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GALAXIES IN X-RAY SELECTED CLUSTERS AND GROUPS IN DARK ENERGY SURVEY DATA II:
HIERARCHICAL BAYESIAN MODELING OF RED-SEQUENCE GALAXY LUMINOSITY FUNCTION


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1. INTRODUCTION

Galaxy clusters are special for both cosmology and astrophysics studies. As the structures that correspond to the massive end of halo mass function, they are sensitive probes of the ΛCDM cosmological model (see reviews in Allen et al. 2011; Weinberg et al. 2013). As the most massive virialized structures in the universe, they provide the sites for studying astrophysical processes in dense environments.

Galaxy clusters are known to harbor red sequence (RS) galaxies, so named because these galaxies rest on a tight relation in the color-magnitude space (Bower et al. 1999). The phenomenon has been employed in finding clusters from optical data (e.g., Gladders & Yee 2000; Miller et al. 2005; Koester et al. 2007; Rykoff et al. 2016; Oguri et al. 2017) and developing cluster mass proxies (e.g., Rykoff et al. 2012). Red sequence galaxies also attract attention in astrophysics studies as they exhibit little star formation activity. Their formation and evolution provide clues to how quenching of galaxy star formation occurs in the cluster environment.

It is well-established that the massive red sequence galaxies form at an early epoch (e.g., Mullis et al. 2005; Stanford et al. 2005; Mei et al. 2006; Eisenhardt et al. 2008; Kurk et al. 2009; Hilton et al. 2009; Papovich et al. 2010; Gobat et al. 2011; Jaffé et al. 2011; Grützbauch et al. 2012; Tanaka et al. 2013), but the formation of faint red sequence galaxies can be better characterized. The latter could be examined through inspecting the luminosity distribution of cluster galaxies, either with the dwarf-to-giant ratio approach (De Lucia et al. 2007), or as adopted in this paper, with a luminosity function (LF) analysis. Results from these analyses are controversial to date, and have been extensively reviewed in literature (e.g., Faber et al. 2007; Crawford et al. 2009; Boselli & Gavazzi 2014; Wen & Han 2015).

To summarize, a few studies have reported a deficit of faint red sequence galaxies with increasing redshift (De Lucia et al. 2007; Stott et al. 2007; Gilbank et al. 2008; Rudnick et al. 2009; Capozzi et al. 2010; de Filippis et al. 2011; Martinet et al. 2015; Lin et al. 2017), indicating later formation of faint red sequence galaxies compared to the bright (and massive) ones. Yet, many other works observe little evolution in the red sequence luminosity distribution up to redshift 1.5 (Andreon 2008; Crawford et al. 2009; De Propris et al. 2013), suggesting an early formation of both faint and bright red sequence galaxies. Differences in these results are hard to interpret given the different methods (see the discussion in Crawford et al. 2009), sample selections and possible dependence on cluster mass (Gilbank et al. 2008; Hansen et al. 2009; Lan et al. 2015), dynamical states (Wen & Han 2015; De Propris et al. 2013), and whether or not the clusters are fossils (Zarattini et al. 2015). Carrying out more detailed analyses, especially in the 0.5 to 1.0 redshift range, may help resolve the differences.

The luminosity distribution of cluster galaxies has also been modeled to connect galaxies with the underlying dark matter distribution. The luminosity function of galaxies in a halo/cluster of fixed mass, entitled the conditional luminosity function (CLF) in the literature (Yang et al. 2003), statistically models how galaxies occupy dark matter halos. Modeling the Halo Occupation Distribution (HOD, Peacock & Smith 2000; Berlind & Weinberg 2002; Bullock et al. 2002) provides another popular yet closely-related approach. Given a dark matter halo distribution, these models (HOD & CLF) can be linked with several galaxy distribution and evolution properties (e.g., Popesso et al. 2005; Cooray 2006; Popesso et al. 2007; Zheng et al. 2007; van den Bosch et al. 2007; Zehavi et al. 2011; Leauthaud et al. 2012; Reddick et al. 2013), including galaxy correlation functions (e.g., Jing et al. 1998; Peacock & Smith 2000; Seljak 2000), galaxy luminosity/stellar mass functions (e.g., Yang et al. 2009), global star formation rate (e.g., Behroozi et al. 2013) and galaxy-galaxy lensing signals (e.g., Mandelbaum et al. 2006).

Furthermore, LF & HOD analyses improve our understanding of the cluster galaxy population. The number of cluster galaxies, especially the number of cluster red sequence galaxies, is a useful mass proxy for cluster abundance cosmology. Deep optical surveys like the Dark Energy Survey (DES1, DES Collaboration 2005) demand refined understanding of the evolution of cluster galaxies to $z = 1.0$ (Melchior et al. 2016).

The Sloan Digital Sky Survey (SDSS2) has enabled detailed analysis of the cluster LFs (or CLFs) with the identification of tens of thousands of clusters to redshift 0.5 (Yang et al. 2008; Hansen et al. 2009). Above redshift 0.5, studies are still limited to small samples containing a handful of clusters.

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1 Yuanyuan Zhang
2 https://www.darkenergysurvey.org
or groups (Andreon 2008; Rudnick et al. 2009; Crawford et al. 2009; De Propris et al. 2013; Martínez et al. 2015; De Propris 2017). However, recent developments of wide field surveys that are more sensitive than SDSS provide an opportunity to reinvigorate such analyses.

In this paper, we constrain the (conditional) red sequence luminosity function (RSLF) with an X-ray selected cluster sample detected in the DES Science Verification (DES-SV) data. Clusters selected with the same approach are used in a cluster central galaxy study in Zhang et al. (2016). The sample contains ~100 clusters and groups in the mass range of $3 \times 10^{13} M_\odot$ to $2 \times 10^{15} M_\odot$, and the redshift range of 0.1 to 1.05. As the clusters are not selected by their red sequence properties, studying RSLF with the sample is not subject to red sequence selection biases. Similar analyses can also be applied to SZ-selected clusters (e.g., clusters discovered from the South Pole Telescope survey: Bleem et al. 2015; Hennig et al. 2017) and clusters selected from optical data. Our paper focuses on cluster red members. The luminosity function of blue galaxies generally deviates from that of the red, but the red cluster members are easier to select photometrically due to the tightness of the color-magnitude relation.

