₃Si₂O₇(OH)₃), phase Egg (AlSiO₃(OH)), and dense high-pressure phases such as δ-AlOOH. Among these various phases, almost nothing is known about the high-pressure behavior of phase-Pi. However, it is likely to play an important role in transporting water into the deep Earth. High-pressure phase relations have shown that phase-Pi is stable between 2 and 7 GPa (Wunder et al., 1993a,b). In this study, we explore the behavior of phase-Pi at high-pressure. We have used first principles simulations to explore how pressure affects the crystal structure and proton environments, equation of state, full elastic constant tensor and elastic anisotropy at high-pressures.

2. Method

In this study, we examined the crystal structure, equation of state, and elasticity of phase-Pi (Al₃Si₂O₇(OH)₃), using static density functional theory calculations using the Vienna ab-initio simulation package (VASP) (Hohenberg and Kohn, 1964; Kohn and Sham, 1965; Kresse and Hafner, 1993; Kresse and Furthmüller, 1996a, b; Kresse and Joubert, 1999). We used generalized gradient approximation (GGA) (PBE:Perdew et al., 1998) and the highly accurate projector augmented wave method (PAW) as implemented in VASP (Kresse and Joubert, 1999). It is known that PBE often describes the energetics and elasticity of hydrous phases better than the local density approximation (LDA) (Mookherjee and Mainprice, 2014; Mookherjee and Tsuchiya, 2015). We performed a series of convergence tests by varying the energy cutoff and k-points. We found that an energy cut-off $E_{\text{cut}} = 800$ eV and a k-point mesh of $3 \times 3 \times 3$ Monkhorst-Pack grid
(Monkhorst and Pack, 1976) with 14 irreducible k-points is sufficient for describing the energetics of phase-Pi (Supplementary Table 1).

We determined the full elastic constant tensor by straining the lattice parameters by 1%, as outlined in previous studies (Chheda et al., 2014), and used symmetric finite displacements (by 0.015Å) to obtain the Hessian matrix and zone-center vibrational frequencies. We computed the single crystal azimuthal anisotropy for compressional (AVp) and shear (AVs) waves in phase-Pi using the formulation for maximum polarization anisotropy.

3. Results

3.1. Crystal Structure

The crystal structure of phase-Pi consists of close packing layers of oxygen atoms along the (110) planes. Perpendicular to the (110) plane, there are layers of distorted eight-membered rings formed by AlO₆ units alternating with layers consisting of SiO₄ tetrahedral units (Figure 1). The distortion of the close packing and the eight-membered rings results in a very low overall space group symmetry (P1). There are six distinct proton environments where each proton is attached to an oxygen atom i.e., O5-H1, O6-H2, O7-H3, O8-H4, O11-H5, and O12-H6. The proton environments for H1, H2, H3, and H4 are characterized by H---H repulsion and H---O attraction. In comparison to our starting guess crystal structure based on single crystal X-ray diffraction (Daniels and Wunder, 1996), the relaxed crystal structure shows lower H---H repulsion and/or slight enhancement of the H---O attraction and hence slightly larger covalent O-H bond lengths for the H1, H2, H3, and H4 protons (Table 1, Figure 1). Upon compression, the O-H…O
hydrogen bonding in H5 and H6 is strong: the covalent bond lengths are 1.03 Å at 0 GPa and increase significantly, eventually resulting in complete symmetrization, i.e., the $d_{O-H}$ become equal to the $d_{H-O}$ at around ~42 GPa. In contrast, the hydrogen bonding, O-H---O in H1, H2, H3, and H4 remains relatively weak. Unlike, O11-H5 and O12-H6, the covalent O-H bond lengths for H1, H2, H3, and H4 remain insensitive to pressures and even decrease slightly, from 0.99-1.00Å at 0 GPa to 0.97Å at 40 GPa, while the hydrogen-bonded H…O distances reduce by only 5-6 % over the same pressure range. In agreement with the proton positions in the crystal structure, the O-H vibron frequencies are comparable to free hydroxyl groups at 0 GPa (3200-3400 cm$^{-1}$) and stiffen further under compression (3260-3510 cm$^{-1}$ at 22 GPa) (Figure 2). The O-H…O angle for H5 and H6 increases and becomes linear (180°) as the hydrogen bond symmetrizes. The O-H vibron modes associated with those protons are much weaker at 0 GPa (2700 cm$^{-1}$) and decrease rapidly under compression (down to 2050 cm$^{-1}$ at 22 GPa), as further indication of significant hydrogen bonding that intensifies with pressure. It is known that hydrogen bond symmetrization often affects the elasticity of hydrous phases including phase-D, phase-H, and δ-AlOOH (Tsuchiya et al., 2002; Tsuchiya et al., 2005; Tsuchiya and Mookherjee, 2015). However, in phase-Pi the hydrogen bond symmetrization occurs at pressures much beyond its thermodynamic stability, similar to other hydrous mineral phases including the 3.65 Å phase and a high-pressure phase of brucite (Mookherjee et al., 2015; Hermann and Mookherjee, 2016). Hence, in the following sections, we will be considering physical properties unaffected by hydrogen bond symmetrization.

