Entailment for Structured Specifications

Citation for published version:

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Peer reviewed version

Published In:
Towards an Encyclopaedia of Proof Systems

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Entailment for Structured Specifications (1988)

\[
SP \vdash \varphi_1 \quad \cdots \quad SP \vdash \varphi_n \quad \{\varphi_1, \ldots, \varphi_n\} \vdash_{\text{Sig}(SP)} \varphi
\]

\[
\frac{}{SP \vdash \varphi} \quad \frac{}{\langle \Sigma, \Phi \rangle \vdash \varphi} \quad \varphi \in \Phi
\]

\[
\frac{SP_1 \vdash \varphi \quad SP_2 \vdash \varphi}{SP_1 \cup SP_2 \vdash \varphi}
\]

\[
\frac{SP \vdash \varphi \quad SP \vdash \sigma(\varphi)}{SP \text{ with } \sigma \vdash \sigma(\varphi) \quad \overline{SP \text{ hide via } \sigma}}\]

Clariﬁcations: INS = \langle \text{Sign}, \text{Sen} : \text{Sign} \to \text{Set}, \text{Mod} : \text{Sign}^{op} \to \text{Cat} \rangle, \langle \models_{\Sigma} \subseteq [\text{Mod}(\Sigma) \times \text{Sen}(\Sigma)]_{\Sigma \in \text{Sign}} \rangle is an institution that deﬁnes the logical system used for speciﬁcations, SP, SP_1 and SP_2 are structured \Sigma-speciﬁcations over INS, where \Sigma is a signature in the category \text{Sign}, \varphi, \varphi_1, \ldots, \varphi_n are \Sigma-sentences, i.e. elements in \text{Sen}(\Sigma), \Phi is a set of \Sigma-sentences, and \sigma(\varphi) denotes \text{Sen}(\sigma)(\varphi), the translation of the sentence \varphi along \sigma : \Sigma \to \Sigma'. Structured speciﬁcations in INS are built from basic speciﬁcations \langle \Sigma, \Phi \rangle, the union of \Sigma-speciﬁcations SP_1 \cup SP_2, the translation “SP with \sigma” of SP along a signature morphism \sigma : \Sigma \to \Sigma', and hiding “SP hide via \sigma” for hiding the symbols in SP not occurring in the image of \sigma : \Sigma' \to \Sigma. Sig[SP] is the signature of SP. Translations of \Sigma-sentences and \Sigma'-models along \sigma : \Sigma \to \Sigma' are required to preserve satisfaction: for any \varphi \in \text{Sen}(\Sigma) and M' \in [\text{Mod}(\Sigma')], M' \models_{\Sigma'} \text{Sen}(\sigma)(\varphi) \iff \text{Mod}(\sigma)(M') \models_{\Sigma} \varphi. Finally, \langle \models_{\Sigma} \subseteq [\text{Pow}(\text{Sen}(\Sigma)) \times \text{Sen}(\Sigma)]_{\Sigma \in \text{Sign}} \rangle is a sound entailment relation for the satisfaction relation \langle \models_{\Sigma} \rangle_{\Sigma \in \text{Sign}}.

The judgement SP \vdash \varphi is meant to capture the property that \varphi is satisﬁed in all models of SP.

History: The ﬁrst systems for proving entailment in structured speciﬁcations were given by Sannella and Burstall [1], Sannella and Tarlecki [2], and Wirsing [3]. The above presentation can be found in [6], Sect. 9.2.

Remarks: The system is sound; completeness is shown in [3] for the ﬁrst-order logic instance and in [5, 6] for an institution INS which is ﬁnitely exact, admits propositional operators, satisﬁes Craig interpolation, and has a complete entailment relation \langle \models_{\Sigma} \rangle_{\Sigma \in \text{Sign}}. [7] shows that this is the most powerful sound proof system that is compositional in the structure of speciﬁcations. [4] provides additional rules for observability operators.