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Entailment for Structured Specifications (1988)

\[
\begin{array}{c}
SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \\
\vdash (\varphi_1, \ldots, \varphi_n) \vdash_{\text{Sig}(SP)} \varphi \\
\frac{SP \vdash \varphi}{(\Sigma, \Phi) \vdash \varphi} & \quad \varphi \in \Phi \\
\frac{SP_1 \vdash \varphi \quad SP_2 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi} & SP \vdash \varphi \\
\frac{SP_1 \vdash \varphi \quad SP_2 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi} & SP \vdash \sigma(\varphi) \\
\frac{SP \vdash \varphi}{\text{SP hide via } \sigma(\varphi)} & SP \vdash \varphi
\end{array}
\]

Clarifications: INS = \langle \text{Sign}, \text{Sen} : \text{Sign} \rightarrow \text{Set}, \text{Mod} : \text{Sign}^{op} \rightarrow \text{Cat} \rangle. \langle \vdash_{\Sigma} \subseteq \text{[Mod}(\Sigma) \times \text{Sen}(\Sigma))_{\Sigma \in \text{Sign}} \rangle is an institution that defines the logical system used for specifications, SP, SP_1 and SP_2 are structured \Sigma-specifications over INS, where \Sigma is a signature in the category Sign. \varphi, \varphi_1, \ldots, \varphi_n are \Sigma-sentences, i.e. elements in \text{Sen}(\Sigma), \Phi is a set of \Sigma-sentences, and \sigma(\varphi) denotes \text{Sen}(\sigma)(\varphi), the translation of the sentence \varphi along \sigma : \Sigma \rightarrow \Sigma'. Structured specifications in INS are built from basic specifications (\Sigma, \Phi), the union of \Sigma-specifications \text{SP}_1 \cup \text{SP}_2, the translation \text{“SP hide via } \sigma(\varphi)\text{” of SP along a signature morphism } \sigma : \Sigma \rightarrow \Sigma', and hiding \text{“SP hide via } \sigma(\varphi)\text{” for hiding the symbols in SP not occurring in the image of } \sigma : \Sigma' \rightarrow \Sigma. \text{Sig}[SP] is the signature of SP. Translations of \Sigma-sentences and \Sigma'-models along \sigma : \Sigma \rightarrow \Sigma' are required to preserve satisfaction: for any \varphi \in \text{Sen}(\Sigma) and M' \in [\text{Mod}(\Sigma')], M' \mid_{\Sigma'} \text{ Sen}(\sigma)(\varphi) \Leftrightarrow \text{Mod}(\sigma)(M') \mid_{\Sigma} \phi. Finally, \langle r_{\Sigma} \subseteq \text{Pow}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} is a sound entailment relation for the satisfaction relation \langle \vdash_{\Sigma} \rangle_{\Sigma \in \text{Sign}}. The judgement SP \vdash \varphi is meant to capture the property that \varphi is satisfied in all models of SP.

History: The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

Remarks: The system is sound; completeness is shown in [3] for the first-order logic instance and in [5][6] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation \langle r_{\Sigma} \rangle_{\Sigma \in \text{Sign}}. [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [4] provides additional rules for observability operators.