3.2. Equation of state and Elasticity
Our results indicate that the static (0 K) zero-pressure unit-cell volume for the triclinic phase-Pi is $V_0 \sim 310.34 (\pm 0.03) \text{ Å}^3$. Unit cell volume predicted by PBE is ~0.7 % larger than the zero-pressure volume based on single crystal X-ray diffraction study, $V_0^{\text{exp}}$ (Wunder et al., 1993; Daniels and Wunder, 1996). A third-order Birch-Murnaghan fit to the energy vs. volume results yield a static (0 K) zero pressure bulk modulus ($K_0$) and pressure derivative of bulk modulus ($K'_0$) as 133.12 (±0.37) GPa and 3.64 (±0.02) respectively (Figure 3). Our static (0 K) results are in very good agreement with the previously reported high-pressure synchrotron X-ray diffraction experiment, $K_0$ of 133.13 GPa (Grevel et al., 2000). The predicted zero pressure lattice parameters are also in good agreement with the single crystal diffraction results with $a_0 > a_0^{\text{exp}}$ by 0.4 %, $b_0 < b_0^{\text{exp}}$ by 0.3 %, $c_0 > c_0^{\text{exp}}$ by 0.03 %, $a_0 < a_0^{\text{exp}}$ by 0.5 %, $\beta_0 < \beta_0^{\text{exp}}$ by 0.6 %, and $\gamma_0 > \gamma_0^{\text{exp}}$ by 0.3 % (Figure 3). The angular lattice parameters $\beta$ and $\gamma$ are closer to 90°. Upon compression, $\beta$ increases from 88.3° towards 90° whereas $\gamma$ decreases from 93.2° towards 90° (Figure 3). The angular lattice parameter $\alpha$ initially decreases from 115.15° to 115.09° upon compression till 7 GPa. At pressure $>7$ GPa $\alpha$ increases (Figure 3). Thus, within its thermodynamic stability, i.e., 2-7 GPa, the angular lattice parameter $\alpha$ and $\gamma$ decrease whereas, $\beta$ increases. It is well known that the static (0 K) zero pressure volume prediction based on PBE is often greater and the predicted bulk modulus often softer in comparison to the experimental results. In contrast, the static (0 K) zero pressure volume prediction based on local density approximation (LDA) is often smaller and the predicted bulk modulus often stiffer compared to the experimental results (Mookherjee and Tsuchiya, 2015). We have incorporated the temperature effects on the bulk modulus using Gibbs2 (Otero-de-la Roza et al., 2011), and find that the predicted bulk modulus at
300 K is ~131 GPa, i.e., 1.6 % softer compared to the experimental results (Figure 3) (Grevel et al., 2000). This softening in bulk modulus is similar to other hydrous phases such as amphibole (Mookherjee and Bezacier, 2012).

