Abstract
Stencil computations are widely used from physical simulations to machine-learning. They are embarrassingly parallel and perfectly fit modern hardware such as Graphic Processing Units. Although stencil computations have been extensively studied, optimizing them for increasingly diverse hardware remains challenging. Domain Specific Languages (DSLs) have raised the programming abstraction and offer good performance. However, this places the burden on DSL implementers who have to write almost full-fledged parallelizing compilers and optimizers.

Lift has recently emerged as a promising approach to achieve performance portability and is based on a small set of reusable parallel primitives that DSL or library writers can build upon. Lift’s key novelty is in its encoding of optimizations as a system of extensible rewrite rules which are used to explore the optimization space. However, Lift has mostly focused on linear algebra operations and it remains to be seen whether this approach is applicable for other domains.

This paper demonstrates how complex multidimensional stencil code and optimizations such as tiling are expressible using compositions of simple 1D Lift primitives. By leveraging existing Lift primitives and optimizations, we only require the addition of two primitives and one rewrite rule to do so. Our results show that this approach outperforms existing compiler approaches and hand-tuned codes.

1 Introduction
Stencils are a class of algorithms which update elements in a multi-dimensional grid based on neighboring values using a fixed pattern. They are used extensively in various domains such as medical imaging (e.g., SRAD), numerical methods (e.g., Jacobi) or machine-learning (e.g., convolution neural networks). Stencils are part of the original "seven dwarfs" [2] and are considered one of the most relevant classes of high-performance computing applications.

However, efficient programming of stencils for parallel accelerators such as Graphics Processing Units (GPUs) is challenging even for experienced programmers. Hand-optimized high-performance stencil code is usually written using low-level programming languages like OpenCL or CUDA. Achieving high-performance requires expert knowledge to manage every hardware detail. For instance, special care is required on how parallelism is mapped to GPUs or how data locality is exploited with local memory to maximize performance.

Domain Specific Languages (DSLs) and high-level library approaches have been successful at simplifying HPC application development. These approaches are based on algorithmic skeletons [9] which are recurring patterns of parallel programming. While these raise the abstraction level, they rely on hard-coded, not performance portable implementations. Alternative approaches are based on code generation, which places a huge burden on the implementers who have to reinvent the wheel for each new application domain.

Lift [38] is a novel code generation approach based on a high-level, data-parallel intermediate language whose central tenet is performance portability. It is designed as a target for DSLs and library authors, and exploits functional
principles to produce high-performance GPU code. Applications are expressed using a small set of functional 
primitives and optimizations are all encoded as formal, semantics-

preserving rewrite rules. These rules define an optimization 

space which is automatically searched for high-performance 
code [42]. This approach liberates programmers and DSL 
implementers from the tedious process of re-writing and 
tuning their code for each new domain or hardware.

This paper shows how stencil code and optimizations 
are expressible in Lift, reusing its existing machinery for 
managing parallelism, memory hierarchy and optimizations. 
Surprisingly, only two new primitives were added to Lift 
for neighborhood gathering and boundary condition han-
dling. By composing these 1D primitives, complex multi-
dimensional stencils are expressible, demonstrating the ex-
tensibility of the Lift approach to new application domains.

The paper also shows how stencil-specific optimizations 
are expressible using existing rewrite-rules in Lift such as 
overlapped tiling. This only requires the addition of one new 
rule that handles the newly introduced primitives. By reusing 
the existing exploration mechanism, we can automatically 
generate high-performance stencil code for AMD, NVIDIA 
and ARM GPUs. Our results show that this approach is highly 
competitive with hand-written implementations and with 
the state-of-the-art PPCG polyhedral GPU compiler.

This paper makes the following contributions:
1. We show how complex multi-dimensional stencils are 
expressible using Lift’s existing primitives with the 
addition of only two new primitives;
2. We formalize and implement a stencil-specific opti-
mization – overlapped tiling – as a rewrite rule;
3. We demonstrate that this approach generates high-
performance code for several stencil benchmarks.

The paper is organized as follows. Section 2 motivates 
our work. Section 3 shows how stencil computations are 
expressed in Lift. Section 4 presents stencil-specific optimizations 
in Lift expressed as rewrite-rules. Section 5 explains 
how Lift expressions are compiled to efficient OpenCL code. 
Section 7 provides experimental evidence that this approach 
produces high-performance code on a selection of GPUs.
Finally, Sections 8 and 9 present related work and conclude.

2 Motivation
The advent of Graphics Processing Units over the past decade 
have been the first sign of an increasing trend of diversity in 
computer hardware. The end of Dennard scaling and Moore’s 
law forces computer architects to specialize their design for 
increased performance and efficiency. Traditional multi-core 
CPUs from Intel and AMD are now challenged by more 
energy-efficient designs by ARM, massively parallel archi-
tectures such as GPUs, and accelerators such as the Xeon 
Phi. This diversity in hardware requires massive changes for 
software as traditional, sequential implementations are hard 
to automatically adapt to this zoo of modern architectures.