The number of member galaxies in low mass clusters is often too low to study LFs for individual systems. It is a common approach to stack the galaxy red luminosity distributions for an ensemble of clusters (e.g., Yang et al. 2009; Hansen et al. 2009). However in this paper, in order to disentangle possible mass dependence and redshift evolution effects, we develop a hierarchical Bayesian modeling technique that simultaneously models these effects. In the rest of the paper, we first introduce our data sets in Section 2 and then describe the methods in Section 3. The results are presented in Section 4. Discussions of the methods and results as well as a summary of the paper are presented in Section 5.

2. DATA

2.1. Dark Energy Survey Science Verification Data

We use the DES Science Verification (DES-SV) data taken in late 2012 and early 2013. The DES collaboration collected this data set with the newly mounted Dark Energy Camera (DECam. Flaugher et al. 2015) for science verification purposes before the main survey began (for details on DES Year 1 operations, see Diehl et al. 2014). In total, the data set covers ~400 deg$^2$ of the sky. For about 200 deg$^2$, data are available in all of the $g$, $r$, $i$, $z$ and $Y$ bands, and the total exposure time in each band fulfills DES full depth requirement (23 to 24 mag in $i$ and 22 to 23 mag in $z$, see more details in Sánchez et al. 2014). A pilot supernovae survey (see Papadopoulos et al. 2015, for an overview) of 30 deg$^2$ sky in $g$, $r$, $i$, $z$ was conducted at the same time, reaching deeper depth after image coaddition (~25 mag in $i$ and ~24 mag in $z$).

The DES-SV data are processed with the official DES data reduction pipeline (Sevilla et al. 2011; Mohr et al. 2012). In this pipeline, single exposure images are assessed, detrended, calibrated and coadded. The coadded images are then fed to the SExtractor software (Bertin & Arnouts 1996; Bertin 2011) for object detection and photometry measurement.

2.2. The DES Photometric Data

We use a DES value-added catalog, the “gold” data set (see the review in Rykoff et al. 2016; Drlica-Wagner et al. 2017)$^4$, based on catalogs produced from the SExtractor software. The detection threshold is set at $1.5\sigma$ ($\text{DETECT_THRESH} = 1.5$) with the default SExtractor convolution filter. The minimum detection area is set at 6 pixels$^5$ ($\text{DETECT_MINAREA} = 6$). The SExtractor runs were performed in dual mode, using the linear addition of $r$, $i$ and $z$ band images as the detection image.

The “gold” data set is subsequently derived with the initial detections, keeping only regions that are available in all of the $g$, $r$, $i$, $z$ bands. Regions with a high density of outlier colors due to the impact of scattered light, satellite or airplane trails, and regions with low density of galaxies near the edge of the survey are removed. Objects near bright stars selected from the Two Micron All Sky Survey (2MASS Skrutskie et al. 2006) are masked. The masking radius scales with stellar brightness in $J$ as $R_{\text{mask}} = 150\sim10/\text{arcseconds}$ with a maximum of 120 arcseconds. Stars of nominal masking radius less than 30 arcseconds are not masked to avoid excessive masking. Coverage of the sample is recorded with the HEALPix$^6$ software (Górski et al. 2005) gridded by $N = 4096$. Photometry are re-calibrated and extinction-corrected using the Stellar Locus Regression technique (SLR: Kelly et al. 2014).

We make use of the SExtractor Kron magnitudes ($\text{mag\_auto}$, Kron 1980) for all detected objects. Since the SExtractor run was performed in dual mode, the Kron aperture and the centroid for different filters are the same, studying RSLF with the sample is not subject to red sequence selection biases. The upper and right histograms respectively show the cluster redshift and mass distribution.

We derive completeness limits for the selected $>5\sigma$ objects. Details of the completeness analyses are provided in Appendix A. In general, the completeness limits are ~0.5 mag brighter than the sample’s 10$\sigma$ depth magnitudes. The selected $>5\sigma$ objects are >99.8% complete above the limits. Because of this high completeness level, we do not correct for incompleteness in this paper.

2.3. The XCS-SV cluster sample

![Fig. 1. — The XCS-SV clusters: redshifts, masses, and mass uncertainties. The upper and right histograms respectively show the cluster redshift and mass distribution.](https://opensource.ncsa.illinois.edu/confluence/display/DESDM/SVA1+GOLD+Catalog)

$^3$ http://des.ncsa.illinois.edu/releases/sva1

$^4$ https://opensource.ncsa.illinois.edu/confluence/display/DESDM/SVA1+GOLD+Catalog

$^5$ DECam pixel scale 0.263"

$^6$ http://healpix.sourceforge.net
The XCS-SV cluster sample is a product from the XMM Cluster Survey (Lloyd-Davies et al. 2011; Mehrten et al. 2012; Viana et al. 2013), which searches for cluster candidates in the XMM-Newton archival data. The XCS-SV sample refers to the confirmed clusters from the DES-SV optical observations. The sample contains galaxy groups, low mass clusters and clusters as massive as $10^{15} M_\odot$ to beyond redshift 1. Selection and confirmation methods of the sample, as well as the cluster photometric redshift measurements are reviewed in Zhang et al. (2016, henceforth referenced as Z16). The sample used in this paper is expanded from that in Z16 after finalizing the input data and selection methods.

Since this paper evaluates luminosity function with the $z$-band photometry, we eliminate clusters above redshift 1.05 for which the rest-frame 4000Å break of RS galaxies have shifted out of DES $z$-band coverage (sensitive to $\sim 8500$ Å). We only use clusters located in DES-SV regions with the analysis magnitude ranges (above characteristic magnitude + 2 mag) above the completeness limits (Section 2.2). The paper works with 94 clusters in total, which are listed in Appendix B, Table 3. In Figure 1, we show the redshifts, masses, and mass uncertainties of the analyzed clusters.

The cluster masses and uncertainties are derived with X-ray temperature based on a literature $T_X - M$ relation (Kettula et al. 2013) (see details also in Z16). $R_{200}$ is derived from $M_{200}$.

2.4. Red Sequence Galaxy Selection

The definition of cluster member galaxies in projected datasets is a difficult challenge. Our method is based on simple color cuts around the cluster red sequence (De Lucia et al. 2007; Stott et al. 2007; Gilbank et al. 2008; Crawford et al. 2009; Martinet et al. 2015). To account for the shifting of the 4000 Å break, we select red sequence galaxies according to $g - r$ color at $z < 0.375$, $r - i$ color at 0.375 $\leq z < 0.775$ and $i - z$ color at 0.775 $\leq z < 1.1$.