Phase-Pi has triclinic space group symmetry i.e., it has 21 independent elastic constants (Nye 1985). All of the principal $C_{11}$, $C_{22}$, $C_{33}$, most of the off-diagonal, $C_{ij}$ (where $i \neq j$) and shear elastic constants, $C_{44}$, $C_{55}$, $C_{66}$ stiffen i.e., $\frac{dC_{ij}}{dP} > 0$ (Figure 4, Table 2). The $C_{11}$ and $C_{22}$ stiffens at a faster rate than $C_{33}$ as noted by their pressure derivaties. The off-diagonal elastic constants such as $C_{16}$, $C_{24}$, $C_{34}$, $C_{45}$, and $C_{56}$ decreases upon compression, i.e., $\frac{dC_{ij}}{dP} < 0$. i.e., (Figure 4, Table 2). The hill averaged bulk ($K_H$) and shear ($G_H$) modulus increase upon compression. Pressure dependence of full elastic constant tensor could be described by finite strain formulations as outlined in previous study (Chheda et al., 2014) and the finite strain fit parameters, such as $C_{ij0}$ and its pressure derivatives $C_{ij0}' \left( \frac{dC_{ij}}{dP} \right)$ and $C_{ij0}'' \left( \frac{d^2C_{ij}}{dP^2} \right)$ are reported in Table 2.

4. Discussion

The high-pressure elasticity of phase-Pi can be understood in terms of the crystal structure. In particular, the compressibility of the individual polyhedral units tends to determine the overall compressibility and elasticity of phase-Pi. There are four distinct crystallographic sites for the silicon atoms. The compressibility of TO$_4$ tetrahedral units for Si(1) and Si(2) are very similar with zero pressure bulk modulus ($K_{0}^{SiO_4}$) of 279 GPa, and pressure derivative of the bulk modulus ($K_{0}^{SiO_4}'$) of 8.6. The compressibility of TO$_4$ tetrahedral units for Si(3) and Si(4) are likewise similar with zero pressure bulk modulus
(\(K_0^{\text{SiO}_4}\)) of 354 GPa, and pressure derivative of the bulk modulus (\(K_0^{\text{SiO}_4}'\)) of 4.9. In contrast to the \(\text{TO}_4\) tetrahedral units, the \(\text{MO}_6\) octahedral sites where \(M=\text{Al}\) are softer. There are six distinct crystallographic sites for the aluminum atoms. The compressibility of the respective \(\text{MO}_6\) octahedral units relate as \(\text{Al}(1) \cong \text{Al}(2), \text{Al}(3) \cong \text{Al}(4), \text{Al}(5) \cong \text{Al}(6)\), with zero pressure bulk modulus (\(K_0^{\text{AlO}_6}\)) and pressure derivative of the bulk modulus (\(K_0^{\text{AlO}_6}'\)) being 131 GPa, 6.3, 189 GPa, 6.4, and 175 GPa, 7.2, respectively. The \(\text{Al}(1)\) and \(\text{Al}(2)\) octahedra are significantly softer compared to the other octahedral units. This could be related to the fact that the \(\text{Al}1-\text{O}12\) and \(\text{Al}2-\text{O}11\) bond lengths are significantly larger, rendering the octahedral sites more distorted. Note that \(\text{O}11\) and \(\text{O}12\) are part of the O-H groups involved in strong hydrogen bonds.

The compressional (\(V_P\)) and shear (\(V_S\)) wave velocities increase upon compression (Figure 5). However, the \(V_P\) and \(V_S\) velocities of phase-Pi are slower than those of topaz-OH (Mookherjee et al., 2016) and the relation persists at higher pressures (Figure 5). The P-wave azimuthal and S-wave anisotropy for phase-Pi is \(\Delta V_P \sim 22.4\%\) and \(\Delta V_S \sim 23.7\%\) at 0 GPa (Figure 5). At pressures \(\sim 0\) GPa, the fast direction of \(V_P\) is consistent with \(C_{33}\) being greater than \(C_{11}\) (Figure 5).

Along a subduction zone significant amounts of sediments could be transported which are often hydrated. A simplified \(\text{Al}_2\text{O}_3-\text{SiO}_2-\text{H}_2\text{O}\) (ASH) ternary is a good representative of the alumina and silica rich layers (Wunder et al., 1993a,b, Schreyer, 1995). Along a typical cold geothermal gradient (~ 4°C/km) hydrous aluminosilicates such as phase-Pi could be stable between a temperature and pressure range of 400 °C-700 °C and 2.0-7.0 GPa respectively. Along the cold geotherm, at lower temperatures of ~400 °C phase-Pi is formed via the reaction:
3 AlOOH (diaspore) + 2 SiO₂ (coesite) = 1 Al₃Si₂O₇(OH)₃ (phase-Pi)  \hspace{1cm} (1)

where as, at the higher pressure limits of 7 GPa, phase-Pi decomposes via a reaction:

2 Al₃Si₂O₇(OH)₃ (phase-Pi) = 3 Al₂SiO₄(OH)₂ (topaz-OH) + 1 SiO₂ (coesite) \hspace{1cm} (2)

Based on the zero pressure density, compressional ($V_p$) and shear ($V_s$) velocity of all the hydrous phases belonging to the ASH ternary (Figure 6, Table 3), we gather that the formation of phase-Pi is likely to be associated with $\Delta V_p$ and $\Delta V_s$ of -0.67 % and -0.95 % respectively. The reduction of velocity is primarily owing to a lower velocity of phase-Pi compared to diaspore. The decomposition of phase-Pi is likely to be associated with $\Delta V_p$ and $\Delta V_s$ of 3.55 % and -0.04 % respectively. This is owing to the fact that $V_p$ of topaz-OH and coesite is greater than phase-Pi whereas the average $V_s$ of topaz-OH and coesite is almost equal or slightly less than phase-Pi. Hence, decomposition of phase-Pi is likely to be associated with an anti-correlation between the $\Delta V_p$ and $\Delta V_s$. However, the presence of other hydrous phases may smear such diagnostic signature. Further information on the effect of temperature, anisotropy, petrographic fabric or lattice preferred orientations on the seismic velocity of hydrous aluminosilicate phases including phase-Pi will certainly aid in refining our prediction.

**Acknowledgements**

Authors thank two anonymous reviewers for their careful and constructive comments.

Y.P., S. B., and M. M. acknowledge the US National Science Foundation grant (EAR-1634422). A. H. acknowledges UK National Supercomputing Service through the UKCP consortium (funded by EPSRC Grant EP/K01465X) and project ID d56 ‘Planetary Interiors’, and by the Condensed Matter Centre for Doctoral Training (funded by EPSRC
Grant 215 EP/L015110/1). AH also acknowledges Royal Society Research Grant RG150247. S. L. acknowledges USTC, China for funding the summer internship.

References


Otero-de-la-Roza, A., Abbasi-Perez, D., and Launa, V., 2011. Gibbs2: A new version of
the quasiharmonic model code. II. Models for solid-state thermodynamics, features and implementation Comp. Phys. Comm., 182, 2232-2248.


Tsuchiya, J., and Mookherjee, M., 2015. Crystal structure, equation of state, and
elasticity of phase H(MgSiO₄H₂) at Earth’s lower mantle pressure. Sci. Rep., 5,
15534.

White, W. M., and Dupre, B., 1986. Sediment subduction and magma genesis in the
Lesser Antilles: isotopic and trace element constraints. J. Geophys. Res., 91,
5927-5941.

and stability of Al₃Si₂O₇(OH)₃ (phase Pi), a hydrous high-pressure phase in the

Wunder, B., Rubie, D. C., RossII, C. R., Medenbach, O., Seifert, F., and Schreyer, W.
1993b. Synthesis, stability, and properties of Al₂SiO₄(OH)₂: A fully hydrated

Figure Captions

Figure 1. Crystal structure of phase-Pi: (a-c) represents the projection down the c-, a-, and
b- crystallographic axes respectively. The light-blue polyhedral units
represent SiO₄ tetrahedral units. The light green polyhedral units represent AlO₆
octahedral units. The red and white spheres are oxygen and hydrogen atoms
respectively. The grey atoms denote the starting guesses for the hydrogen atom
positions H1’, H2’, H3’, and H4’ based on single crystal X-ray diffraction
(Daniels and Wunder, 1994). The flip in positions from H1’, H2’, H3’, and H4’ to
the relaxed positions of H1, H2, H3, and H4 is shown by curved and dashed
arrows. The protons and related oxygen atom pairs are shown by the dashed
rectangles, there are six such O-H...O units, i.e., O5-H1---O10; O6-H2---O11; O7-H3---O12; O8-H4---O11; O11-H5---O10; and O12-H6---O9.

