Incidence Calculus

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Incidence Calculus is a mechanism for automatic reasoning about uncertain knowledge. Most automatic reasoning mechanisms represent knowledge using logical formulae and manipulate these formulae with rules of inference. When the knowledge is uncertain an *uncertainty value* is associated with each formula and a propagation rule is associated each rule of inference. Probabilities are often used as uncertainty values. The propagation rule calculates the new uncertainty for the conclusion of the rule of inference from the old uncertainty values of its premises. More generally, it may only be possible to place upper and lower bounds on the uncertainty values of formulae, so that the propagation rule generates upper and lower bounds for the conclusion from those on the premises.

One problem with using probabilities as uncertainty values is that the probability of a compound formula cannot be calculated solely from the probabilities of its sub-formulae. Consider, for instance, a formulae P with probability 0.5. Note that its negation, $\neg$P, will also have probability 0.5. Now we would like P$\wedge$P to also have probability 0.5, but P$\wedge\neg$P to have probability 0. But both these compound formulae are conjunctions of two sub-formulae each with probability 0.5. One consequence of this is that either the uncertainty propagation rule has to appeal to information additional to probabilities or it will calculate bounds that are much too loose. For instance, without additional information the probability of P $\wedge$ Q can be anything between 0 and the minimum of the probabilities of P and Q.

Incidence calculus solves this problem by appealing to the foundations of probability theory. Instead of associating probabilities directly with formulae, sets of possible worlds (called *outcomes* in probability and *points* in statistics) are associated with formulae. Probabilities are then associated with possible worlds and the probabilities of formulae are calculated by summing the probabilities of their associated possible worlds. The set of possible worlds associated with a formula is called its *incidence*. To see how this solves the problem suppose there are two possible worlds $\mathfrak{a}$ and $\mathfrak{b}$ with equal probability. The incidence of P might be $\{\mathfrak{a}\}$. Then, be definition, the incidence of $\neg$P will be $\{\mathfrak{b}\}$, that of P $\wedge$ P will be $\{\mathfrak{a}\}$ and that of P $\wedge\neg$P will be $\{\ \}$, as required to give the desired probabilities. An algorithm called the *legal assignment finder* is provided in incidence calculus to propagate incidence bounds between propositional formulae and their sub-formulae and hence infer new probability bounds from old.

Introductions to incidence calculus can be found in [Bundy, 1992, Bundy, 1985].

References

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