2.1 A Solved Problem: High-Level Programming 
Abstractions for Stencils
Domain specific languages (DSLs) and libraries help application 
developers target modern hardware, shielding them from the ever changing landscape. They are commonly ac-
cepted as being part of the solution to address the perfor-
mance portability challenge. They are widely used in stencil 
computations, which has been extensively – and success-
fully – studied in terms of application-specific optimizations in the high performance computing community. High-level 
framework such as Halide [33] are designed specifically to 
express stencil computations in a functional style, fuse mul-
iple operations and generate parallel GPU code automatically. Similarly, PolyMage [31] fuses multiple stencil operations and uses the polyhedral model to produce parallel CPU code. These approaches are particularly good at optimizing long pipelines of stencil operations typically found in image processing applications.

While the use of DSLs provides a nice solution for the 
end user, they are costly in terms of compiler development. 
Each new DSL needs to implement its own backend compiler and optimizer with its own approach to parallelization. This is clearly not sustainable given the number of application domains and the ever growing hardware diversity.

2.2 The Real Challenge: Universal High 
Performance Code Generation
What is needed is a compiler approach which can be reused 
over a wide range of domains and deliver high performance 
on a broad set of devices. Figure 1 shows the vision of a uni-
versal compiler between DSLs and hardware which was first proposed by Delite [45]. Delite advocates the use of a small 
set of parallel functional primitives upon which DSL are 
implemented. A single backend takes care of compiling and 

optimizing these primitives down to high performance GPU 
code, enabling all the DSLs implemented on top of Delite to 
benefit from these optimizations. This type of approach can 
lead to good performance for many domains on a specific 
parallel device and in particular for stencil code.

Lift [38, 41, 42] has recently emerged as a novel approach 
to address the performance portability challenge. It follows
a similar philosophy as Delite by offering a small set of data-
parallel patterns used to implement higher-level abstrac-
tions. In contrast to Delite, Lift generates high performance
code by encoding algorithmic choices and device-specific
optimizations as provably correct rewrite rules. This design
makes it easy to extend and add new optimizations into
the compiler, in contrast to Delite where optimizations are
hard-coded for each backend.

Lift has demonstrated that high performance is achiev-
able for linear algebra operations [41]. This paper takes the
Lift approach a step further and shows how it can also be
applied, with few modifications, to stencil computations. In
particular, we show how complex multi-dimensional stencils
are expressible by composing a handful of simple 1D primit-
ives. Additionally, we strive to leverage existing functional-
ity in Lift, inheriting the benefits of automatic exploration
of algorithmic and device-specific optimizations.

3 Extending Lift for Stencil Computations

Figure 2 shows the extension to Lift for supporting stencil
computations which we describe in this and the following
section. Only minor additions are required to support stencils
and generate high-performance code across multiple parallel
devices. We begin by describing the existing Lift primitives
we reuse, before introducing the two new primitives slide
and pad which allow us to express stencil computations in
a functional style. After discussing a 1D example, we intro-
duce the handling of multi-dimensional stencils which are
expressed by composition of the fundamental 1D primitives.

3.1 Existing High-Level Lift Primitives

Lift was introduced in [38], offering a collection of data-
parallel functional primitives. Prior work has shown that
it is possible to compile these effectively to the GPU [42]
using rewrite-rules. The most relevant primitives for stencil
applications are shown below with their types which explain
how these primitives can be composed.

- **map**: 
  \[ (f : T \rightarrow U, \text{in} : [T]^n) \rightarrow [U]^n \]

- **reduce**: 
  \[ (\text{init} : U, f : (U, T) \rightarrow U, \text{in} : [T]^n) \rightarrow [U] \]

- **zip**: 
  \[ (\text{init1} : [T]^n, \text{init2} : [U]^m) \rightarrow [(T, U)]^n \]

- **iterate**: 
  \[ (\text{init} : [T]^n, f : [T]_m \rightarrow [T]_m, \text{m} : \text{Int}) \rightarrow [T]^n \]

- **split**: 
  \[ (\text{m} : \text{Int}, \text{in} : [T]^n) \rightarrow [[T]^n]_m \]

- **join**: 
  \[ (\text{in} : [[T]^m]_n) \rightarrow [T]_{mn} \]

- **at**: 
  \[ (\text{i} : \text{Cst}, \text{in} : [T]^n) \rightarrow T \]

- **get**: 
  \[ (\text{i} : \text{Cst}, \text{in} : \{T_1, T_2, \ldots\}) \rightarrow T_i \]

- **array**: 
  \[ (\text{n} : \text{Int}, f : (\text{i} : \text{Int}, \text{n} : \text{Int}) \rightarrow T) \rightarrow [T]^n \]

- **userFun**: 
  \[ (s_1 : \text{ScalarT}, s_2 : \text{ScalarT'}, \ldots) \rightarrow \text{ScalarU} \]

We write \([T]^n\) for an array with \(n\) elements of type \(T\). Note
that arrays can be nested and carry their size in their type.
We write \(\{T_1, T_2, \ldots\}\) for a tuple with component types \(T_i\).
Finally, \(T \rightarrow U\) denotes a function type expecting a value
of type \(T\) and returning a resulting value of type \(U\).