Figure 2. There are six distinct wyckoff sites for the hydrogen atoms (Table 1). The left panel (a, b, and c) shows the evolution of O-H (filled diamonds) and H...O bonds (filled circles), and the O-H...O angles (filled triangles) upon compression. The right panel (d, e, and f) shows the evolution of the corresponding O-H stretching frequencies as a function of pressure. The magnitude and the pressure dependence of O5-H1, O7-H3, and O11-H5 are similar to O6-H2, O8-H4, and O12-H6 respectively and hence are not shown here. Two hydrogen sites H5 and H6 exhibit symmetrization of hydrogen bonding at pressures greater than 35 GPa (shaded region in light grey), i.e., beyond the thermodynamic stability of phase-Pi. As the hydrogen bond symmetrizes the O-H...O angle also becomes linear, i.e., 180°.

Figure 3. (a) Pressure-volume results of phase-Pi. Inset shows the temperature dependence of the bulk modulus (K_T) (filled light blue symbol) using Gibbs2 (Otero-de-la-Roza et al., 2011) the grey filled symbol represents experimental result (Grevel et al., 2000) (b) Lattice parameters, a-, b-, and c- axes as a function of pressure. (c) Angular lattice parameters, α, (d) β, and γ as a function of pressure. Predictions from this work (light blue filled symbols) are compared to experiment (filled light grey symbols) (Daniels and Wunder, 1996).

Figure 4. Full elastic constant tensor (C_{ij}) components, for phase-Pi as a function of pressure (Table 2). There are 21 sub plots, representing the 21 independent elastic constants for the triclinic symmetry.
Figure 5. (a) Pressure dependence of compressional wave ($V_p$) and shear wave velocity ($V_s$) for phase-Pi, topaz-OH (both orthorhombic and monoclinic structure), and topaz-F are shown. (b) Plot of elastic anisotropy $AV_p$ and $AV_s$ for phase-Pi as a function of pressure. (c-d) The stereographic projection down the X3 axes for $AV_p$ and $AV_s$ at 0 GPa. The velocity ($V_p$) and difference ($dV_s$) color contours are also shown for reference.

Figure 6. Plot of $V_p$ and $V_s$ as a function of density for the mineral phases that are stable in subducted sediments. The blue and red dashed line represents linear regression fits for compressional wave and shear wave respectively. Inset shows a ternary plot of the Al$_2$O$_3$-SiO$_2$-H$_2$O components that are relevant for the mineral phases that are stable in subducted sediments (Table 3).
Symmetric H-bonding

OH Stretching frequency [cm$^{-1}$]

Pressure [GPa]

Hydrogen bond angle, O-H...O [°]

Pressure [GPa]

Hydroxyl (O-H) and Hydrogen-bond (H...O)

\[ V_p = 2.75 \rho + 0.04 \]
\[ V_s = 1.67 \rho - 0.11 \]
Table 1. Fractional coordinates of oxygen and hydrogen atoms for the starting configuration and fully relaxed structure.