For a cluster at redshift $z$, we first apply a K-correction term (Blanton & Roweis 2007) to all the objects in the cluster field. These objects are band-shifted to a reference redshift (depending on the color choice), assuming the cluster redshift to be their original redshifts. We compare the corrected colors to a model color with the following standard:

$$(g-r)_{z=0.25} - (g-r)_{\text{model at } z=0.25} < \sqrt{\Delta_{g-r}^2 + \Delta_{g-r}^2},$$

$$(r-i)_{z=0.55} - (r-i)_{\text{model at } z=0.55} < \sqrt{\Delta_{r-i}^2 + \Delta_{r-i}^2},$$

$$(i-z)_{z=0.9} - (i-z)_{\text{model at } z=0.9} < \sqrt{\Delta_{i-z}^2 + \Delta_{i-z}^2}.$$

In these equations, the model colors ($g-r$, $r-i$, or $i-z$, details explained below) are the mid-points of a selection window at a reference redshift. $\Delta_{g-r}$, $\Delta_{r-i}$, and $\Delta_{i-z}$ are the photometry uncertainties. $\Delta_{g-r}$, $\Delta_{r-i}$, and $\Delta_{i-z}$ are the widths of the selection windows.

We set $\Delta_{g-r}$ to be 0.2 mag. The clipping width is chosen to be larger than the combination of the intrinsic scatter and the slope of red sequence color-magnitude relations, while avoiding a significant amount of blue galaxies. $\Delta_{r-i}$ is adjusted to be 0.15 through matching the number of selected cluster galaxies (after background subtraction, see Section 3.2 for details) to fiducial $g - r$ selections at $0.3 \leq z < 0.5$. $\Delta_{i-z}$ is adjusted to be 0.12 through matching the number of selected cluster galaxies (after background subtraction, see Section 3.2 for details) to fiducial $r - i$ selections at $z \geq 0.7$.

The model colors of $g - r$ at $z = 0.25$, $r - i$ at $z = 0.55$ and $i - z$ at $z = 0.9$ are based on a simple stellar population template from Bruzual & Charlot (2003), assuming a single starburst of metallicity $Z = 0.008$ at $z = 3.0$, computed with the EZGAL package 7 (Mancone & Gonzalez 2012). In Figure 2, we show the red sequence model, over-plotting the observer frame colors of the selected objects. Overall, the colors of the selected RS candidates match template well. The template also matches the colors of cluster red sequence defined by the RedMaPPer method (Rykoff et al. 2016).

For RS candidates selected with the above criteria, we employ a statistical background subtraction approach (see details in Section 3) to eliminate background objects, which on average constitute 50% of the cluster region galaxies brighter than $m^*+2$ mag.

The performance of star-galaxy classifiers applied to the DES SVA1 “gold” sample (Section 2.2) depends on the object’s apparent magnitude. The classifiers become indefinite for objects fainter than $\sim 22$ mag in $z$-band. Since it is possible to eliminate the stellar contamination with the background subtraction procedure (we estimate the background locally assuming a constant Galactic stellar density), we do not attempt to separate stars and galaxies among the RS candidates (above 21 mag in $z$, stars make up $\sim 10\%$ of the sample). We nevertheless refer to all objects as “galaxies”.

3. METHODS

The main results in this paper are derived with a hierarchical Bayesian method (application examples to cosmology can be found in Loredo & Hendry 2010). We constrain the RSLF with a single Schechter function (Schechter 1976) to the magnitude limit of $m^*+2$ mag, and simultaneously model the mass and redshift dependence of parameters (Section 3.1: a hierarchical Bayesian method). To test the method, we compare the constraints to results derived from stacking cluster galaxy number counts in luminosity bins (Section 3.2: alternative histogram method).

Generally, the input to both methods includes the observed magnitudes, $\{m_i\}$, of objects inside clusters or in a “field”

7 http://www.baryons.org/ezgal/
region ($m_i$ is the apparent magnitude of the $i$th object). We define the cluster region as enclosed within $0.6 \, R_{200}$ of the cluster centers (X-ray object centers). The contrast between cluster and background object densities (about 1:1 for most of the clusters) is large with this choice, and the amount of retained cluster galaxies is reasonable. We choose the field region to be annular, centered on the cluster, with the inner and outer radii being $3 \, R_{200}$ and $8 \, R_{200}$ respectively. The choice helps eliminating RSLF contributions from cluster-correlated large scale structures along the line-of-sight. The cluster central galaxies selected according to the criteria in Z16 are eliminated from the analysis. Central galaxies are known to be outliers to a Schechter function distribution. Their properties and halo occupation statistics are investigated in Z16.

The area of these regions are traced with random points that cover the “gold” sample footprint. For each cluster, we generate $\sim 1.5$ million random points within $10 \, R_{200}$. The number density is high enough that the resulting uncertainty is negligible ($\sim 1\%$ in the luminosity distribution measurements). We ignore the uncertainties from using random points.

3.1. A Hierarchical Bayesian Method

Given a model with a set of parameters $\Omega$ that describe the distribution of observables, Bayesian theory provides a framework for inferring $\Omega$ with a set of observed quantities $\{x\}$. In this sub-section, we describe methods developed in this framework.

Denoting the probability of observing $\{x\}$ in model $\Omega$ to be $P(\{x\}|\Omega)$, and the prior knowledge about the model parameters to be $P(\Omega)$, after observations of $\{x\}$, the Bayes’ theorem updates the knowledge about model parameters, namely the posterior distribution, to be:

$$P(\Omega|\{x\}) \propto P(\{x\}|\Omega)P(\Omega).$$  (2)

The above equation uses a proportional sign instead of an equal sign as a probability function needs to be normalized to 1. The normalization factor is un-interesting when the posterior probability is sampled with Markov Chain Monte Carlo.

In our application, the observables include the observed magnitudes of objects in the cluster or field region. A major component of our model is the Schechter function. The parameters of the Schechter function vary for clusters of different masses and redshifts. Our model, as the hierarchical model, assumes redshift and mass dependences for the faint end slope and the characteristic magnitude.