**Map, Reduce, Iterate** The map primitive applies a function
\(f\) to all elements of an array and produces a new array of
the same length. In Lift, this is the only primitive that expresses
data parallelism. reduce applies a reduction operator \(f\) to an
array by traversing it, applying \(f\) to the elements and an
accumulator variable initialized with the given init value.
iterate performs \(m\) iterations of a function \(f\) reusing the
output produced as an input for the next iteration. While this
paper purely evaluates single iteration stencils, the iterate
primitive can be used to perform multiple iterations.

**Zip, Split, Join** zip creates an array of tuples \([T, U]\) by
combining two input arrays of the same length. split intro-
duces an additional dimension, by splitting the input array
into chunks of size \(m\), where \(m\) is a positive number evenly
dividing the input size \(n\). join performs the opposite opera-
tion.

**Array and Tuple accesses** The at primitive enables the
indexing of arrays with constant (literal) indices. For stencils
this is useful for accessing the elements which define the
stencil shape. For the rest of this paper, we will write \(\text{in}[3]\)
as syntactic sugar for \(\text{at}(3, \text{in})\). Similarly, the get primitive
indexes into the components of a tuple. For instance, get(2, \text{in})
returns the second component of tuple \text{in}. For the rest of this
paper, we will write \(\text{in.2}\) as syntactic sugar for get(2, \text{in})

**Array Constructor** This primitive constructs array ele-
ments lazily by invoking the function \(f\) with index \(i\) and
array length \(n\). Later we will show this primitive being used
for creating masks which can be useful for certain stencils.

**UserFun** Finally, userFuns define arbitrary functions which
operate on scalar values. These functions are written in C
and are embedded in the generated OpenCL code.
3.2 Extensions for Supporting Stencils

It is not possible to express stencil computations in Lift using solely the primitives just described as they are too restrictive. Instead of expressing stencil computations using a single high-level stencil primitive, as often seen in other high-level approaches, e.g. [7, 39], in Lift we aim for composability and instead express stencil computations using smaller fundamental building blocks. Using an example, we can show that stencil computations can be decomposed in three steps: consider the 3-point stencil shown in Listing 1 applied on a 1D array $A$ of length $N$ that sums the elements of each neighborhood. As denoted in the comments, stencil computations consist of three fundamental parts:

(a) for every element of the input, a neighborhood is accessed specified by the shape of the stencil (line 3);
(b) boundary handling is performed which specifies how to handle neighboring values for elements at the borders of the input grid (lines 5 and 6);
(c) finally, for each neighborhood their elements are used to compute an output element (line 7).

We have added new primitives to perform the first two steps. Following Lift’s design goal, each primitive expresses a single concept and complex functionality is achieved by composition. The first new primitive handles boundary conditions and the second one expresses element grouping.

**Boundary Handling with pad.** pad adds $l$ and $r$ elements at the beginning and end of the input array $in$, respectively. A first variant reindexes into the input array, a second variant appends values computed by a user-specified function. For stencil computations, these primitives are used to express what happens when we reach the edge of the data boundary.

Step 1 in Figure 3 visualizes boundary handling with pad. The input array on the top is enlarged with one element on each side as highlighted with dashed lines.

The pad primitive for reindexing has the following type:

$$pad : ( l : \text{Int}, r : \text{Int}, h : (i : \text{Int}, \text{len} : \text{Int}) \rightarrow \text{Int}, \text{in} : [T]n \rightarrow [T]l+n+r)$$

It uses the index function $h$ to map indices from the range $[0, l+n+r]$ into the smaller range of the input array $[0, n]$. The elements added at the boundaries are, thus, elements of the input array and $h$ is used to determine which elements this will be. For instance, by defining the following function:

$$\text{clamp}(i, n) = (i < 0) \ ? \ 0 : ((i >= n) \ ? \ n-1 : i)$$

it is possible to express a clamping boundary condition which artificially extends the original input array by two elements to the left and three to the right by repeating the value at the boundary. In the extended version of Lift we write: $pad(2, 3, \text{clamp}, \text{input})$.

Indexing functions implementing mirroring or wrapping are similarly defined. Indexing functions must not reorder the elements of the input array, but only map indices from outside the array boundaries into a valid array index.

The pad primitive which appends values has a very similar type (not shown for brevity), where the function $h$ produces a value which is added to the edges of the array. This variation of pad is used to implement constant boundary conditions, or dampening boundary conditions where the out-of-bound value decreases with the distance to the boundary.

**Creating Neighborhoods with slide.** The slide primitive applies a sliding window of length size which traverses past step elements. For a one-dimensional 3-point stencil we write: $slide(3, 1, \text{input})$.

This creates a nested array, as shown after step 2 in Figure 3, where each element of the outer array is itself an array of three elements. The second element of the first inner array is hereby also the first element of the second array, which corresponds to the notion that we group the first three elements together before we move the sliding window by one element. The type of slide can be described as:

$$slide : (\text{size} : \text{Int}, \text{step} : \text{Int}, \text{in} : [T]n) \rightarrow [[T]n \times size + step]$$

It can produce an output of multi-dimensional neighborhoods.

---

**Listing 1.** Simple 3-Point Jacobi Stencil in C

```c
for(int i = 0; i < N; i++) {
    int sum = 0;
    for(int j = -1; j <= 1; j++) { // (a)
        int pos = i+j;
        pos = pos < 0 ? 0 : pos; // (b)
        pos = pos > N-1 ? N-1 : pos;
        sum += A[pos];
    }
    B[i] = sum;
}
```

**Figure 3.** Expressing a stencil in Lift using pad for boundary handling, slide for creating the neighborhood and map to compute the output elements. These three logical steps are compiled into a single efficient OpenCL kernel by Lift.
High Performance Stencil Code Generation with Lift

Computing the Stencil for each Neighborhood with map
The map primitive is the only way in Lift to express data parallelism. As stencils are naturally data-parallel, we express the last step of the stencil computation using the map primitive. This step takes arrays of neighborhood as its input and performs the stencil computation to produce a single output value for each neighborhood.

3.3 One-dimensional Stencil Example in Lift
Listing 2 shows a simple 3-Point Jacobi Stencil expressed in Lift. This is the same example we saw as C code in Listing 1. Due to the functional style of nested function calls, the Lift expression reads from bottom to top. We can see the decomposition in three logical steps: first, boundary handling is performed (line 7) using pad; then, the neighborhoods are created (line 6) using slide; finally, map is used (line 5) to perform the computation for every created neighborhood. The computation is defined as function sumNbh in line 1.

It is important to emphasize that the logical distinction of these three steps will not be echoed in the generated OpenCL code. The boundary handling and creation of neighborhoods are not performed by copying elements in memory, but are combined with map in a single step by creating a compiler-internal data structure, called view in Lift [42], which influences how data will be read from memory. This is discussed in more detail in Section 5.

3.4 Multi-Dimensional Stencils in Lift
One of the crucial concepts in this paper is the ability to express complex multi-dimensional stencils as compositions of simple one-dimensional primitives. We will now show how we define n-dimensional versions of pad_n and slide_n as compositions of the simple pad, slide, and map primitives we have just seen.

Multi-dimensional stencils are then expressed following the same structure as one-dimensional stencils:

\[
map_n(f, \text{slide}_n(size, step, pad_n(l, r, h, input)))
\]

Boundary handling is performed via pad_n using the function h. Here we present the simple case where the same boundary handling strategy is performed in each dimension. It is straightforward – and supported by our implementation – to do different boundary handlings in each dimension. The slide_n creates a n-dimensional neighborhood, which is then processed by map_n.

Multi-Dimensional Boundary Handling
Boundary handling in multiple dimensions follows the same idea as in the one-dimensional case. Using nested maps, we apply pad to inner dimensions. Thus, pad_n is defined recursively:

\[
\begin{align*}
\text{pad}_1(l, r, h, \text{input}) &= \text{pad}(l, r, h, \text{input}) \\
\text{pad}_n(l, r, h, \text{input}) &= \text{map}_{n-1}(\text{pad}_n(l, r, h), \text{pad}_{n-1}(l, r, h, \text{input}))
\end{align*}
\]

where map_n are n nested maps:

\[
\begin{align*}
\text{map}_1(f, \text{input}) &= \text{map}(f, \text{input}) \\
\text{map}_n(f, \text{input}) &= \text{map}_{n-1}(\text{map}(f, \text{input}))
\end{align*}
\]

While the base case is the one-dimensional pad, for each higher dimension a pad primitive is added where nested maps are used to apply it to the innermost dimension.

We provide a simple 2D example for pad_2 using the clamp boundary handling which repeats the values at the boundary:

\[
\text{pad}_2(1, 1, \text{clamp}, \begin{bmatrix} [a, b], [c, d] \end{bmatrix}) =
\text{map}(\text{pad}(1, 1, \text{clamp}, \begin{bmatrix} [a, b], [c, d] \end{bmatrix})) =
\begin{bmatrix}
[a, a, b, b], \\
[a, a, b, b], \\
[c, c, d, d], \\
[c, c, d, d]
\end{bmatrix}
\]

After expanding the definition of pad_2, we first apply pad to the outer dimension of the two-dimensional array, resulting in an enlarged array where the first and last element – themselves both arrays – are prepended and appended. Then, in the second step, pad is applied to the nested dimension using the map primitive, which applies pad to every nested array resulting in the final two-dimensional array.

Multi-Dimensional Neighborhood Creation
The creation of multi-dimensional neighborhoods is more complex than the boundary handling, but follows a similar idea.

For the two-dimensional case, slide_n is defined as:

\[
\text{slide}_n(size, step, \text{input}) =
\begin{cases}
\text{map}(\text{transpose}, \\
\text{slide}(size, step))
\end{cases}
\text{map}(\text{slide}(size, step, \text{input}))
\]

We explain this definition using an example:

\[
\text{slide}_2(2, 1, \begin{bmatrix} [a, b, c], [d, e, f] \end{bmatrix}) =
\begin{bmatrix}
[a, b, c], \\
[d, e, f], \\
[g, h, i]
\end{bmatrix}
\]

\[
\text{map} (\text{slide}(2, 1, \begin{bmatrix} [a, b, c], [d, e, f], [g, h, i] \end{bmatrix})) =
\begin{bmatrix}
[a, b, c], [d, e, f], [g, h, i]
\end{bmatrix}
\]
We first recursively apply the sliding in one inner dimension, which span three dimensions for physical space and one for time. The first grid is taken point-by-point, however in lines 1-2) indicate previous and current time steps in an equation to model the sound (lines 8–11).

A difficult problems for wave-based simulations is accurate handling of physical obstacles in the room. The variable coefficients (a.k.a. "loss") at the obstacles boundary are handled through the use of a mask, which returns a different value depending on whether it is on an obstacle or not. In Lift, this mask is calculated on the fly using the array3d generator and contains a value at each point in the grid.

3.6 Summary

In this section, we have demonstrated how stencils are expressed in Lift by extending the set of primitives by two new additions: pad and slide. Together with existing Lift primitives, this allows for expressing multi-dimensional stencils which are built from the one-dimensional building blocks.

Crucially, the parallelism found in stencil applications is expressed using the existing map primitive, without introducing a special case for stencils. Rewrite rules explaining how to optimally leverage OpenCL hardware using map are then reusable for stencil applications as we show next.

4 Expressing Optimizations

This section discusses how stencil-specific optimizations can be expressed as rewrite rules. These new rules are then used together with Lift’s existing rules to explore the implementation space of stencil applications. By applying different rewrites, we can tailor programs to target different architectures, thus, achieving performance portability.

4.1 Exploiting Locality through Tiling

Stencil applications involve local computations which only access elements in a neighborhood. Furthermore, neighboring elements in a grid share large parts of their neighborhoods. Exploiting this locality is the most commonly used and most successful optimization for stencil computations.
Figure 4. Overlapped tiling for a 3-point Jacobi stencil.