<table>
<thead>
<tr>
<th>Atom/Label</th>
<th>Structural Parameters: Starting Guess</th>
<th>Structural Parameters: Fully Relaxed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>fractional coordinates d</td>
<td>d &lt;sub&gt;O-H&lt;/sub&gt; d &lt;sub&gt;H---H&lt;/sub&gt; d &lt;sub&gt;H---O&lt;/sub&gt; [Å]</td>
</tr>
<tr>
<td>5</td>
<td>0.6261 0.1399 0.1535 0.9840 0.1528 0.9980</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.3478 0.8647 0.8482 0.3512 0.8616 0.8462</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.4948 0.9130 0.8203 0.9990 0.0069 0.8682</td>
<td></td>
</tr>
<tr>
<td>1---O3</td>
<td>1.9430</td>
<td></td>
</tr>
<tr>
<td>1---O10</td>
<td>1.6330</td>
<td></td>
</tr>
<tr>
<td>2---O4</td>
<td>1.9250</td>
<td></td>
</tr>
<tr>
<td>2---O11</td>
<td>1.6330</td>
<td></td>
</tr>
<tr>
<td>1---H2</td>
<td>2.5026 2.5592</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.1268 0.6397 0.1496 0.0983 0.4916 0.1292</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.9855 0.5901 0.1762 0.0983 0.4916 0.1292</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0.8487 0.3642 0.8463 0.8534 0.3609 0.8460</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.9936 0.4112 0.8222 0.9740 0.5043 0.8699</td>
<td></td>
</tr>
<tr>
<td>3---O1</td>
<td>1.9850</td>
<td></td>
</tr>
<tr>
<td>3---O12</td>
<td>1.6940</td>
<td></td>
</tr>
<tr>
<td>4---O2</td>
<td>1.9770</td>
<td></td>
</tr>
<tr>
<td>4---O11</td>
<td>1.6940</td>
<td></td>
</tr>
<tr>
<td>3---H4</td>
<td>2.4669 2.4433</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.9203 0.7554 0.9038 0.9206 0.7507 0.9045</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.7980 0.7508 0.9689 0.8970 0.7602 0.9895 1.0320</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.0559 0.2424 0.0999 0.0651 0.2451 0.0945</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.1821 0.2412 0.0308 0.9250 0.2015 0.2357 0.0095 1.0320</td>
<td></td>
</tr>
</tbody>
</table>

# Single Crystal X-ray Diffraction (Daniels and Wunder, 1994), *cif files for the relaxed crystal structures are in supplementary files.
Table 2. The full elastic constant tensor ($C_{ij}$), bulk ($K_H$), and shear ($G_H$) modulus for phase-Pi as a function of pressure. The finite strain fit results are also tabulated. Subscript ‘$H$’ refers to the “Hill” average.