3.1.1. Basic Components of the Model

For one cluster galaxy, we assume that the probability of observing it with magnitude $m$ follows a Schechter function:

$$f(x) = \psi_f(0.4 ln10)10^{0.4(m^* - x)(\alpha + 1)}exp(-10^{0.4(m^* - x)})$$  (3)

In this equation, $\psi_f$ is the normalization parameter that normalizes $f(x)$ to 1. $\alpha$ and $m^*$ are the faint end slope and the characteristic magnitude, treated as free parameters of the model.

For one object in the cluster region, it can be either a cluster galaxy or a field object. For a field object, we denote the probability of observing it with magnitude $m$ to be $g(m)$. $g(m)$ is approximated with a normalized histogram of the object magnitude distribution in the field region.

The probability of observing one object in the cluster region is the combination of observing it as a field object and observing it as a cluster galaxy. The probability writes

$$h(m) = \psi_h[N_{cl}f(m)+N_{bg}g(m)]$$  (4)

In this equation, $N_{cl}$ is the number of cluster galaxies in the cluster region, and $N_{bg}$ is the number of field galaxies in the cluster region. Again, there exists a normalization factor $\psi_h$ that normalizes the probability function to 1.

We treat the sum of $N_{bg}$ and $N_{cl}$ as a Poisson distribution. The expected value of $N_{bg}$ can be extrapolated from the field region and the area ratio between the cluster and the field regions. Equation 4 introduces one free parameter, $N_{cl}$, which controls the relative density between cluster and field galaxies in the cluster region. $N_{cl}$ can be further related to the amplitude of the Schechter function, $\phi^*$ (in unit of total galaxy count), as the integration of the Schechter function over the interested magnitude range, written as

$$N_{cl} = \int \frac{\phi^* f(m)}{\psi_f} dm = \frac{\phi^*}{\psi_f} \int f(m) dm.$$  (5)

Thus far, the free parameters in our models are $\alpha$, $m^*$ from Equation 3 and $\phi^*$. Note that, in this section, we only perform analyses with galaxies brighter than the completeness magnitude limit (galaxies are considered to be more than 99.8% complete throughout the analyzed magnitude range, according to Section A).

We constrain $\phi^*$ with the number count of observed objects in the cluster region ($N$), assuming a Poisson distribution:

$$N \sim \text{Poisson}(N_{cl}+N_{bg})$$  (6)

The log-likelihood is explicitly written as:

$$\log P(N) \propto N \log (N_{cl}+N_{bg}) - (N_{cl}+N_{bg}).$$  (7)

For one cluster, we take the observables to be the observed magnitudes of cluster region objects, $\{m_j\}$, the object number count and $N$ and the background object number count. $N_{bg}$ is treated as a known quantity. The log-likelihood of observing these quantities is:

$$\log P(\{m_j\}, N | \alpha, m^*, \phi^*) \propto \log P(N | \phi^*, \alpha, m^*) + \sum_i \log P(\{m_i\} | \alpha, m^*, \phi^*)$$  (8)

$$\propto \log P(N) + \sum_i \log h(m_i).$$

3.1.2. Hierarchical Model

The Bayesian approach makes it possible to add dependences to $\alpha$ and $m^*$. We rewrite $\alpha$ and $m^*$ with redshift or mass dependences:

$$\alpha_j = A_\alpha \log(1+z_j) + B_\alpha (\log M_{true,j} - 14) + C_\alpha,$$

$$m^*_{cl,j} = A_{m^*} \log M_{true,j} - 14 + C_{m^*}.$$  (9)

Here, we distinguish between true and observed $M_{200}$ of clusters. $\log M_{true,j}$ represents the true $M_{200}$ mass of the $j$th cluster, while we use $\log M_{obs,j}$ to represent the $M_{200}$ mass derived from X-ray temperature for the $j$th cluster. $\log M_{true,j}$ for different clusters are treated as fixed parameters in the analysis, but we use observational constraints on $\log M_{200}$...
from X-ray data as priors (Gaussian distributions): \(\log \mathcal{M}_{\text{true,}j} \sim \mathcal{N}(\log \mathcal{M}_{\text{obs,}j}, \sigma_M^2)\). \(\sigma_M\) is the measurement uncertainty of \(\log \mathcal{M}_{\text{obs,}j}\) from X-ray data. The assumption about \(\log \mathcal{M}_{\text{true,}j}\) allows us to incorporate mass uncertainties into the analysis. Furthermore, we constrain \(m^*\) at \(z = 0.4\), but for each cluster, we extrapolate the \(m^*\) at its observed \(z\) from \(z = 0.4\) assuming a simple stellar population from Bruzual & Charlot (2003) with a single star burst of metallicity \(Z = 0.008\) at \(z = 3.0\) (the red sequence galaxy template used in Section 2.4).

\(\phi^*\) for each cluster is constrained separately. We assume a Gaussian distribution of \(\{\phi^*_j\}\) given the values predicted by the relation: \(\phi^*_j \sim \mathcal{N}(\log \phi_{\text{mean,}j}, \sigma_{\log \phi}^2)\). \(\sigma_{\log \phi}\) is the intrinsic scatter of the relation, fixed at 0.5 (Allowing the parameter to vary gives a scatter of ~0.2 to 0.3) to reduce the number of free parameters. We further assume a power law relation between \(\mathcal{M}_{\text{true,}j}\) and \(\phi_{\text{mean,}j}\):

\[
\log \phi_{\text{mean,}j} = B_\phi \times \log \mathcal{M}_{\text{true,}j} + C_\phi. \tag{10}
\]

The log likelihood of having \(\phi^*_j\) given \(\mathcal{M}_{\text{true,}j}\) writes:

\[
g_j(\phi^*_j) \propto \left(\frac{(B_\phi \times \log \mathcal{M}_{\text{true,}j} + C_\phi) - \log \phi^*_j}{2\sigma_{\log \phi}}\right)^2. \tag{11}
\]