Listing 4. 3-Point Jacobi Stencil using overlapped tiling

```
fun(A: Array(Float, N) =>
  map(tile => map(sumNbh, slide(3, 1, tile),
    1
    fun(A: Array(Float, N) =>
      with 2 elements overlapping with the next tile.
    1
    fun(A: Array(Float, N) =>
      Figure 5 shows the creation of tiles in the first step (by using `slide(5, 3)`), then for each tile we create the local neighborhoods using the `slide` primitive again.

Tiling as a Rewrite Rule  Phrasing tiling as a rewrite rule makes it accessible to LIFT's automatic exploration process.

Tiling in one dimension is expressible as follows:

\[
\text{map}(f \cdot \text{slide}(size, step, input)) \mapsto \text{join}((\text{map}(f), \text{slide}(size, step, tile)), \\
(\text{slide}(u, v, input)))
\]

The parameters \(u\) and \(v\) have to be selected appropriately, i.e., the difference between the `size` and `step` has to match the difference of \(u\) and \(v\): \(size - step = u - v\). Figure 4 visualizes this constraint for the one-dimensional 3-point Jacobi where the neighborhood `size` is 3 (and the `step` is 1). When choosing the size of the tile \(u\), e.g. 5 in the example, \(v\) has been selected so that it matches the formula (i.e. 3 in this case) as \(3 - 1 = 5 - 3\). This is the only valid choice for \(v\) as it determines the overlap created between the tiles which corresponds with the size of the original neighborhood. Choosing \(u\) and \(v\) according to the formula ensures that we end up with the same number of neighborhoods on both sides of the rewrite rule.

To see that this rule is semantics preserving, we can decompose it into two smaller rules:

\[
\text{map}(f \cdot \text{join(input))} \mapsto \text{join}((\text{map}(f), \text{input})) \mapsto \\
\text{slide(size, step, input)} \mapsto \\
\text{join}((\text{tile} \mapsto \text{map}(\text{slide}(size, step, tile)), \\
\text{slide}(u, v, input))
\]

Here it can be seen that the first rule is semantics preserving, as on both sides function \(f\) is applied to each element of the two-dimensional `input`. On the left-hand side, this is done by flattening the input and then applying the function, whereas on the right-hand side the function is first applied to each element of the input and then flattened afterwards.

Assuming that \(u\) and \(v\) are valid parameter choices as described above, the correctness of the second rule is also straightforward. Starting on the right-hand side, we create tiles using the first `slide` primitive. Then, we perform the second `slide` for each created tile, before the `join` removes the outermost dimension and, therefore, resolves the tiles, leaving us with a two-dimensional array equivalent to the array produced by only applying the second `slide`.

Overlapped Tiling in LIFT in Multiple Dimensions  Our extension to LIFT fully supports tiling in higher dimensions. Figure 6 visualizes overlapped tiling in two dimensions.

The optimization rules for tiling higher-dimensional stencils are expressed by reusing the one-dimensional primitives. The rewrite rule covering two-dimensional tiling looks similar to the one-dimensional case when written with the `map`
and \( \text{slide}_2 \) primitives introduced previously:
\[
\text{map}_2(f, \text{slide}_2(\text{size}, \text{step}, \text{input})) \mapsto \\
\text{map}(\text{join}(\text{map}(\text{transpose},
\text{map}(\text{tile} \Rightarrow \text{map}_2(f, \text{slide}_2(\text{size}, \text{step}, \text{tile}),
\text{slide}_2(a, v, \text{input}))))))
\]

However, it is difficult to ensure that the correct dimensions are combined at the very end on the right-hand side.

### 4.2 Usage of Local Memory
Tiling is used to exploit locality. Modern GPUs have fairly small caches and rely on the programmer explicitly using the fast scratchpad memory, called local memory in OpenCL. This can be cumbersome and does not always provide better performance. Whether or not the local memory is beneficial depends on the hardware architecture and the amount of data reuse in the stencil application.

In Lift, we address these issues by expressing the local memory usage as a rewrite rule. When exploring the optimization space, this rule will be one of many optimization choices applied in the automatic optimization process.

Besides the high-level primitives introduced in Section 3, Lift also defines OpenCL-specific low-level primitives [38] to exploit particular features of OpenCL, such as the use the local memory. The \( \text{toLocal} \) primitive wraps around a function to indicate that this function should write its result into local memory. To copy a single scalar value into local memory, we can use the identity user function \( id \), as in: \( \text{toLocal}(id) \). For copying arrays, we wrap the \( \text{map}(id) \) function in \( \text{toLocal} \).

As copying data into local memory is always legal inside an OpenCL workgroup, we introduce this rewrite rule:
\[
\text{map}(id) \mapsto \text{toLocal}(\text{map}(id))
\]

Together with a rule which introduces \( \text{map}(id) \) at any position, this allows the exploration of copying to local memory as an optimization. Currently, heuristics are used to prevent applying this rule at unfavorable places.

### 4.3 Loop Unrolling
Unrolling loops is a traditional low-level optimization which can greatly increase performance for certain cases. To explore loop unrolling as an optimization for stencil applications, we make use of a variation of the \( \text{reduce} \) primitive which is unrolled by the Lift compiler. As we saw in the 3-Point Jacobi example in Listing 2, the \( \text{reduce} \) pattern is often used in stencil computations to sum up the values in a neighborhood. The unrolled variation of the reduction is called \( \text{reduceUnroll} \) and has a matching rewrite rule providing it as an optimization choice during exploration. Unrolling is only legal if the size of the input array has a length which is known at compile time. For stencils this is usually the case, as the reduction is applied to a neighborhood which almost always consists of a fixed number of elements.

### 4.4 Summary
In this section we have shown how stencil optimizations are expressed as rewrite rules, which are then applied by the Lift exploration process. Overlapped tiling in multiple dimensions is expressed by reusing ideas of the simple one-dimensional case. Together with low-level optimizations, such as usage of local memory, we can automatically explore a variety of optimizations for stencil applications.

### 5 Code Generation
A stencil program expressed using \( \text{pad} \), \( \text{slide} \), and \( \text{map} \) is rewritten into a Lift expression of low-level, OpenCL-specific primitives which explicitly encode implementation and optimization choices like the use of local memory. The OpenCL code generation is largely unchanged from the compilation of Lift programs as described in [42].

Lift uses so-called views [42] when implementing primitives, which modify data layout without performing computations themselves. These operations are not performed in memory, but define how primitives read input data.

\( \text{Pad} \) and \( \text{slide} \) are implemented using this same approach. These primitives are integrated with Lift’s view system and are not directly compiled to OpenCL code. Instead, the reindexing of computations introduced with \( \text{pad} \) are performed when the padded array is read for the first time. Similarly, the \( \text{slide} \) primitive does not physically copy created neighborhoods into memory. \( \text{Slide} \) guides accesses to elements in a neighborhood to the original array, so that accesses to the same element in different neighborhoods result in memory accesses from the same physical location.

This technique means Lift can build complex - potentially multi-dimensional - abstractions which simplify the implementation of stencil applications compiled to efficient OpenCL code.

### 6 Experimental Setup

**Platforms and Measurement**
Experiments are conducted using single precision floats on: a Tesla K20c with CUDA 8.0 driver version 367.48; an AMD Radeon HD 7970 with OpenCL version 1.2 AMD-APP (1912.5); and the SAMSUNG Exynos 5422 ARM Mali GPU with OpenCL 1.2 v1.r17p0. The medians of 100 executions are reported measured using the OpenCL profiling API. Data transfer times are ignored since the focus is on the quality of the generated kernel code.
with their key characteristics. This section presents the results of the exploration and auto-tuning process. It also shows the performance achieved by hand-written optimized kernels from the benchmark suites or from HPC experts, as explained previously. Performance is expressed in elements updated per second, which we define simply as the output size divided by the execution time.

<table>
<thead>
<tr>
<th>Benchmark</th>
<th>Dim</th>
<th>Pts</th>
<th>Input size</th>
<th>#grids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stencil2D</td>
<td>2D</td>
<td>9</td>
<td>4098×4098</td>
<td>1</td>
</tr>
<tr>
<td>SRAD1</td>
<td>2D</td>
<td>5</td>
<td>504×458</td>
<td>1</td>
</tr>
<tr>
<td>SRAD2</td>
<td>2D</td>
<td>3</td>
<td>504×458</td>
<td>2</td>
</tr>
<tr>
<td>Hotspot2D</td>
<td>2D</td>
<td>5</td>
<td>8192×8192</td>
<td>2</td>
</tr>
<tr>
<td>Hotspot3D</td>
<td>3D</td>
<td>7</td>
<td>512×512×8</td>
<td>2</td>
</tr>
<tr>
<td>Acoustic</td>
<td>3D</td>
<td>7</td>
<td>512×512×404</td>
<td>2</td>
</tr>
<tr>
<td>Gaussian</td>
<td>2D</td>
<td>25</td>
<td>4096² / 8192²</td>
<td>1</td>
</tr>
<tr>
<td>Gradient</td>
<td>2D</td>
<td>5</td>
<td>4096² / 8192²</td>
<td>1</td>
</tr>
<tr>
<td>Jacobi2D</td>
<td>2D</td>
<td>5/9</td>
<td>4096² / 8192²</td>
<td>1</td>
</tr>
<tr>
<td>Jacobi3D</td>
<td>3D</td>
<td>7/13</td>
<td>256³ / 512³</td>
<td>1</td>
</tr>
<tr>
<td>Poisson</td>
<td>3D</td>
<td>19</td>
<td>256³ / 512³</td>
<td>1</td>
</tr>
<tr>
<td>Heat</td>
<td>3D</td>
<td>7</td>
<td>256³ / 512³</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1. Benchmarks used in the evaluation.

**Benchmarks** The LIFT-generated kernels are compared against hand-tuned and automatically-generated kernels from the PPCG compiler. We collected hand-written kernels from SHOC (v1.1.5), Rodinia (v3.1) and an OpenCL version of the acoustics simulation code discussed in Section 3.5. We hard-coded each benchmark to perform a single iteration of the stencil computation. We also collected a series of single-kernel C codes that work with the PPCG compiler from a recent study [35, 36], provided by the authors. Table 1 lists these benchmarks along with their key characteristics.

**Auto-Tuning** LIFT exposes optimization choices via rewrite rules which leads to several low-level LIFT expressions per benchmark. Each low-level expression contains many parameters that are tunable, controlling for instance: local/global thread counts, tile sizes, how much work a thread performs or how memory accesses are reordered. The parameters of each LIFT expression are fine-tuned using the ATF auto-tuning framework [34] which builds on top of OpenTuner [1] and additionally allows constraint specification in the parameter space. The auto-tuner was used for a maximum of three hours for a single program for tuning all expressions. The PPCG compiler used in our comparison exposes global/local thread counts and tile sizes as tunable parameters in each dimension. We also use ATF and OpenTuner for finding the best combination of these parameters, with again a maximum tuning time of three hours per benchmark. For both LIFT and PPCG, the auto-tuner has been enhanced to take into account OpenCL specific constraints (e.g., global thread counts should be a multiple of local thread counts).

7 Evaluation

7.1 Performance Results

This section presents the results of the exploration and auto-tuning process. As can be seen, in most cases the LIFT generated kernels are comparable to their hand-written counter parts, showing that our compiler approach generates high-performance kernels.

The benchmarks sr ad1 and sr ad2 seem to under-perform compared to the other benchmarks on the AMD and Nvidia platforms. This is due to the input sizes being too small to saturate these large GPUs (on the smaller ARM GPU, these benchmarks perform as good as the others).

The Hotspot2D benchmark is also a clear outlier on the AMD and ARM platforms. On the ARM GPU, the LIFT generated version is 2× faster than the hand-written version. On the AMD platform, the performance of the hand-written version is clearly under-performing, especially compared to the performance of the other benchmarks. The LIFT generated kernel achieves similar performance than the other benchmarks while being 15× faster than the hand-written version which was originally written for an Nvidia platform. This clearly illustrates the need for code-generation techniques which compile generated code specific to a particular device.

7.2 Performance Comparison of LIFT versus PPCG

This section compares LIFT with the state-of-the-art PPCG polyhedral GPU compiler [48]. Similar to LIFT, PPCG is an approach for compiling data-parallel algorithms starting from a single program and generating optimized code.

Figure 7 shows the performance for the six benchmarks of which we have hand-written implementations. As can be seen, the best PPCG-generated kernels are comparable to their hand-written counter parts, showing that our compiler approach generates high-performance kernels.

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On Nvidia, many benchmarks achieve a speedup of up to 4× over PPCG, such as the Heat program with large size, where LIFT is 4.3× faster. In this case, the best LIFT kernel performs no tiling and each thread only computes 2 elements. On the contrary, the PPCG version looks very different and uses tiling, with each thread processing 512× more elements sequentially than LIFT. For Gradient on the small size, the PPCG version looks very different and uses tiling, with each thread processing 512× more elements sequentially than LIFT. On the large size, the PPCG performance is almost as good as LIFT. In this case, both versions are similar, use tiling and the difference between the amount of sequential work is only 4×.

On AMD, the results look more uniform, with the exception of the Poisson benchmark on the large input. Here again, the best LIFT kernel does not use tiling, while the PPCG compiler generates a tiled version of the benchmarks. On the ARM GPU, the results of LIFT and PPCG are much closer than on the other platforms, with most of the gain coming again from not using tiling.

Interestingly, none of the LIFT kernels generated for the ARM or AMD GPU use tiling, however on Nvidia 33% of the best LIFT versions use the tiling optimization. This confirms that different optimization strategies are required for varying program/input sizes as well as for different hardware.
8 Related Work

Stencil-Specific High-Level Programming Approaches

There exist many approaches aiming to simplify the programming of stencils. These include stencil-specific DSLs (Domain Specific Languages) or EDSLs (Embedded DSLs) like HLSF [12], Pochoir [46], Halide [33], PolyMage [31], and many others [3, 8, 20, 22, 30]. [35, 36] discuss a stencil-specific compiler with a focus on fusion of stencil operations to minimize data movements. Even more specialized solutions exist for Partial Differential Equations [4, 5] and image processing [14]. Skeleton libraries providing stencil skeletons include SkePU [13], SkelCL [40], MUESLI [24], and PASTHA [25]. Most of these approaches rely on hard-coded and stencil specific implementations.

Optimizations for Stencil Computations

There are also many works detailing stencil optimization strategies. These include blocking [32, 50, 51] and tiling approaches [17–19, 23, 26, 37], and other collections of optimizations [11, 15, 28, 44]. Furthermore, multiple auto-tuning frameworks aim to automatically optimize stencils [16, 21, 27].

High Performance Code Generation

Languages like Accelerate [29], StreamIt [47] or Halide [33] aim to simplify the programming of GPUs through parallel patterns. Delite [45] is the closest related work. A small set of parallel patterns is compiled and optimized by a single backend into high-performance code. This approach lacks performance portability as device-specific optimizations have to be implemented separately for each platform. In contrast, Lift goes a step further by encoding optimizations in an extensible system of rewrite rules.

9 Conclusions

This paper has shown how stencils and their optimizations are expressible in the data-parallel, hardware-agnostic intermediate language Lift. The language has been extended by two primitives to gather neighboring elements (slide) and define boundary conditions (pad). Lift can now express complex stencils (like acoustic simulations), which will allow higher-level DSLs to be defined on top of these primitives.

Stencil-specific optimizations are encoded as rewrite rules. Due to the general nature of Lift, we are able to leverage existing Lift optimizations which are also directly applicable to stencil computations. This demonstrates that Lift is easily extensible to new domains with little effort required.

Finally, experimental results provide evidence that this approach generates high-performance stencil code on GPUs. On three platforms, we see that performance is on par with hand-optimized reference implementations. We also compare our approach to the PPCG polyhedral GPU compiler, showing that Lift outperforms it in many cases.

Acknowledgments

We would like to thank the Lift team; Prashant Singh Rawat for help with the PPCG comparison; Ari Rasch and students of the University of Münster for help with the ATF framework and its integration with Lift. The first author was supported by a HiPEAC collaboration grant. This work was also supported in part by the EPSRC Centre for Doctoral Training in Pervasive Parallelism, funded by the UK Engineering and Physical Sciences Research Council (grant EP/L01503X/1) and the University of Edinburgh.
References


A Artifact Description
Submission and reviewing guidelines and methodology: http://cTuning.org/ae/submission.html

A.1 Abstract
The artifact contains the high performance stencil code generator Lift which is described in the CGO 2018 paper High Performance Stencil Code Generation with Lift. Furthermore, this artifact contains the scripts and reference implementations required to reproduce the performance results presented in the paper. To validate the results build Lift and the software used as comparison with the provided scripts, run the benchmarks and, finally, the plotting script to reproduce the results from Figure 7 and 8 in the paper. We also provide scripts to generate OpenCL kernels from high-level Lift expressions, as well as scripts for performing a time-intensive auto-tuning for finding the best performing OpenCL kernel and its numerical parameters on the target platform.

A.2 Description
A.2.1 Check-List (Artifact Meta Information)
- Program: The Lift stencil code generator implemented in Scala; The PPCG polyhedral compiler; Benchmark programs implemented in C/C++ using OpenCL
- Compilation: With provided scripts
- Data set: Provided with the corresponding benchmarks
- Run-time environment: Linux with OpenCL
- Hardware: Any OpenCL enabled GPU
- Output: Runtime in CSV files and plots as PDFs
- Experiment workflow: Git clone; build software; run benchmarking scripts; observe performance results
- Publicly available?: Yes

A.2.2 How Delivered
The artifact is publicly available and hosted on gitlab at: https://gitlab.com/larisa.stoltzfus/liftstencil-cgo2018-artifact/

A.2.3 Hardware Dependencies
An OpenCL enabled GPU is required. In the paper we used a Nvidia Tesla K20c, an AMD Radeon HD 7970, and a ARM Mali GPU on an ODROID-XU4 developer board.

A.2.4 Software Dependencies
Lift requires Java 8, OpenCL, OpenTuner, the Auto Tuning Framework (ATF), and PPCG as its main dependencies.

The software dependencies are listed on the gitlab page.

A.2.5 Datasets
Datasets are provided as part of the artifact.

A.3 Installation
After cloning the repository, build scripts are provided for Lift, OpenTuner, the ATF, PPCG, and the used benchmarks.

Detailed installation descriptions are given on gitlab.

A.4 Experiment Workflow
After the installation, provided scripts should be used for running all benchmarks and plotting the results.

Detailed descriptions for the experiment workflow are provided on the gitlab page.

A.5 Evaluation and Expected Result
The main results of the artifact evaluation is to reproduce the performance comparison given in Figure 7 and 8. Depending on the precise GPU used for evaluation we expect the results to show a similar performance trend as reported in the paper between the Lift generated OpenCL kernels compared to the reference implementations.

The reviewers are invited to investigate the implementation of the Lift compiler, re-generate OpenCL kernels with Lift and perform the auto-tuning process for finding the best OpenCL kernel and parameters for a target hardware.