<table>
<thead>
<tr>
<th>V</th>
<th>315</th>
<th>310</th>
<th>305</th>
<th>300</th>
<th>295</th>
<th>290</th>
<th>285</th>
<th>280</th>
<th>finite strain fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0</td>
<td>2.2</td>
<td>4.6</td>
<td>7.2</td>
<td>10.0</td>
<td>13.1</td>
<td>16.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C_{11}</td>
<td>221.7</td>
<td>234.8</td>
<td>250.2</td>
<td>266.5</td>
<td>282.9</td>
<td>299.1</td>
<td>314.5</td>
<td>329.1</td>
<td>234.6</td>
</tr>
<tr>
<td>C_{12}</td>
<td>66.0</td>
<td>73.4</td>
<td>81.5</td>
<td>89.8</td>
<td>98.7</td>
<td>107.5</td>
<td>116.4</td>
<td>125.8</td>
<td>73.4</td>
</tr>
<tr>
<td>C_{13}</td>
<td>72.4</td>
<td>77.9</td>
<td>84.4</td>
<td>91.1</td>
<td>98.7</td>
<td>106.2</td>
<td>113.8</td>
<td>121.7</td>
<td>77.8</td>
</tr>
<tr>
<td>C_{14}</td>
<td>5.0</td>
<td>6.2</td>
<td>6.9</td>
<td>7.0</td>
<td>6.7</td>
<td>6.1</td>
<td>5.3</td>
<td>4.5</td>
<td>6.2</td>
</tr>
<tr>
<td>C_{15}</td>
<td>-30.5</td>
<td>-29.4</td>
<td>-27.3</td>
<td>-25.0</td>
<td>-22.9</td>
<td>-21.8</td>
<td>-21.4</td>
<td>-21.7</td>
<td>-29.6</td>
</tr>
<tr>
<td>C_{16}</td>
<td>14.6</td>
<td>14.6</td>
<td>14.3</td>
<td>14.0</td>
<td>13.8</td>
<td>14.0</td>
<td>14.3</td>
<td></td>
<td>14.6</td>
</tr>
<tr>
<td>C_{22}</td>
<td>279.0</td>
<td>292.0</td>
<td>305.5</td>
<td>319.4</td>
<td>333.9</td>
<td>348.6</td>
<td>364.0</td>
<td>380.2</td>
<td>291.5</td>
</tr>
<tr>
<td>C_{23}</td>
<td>54.2</td>
<td>60.7</td>
<td>67.6</td>
<td>74.8</td>
<td>82.6</td>
<td>90.4</td>
<td>98.7</td>
<td>107.6</td>
<td>60.6</td>
</tr>
<tr>
<td>C_{24}</td>
<td>17.7</td>
<td>17.2</td>
<td>16.8</td>
<td>16.2</td>
<td>15.5</td>
<td>14.7</td>
<td>13.8</td>
<td>13.1</td>
<td>17.3</td>
</tr>
<tr>
<td>C_{25}</td>
<td>1.5</td>
<td>2.4</td>
<td>3.4</td>
<td>4.2</td>
<td>4.8</td>
<td>5.0</td>
<td>4.9</td>
<td>4.6</td>
<td>2.4</td>
</tr>
<tr>
<td>C_{26}</td>
<td>1.2</td>
<td>1.3</td>
<td>1.3</td>
<td>1.4</td>
<td>1.4</td>
<td>1.5</td>
<td>1.5</td>
<td></td>
<td>1.3</td>
</tr>
<tr>
<td>C_{33}</td>
<td>256.2</td>
<td>266.0</td>
<td>275.9</td>
<td>285.5</td>
<td>295.9</td>
<td>306.0</td>
<td>316.1</td>
<td>326.9</td>
<td>265.6</td>
</tr>
<tr>
<td>C_{34}</td>
<td>-12.5</td>
<td>-12.8</td>
<td>-13.4</td>
<td>-14.3</td>
<td>-15.5</td>
<td>-16.9</td>
<td>-18.6</td>
<td>-20.3</td>
<td>-12.8</td>
</tr>
<tr>
<td>C_{35}</td>
<td>-15.9</td>
<td>-14.8</td>
<td>-13.5</td>
<td>-12.3</td>
<td>-11.3</td>
<td>-10.7</td>
<td>-10.4</td>
<td>-10.2</td>
<td>-14.7</td>
</tr>
<tr>
<td>C_{36}</td>
<td>5.2</td>
<td>5.5</td>
<td>5.6</td>
<td>5.8</td>
<td>5.8</td>
<td>5.8</td>
<td>5.4</td>
<td></td>
<td>5.5</td>
</tr>
<tr>
<td>C_{44}</td>
<td>82.2</td>
<td>86.1</td>
<td>90.0</td>
<td>94.0</td>
<td>98.1</td>
<td>102.2</td>
<td>106.4</td>
<td>110.8</td>
<td>86.0</td>
</tr>
<tr>
<td>C_{45}</td>
<td>3.8</td>
<td>3.2</td>
<td>2.5</td>
<td>2.1</td>
<td>1.5</td>
<td>0.4</td>
<td>-0.5</td>
<td>-1.3</td>
<td>-2.0</td>
</tr>
<tr>
<td>C_{46}</td>
<td>0.6</td>
<td>1.1</td>
<td>1.6</td>
<td>2.1</td>
<td>2.6</td>
<td>3.1</td>
<td>3.3</td>
<td>3.4</td>
<td>1.0</td>
</tr>
<tr>
<td>C_{55}</td>
<td>87.3</td>
<td>92.8</td>
<td>99.6</td>
<td>106.8</td>
<td>114.0</td>
<td>120.6</td>
<td>126.7</td>
<td>132.3</td>
<td>92.4</td>
</tr>
<tr>
<td>C_{56}</td>
<td>3.5</td>
<td>3.4</td>
<td>2.8</td>
<td>2.0</td>
<td>1.1</td>
<td>0.4</td>
<td>-0.3</td>
<td>-0.8</td>
<td>3.3</td>
</tr>
<tr>
<td>C_{66}</td>
<td>84.9</td>
<td>87.3</td>
<td>89.8</td>
<td>92.4</td>
<td>94.9</td>
<td>97.5</td>
<td>100.1</td>
<td>102.9</td>
<td>87.2</td>
</tr>
<tr>
<td>K_H</td>
<td>124.6</td>
<td>133.2</td>
<td>142.7</td>
<td>152.4</td>
<td>162.6</td>
<td>172.6</td>
<td>182.7</td>
<td>193.0</td>
<td>133.2</td>
</tr>
<tr>
<td>G_H</td>
<td>86.4</td>
<td>90.1</td>
<td>94.1</td>
<td>98.1</td>
<td>102.1</td>
<td>105.8</td>
<td>109.5</td>
<td>113.0</td>
<td>90.1</td>
</tr>
</tbody>
</table>
Table 3. Density, compressional ($V_p$), and shear ($V_s$) wave velocity of mineral phases in Al$_2$O$_3$-SiO$_2$-H$_2$O ternary.