The free parameters of this model are \(A_\alpha, B_\alpha, C_\alpha, B_m, C_m, B_\phi, C_\phi, \{\phi^*_j\}\) and \(\{\mathcal{M}_{\text{true,}j}\}\). The observed quantities are \(\{m_{ij}\}\) and \(\{N_j\}\) of all clusters. \(\{\log \mathcal{M}_{\text{obs,}j}\}\) are treated as priors for \(\{\log \mathcal{M}_{\text{true,}j}\}\). \(\{z_j\}\) as well as \(N_{bg, j}\) are treated as known quantities for each of the clusters. We summarize the model dependences with a schematic diagram in Figure 3. The log-likelihood of observing these quantities is:

\[
\log L(\{m_{ij}\}, \{N_j\}|A_\alpha, B_\alpha, C_\alpha, B_m, C_m, B_\phi, C_\phi, \{\phi^*_j\}, \{\mathcal{M}_{\text{true,}j}\}) = \log L(\{m_{ij}\}, \{N_j\}|\{\phi^*_j\}, \{\mathcal{M}_{\text{true,}j}\}) + \sum_i \log \mathcal{P}(\{m_{ij}\}|\{\alpha_j, m^*_{ij}, \phi^*_j\})
\]

\[
+ \sum_j \log \mathcal{L}(\phi^*_j|\mathcal{M}_{\text{true,}j})
\]

\[
\propto \sum_i \log \mathcal{P}(\{m_{ij}\}) + \sum_j \log h_j(m_{ij}, j) + g_j(\phi^*_j). \tag{12}
\]

Finally, the parameter posterior likelihood is

\[
\log L(A_\alpha, B_\alpha, C_\alpha, B_m, C_m, B_\phi, C_\phi, \{\phi^*_j\}, \{\mathcal{M}_{\text{true,}j}\}|\{m_{ij}\}, \{N_j\}) = \log L(\{m_{ij}\}, \{N_j\}|\{\phi^*_j\}, \{\mathcal{M}_{\text{true,}j}\}) + \sum_i \log \mathcal{P}(\{m_{ij}\}|\{\alpha_j, m^*_{ij}, \phi^*_j\})
\]

\[
+ \sum_j \log \mathcal{L}(\phi^*_j|\mathcal{M}_{\text{true,}j})
\]

\[
+ \log \mathcal{L}_{\text{prior}}(A_\alpha, B_\alpha, C_\alpha, B_m, C_m, B_\phi, C_\phi, \{\phi^*_j\}, \{\mathcal{M}_{\text{true,}j}\})
\]

\[
\propto \sum_j \log \mathcal{P}(\{N_j\}) + \sum_i \log h_j(m_{ij}, j) + g_j(\phi^*_j)
\]

\[
+ \log \mathcal{L}_{\text{prior}}(A_\alpha, B_\alpha, C_\alpha, B_m, C_m, B_\phi, C_\phi, \{\phi^*_j\}, \{\mathcal{M}_{\text{true,}j}\}). \tag{13}
\]

We assume flat priors for most of the model parameters except \(C_m\). For \(C_m\), we assume a Gaussian distribution as the prior, with the measurement from Hansen et al. (2009) as the mean, and 1 mag\(^2\) as the variance. These priors are listed in Table 1. Sampling from the parameter posterior likelihood is performed with the PyMC package (Fonnesbeck et al. 2015).

### 3.2. Alternative Histogram Method

We develop a separate method to test the fore-mentioned technique. This method starts with counting galaxies in magnitude bins. We use 150 bins from 15mag to 30mag spaced by 0.1mag. We do not see change of the results when adjusting the bin size from 0.2mag to 0.05mag.

The histogram counting is performed for the cluster region, \(N(m)\), and the field region, \(N(m)_{\text{background}}\). To estimate the contribution of field galaxies to the cluster histogram, we weight the number count of objects in the field region, with the random number ratio:

\[
N_{\log}(m) = N(m)_{\text{background}} \times \frac{N_{\text{random, cluster}}}{N_{\text{random, background}}}. \tag{14}
\]
The histograms are then averaged for both the cluster region red sequence galaxy template used in Section 2.4 and 3.1.2. Cluster mass, and also record the number count of clusters in each magnitude bin, \( C(m) \). During the summing process, we shift \( m \) by the apparent magnitude difference between the cluster redshift and a reference redshift (depending on the cluster redshift and mass binning) of a simple passively-evolving stellar population from Bruzual & Charlot (2003) with a single star burst of metallicity \( Z = 0.008 \) at \( z = 3.0 \) (the same red sequence galaxy template used in Section 2.4 and 3.1.2). The histograms are then averaged for both the cluster region and the field region to obtain \( N(m) \) and \( N_{bg}(m) \). Subtracting \( N_{bg}(m) \) from \( N(m) \) yields the luminosity distribution of cluster galaxies (Figure 4 in redshift bins and Figure 5 in mass bins).

We assume a Schechter function distribution for cluster galaxies:

\[
S(m) = \phi(0.4\ln10)10^{0.4(m^* - m)}(m^{*+1})\exp(-10^{0.4(m^* - m)}),
\]

therefore the expected number of galaxies in each magnitude bin in the cluster region is

\[
E(m) = S(m) + N_{bg}(m).
\]

Assuming Poisson distributions for the number of galaxies in each bin, we sample from the following likelihood:

\[
\log L \propto \sum_m N(m)C(m)\log[E(m)C(m)] - E(m)C(m).
\]

Sampling from the likelihood is performed with the EMCEE package (Foreman-Mackey et al. 2013).

4. RESULTS

4.1. Results from Hierarchical Bayesian Modeling

The Hierarchical Bayes model (Section 3.1.2) simultaneously constrains the redshift evolution and mass dependence of \( \alpha \) and \( m^* \):

\[
\begin{align*}
\alpha &= A_\alpha \log(1 + z) + B_\alpha (\log M_{200} - 14) + C_\alpha, \\
\log m^* &= B_m (\log M_{200} - 14) + C_m.
\end{align*}
\]

For each cluster, we only make use of the \([m^*-2, m^*+2]\) magnitude range. Galaxy members of the analyzed clusters are complete within this range by selection (see details in Section 2.3). The constraints of the \( \alpha \) and \( m^* \) relations are Gaussian-like according to visual checks. In Figure 6, we plot the \( \alpha \) and \( m^* \) relations as well as their uncertainties. For comparison, we show constraints from the alternative histogram approach (discussed in the following section).

