Entailment for Structured Specifications (1988)

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\begin{align*}
SP \vdash \varphi_1 & \quad \cdots \quad SP \vdash \varphi_n \quad \{\varphi_1, \ldots, \varphi_n\} \vdash_{\text{Sig}(SP)} \varphi \\
\hline
SP \vdash \varphi \\
\hline
(\Sigma, \Phi) \vdash \varphi \quad & \varphi \in \Phi \\
SP_1 \vdash \varphi & \quad SP_2 \vdash \varphi \\
SP_1 \cup SP_2 \vdash \varphi \\
SP \vdash \varphi \\
SP \vdash \sigma(\varphi) \\
SP \text{ with } \sigma \vdash \varphi \\
\hline
\end{align*}
\]

Clarifications: INS = \langle \text{Sign}, \text{Sen} : \text{Sign} \to \text{Set}, \text{Mod} : \text{Sign}^{op} \to \text{Cat}, \langle \models_{\Sigma} \subseteq [\text{Mod}(\Sigma)] \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}} \rangle is an institution that defines the logical system used for specifications, \(SP, SP_1\) and \(SP_2\) are structured \(\Sigma\)-specifications over INS, where \(\Sigma\) is a signature in the category \text{Sign}, \(\varphi_1, \varphi_2, \ldots, \varphi_n\) are \(\Sigma\)-sentences, i.e. elements in \text{Sen}(\Sigma), \Phi is a set of \(\Sigma\)-sentences, and \(\sigma(\varphi)\) denotes \text{Sen}(\sigma(\varphi)), the translation of the sentence \(\varphi\) along \(\sigma : \Sigma \to \Sigma'\). Structured specifications in INS are built from basic specifications \((\Sigma, \Phi)\), the union of \(\Sigma\)-specifications \(SP_1 \cup SP_2\), the translation “SP with \(\sigma\)” of \(SP\) along a signature morphism \(\sigma : \Sigma \to \Sigma'\), and hiding “SP hide via \(\sigma\)” for hiding the symbols in \(SP\) not occurring in the image of \(\sigma : \Sigma' \to \Sigma\). \(\text{Sig}(SP)\) is the signature of \(SP\). Translations of \(\Sigma\)-sentences and \(\Sigma'\)-models along \(\sigma : \Sigma \to \Sigma'\) are required to preserve satisfaction: for any \(\varphi \in \text{Sen}(\Sigma)\) and \(M' \in [\text{Mod}(\Sigma')]\), \(M' \models_{\Sigma'} \text{Sen}(\sigma(\varphi)) \iff \text{Mod}(\sigma(M')) \models_{\Sigma} \varphi\). Finally, \(\langle \models_{\Sigma} \subseteq [\text{Pow}(\text{Sen}(\Sigma))] \times \text{Sen}(\Sigma) \rangle_{\Sigma \in \text{Sign}}\) is a sound entailment relation for the satisfaction relation \(\langle \models_{\Sigma} \rangle_{\Sigma \in \text{Sign}}\).

The judgement \(SP \vdash \varphi\) is meant to capture the property that \(\varphi\) is satisfied in all models of \(SP\).

History: The first systems for proving entailment in structured specifications were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

Remarks: The system is sound; completeness is shown in [3] for the first-order logic instance and in [5] for an institution INS which is finitely exact, admits propositional operators, satisfies Craig interpolation, and has a complete entailment relation \(\langle \models_{\Sigma} \rangle_{\Sigma \in \text{Sign}}\). [7] shows that this is the most powerful sound proof system that is compositional in the structure of specifications. [4] provides additional rules for observability operators.