<table>
<thead>
<tr>
<th>Mineral</th>
<th>Abbreviation</th>
<th>Formula</th>
<th>$\rho$ [g/cm$^3$]</th>
<th>$V_p$ [km/s]</th>
<th>$V_s$ [km/s]</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>corundum</td>
<td>cor</td>
<td>Al$_2$O$_3$</td>
<td>3.95</td>
<td>10.94</td>
<td>6.41</td>
<td>Ohno et al. (1986)</td>
</tr>
<tr>
<td>quartz</td>
<td>qz</td>
<td>SiO$_2$</td>
<td>2.64</td>
<td>6.09</td>
<td>4.12</td>
<td>Ohno et al. (2006)</td>
</tr>
<tr>
<td>oesite</td>
<td>cs</td>
<td>SiO$_2$</td>
<td>2.93</td>
<td>8.17</td>
<td>4.58</td>
<td>Weidner and Carleton (1977)</td>
</tr>
<tr>
<td>ice</td>
<td>ice-iii</td>
<td>H$_2$O</td>
<td>1.16</td>
<td>3.66</td>
<td>2.01</td>
<td>Tulk et al. (1994)</td>
</tr>
<tr>
<td>andalusite</td>
<td>and</td>
<td>Al$_2$SiO$_5$</td>
<td>3.15</td>
<td>9.76</td>
<td>5.65</td>
<td>Vaughan and Weidner (1978)</td>
</tr>
<tr>
<td>illmanite</td>
<td>sil</td>
<td>Al$_2$SiO$_5$</td>
<td>3.24</td>
<td>9.65</td>
<td>5.42</td>
<td>Vaughan and Weidner (1978)</td>
</tr>
<tr>
<td>yinite</td>
<td>ky</td>
<td>Al$_2$SiO$_5$</td>
<td>3.76</td>
<td>9.68</td>
<td>5.87</td>
<td>Winkler et al. (2001)</td>
</tr>
<tr>
<td>iaspore</td>
<td>dia</td>
<td>AlOOH</td>
<td>3.38</td>
<td>9.42</td>
<td>5.83</td>
<td>Jiang et al. (2005)</td>
</tr>
<tr>
<td>aolinite</td>
<td>kl</td>
<td>Al$_2$SiO$_5$(OH)$_4$</td>
<td>2.52</td>
<td>6.23</td>
<td>3.55</td>
<td>Katahara (1996)</td>
</tr>
<tr>
<td>opaz</td>
<td>mon-top-OH</td>
<td>Al$_2$SiO$_4$(OH)$_2$</td>
<td>3.43</td>
<td>9.75</td>
<td>5.81</td>
<td>Mookherjee et al. (2016)</td>
</tr>
<tr>
<td>opaz</td>
<td>orth-top-OH</td>
<td>Al$_2$SiO$_4$(OH)$_2$</td>
<td>3.39</td>
<td>9.51</td>
<td>5.51</td>
<td>Mookherjee et al. (2016)</td>
</tr>
<tr>
<td>phase-Pi</td>
<td>pi</td>
<td>Al$_2$Si$_2$O$_7$(OH)$_3$</td>
<td>3.21</td>
<td>8.86</td>
<td>5.28</td>
<td>this study</td>
</tr>
</tbody>
</table>
**Highlights**: Elasticity of phase-Pi (Al₃Si₂O₇(OH)₃) - a hydrous aluminosilicate phase
- First report of equation of state of phase-Pi (Al₃Si₂O₇(OH)₃)
- First report of elasticity of phase-Pi at high pressure
- Velocity density systematics of hydrous in subducted sedimentary layers