The RSLF faint end slope, \( \alpha \), displays a weak evidence of redshift evolution. The \( A_\alpha \) parameter that controls the redshift evolution effect deviates from 0 at a significance level of 1.9\( \sigma \). For clusters of \( \log M_{200} = 14.1 \) (median mass of the cluster sample), \( \alpha \) is constrained to be \(-0.69 \pm 0.13 \) at \( z = 0.2 \), rising to \(-0.52 \pm 0.14 \) at \( z = 0.6 \). The mass dependence of \( \alpha \) is ambiguous. The \( B_\alpha \) parameter that controls this feature deviates from 0 by 0.9\( \sigma \). The effect is likely degenerate with the mass dependence of \( m^* \). When removing \( m^* \) mass dependence from the method (setting \( B_m \) to be 0), \( B_\alpha \) is consistent with 0.

We assume passive evolution to the RSLF characteristic magnitude \( m^* \). We do not notice deviations of \( m^* \) from the assumption (the \( m^* \) results in redshift and mass bins agree with the model). Although the method models \( m^* \) as mass dependent, the effect appears to be insignificant (\( B_m \) deviates from 0 by 1.0\( \sigma \)).

The hierarchical Bayesian method also constrains the RSLF amplitudes, \( \phi^* \), and the relations between \( \phi^* \) and \( \log M_{200} \). \( \phi^* \) scales with the total number of cluster galaxies. Our result shows a strong correlation between \( \phi^* \) and cluster mass (Figure 7).

4.2. Results in Redshift/Mass Bins

We divide the clusters into two redshift bins: \( 0.1 \leq z < 0.4 \) and \( 0.4 \leq z < 1.05 \) and apply the alternative histogram method (Section 3.2). The median cluster masses in each of the bins are \( 10^{14}hM_\odot \) and \( 10^{14.9}hM_\odot \) respectively. The fitted parameters are listed in Table 2. Results are also shown in Figure 4 and 6. Again, the RSLF faint end slope, \( \alpha \), displays a hint of redshift evolution. The measurements in two redshift bins differ by \( \sim 1.2\sigma \).

We divide the clusters into two mass bins: \( 13.2 \leq \log M_{200} < 14.4 \), \( 14.4 \leq \log M_{200} < 15.1 \) and apply the altema-
8

Fig. 6.— (Panels a and b) Redshift evolution of the faint end slope, $\alpha$, and the characteristic magnitude, $m^*$ (assuming passive redshift evolution of a simple stellar population from Bruzual & Charlot (2003) with a single star burst of metallicity $Z = 0.008$ at $z = 3.0$). (Panels c and d) Mass dependence of the faint end slope, $\alpha$, and the characteristic magnitude, $m^*$ (assuming passive redshift evolution). Solid red lines and shades indicate results derived with the hierarchical Bayesian method (Section 3.1). Solid red circles indicate results derived with the alternative histogram method (Section 3.2). Literature reports of the $\alpha$ and $m^*$ parameters are over-plotted.

4.3. Comparison to Literature

In Figure 6, we over-plot literature measurements of the RSLF $\alpha$ and $m^*$ parameters.

At low redshift, RSLF analyses based on SDSS data are available from Hansen et al. (2009, $z \sim 0.25$) and Lan et al. (2015, $z < 0.05$). The SDSS faint end slope measurements (Hansen et al. 2009) appear to be consistent with our results. The SDSS characteristic magnitudes appears to be slightly fainter than the values constrained in this paper, but still consistent within this paper’s $1\sigma$ uncertainties ($M^*_z$ at redshift 0.4 is $-22.0$ from Lan et al. or $-22.13$ from Hansen et al., comparing to $-22.19 \pm 0.19$ in this paper). Note that the SDSS results are derived with $r$ (Lan et al. 2015, $z < 0.05$) or $i$ (Lan et al. 2015, $z < 0.05$) band data and we assume a red sequence model in Section 2.4 when comparing the characteristic magnitudes.

In terms of parameter mass dependence, the $\alpha$ and $m^*$ measurements from Lan et al. (2015, $z < 0.05$) in different cluster mass ranges match well with our constraints. In Hansen et al. (2009), the mass dependence results for cluster RS galaxies are not explicitly listed, but there is a trend of $\alpha$ steepening in the mass range of $[10^{12}M_\odot, 10^{14}M_\odot]$, and then stabilizing beyond $10^{14}M_\odot$. The quantity $m^*$ displays a trend of brightening in the mass range of $[10^{13}M_\odot, 5 \times 10^{14}M_\odot]$, and then stabilizing beyond $5 \times 10^{14}M_\odot$. These measurements qualitatively agree to our result.

At intermediate to high redshift, measurements of RSLF are still scarce. Sample sizes used in previous works are much smaller than those in SDSS-based studies. Any mass depen-
shift and mass and stack the red sequence galaxies to increase the signal-to-noise of the RSLF. The stacked RSLF parameters are consistent with the Bayesian results. Our work represents the largest RSLF study to date that goes to redshift \( \sim 1.0 \).

A particularly interesting by-product of this study is that our model allows us to improve cluster mass estimation. This is because our Bayesian model constrains cluster masses, \( \log M_{\text{true}} \), from the correlation between \( \phi^* \) and \( \log M_{\text{true}} \); using priors inferred from X-ray measurements, \( \log M_{\text{obs}} \). While the posterior values of \( \log M_{\text{true}} \) agree to the values of \( \log M_{\text{obs}} \) (top panel of Figure 8), the uncertainties of \( \log M_{\text{true}} \) appear to be improved. The improvements are especially noticeable when the uncertainties of \( \log M_{\text{obs}} \) is higher than 0.3 dex (middle panel of Figure 8).

Based on the improved uncertainties of \( \log M_{\text{true}} \), and assuming \( \phi^* \) and X-ray measurements contribute independent Gaussian-like uncertainties to \( \log M_{\text{true}} \), we estimate the uncertainties of inferring \( \log M_{\text{true}} \) from only \( \phi^* \) as:

\[
\sigma(\log M_{\text{true}}) \text{ from } \phi^* = \frac{\sigma(\log M_{\text{true}})}{\sqrt{1.0 - \frac{\sigma^2(\log M_{\text{true}})}{\sigma^2(\log M_{\text{true}})}}}
\]

These estimations are shown in the bottom panel of Figure 8, which range from 0.2 to 0.4, with an average of 0.34. Comparatively, optical mass proxies derived from the numbers of cluster galaxies have intrinsic mass scatters between 0.2 to 0.5 dex (Rozo et al. 2009; Saro et al. 2015). This analysis demonstrates the potential of \( \phi^* \) as a cluster mass proxy.

Since the redshift evolution of the RSLF is only detected at a significance level of 1.9\( \sigma \), it is worthwhile to apply the analysis to a larger cluster sample. We expect the XCS to find over 1000 clusters within the DES final data release. We may also utilize new and large optical cluster catalogs such as RedMaPPer. However, optically characterized clusters will add new challenges from the covariance between the richness-inferred cluster masses and the red-sequence luminosity functions. An evolving abundance of faint RS galaxies will also introduce a redshift evolution component into the cluster mass-richness scaling relation. Assuming the \( \alpha \) evolution reported in this paper, we expect the number of RS galaxies above \( m^* + 2 \text{ mag} \) to decrease by \( \sim 20\% \text{ from } z = 0 \) to \( z = 1.0 \). Using the parameterization of cluster mass-richness scaling relation in Melchior et al. (2016), we expect the mass-to-richness ratio to change with redshift as \( (1+z)^{0.26} \) (constrained as \( (1+z)^{0.18/0.75(\text{stat})/0.24(\text{sys})} \)) in the fore-mentioned weak lensing study. Of course there could be additional effects on the mass-richness relation if there is redshift evolution in \( m^* \) and \( \phi^* \) or if the mass dependence of the RSLF is not properly accounted.

Regardless, we expect to increase the X-ray cluster sample size by at least a factor of 10 by the end of DES. Using catalog-level simulations of RSLF similar to the ones observed here, we expect to increase our sensitivity on the evolution of \( \alpha \) by a factor of three.

If there is redshift evolution in the faint-end slope of the red sequence galaxies, we can explain it through formation times and growth histories of galaxies. For instance, bright and faint cluster red sequence galaxies may have different formation times. It is possible that fainter galaxies are quenched during, rather than before, the cluster infall process. Hence the fraction of faint red sequence galaxies gradually increase with time. Astrophysical processes that slowly shut down
galaxy star formation activities, e.g., strangulation (sometimes called starvation) (Larson et al. 1980; Balogh & Morris 2000; Balogh et al. 2000; Peng et al. 2015) and hence gradually increase the fraction of faint red sequence galaxies, will be preferred over more rapid processes such as ram pressure stripping (Gunn & Gott 1972; Quilis et al. 2000). Combining the observational constraints on the evolution of the faint-end slope together with the cluster accretion history in simulations should help us place good constraints on the formation and transition times of cluster red sequence galaxies (McGee et al. 2009).

In summary, we constrain the relation between RSLF amplitudes and cluster masses, and the correlation improves the estimation of cluster masses. We find a hint that the Schechter function faint-end slope becomes less negative for clusters at higher redshifts, indicating an increasing abundance of faint red sequence galaxies with time. The redshift evolution of RSLF parameters may also impact the accuracy of optical cluster cosmology analyses.

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**Fig. 8.** In the hierarchical Bayesian method, we constrain cluster masses using X-ray temperature-inferred measurements as priors. (Panel a) The posterior estimations of cluster masses, logMtrue, agree with the priors logMobs, (Panel b) The assumption in the hierarchical Bayes model that cluster masses scale with RSLF amplitudes, $\phi^*$, helps improving the accuracy of cluster mass estimations. The posterior uncertainties of the mass estimations, $\sigma(\log M_{\text{true}})$, appear to be decreased, especially when the prior uncertainties, $\sigma(\log M_{\text{obs}})$, are higher than 0.2 dex. (Panel c) Based on the improved mass uncertainties, we estimate the uncertainties of inferring cluster masses from only $\phi^*$, which range from 0.2 to 0.4 dex (see details in Section 4.1).
The completeness function models the detection probability of objects in terms of their apparent magnitude. In this paper, the completeness function is defined as the ratio between the numbers of observed and true objects at magnitude $m$.

We model the completeness function with a complementary error function of three parameters:

$$p(m) = \frac{1}{2} \text{erfc}(\frac{m - m_{50}}{\sqrt{2w}}).$$  \hfill (A1)

In the above equation, $m_{50}$ is the 50% completeness magnitude, $w$ controls the steepness of the detection drop-out rate and $\lambda$ is the overall amplitude of the completeness function. We further assume linear dependence of $m_{50}$ and $w$ on the depth of the image, which is characterized by the 10 $\sigma$ limiting magnitude. In this paper, we evaluate the $z$-band completeness function, which is related to image depth in $z$.

The $m_{50} - m_{10\sigma}$ and $w - m_{10\sigma}$ relations are evaluated with simulated DES images and real data. The relations used in this paper are derived from the UFiG simulation (Bergé et al. 2013; Chang et al. 2015)(also see Leistedt et al. 2016, for an application), which is a sky simulation that is further based on an N-body dark matter simulation. The dark matter simulation is populated with galaxies from the Adding Density Determined GAlaxies to Lightcone Simulations (ADDGALS) algorithm (Wechsler et al. in prep., also see Lin et al. 2010; Busha et al. 2013; Dietrich et al. 2014).

We use the UFiG product that matches the footprint of the “gold” sample in Section 2.2. The simulation is divided into fields of 0.53 deg$^2$, with characteristic quantities like the image depth and seeing matching those of the DES-SV patches. SExtractor is applied to the simulated images with identical DES-SV settings. We select objects with $\text{magerr}_{\text{auto}} < 0.218$ mag in $z$ ($5\sigma$ significance), derive their observed magnitude distribution, and compare it to the truth magnitude distribution of all input truth objects (see illustration about the procedure in Figure 9). The ratio between the two is well described by Equation A1. The derived $m_{50}$ and $w$ are tightly related to the depth of the image as shown in Figure 10.

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APPENDIX

A. COMPLETENESS FUNCTION

A.1. The Completeness Function Model

The completeness function models the detection probability of objects in terms of their apparent magnitude. In this paper, the completeness function is defined as the ratio between the numbers of observed and true objects at magnitude $m$.

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A.2. Relations between Model parameters and Image Depth

The $m_{50} - m_{10\sigma}$ and $w - m_{10\sigma}$ relations are evaluated with simulated DES images and real data. The relations used in this paper are derived from the UFiG simulation (Bergé et al. 2013; Chang et al. 2015)(also see Leistedt et al. 2016, for an application), which is a sky simulation that is further based on an N-body dark matter simulation. The dark matter simulation is populated with galaxies from the Adding Density Determined GAlaxies to Lightcone Simulations (ADDGALS) algorithm (Wechsler et al. in prep., also see Lin et al. 2010; Busha et al. 2013; Dietrich et al. 2014).

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8 Magnitude with $\text{magerr}_{\text{auto}} = 0.108$. For a flux measurement at a significance level of 10 $\sigma$, the corresponding magnitude uncertainty is $2.5/\ln10/10 = 0.108$. 
We also perform the analysis with the BALROG simulation (Suchyta et al. 2015), which inserts simulated objects into real DES-SV images. The results are similar.

To further verify the derived relations, we stack high quality images from the DES Supernovae survey (with a total exposure time of $\sim 1000$ s) to mimic main survey depth. The $z$-band depth of the stacks ranges from 21.5 mag to 22.5 mag, comparing to > 24 mag coadding all eligible exposures. We compare the object counts in this set of coadds and the full coadds to evaluate $m_{50}$ and $w$ (also shown in Figure 10).

The $m_{50}$ appears to be $0.1 - 0.4$ mag deeper than the simulation relations. The effect is consistent with the $mag\_auto$ biases shown in Z16. In this test, we compare to the observed Kron magnitudes rather than the "truth" magnitudes (which is not known) from the deeper stack. Z16 shows that the observed Kron magnitudes are fainter by 0.1 to 0.4 mag comparing to the "truth" magnitudes at < 24 mag.

Figure 10 indicates that the amplitude of the complementary error function is lower than 1 in UFIG and BALROG. This is mostly caused by the same photometry measurement bias discussed above (another effect is the blending of truth objects, which causes incompleteness at a < 2% level). Objects are measured fainter by the Kron magnitude. Compared to the truth magnitude distribution, the observed magnitude distribution is systematically shifted to the fainter side (see this effect in Figure 9). The result is that the observed magnitude distribution is always lower than the truth distribution, and the amplitude of the fitted completeness function is below 1. This shift and the resulting amplitudes of the completeness function are not of interest in this paper. We explicitly assumes the amplitudes of the completeness function to be 1.

Figure 11 indicates that the completeness function in galaxy clusters are different from that of the fields, possibly because of blending and larger galaxy sizes. We test the effect with simulated objects (BALROG simulation, Suchyta et al. 2015) inserted into RedMaPPer clusters (Rykoff et al. 2016) selected in DES-SV data. We see evidence that the $m_{50}$ inside galaxy clusters shift by $\sim 0.1$ mag comparing to fields of equivalent depth (Figure 11). As the sample of simulated galaxies is small, we are unable to characterize the distribution of the shifts and hence do not attempt to correct $m_{50}$ in this paper.

We notice hints that the completeness function in galaxy clusters are different from that of the fields, possibly because of blending and larger galaxy sizes. We test the effect with simulated objects (BALROG simulation, Suchyta et al. 2015) inserted into RedMaPPer clusters (Rykoff et al. 2016) selected in DES-SV data. We see evidence that the $m_{50}$ inside galaxy clusters shift by $\sim 0.1$ mag comparing to fields of equivalent depth (Figure 11). As the sample of simulated galaxies is small, we are unable to characterize the distribution of the shifts and hence do not attempt to correct $m_{50}$ in this paper.

A.3. Completeness Limits of the RSLF Analyses

We determine the magnitude limits of the RSLF analyses according to the completeness functions. We perform the analyses only with galaxies brighter than the following limit: $m_{50} = m_{50} - 2\sqrt{2}w$. The cut ensures detection probability above 99.8% $\times \lambda$ for the selected galaxies.
Fig. 11.— We evaluate the $m_{50}$ parameters (50% completeness magnitudes) for cluster and for field regions of the same depth with the BALROG simulation. The $m_{50}$ of a cluster region is potentially shallower by $\sim 0.1$ mag compared to a same-depth field region potentially because of blending in the cluster region.

Fig. 12.— For each cluster, we derive a completeness limit, $m_{\text{lim}}$, from the completeness function. At $z < 0.4$, all of the DES XCS-SV clusters are complete to $m^* + 2$ mag and beyond. This is also true for more than 2/3 of the clusters at $z > 0.4$. Incomplete clusters of $m_{\text{lim}}$ below $m^* + 2$ mag are not included in this paper’s analyses. The scatters of $m_{\text{lim}}$ are caused by DES depth variations in different parts of the sky.

If the cluster region completeness functions follow different relations as discussed above, the magnitude cut still ensures high detection probability (lower limit of 99%×λ instead of 99.8%×λ).

For all of the $z < 0.4$ clusters, $m_{\text{lim}}$ is more than 2 mag fainter than the characteristic magnitude measured in Hansen et al. (2009). This is also true for more than 2/3 of the clusters at $z > 0.4$. The cluster sample size drops steeply above redshift 0.7, and most of the complete clusters are located in the DES deep supernovae fields. As the galaxy samples are highly complete, we do not correct detection probability in this paper.

Because the $g$, $r$, $i$, $z$-band observations are performed independently, one may wonder if the image depth in the bluer bands is sufficient for computing colors. For example, the $i$-band band observation of an object detected in $z$ may be too shallow that it does not have valid $i$-band photometry measurement. We confirm that after applying the $z$-band magnitude limit cut ($\text{mag}_{\text{auto}}(z) < m_{\text{lim}}$), 99.5% and 99.6% of the cluster region objects are detected in $r$ and $i$ respectively. 98.3% or 99.2% of the objects have good $r$ or $i$-band photometry measurement ($\text{magerr}_{\text{auto}}$ above 3σ, i.e., $\text{magerr}_{\text{auto}} < 2.5/\ln10/3$). We conclude that the DES multi-band data are sufficiently deep for red galaxy selection.

B. CLUSTER INFORMATION

<table>
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<th>Cluster Name</th>
<th>R.A.</th>
<th>Decl.</th>
<th>log($M_{500}$/$M_{\odot}$)</th>
<th>Cluster Redshift</th>
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<td>8.202084</td>
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<td>-43.376720</td>
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<td>0.3928a</td>
</tr>
<tr>
<td>XCSJ003428.0-431854.2</td>
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