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Two-Stage Admission and Scheduling Mechanism for Electric Vehicle Charging

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Abstract—In order to provide an efficient charging service to future electric vehicles, it is important for the charging station to estimate its available capacity. As the charging station usually has a fixed number of chargers and a limited charging capacity, it cannot accommodate all the arriving cars. Thus, a decision has to be made by the charging station whether a new arriving car can be admitted and then be scheduled for charging. This not only depends on the charging station’s physical constraints but also depends on how certain the charging station is towards its energy supply. We study the problem of EV charging using various energy sources (such as energy from the grid, renewable energy such as solar photovoltaic panels, and local energy storage) and their impacts on the charging station’s performance. Unlike the energy from the grid, solar generation varies with time and cannot be predicted precisely. To this end, we introduce a multiplier $k$ to measure the effect of solar prediction error and a composite performance index (the Figure of Merit) to capture the charging station’s utility, also taking account of EVs’ charging requirements. We further propose a two-stage admission and scheduling mechanism to find the optimal trade-off between accepting EVs and missing charging deadlines by determining the best value of $k$ under various energy supply scenarios.

Index Terms—Electric Vehicles, Admission control, Scheduling, Solar energy, Energy storage, Stochastic optimization.

I. INTRODUCTION

ELECTRIC Vehicle (EV) technology has attracted a growing interest from the public in recent years. EVs, as an alternative to the traditional internal combustion engine vehicles, are widely perceived as a green solution to improve energy efficiency and reduce carbon emissions [1]. EVs typically have a high energy requirement with a high charging rate, hence their rapid growth under the notion of a smarter grid can place a considerable amount of stress on the existing power grid without effective scheduling mechanisms.

There are some existing papers studying the EV charging and scheduling problem that can help to balance the load and avoid overload on the grid, such as in [2]–[5]. These papers mainly consider the case that the charging energy is drawn from the grid and focus on a deterministic setting where the scheduling problem can be solved using Mixed Integer Linear Programming (MILP) or various congestion management approaches [6]. However, if an EV is entirely charged by coal-fired power plants, the CO2 produced is generally more than for an alternative fuel-driven vehicle [7], which will lose the environmental benefits of introducing EVs. Thus, integrating renewable energy in coordination with EV charging is promising. Due to the time-varying and unpredictable nature of renewable energy generation, it becomes more challenging than using the energy from the main power grid.

In this paper, we study the problem of EV charging and scheduling using renewable energy at a microgrid-like charging station, where the upcoming EVs are charged by the energy from the solar panels as well as the local energy storage.

A. Related Works

Although EV charging scheduling involving renewable energy is now being studied such as in [8]–[14], they fail to address the necessity of introducing an effective admission control prior to real-time scheduling. Rather, these works assume that the charging station has the capacity to charge all arrivals. Although prior work in [15] has addressed the admission issue prior to charging scheduling, it did not consider the case of how to use renewable sources when only a forecast of the expected energy is available.

On the other side, many papers have focused on the benefits to either the EVs or the charging station, but not both simultaneously. References [2], [3], [16] have mainly focused on EV users’ perspective. In [2], the EV users are incentivized to report their charging requirements to the charging station truthfully, thus the quality of service is guaranteed. Other work in [9], [17]–[19] has mainly focused on optimizing the system’s cost, but they did not pay much attention to the EV owners’ satisfaction level. A pricing signal has also been leveraged to schedule, guide and coordinate EV charging such as work in [20]–[22]. Some papers have considered a deterministic energy availability setting in a charging scenario such as in [9], [13], [23], but their approaches cannot cope well with the uncertainty of renewable energy prediction.

In this work, we aim to fill the gap using renewable energy effectively to charge EVs by introducing a stochastic solar generation model, an effective admission control prior to the charging process, and a composite performance index to reflect both the charging station’s utility and EVs’ satisfaction level.

B. Main Contributions

We consider a commercial charging station powered by on-site solar panels and a local energy storage unit, that provides the charging service in the daytime. We first develop a stochastic solar generation model by introducing a multiplier $k$ for the solar energy prediction to account for the effect of prediction error. A performance index (the Figure of Merit) is proposed to measure the charging station’s utility. It takes into account the EV users’ charging requirements as well as
a penalty factor for turning away new arrivals and missing charging deadlines. The aim is then to maximize the Figure of Merit (FoM) by deciding which arriving EV to admit based on the uncertain solar supply and EV requirements. Thus, a priority-based two-stage admission and scheduling mechanism is further proposed to find the optimal value of $k$ that maximizes the charging station’s FoM.

We extend the work presented previously in [24] by optimizing the multiplier $k$ over time as well as investigating the impact of a local energy storage on the charging station’s FoM. The main contributions can be summarized as follows.

- Introduce a multiplier $k$ to account for the effect of solar generation prediction error.
- Propose the Figure of Merit (FoM) to measure the charging station’s performance.
- Propose a priority-based two-stage mechanism to solve FoM maximization problem.
- Investigate the impact of integrating energy storage and the cost of using various energy sources.

The remainder of this paper is organized as follows. Section II presents the system description and the mathematical models for the charging problem. Section III shows the proposed two-stage mechanism. Section IV illustrates the numerical evaluations. The conclusions are given in Section V. Table I further shows the main notation used in this paper.

## II. Problem Formulation

The charging station, such as a shopping mall, is equipped with on-site solar panels and energy storage units. It can be assumed that EVs can wirelessly contact the station in advance to try to reserve a charging spot and reduce risk. In practice, there may also be other conventional grid-connected or similar islanded stations within the same area that a rejected car can try. Web services\(^1\) can also provide information about car charging to consumers and help them to plan their journey. Such services can process EV charging requests and estimate each station’s capacity, in order to divert the on-the-move EVs to the best station, such as in the work proposed in [16], [25], [26]. Thus, a rejected EV will not become immobile due to low battery state, as long as another station is available. In this article, we evaluate one specific charging station’s performance and suppose that the charging station has $M$ charging points installed and operates in the daytime, as shown in Fig. 1.

### A. EV Arrival Model

Suppose that EVs arrive at the charging station following a Poisson distribution with an average arrival rate $\lambda$\(^2\) and a charging task:

\[
(E_i, t_i^a, t_i^d, a_i^m),
\]

\(^1\)Such as www.chargeyourcar.org.uk in the United Kingdom

\(^2\)The average arrival rate $\lambda$ is normally assumed as a constant variable in many papers, such as [14], [15], [27]. The arrival rate can be also modelled as a time-dependent variable $\lambda(t)$ for a potential higher accuracy, such as work in [28], [29]. Considering our main focus is to investigate the admission and scheduling mechanism, for simplicity, we assume a stationary Poisson arrival of the EVs in this study.

where $E_i$ is the charging energy requirement [kWh], $t_i^a$ is the arrival time, $a_i^m$ is the maximum charging rate [kW], and the deadline urgency $t_i^d$ is the maximum number of timeslots\(^3\) within which EV $i$ requires to be charged. Hence the active charging interval for $i$ is $[t_i^a, t_i^a + t_i^d]$. The energy level of EV $i$’s battery can be increased by $\Delta E_i(t) = \tau a_i(t)$ within each timeslot, where $\tau$ is the duration of a timeslot which is assumed to be 10 minutes. The variable $a_i(t)$ is the actual charging rate with $a_i(t) \in [0, a_i^m], \forall t$ and can be assumed as a constant for each timeslot. The remaining energy requirement for EV $i$ at $t$ can be written as:

\[
R_i(t) = E_i - \sum_{x=1}^{t} \Delta E_i(x).
\]

EVs can be put in different priority orders based on their battery states and the remaining charging times. A priority factor is further defined to capture this relation as:

\[
\omega_i(t) = \frac{t_i^a + t_i^d - t}{R_i(t)}.
\]

EV $i$ is more patient if it has a longer deadline with a lower energy requirement. The lower $\omega_i(t)$ is, the more urgent it is.

\(^3\)One timeslot has a duration of 10 minutes.
for EV $i$ to be charged, so a highest possible charging rate $a_i(t)$ is preferred. This relation can be further captured to reflect EV users’ satisfaction level as:

$$SAT_i(t) = \frac{1}{\omega_i(t)(a^m_i - a_i(t))^2}.$$  

(4)

For an EV to have a higher satisfaction level, a higher $SAT_i(t)$ is preferred. This principle is also accounted for in the design of the admission and scheduling mechanism, that EVs are admitted by the order of their priority factor and are allocated highest possible charging rates in turn.

### B. Energy Supply Model

As shown in Fig. 1, the energy used to charge EVs can be bought from the grid with a price $p_g$ as in [15]. Alternatively, the energy can be generated from on-site solar panels with a lower maintenance fee $p_m$ and stored for later use. At each considered timeslot, the storage can be charged from the solar panels or discharged to serve the EVs.

**Solar Generation:** Energy generated from solar panels is time-varying and limited, denoted as $S(t)$. Based on historical data and the weather forecast, $S(t)$ can be predicted, though the prediction is not always accurate. An effective scheduling mechanism should be able to adjust to this prediction uncertainty and be able to estimate the total capacity of the solar generation, since the actual values cannot be predicted precisely. The prediction uncertainty can be assumed as independent and identically distributed Gaussian samples with standard deviation $\sigma$ and mean zero, denoted as $e(t) \sim N(0, \sigma)$, since any correlations can be removed by applying various techniques from time series analysis. Thus $S(t)$ can be represented as:

$$S(t) = \hat{S}(t) + e(t),$$

(5)

where the prediction profile of the solar energy generation $\hat{S}(t)$ and its standard deviation $\sigma$ are extracted and estimated from the national grid database [30].

A prediction interval gives a range within which we expect the future observations to lie. The system should not be too optimistic as that will lead to over-admission of EVs and the system will miss many charging deadlines. Conversely, it should not be too pessimistic which will lead to charging less cars than the system will allow and losing revenue. Hence a multiplier $k$ is introduced to capture the effect of the prediction error $e(t)$. We expect the future solar energy observation $S(t)$ in Eq. (5) to lie within a prediction interval:

$$[\hat{S}(t) - k\sigma, \hat{S}(t) + k\sigma], \ k \in \mathbb{R},$$

(6)

with a specified probability depending on $k$. For instance,

1) if $k = 0$, $S(t) = \hat{S}(t)$;
2) if $k = 1$, $S(t)$ has a probability of 0.68 to lie within $[\hat{S}(t) - \sigma, \hat{S}(t) + \sigma]$.

Let us denote the lower bound of the prediction interval as:

$$S_{lb}(t, k) = \hat{S}(t) + k\sigma, \ k \in \mathbb{R},$$

(7)

where $S(t)$ has a specified probability to lie above $S_{lb}(t, k)$. For instance,

1) if $k = -1$, $S(t) > \hat{S}(t) - \sigma$ with probability 0.84;
2) if $k = +1$, $S(t) > \hat{S}(t) + \sigma$ with probability 0.16.

**Storage:** A local energy storage unit can be introduced to hedge against the uncertainty of the random variables such as the solar energy generation and EVs’ charging requirements. By providing sufficient flexibility, storage can help increase the station’s $FoM$. At time $t$, the excess solar energy after charging EVs can be stored for later use. However, introducing energy storage will also introduce an extra cost to the charging station. An operational cost $p_b$ of the storage is yielded per unit energy charged or discharged, that is every time when the energy state of the storage $B(t)$ changes. The cost $p_b$ can be assumed to be proportional to the storage capacity $B$. In this paper, we exclude the installation cost of the storage but to exploit the appropriate capacity of the storage to fulfil this task as an energy buffer. Thus, we assume the storage is controlled by the charging station which decides when to charge or discharge the storage unit in order to maximize the $FoM$.

Therefore, we aim to find the optimal value of $k$ where the charging station can achieve the best trade-off between admitting cars and failing to meet their deadlines. To this end, a performance index is proposed next to measure the charging station’s utility and EV owners’ satisfaction level.

### C. The Figure of Merit (FoM)

The revenue of the charging station is related to providing charging service to the admitted EVs. If an admitted EV is not charged as promised, the charging station will pay a penalty. This fits into a scenario where there is a trade-off between the admission rate and the successful service rate. Hence only evaluating the admission rate is not a fair measurement of the system performance. A composite performance index is further defined to better capture this relation, called the Figure of Merit ($FoM$):

$$FoM = U \cdot \frac{N_{adm} - \gamma N_{ms}}{N_{tar}}.$$  

(8)

The terms in this equation are defined as follows:
1) $U$: is the utilization of the energy supply and defined as the ratio of energy usage of the day. Depending on the capacity of the storage, there will possibly be some solar energy wasted and some energy left in the storage unit. $U$ can be written as

$$U(k) = 1 - \frac{\sum_{t=1}^{T} S_r(t,k) + E_b(k)}{\sum_{t=1}^{T} S(t)}, \quad (9)$$

where $S_r$ is the excess solar energy, $E_b$ is the amount of energy left in the storage at the end of the day at 6pm, and $S$ is the actual solar generation.

2) $\gamma$: is the penalty factor and can be defined as

$$\gamma = \frac{\gamma_{ms}}{\gamma_{pd}}, \quad (10)$$

where $\gamma_{ms}$ and $\gamma_{pd}$ represent the penalty weights placed on missing charging deadlines and rejecting cars, respectively.

3) $N_{ad}, N_{ms}, N_{ar}$: represent the number of admitted EVs, the number of EVs missing their charging deadlines, and the number of new arrivals. Further, Eq. (8) can be written as

$$FoM(k) = U(k) \cdot \left(1 - \frac{N_{ms}}{N_{ad}(k)}\right) \cdot \frac{N_{ad}(k)}{N_{ar}} \cdot \frac{N_{ad}(k)}{N_{ar}} \cdot \frac{N_{ad}(k)}{N_{ar}}, \quad (11)$$

where $R_s$ is the ratio of missing charging deadlines and $P_d$ is the probability of declining cars upon arrival.

The proposed composite performance index $FoM$ is able to reflect the benefit for both the charging station’s utility and EVs’ satisfaction. By the definition of $FoM$ in Eq. (8), some factors are considered to reflect the station’s performance, such as the utilization of the energy supply $U$, the penalty factor $\gamma$, the number of EVs being admitted $N_{ad}$ and the number of EVs missing their charging deadlines $N_{ms}$. However, the values of $U$, $N_{ad}$ and $N_{ms}$ are determined by taking account of EV owners’ satisfaction level defined in Eq. (4). The objective for EV $i$ is to maximize its satisfaction subject to its priority factor $\omega_i(t)$ and its allocated charging rate $a_i(t)$.

**D. Optimization Problem Formulation**

In the proposed system, if the solar generation and the EVs’ charging tasks are known, the optimization problem can be solved using mixed integer programming. However, the charging station does not know precisely how much solar energy will be generated from the solar panels in the future, and needs to make decisions under uncertainty. The objective of the charging station is to maximize its $FoM$ by finding the best value of $k$, that is to maximize the utilization of the energy supply and its admission rate but to minimize the rate of missing charging deadlines as in Eq. (11). The value of $FoM$ can be further calculated by determining $P_d$ based on the estimated solar generation at the admission stage, $R_s$ based on the actual solar generation and $S_r, E_b$ based on the allocated charging rate $a_i(t)$ at the scheduling stage. Therefore, the optimization problem can be formulated as

$$\begin{align*}
\text{maximize} & \quad FoM(t,k) \\
\text{subject to} & \quad a_i(t) \in [0, a_i^m]; \\
& \quad \sum_{t \in N_c(t,k)} a_i(t) \leq S(t) + B(t); \\
& \quad |N_c(t,k)| \leq M; \\
& \quad N_c(t,k) \subseteq N(t,k);
\end{align*} \quad (12)$$

where $N_c(t,k)$ is the charging set, $|\cdot|$ represents the number of elements in a set, and $N(t,k)$ is the admission set. The proposed model has an inherent two-stage decision process, since the objective function $FoM$ is determined by decisions made in both stages. In the first admission stage (Section III-A), we use a stochastic model to represent PV generation to decide the number of admissions $N_{ad}$. In the second scheduling stage (Section III-B), we use the currently available data to decide the number of missing charging deadlines $N_{ms}$ and the utilization of the energy supply $U$. Thus, this is a coupled unseparated stochastic optimization problem. Since the expression of the objective function $FoM$ is non-differentiable, traditional optimization methods such as Lagrange and Dual decomposition are not able to solve such a problem. Furthermore, the optimal decision made at a certain timeslot requires the full knowledge of future information, such as future solar energy generation, which is hard to acquire in practice. A Markov Decision Process (MDP) approach could yield better performance. However, we have not evaluated this approach in the paper as the number of states required to adequately capture the dynamics of the system (such as the number of cars charging, battery states, charging deadlines, etc.) would be very large. This would make the MDP very complex to process. Our objective is to use a single parameter $k$ to adjust the stochastic estimate of solar energy to maximize the $FoM$. Therefore, we design a heuristic two-stage mechanism to tackle this complicated problem based on current available information to achieve the highest utility $FoM$ at each timeslot, as discussed in the next section.

**III. TWO-STAGE MECHANISM**

In order to solve the optimization problem in Eq. (12), we propose a priority-based two-stage Admission and Scheduling mechanism, as described in this section.

**A. Admission Control Algorithm (ACA)**

Suppose that EV $i$ arrives at the station at time $t$ with a charging task $(E_i, t_i^e, t_i^r, a_i^m)$. For the charging station, the available solar supply at $t$ is known while the future solar generation is only available as an estimate $\hat{S}(t)$. Along with the energy state $B(t)$ in the local energy storage at $t$, the admission control is to decide which new arrivals to admit, given that there are $N_{ad}(t)$ new arrivals during $t$, where $i \in N_{ad}(t)$.
Algorithm 1 Admission Control Algorithm (ACA).

1. Input: \((E_i, t_i^a, t_i^d, a_i^m), t, N_{ex}(t, k), B(t), S_{lb})\)
2. Output: \(N(t, k)\)
3. procedure ACA \((E_i, t_i^a, t_i^d, a_i^m), t, N_{ex}, B(t), S_{lb}\)
4. Define \(j \in \{(i, \forall i \in N_{ad}(t)), N_{ex}(t, k)\}\)
5. Compute \(A_w(t)\) from eq. (13).
6. for \(T = t\) to \(A_w(t)\) do
7. Compute priority \(\omega_j(T)\) from eq. (3).
8. Rank \(\omega_j(T)\) in ascending order as \(\omega_j^a(T)\).
9. \(\omega_j^a(T) \leftarrow \omega_j^a(T)(1 : \min\{\omega_j^a(T), M\})\)
10. for \(j\) in the order of \(\omega_j^a(T)\) do
11. \(\Delta_j^1(T) = \min\{\tau a_j^m, R_j(T), S_{lb}(T, k)\}\).
12. \(\Delta_j^2(T) = \min\{\tau a_j^m - \Delta_j^1(T), R_j(T), B(t)\}\).
13. \(\Delta_j(T) = \Delta_j^1(T) + \Delta_j^2(T)\).
14. \(\text{Update } R_j(T)\).
15. if \(R_j(T) = 0\) then
16. \(t_j^f = T;\)
17. Admit \(i\) into set \(N_{ad}(t, k)\) if \(t_j^f \leq t_i^a + t_i^d\).
18. Output \(\rightarrow N(t, k) = (N_{ex}(t, k), N_{ad}(t, k))\).

Among those, only EVs in set \(N_{ad}(t, k)\) are admitted, while the rest are declined. We first define an admission window as in Definition 1.

Definition 1. An admission Window is the maximum remaining time before the deadlines of both the existing and the arriving EVs at \(t\), within which the admission decision to EV \(i \in N_{ar}\) is made. It can be written as

\[
A_w(t) = \max\{t_i^a + t_i^d - t, t_i^d\}, \quad i \in N_{ex}(t, k), \quad j \in N_{ar}(t),
\]

where \(N_{ex}(t, k)\) is the set of previously admitted EVs that are still in the charging station at \(t\).

Definition 2. The Admission Rule: A new arrival \(i \in N_{ar}\) is virtually scheduled from \(t\) to \(A_w(t)\). If its virtual finishing time \(t_i^f\) is earlier than its deadline \(t_i^a + t_i^d\), then \(i\) is admitted.

Along with \(N_{ex}(t, k)\), the set of cars in the system (being the updated admission set) can be expressed as \(N(t, k) = N_{ex}(t, k), N_{ad}(t, k)\), where \(N_{ad}(t, k) \subset N_{ar}(t)\). This procedure is further shown in Algorithm 1. Every time Algorithm 1 is executed, the admission set \(N(t, k)\) is updated.

As discussed in Section II-B, we know that if the charging station is too conservative towards the prediction of solar generation, i.e., the lower bound \(S_{lb}(t, k)\) is lower than the actual solar generation \(S(t)\), the admission control would admit fewer EVs than its actual capacity, hence the charging station will lose revenue. If the charging station is too optimistic in its prediction, the admission control would let more EVs in than its actual capacity, hence some admitted cars will not be able to meet their charging deadlines and the charging station will pay a penalty to those cars’ owners. In order to compute the Figure of Merit (FoM) of the charging station, \(N_c(t, k)\)

Algorithm 2 Charging Scheduling Algorithm (CSA).

1. Input: \(N(t, k), t, S(t), B(t)\)
2. Output: \(N_c(t, k), S_t, E_b\)
3. procedure CSA \((N(t, k), t, S(t), B(t)\) )
4. Define \(i \in N(t, k)\).
5. Compute priority \(\omega_i(t)\) from eq. (3).
6. Rank \(\omega_i(t)\) in ascending order as \(\omega_i^a(t)\).
7. \(\omega_i^a(t) \leftarrow \omega_i^a(t)(1 : \min\{\omega_i^a(t), M\})\).
8. for \(i\) in the order of \(\omega_i^a(t)\) do
9. \(\Delta_i^1(t) = \min\{\tau a_i^m, R_i(t), S_t(t)\}\).
10. \(\Delta_i^2(t) = \min\{\tau a_i^m - \Delta_i^1(t), R_i(t), B(t)\}\).
11. \(\Delta_i(t) = \Delta_i^1(t) + \Delta_i^2(t)\).
12. Allocate \(\Delta_i(t)\) to \(i\).
13. \(B(t) = B(t) + |B - B(t), S(t)\)\).
14. Output \(\rightarrow N_c(t, k), S_t, E_b\) and \(S\).

(B) Charging Scheduling Algorithm (CSA)

In Section III-A, all the admitted EVs, i.e., \(i \in N(t, k)\), should be scheduled for charging at this stage. According to the admitted EVs’ priority order \(\omega_i(t)\), the available solar energy \(S(t)\), the number of available chargers \(M\), the charging rate limit \(a_i^m\) and the available energy in the storage \(B(t)\), the charging station needs to decide which EV to charge for the current timeslot. The procedure is shown in Algorithm 2.

The charging rate \(a_i(t)\) is allocated as shown in Lines 8-14 of Algorithm 2 based on the current solar generation and the energy in the storage. Then the charging set \(N_c(t, k)\), the cumulative excess solar energy \(S_t\), and the total solar energy generation \(S\) are computed and determined accordingly from Algorithm 2.

In the next section, we use numerical evaluations to show how the proposed two-stage Admission Control Algorithm (ACA) and Charging Scheduling Algorithm (CSA) are implemented to solve the charging station’s FoM maximization problem in eq. (12) for different energy supply scenarios.

IV. Numerical Evaluations

We investigate a microgrid-like charging station using various energy sources to charge the arriving EVs and show how the proposed two-stage admission and scheduling mechanism affects the charging station’s Figure of Merit and the corresponding optimal value of \(k\). In Table II, the charging rate considered is at 50 kW level, so the considered charging scenario is within the range of fast charging. Since our main focus is to investigate the impact of the uncertain solar

4Among \(N(t, k)\), only EV \(i \in N_c(t, k)\) can be charged with \(N_c(t, k)\) \(\subset N(t, k)\)
A. Energy from the Grid

In this section, we assume the energy supply is drawn from the power grid and is sufficient to meet all charging requirements. A First In First Out (FIFO) scheme is implemented as the benchmark for the proposed priority-based admission (AD) scheme. The performance of AD and FIFO is compared in terms of rejection probability, successful service rate, delay times, and the number of EVs in the system, as shown in Fig. 2. The simulation parameters follow Table II with EVs’ arrival times and the number of EVs in the system, as shown in Fig. AD scheme. The performance of AD and FIFO is compared with other variables. Specifically, EVs’ arrival times and the number of EVs in the system are limited due to the physical constraints such as the total number of chargers, each charger’s capacity, and individual EV’s maximum charging rate, even though the energy supply from the grid is sufficient. Thus, introducing an effective admission and scheduling scheme is important for the charging station to gain revenue. Next, we increase the complexity of the system by replacing the energy supply from the grid with energy from solar panels.

B. Energy from the Solar Panels

This is the case where the charging station uses solar energy to charge EVs. Suppose that we know precisely how much energy is generated precisely from the solar panels, so the true value $S(t)$ equals the prediction $\hat{S}(t)$ in (5). The solar energy prediction profile is shown in Fig. 3 (a). We can see in Fig. 3 (b) that EVs’ rejection probability is massively influenced by the availability of the solar energy when compared to using the energy from the power grid. In practice, the prediction is not always equal to the true value as we have discussed in Section II-B. The prediction profile of the solar generation $\hat{S}(t)$ and the standard deviation of the prediction error $\sigma$ are extracted and estimated from the national grid database [30]. We then solve the formulated FoM maximization problem by finding the best value of $k$. If $k = -0.5$, the relation of the solar prediction profile, the true values, and the defined lower bound of the prediction (Eq. (5), (6), (7)) is shown in Fig. 3 (a).

1) Optimize $k$: We start to optimize $k$ as a constant value. The objective is to find the best choice of $k$ that maximizes the value of FoM subject to the energy constraints and the solar prediction uncertainty. We conduct a Monte Carlo simulation for 500 runs, using the parameters in Table II. The value of FoM is computed when EVs arrive at a rate from 0.5 to 2 per timeslot, and the results are shown in Fig. 4. The optimal value of $k$ for various arrival rates $\lambda$ and penalty factors $\gamma$ is summarized in Table III. The impact of the penalty factor $\gamma$ on the final optimal value $k^*$ for various arrival rates $\lambda$ is further shown in Fig. 5. We can see that the optimal $k^*$ to maximize the FoM decreases when the penalty $\gamma$ increases from 1, 3 to 6. This indicates that the larger the penalty factor is, the more conservatively the charging station is towards the solar prediction.

Shorter and longer deadline urgencies: We further investigate the impact on the optimal value of $k$ and FoM if the arriving EVs have shorter (e.g., 7 timeslots being half of the baseline of 15 slots) and longer (e.g., 30 timeslots) deadline urgencies. Simulation results show the comparison in terms of FoM for $t_d = 7, 15, 30$ in Fig. 6. We can
see that the optimal value of $k$ is reduced (being more conservative) by 2.5% 6 under the shorter deadline scenario and increased (being less conservative) by 2.5% under the longer deadline scenario. However, when compared to the corresponding changes in FoM* (49.3% and 44.6%), these variations in $k$ are small enough to be neglected. This indicates that the deadline urgency $t_d$ will not affect the optimal value of $k$, thus the proposed scheme could work well for EVs with various deadline urgencies. In addition, longer deadline urgency means that the charging station has more available time to successfully schedule more EVs, hence the value of FoM is higher.

**Stronger Uncertainty (2σ):** We show how the proposed scheme could work well for systems with stronger uncertainty.

6A normalized degree of variation of a vector $a$ is defined to better measure the changes in values of $k$ and FoM as: $D(a^*) = \frac{\Delta a^*}{\max(a^*) - \min(a^*)}$ where $a^*$ is the value of interest.

In one-step ahead forecasting, we know that the standard deviation of the forecast distribution is almost the same as the standard deviation of the residuals. Hence the standard deviation of solar prediction error $\sigma$ can be estimated from the prediction profile $\hat{S}$. If a less accurate forecasting method is used, we would expect that the standard deviation of the residuals is higher, hence the system has stronger uncertainty. We conduct simulations with the parameters in Table II but replace the standard deviation with $\sigma = 2\sigma$ to find the optimal value of $k$ that maximizes FoM. The comparison to $\sigma$ in terms of FoM is shown in Fig. 7. It can be observed that a lower arrival rate ($\lambda = 1.5$) yields a higher FoM. This is because more arrivals will lead to more EVs being rejected due to the charging station’s limited capacity. Since a higher penalty is paid in total, the FoM is lower. We can also see that when the uncertainty is stronger (2σ), having an overly conservative prediction ($k < -1.3$) will lead to no EVs being accepted at all. In addition, it is shown that the optimal value of $k$ increases by 2.5% when $\lambda = 1.5$ and by 5% when $\lambda = 2$. 

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**Fig. 3.** EVs’ rejection probability over different arrival rates in (b) when using solar energy in (a).

**Fig. 4.** Figure of Merit (FoM) with various arrival rates $\lambda = 0.5, 1, 1.5, 2$ vehicles per timeslot and penalty factors $\gamma = 1, 3, 6$, and the highlighted optimal value of $k$.

**Fig. 5.** The impact of the penalty factor $\gamma$ on the final optimal value $k^*$ over various arrival rates $\lambda$.

**Fig. 6.** Figure of Merit (FoM) with shorter and longer deadline urgencies.
thus a stronger uncertainty allows the charging station to make a less conservative prediction. However, when compared to the corresponding changes in FoM (58.6% and 80.1%), these variations in $k$ are small enough to be ignored. This indicates that a stronger uncertainty (2$\sigma$) will not significantly affect the optimal value of $k$, thus the proposed scheme could work well for systems with a stronger uncertainty.

2) Optimize $k(t)$: Previously, $k$ is assumed as a constant. Intuitively, the charging station should be more certain about the solar prediction as the time approaching to the end of the day. Hence, we would expect $k$ varies over time. We take the case where $\lambda = 1.5, \gamma = 3$ for example. From Table III, we know that the optimal value of $k$ is -0.3. We assume that $k$ is a linear function of time $t$, and can be written in the form as

$$k(t) = k^* + m_r t,$$

where $m_r$ is the gradient and is a constant. Re-arrange Eq. (14) as

$$k(t) = -0.3 + \frac{k_r + 0.3}{72} t, \quad k_r \in [k_{\min}, k_{\max}].$$

where $k_{\min} = -3$, $k_{\max} = 3$, according to the 3$\sigma$ rule. The objective is transformed to find which one from this set of lines in $k(t)$ can make $FoM(k(t))$ achieve its maximum value. We compute the value of FoM when $k_r$ is from -3 to 3. As shown in Fig. 8, FoM reaches its optimal value 0.3014 when $k_r = 0.4$. From Eq. (15), the optimal $k(t)$ can be written as

$$k(t) = -0.3 + 0.00972 t,$$

where $FoM$ is improved by 5.5% when compared to the case where $k$ has a constant value in Section IV-B1. The comparison is also shown in Fig. 8.

The set of lines in $k(t)$ is demonstrated in Fig. 9. We can see that the optimal line $k'(t)$ is increasing over time, meaning that the charging station is making a less conservative prediction of the solar generation. This is because with less time left till the end of the day, the impact of paying penalties on the value of the FoM is getting less significant. This is in line with the result in Fig. 8, where $FoM(k'(t))$ outperforms $FoM(k^*)$ by 5.5%.

### C. Energy from Storage

A high rejection rate of the arriving cars will cause a reputation loss to the station over time. Introducing local energy storage can not only reduce the fluctuation of solar prediction uncertainty but can also increase EVs’ admission rate by leveraging stored energy for charging. In this section, we show the impact of the storage’s capacity and its initial state on system’s performance in terms of $FoM$. A low storage capacity is similar to using the energy from the solar panels while a high storage capacity might be more similar in terms of reliability to using the energy from the power grid.

### Table III

<table>
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<tr>
<th>$\lambda$</th>
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<th>3</th>
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<td>$-0.4$</td>
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### Table IV

<table>
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<tr>
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<th>$\gamma$</th>
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<tbody>
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<tr>
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</table>
Suppose that the energy storage has a sufficient capacity to accommodate all excess solar energy. First, we conduct a Monte Carlo simulation for 500 times to compute the optimal value of $k$ as a constant value, as shown in Table IV. We further show the comparison of FoM under the optimal $k$ values without (Table III) and with (Table IV) an energy storage in Fig. 10. It is worth noting that when comparing the results in Table III and Table IV, the optimal value of $k$ decreases slightly in four scenarios, i.e. when $(\lambda = 1.5, \gamma = 1), (\lambda = 2, \gamma = 1), (\lambda = 2, \gamma = 3)$ and $(\lambda = 2, \gamma = 6)$, respectively. However, when compared to the corresponding changes in $FoM^*$ from Fig. 10 (being 63.5%, 63.7%, 68.4% and 58.4%), the variations in $k^*$ (being 2.5%, 2.5%, 5% and 2.5%) are small enough to be neglected. This indicates that the implementation of storage units will not affect the optimal value of $k$, but can increase the $FoM$ significantly.

We still take the case where $\lambda = 1.5, \gamma = 3$ for example. Following the steps in Section IV-B2, the $k$ constant can be optimized over time $t$. The corresponding $FoM$ is computed as shown in Fig. 11, where the optimal value of $FoM$ and $k(t)$ coincide with the case when $k$ has a constant value, i.e. $k^*(t) = k^* = -0.3$. This is because sufficient storage capacity is able to balance the time-varying solar generation. We then discuss the impact of the storage’s capacity and its initial state on charging station’s performance.

1) Insufficient Storage Capacity: Define the storage capacity as $B$. 7 We investigate when the charging station implements a smaller storage: 1/4, 1/2 and 3/4 of $B$. We still take the case where $\lambda = 1.5, \gamma = 3$ for example. Following the steps in Section IV-B2, the optimal value of $FoM$ is computed as shown in Fig. 12. When the storage has a small capacity ($B/4$), the optimal value of $k$ coincides with the case when there is no storage installed, as in Eq. (16). As the capacity increases ($B/2, 3B/4, B$), the optimal value of $k$ remains as a constant -0.3 as shown in Fig. 11. From Fig. 12 we can see that the performance gain in terms of $FoM$ when compared to a zero storage case is 43%, 69%, 90%, 96% for a storage with capacity $B/4, B/2, 3B/4, B$ respectively. If the operational cost of the energy storage is proportional to its capacity, implementing energy storage with capacity $B/4$ with a $FoM$ gain of 43% is most economically beneficial. If we aim to be able to store more excess solar energy, implementing three storage units with capacity of $B/4$ each can not only achieve a $FoM$ gain of 129% but can also save 1/4 of the cost compared to implementing a full capacity storage system.

2) Initial State of Storage: In the previous discussion in Section IV-C, we assume that the energy storage starts empty at 6am. Here, we further investigate how the initial state affects the system and the proposed scheme. Suppose that the storage’s capacity is 193 kWh, which accounts for three $B/4$ storage units. We still take the case where $\lambda = 1.5, \gamma = 3$ for example. Following the steps in Section IV-B2, the optimal value of $k$ remains a constant -0.3 when the storage starts

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7Through 500 Monte Carlo simulations when a sufficient storage is assumed, the average storage capacity limit is $X = 257$ kWh. Thus we assume the full storage capacity is $B = X$. 

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Fig. 10. Comparison of $FoM$ with (w/S) and without (w/o) energy storage under optimal $k$ values from Table IV and Table III.

Fig. 11. Figure of Merit ($FoM$) when multiplier $k$ is a function of time $t$ with sufficient storage capacity.

Fig. 12. Figure of Merit ($FoM$) with different storage capacities under their corresponding optimal values of $k(t)$. 
empty, 1/4 full, 1/2 full, and 3/4 full, while the optimal value of $FoM$ increases from 0.5775 to 0.6123, 0.6488 and 0.6869, respectively. Further, the amount of energy in the storage with a different initial state is shown in Fig. 13. It is straightforward to see that the more energy to start with, the more EVs the charging station is able to charge in the beginning of the day (when solar is low), hence the value of $FoM$ is increasing with the storage’s initial state, but the optimal value of $k$ stays the same at –0.3. This indicates that the starting state of the storage will not affect the optimal value of $k$, thus the proposed scheme could work well regardless the storage’s initial state.

D. Discussion of the cost of using various energy sources

In this section, we discuss the cost paid by the station for the energy consumed in order to meet EVs’ charging requirements when use the energy from the grid, on-site solar panels and storage units, respectively.

The connected operation of the charging station indicates that energy is supplied from energy providers based on some contract. We take the conventional grid-connected operation as a benchmark to give an estimate of the charging station’s maximum charging capacity, subject to EVs’ requirements. For simplicity, we assume that the charging station is contracted with an energy provider under three different price plans:

1) A fixed tariff of 10 pence per kWh energy.
2) A time-of-use (TOU) rate of 15 pence for on-peak (6am-10am, 5pm-6pm) and 5 pence for off-peak (10am-5pm) per unit kWh energy [31], denoted as Grid-TOU-plan1.
3) A TOU rate of 13 pence per kWh for on-peak hours and 8 pence per kWh for off-peak hours, denoted as Grid-TOU-plan2.

First, we have computed the total energy consumed to charge EVs from supplier, PV, and PV storage system under the optimal operation point of the proposed approach over 500 different scenarios via Monte Carlo simulations, as shown in Figure 14 (a).

Let $p_s$ denote the price of per unit energy from supplier, $p_b$ denote the operational cost of the storage that is yielded per unit energy charged or discharged, and $p_s$ denote the maintenance fee of solar panels per unit energy output. The costs of using energy from supplier under these three price plans are calculated, along with the costs of using PV and PV storage system.

Following the simulation parameters in Table II with EVs’ arrival rate $\lambda$ varying from 0.5 to 2 per timeslot and the penalty factor $\gamma = 3$, the comparison of the costs is shown in Figure 14 (b). If energy is bought from supplier, following the steps in Section IV-A, EVs are admitted based on their priority orders. If energy is drawn from PV and storage units, following the steps in Section IV-B2, the optimal value of $k(t)$ to maximize $FoM$ is first computed, then used to calculate the corresponding cost when the station operates under these optimal points (i.e., at the maximal $FoM$).

It can be observed that depending on the actual tariff and energy consumption, the TOU hours tariff does not always outperform a fixed price plan, i.e., the costs are shown as $Cost(\text{TOU-plan2}) > Cost(\text{fixed}) > Cost(\text{TOU-plan1})$. Nevertheless, energy from supplier is the most expensive source with energy from PV being the cheapest option. Implementing storage units can hedge against the uncertainty of the station and increase the utilization of PV, so that the energy consumed to charge EVs increases in Figure 14 (a), as well as the performance index $FoM$, but it will bring extra cost as shown in Fig. 14 (b).

In practice, depending on the wholesale market situation, balancing requirements from the system operator, the costs of increasing on-site PV and storage’s capacity, the charging station might be able to arbitrage using storage. However, by applying our proposed admission and scheduling mechanism, the charging station can obtain a comprehensive evaluation of its self-consumption pattern before negotiating any deals with aggregators and system operator.

\footnote{In practice, the prices and peak times also vary based on the season and day of the week.}
V. CONCLUSION

We have studied the problem that a charging station needs to decide which arriving EVs to admit and schedule according to its limited energy capacity in order to achieve an optimal utility. To this end, we first introduce a multiplier \( k \) to account for the effect of solar energy prediction uncertainty. Then we propose a performance index (the Figure of Merit, FoM) to measure the charging station’s utility, and formulate the problem as FoM maximization. A two-stage admission and scheduling mechanism is then proposed to solve the optimization problem. The multiplier \( k \) is first optimized as a constant (where FoM has a 29.5% gain compared to when \( k \) is not optimized) and then as a function of time (where FoM has a 5.5% gain compared to when \( k \) is optimized but as a constant) by finding the maximum value of FoM under various energy supply scenarios. Through Monte Carlo simulations, we have shown the solution to the optimization problem and discussed the impact of some of the key factors (such as shorter and longer deadline urgencies, stronger uncertainty of the prediction error, the storage capacity and its initial state, and the cost of various energy sources) on the charging station’s performance in terms of FoM and the corresponding optimal value of \( k \).

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REFERENCES


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