Accelerating T2* Mapping with Maximum Likelihood Estimation (MLE) and Parallel Imaging (PI)

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Accelerating T2* Mapping with Maximum Likelihood Estimation (MLE) and Parallel Imaging (PI)

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SYNOPSIS (100 words):
The utility of MR parametric mapping is limited due to the lengthy acquisition time. A Maximum Likelihood Estimation (MLE) and Parallel Imaging (PI) method is presented for MR parameter mapping. The approach is based on a high Signal to Noise (SNR) assumption such that the noise can be modelled as Gaussian and estimates the parameters that maximizes the signal from a multichannel coil. The method was tested on a multi-echo gradient-echo T2* mapping experiment in a phantom and a human brain. Accurate T2* maps were reconstructed up to an acceleration factor of 6 with a small error for phantom and human brain.

(750 words)

Background:
The estimation of MR parameters such as relaxation times (T1, T2, T2*) requires the acquisition of multiple images at different sequence parameters. Parameter estimation is achieved by fitting the signal evolution with a parameter-dependent model on a pixel-wise basis. It has been demonstrated that the accurate estimation of relaxation times can be done over a wide range of SNR and phased array coil configurations with the MLE technique 1, 2. The aim of this study is to present a method based on MLE that can estimate the relaxation times in conjunction with PI methods.

Theory:
The mono-exponential model applied to decay of a multi-echo sequence is given by

\[ M(TE)_i = PD \cdot \exp\left(\frac{-TE_i}{T}\right) \]  

(1)

Here, pseudo density PD represents the signal amplitude for an echo time \( TE = 0 \) and \( T \) is the relaxation time. Because the repetition time is fixed, PD is the product of three unknown factors: T1 weighting, PD and receiver coil response2. Conventionally parameters are estimated by least squares which minimizes the residual sum of squares between observed data \( y \) and exponential model at the \( ith \) TE over N echoes:

\[ X^2 = \sum_{i=1}^{N} [y_i - PD \cdot \exp\left(\frac{-TE_i}{T}\right)] \]  

(2)
Multichannel coils are used to enhance SNR and to accelerate the image acquisition by employing PI\(^4,5\). In the case of images acquired with parallel methods, the distribution of noise in the image is dependent upon the coil sensitivity profiles and varies spatially as well as with the reconstruction method.

According to MLE, for a given number of coil channels \(C\), the best estimate of relaxation time \(T\) will be derived by maximizing with respect to \(PD_C\) and \(T\) the joint probability distribution:

\[
\ln P_C(y_C; PD, T) = \prod_{i=1}^{N} P\{y_C(TE_i); PD, T\} = \sum_{j=1}^{C} \sum_{i=1}^{N} \left[ y_{i,j} - PD_j \cdot \exp\frac{TE_i}{T} \right]
\]

where \(P\{y_C(TE_i); PD, T\}\) is the joint signal probability distribution. In the case of higher SNR when the noise distribution can be considered normal, the approximation to the ML is equivalent to the weighted least squares solution because the variance of noise in case of multichannel coils is uncorrelated across the coils.

Our MLE approach estimates the proton density which is weighted according to the sensitivity information of the coils and a relaxation time which maximizes the signal from all the coils according to the equation (3). For parallel imaging, SPIRiT operator \((G_p)\) with acquisition parameter \(p\) that multiplies SPIRiT kernel in image space is used \(^6\). To accelerate the parametric mapping, the problem can be solved using a projection onto convex sets (POCS) algorithm applying MLE and SPIRiT iteratively (Figure1) to undersampled data in order to impose both the exponential relaxation model and the SPIRiT multi-coil model.

Methods:

The proposed approach was demonstrated for T2* mapping in the phantom comprising nine compartments (T2* values 65-250 ms). Furthermore, an in-vivo experiment was performed in a healthy human brain. All measurements were performed on a 1.5T clinical scanner (GE Healthcare, Waukesha, WI, USA) using a 3D-enhanced fast gradient-recalled echo sequence with the following parameters (16 echoes, TR = 87 ms, FOV = 256mm, 256x256 matrix, readout bandwidth=31.56 KHz, flip angle 15°, 2mm slice thickness). The total acquisition time for acquiring 32 slices was approximately 18 min. The dataset was retrospectively undersampled by factors of 2, 3, 4, 5, and 6 with variable density Poisson disk patterns including an autocalibration region of 24 x 24 in the ky–kz plane. A 7x7 SPIRiT kernel was calibrated from the autocalibration region and the undersampled data was reconstructed by SPIRiT and MLE using proposed POCS reconstruction. To determine the error of the estimated T2* maps, the normalized root mean square error (nRMSE) was calculated for all the accelerated and fully sampled dataset.

Results:

Figure 2 (a&b) show results for the T2* maps obtained from the fully sampled data and the maps from undersampled data for the phantom and in-vivo measurements, respectively. The number of iterations needed for the reconstruction was between 10 and 15. The NRMSE with respect to the fully sampled map is given below each map. T2* maps from phantom and the volunteer showed the same trend with increased nRMSE with increasing acceleration factor.
Conclusion:
The proposed MLE-SPIRiT reconstruction allows reconstruction of T2* maps with a small number of iterations. The method allows significant reduction of the required data without compromising the quality of the parameter maps. This could be used to accelerate MR parameter mapping, which is important for applications in which scan time is limited, and further contributes to increasing patient comfort.

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Iterative POCS ALGORITHM FOR SPIRiT-MLE

INPUTS: $y_p$—k-space measurements with acquisition parameter $p$
$D_p$—subsampling operator selecting acquired k-space with acquisition parameter $p$
$G_p$—SPIRiT operator with acquisition parameter $p$

OPTIONAL PARAMETERS:

MaxIter—stopping criteria by number of iterations (default=30)
TolDiff—stopping criteria by reconstruction difference between two iterations (default=$10^{-4}$)

OUTPUTS: $T$, $PD_j$—Parameters estimates from Eq. [3]

% initialization
$$k = 0, x_p^{(k)} = F^{-1} D_p^T y_p, p = 1, 2, 3, \ldots n_p$$

%iterations
$$\text{while } (k < \text{MaxIter} \text{ and } \| x^{(k)} - x^{(k-1)} \|_2^2 / \| x^{(0)} \|_2^2 > \text{TollDiff})$$
$$k = k + 1$$

%SPIRiT operation
$$x_p^{(k)} = G_p x_p^{(k-1)}, p = 1, 2, 3, \ldots n_p$$

%MLE
$$\min x \text{ s.t. } \sum_{j=1}^{C} \sum_{i=1}^{N} [y_{i,j} - PD_j \cdot \exp \frac{TE_i}{T}]$$

%Data Consistency
$$x_p^{(k)} = F^{-1} [(1 - D_p^T D_p) F x_p^{(k)} + D_p^T y_p], p = 1, 2, 3, \ldots n_p$$

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Figure 1: The convex optimization problem in Eq. [3] can be solved using a POCS algorithm, in which SPIRiT, the MLE and consistency with the data acquisition are enforced sequentially. The SPIRiT kernel is first calculated from the fully sampled central k-space before the POCS algorithm is carried out.
Figure 2: T2* maps of the fully sampled dataset of phantom (a) and human brain (b) and from the MLE-SPIRiT with acceleration factor of 2, 3, 4, 5, 6 are shown in the top row. The corresponding difference of the T2* maps between fully sampled and the undersampled data are shown in the bottom row with the nRMSE. The colorbar shows the T2* values in ms.
References: