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Citation for published version:
Liu, W, Bundy, A & Robertson, D 1993, On the Relation Between Incidence Calculus and ATMS. in M Clarke, R Kruse & S Moral (eds), Symbolic and Quantitative Approaches to Reasoning and Uncertainty: European Conference ECSQARU '93 Granada, Spain, November 8–10, 1993 Proceedings. vol. 747, Lecture Notes in Computer Science, vol. 747, Springer-Verlag GmbH, pp. 249-256. DOI: 10.1007/BFb0028207

Digital Object Identifier (DOI):
10.1007/BFb0028207

Published In:
Symbolic and Quantitative Approaches to Reasoning and Uncertainty

On the Relations between Incidence Calculus and ATMS

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Abstract. This paper discusses the relationship between incidence calculus and the ATMS. It shows that managing labels for statements in an ATMS is similar to producing the incidence sets of these statements in incidence calculus. We will prove that a probabilistic ATMS can be implemented using incidence calculus. In this way, we can not only produce labels for all nodes in the system automatically, but also calculate the probability of any of such nodes in it. The reasoning results in incidence calculus can provide justifications for an ATMS automatically.

1 Introduction

The ATMS is a symbolic reasoning technique used in the artificial intelligence domain to deal with problems by providing dependent relations among statements during inference normally. This technique can only infer results with absolutely true or false. It lacks the ability to draw plausible conclusions such as that a conclusion is true with some degree of belief. However in many cases, pieces of information from a knowledge base provide assumptions and premises with uncertainties. It is necessary to let the ATMS have the ability to cope with uncertainty problems.

In order to overcome this problem, some research on the association of numerical uncertainties with ATMS has been carried out. In [8], De Kleer and Williams use probability theory to deal with such associated with assumptions. In [11, 15], the authors use possibilistic logic to handle this problem. In [11] both assumptions and justifications are associated with uncertainty measures. The uncertainty values associated with justifications are used to select the path for deriving a node. Only those pathes with strong supporting relations are used to infer the corresponding nodes. [15] continues the work carried out in [11] and extends it to deal with a military data fusion application. [5, 6, 14, 16, 19, 20] all use Dempster-Shafer theory of evidence to calculate beliefs in statements. Among them [16] studies a formal relation between DS theory and ATMS. It is proved in [16] that any belief network in DS theory can be translated into an ATMS structure. In such a system, the inference is performed based on ATMS techniques with a probability model on assumptions. One common limitation in all these extensions of the ATMS[1] is that the probabilities assigned to assumptions must be assumed probabilistically independent in order to calculate the degree of belief in a statement. In this paper, we continue this research and

[1]Except the discussion in [11, 15] in which the topic was not discussed.

intend to provide a general basis for constructing a probabilistic ATMS. The uncertainty technique we have chosen is incidence calculus.

The main contributions of this paper are: We prove that incidence calculus and the ATMS are equivalent at both the symbolic reasoning level and numerical inference level if we associate proper probabilistic distributions on assumptions. We show that the integration of symbolic and numerical reasoning patterns are possible and incidence calculus itself is a typical example of this unification. The result of investigating the relationship between incidence calculus and ATMS can provide a theoretical basis for some results in [16]. We will show that incidence calculus can be used to provide justifications for nodes automatically without human involvement. Therefore a complete automatic ATMS system is constructible.

The paper is organized as follows. Section 2 introduces the basics of incidence calculus. In section 3 we introduce the ATMS notations and extend it by adding probabilities to assumptions. In section 4 we will explore how to manipulate labels of nodes and calculate degrees of belief in nodes in incidence calculus. In the concluding section, we summarize our results.

2 Incidence Calculus

Incidence calculus [1, 2] starts with two sets, the set P contains propositions and the set $\mathcal{W}$ consists of possible worlds with a probability distribution on them. For each element w of $\mathcal{W}$, the probability on w, $\varrho(w)$, is known and $\Sigma\varrho(w) = 1$. From the set P, using logical operators $\wedge, \vee, \neg, \rightarrow$, a set of logical formulae are formed which is called the language set of P, denoted as $\mathcal{L}(P)$. The elements in the set $\mathcal{W}$ may make some formulae in $\mathcal{L}(P)$ true. For any $\phi \in \mathcal{L}(P)$, if every element in a subset W_1 of $\mathcal{W}$ makes ϕ true and W_1 is the maximal subset of this kind, then W_1 is represented as $i(\phi)$ in an incidence calculus theory and it is called the incidence set of ϕ. Therefore, the supporting set of a formula ϕ is $i(\phi)$ and its probability is $p(\phi) = wp(W_1)$ where $wp(W_1) = \Sigma_{w \in W_1}\varrho(w)$. It is assumed that $i(\perp) = \{\}$ and $i(T) = \mathcal{W}$ where $\perp, T$ represent *false* and *true* respectively.

Definition 1: Incidence calculus theories: an incidence calculus theory is a quintuple $< \mathcal{W}, \varrho, P, \mathcal{A}, i >$ where $\mathcal{W}$ is a set of possible worlds with a probability distribution ϱ, P is a set of propositions and $\mathcal{A}$ is a subset of $\mathcal{L}(P)$ which is called a set of axioms. The function i assigns an incidence set to every formula in $\mathcal{A}$. For any two formulae in $\mathcal{A}$, we have $i(\phi \wedge \psi) = i(\phi) \cap i(\psi)$.

Based on this definition, given two formulae $\phi, \psi \in \mathcal{A}$, we have $i(\phi) \subseteq i(\psi)$ if $\phi \rightarrow \psi = T$. For any other formula $\phi \in \mathcal{L}(P) \setminus \mathcal{A}$, it is possible to get the lower bound $i_*(\phi)$ of its incidence set as $i_*(\phi) = \bigcup_{\psi \rightarrow \phi = T} i(\psi)$ where $\psi \in \mathcal{A}$ and $\psi \rightarrow \phi = T$ iff $i(\psi \rightarrow \phi) = \mathcal{W}$. The degree of our belief in a formula is defined as $p_*(\phi) = wp(i_*(\phi))$.

Definition 2: Semantic implication set and essential semantic implication set: for any formula $\phi \in L(P)$, if $\psi \rightarrow \phi = T$ then ϕ is said to be semantically implied by ψ, denoted as $\psi \models \phi$. Let $SI(\phi) = \{\psi \mid \psi \rightarrow \phi = T, \forall\psi \in \mathcal{A}\}$, set

$SI(\phi)$ is called a semantical implication set of ϕ. Furthermore, let $ESI(\phi)$ be a subset of $SI(\phi)$ which satisfies the condition that a formula ψ is in $ESI(\phi)$ for any ψ' in $SI(\phi)$ $\psi \to \psi' \neq T$, then $ESI(\phi)$ is called an essential semantical implication set of ϕ. This is denoted as $ESI(\phi) \models \phi$.

Proposition 1 *If $SI(\phi)$ and $ESI(\phi)$ are a semantic implication set and an essential semantic implication set of ϕ, then the following equation holds: $i_*(\phi) = i_*(SI(\phi)) = i_*(ESI(\phi))$ where $i_*(SI(\phi)) = \bigcup_{\phi_j \in SI(\phi)} i(\phi_j)$.*

This proposition can be proved based on the definitions of lower bound of incidence set i_* and $SI(\phi)$ and $ESI(\phi)$ above. It will be proved later that the essential semantic implication set of a formula is exactly the same as the set of justifications of that formula in an ATMS.

When two incidence calculus theories are given on different sets of possible worlds and the two sets are probabilistically independent (or DS-Independent[2]), the combination can be performed using the Corollary 1 in [3]. Given that $< \mathcal{W}_1, \varrho_1, P, \mathcal{A}_1, i_1 >$ and $< \mathcal{W}_2, \varrho_2, P, \mathcal{A}_2, i_2 >$, applying Corollary 1 we get a combined theory $< \mathcal{W}_3, \varrho_3, P, \mathcal{A}_3, i_3 >$ where

$$\mathcal{W}_0 = \bigcup_{\phi \wedge \psi = \perp} i_1(\phi) \otimes i_2(\psi) \qquad \phi \in \mathcal{A}_1, \psi \in \mathcal{A}_2$$

$$\mathcal{W}_3 = \mathcal{W}_1 \otimes \mathcal{W}_2 \setminus \mathcal{W}_0$$

$$\varrho_3(w) = \varrho_3((w_{1i}, w_{2j})) = \frac{\varrho_1(w_{1i})\varrho_2(w_{2j})}{1 - \sum_{(w'_{1i}, w'_{2j}) \in \mathcal{W}_0} \varrho_1(w'_{1i})\varrho_2(w'_{2j})}$$

$$\mathcal{A}_3 = \{\varphi \mid \varphi = \phi \wedge \psi, where \phi \in \mathcal{A}_1, \psi \in \mathcal{A}_2, \varphi \neq \perp\}$$

$$i_3(\varphi) = \bigcup_{(\phi \wedge \psi \to \varphi) = T} (i_1(\phi) \otimes i_2(\psi)) \setminus \mathcal{W}_0 \qquad \phi \in \mathcal{A}_1, \psi \in \mathcal{A}_2$$

In general a pair (w_{1i}, w_{2j}) is an element of $\mathcal{W}_1 \otimes \mathcal{W}_2 \setminus \mathcal{W}_0$. It is required that T is automatically added into a set of axioms $\mathcal{A}$ if $\cup_{\phi \in A} i(\phi) \subset W$.

3 The ATMS

The truth maintenance system (TMS) [9] and later the ATMS [7] are both symbolic approaches to producing a set of statements in which we believe. The basic and central idea in such a system is that for each statement we believe in, a set of supporting statements (called labels or environments generally in the ATMS) is produced. A set of supporting statements is, in turn, obtained through

[2] See definition and explanation in [3]. In the analysis [3], two sets of possible worlds are probabilistically independent cannot guarantee they are DS-Independent when their original source is known. In the case that original source is the set product of these two sets, their probabilistic independence also implies their DS-Independence. In this paper, as we only consider the latter case, we will use term *probabilistically independent* to name the relations among two sets.

a set of arguments attached to that statement (called justifications). In an ATMS, a justification of a statement (or called node) contains other statements (or nodes) from which the current statement can be derived. Justifications are specified by the system designer. For instance, if we have two inference rules as: $r_1 : p \rightarrow q$ and $r_2 : q \rightarrow r$, then logically we can infer that $r_3 : p \rightarrow r$. In an ATMS, if r_1, r_2 and r_3 are represented by $node_1$, $node_2$ and $node_3$ respectively, then $node_3$ is derivable from the conjunction of $node_1$ and $node_2$ and we call (r_1, r_2) a justification of $node_3$. Normally a rule may have several justifications. Further more if r_1 and r_2 are valid under the conditions that A and B are true respectively, then rule r_3 is valid under the condition that $A \wedge B$ is true, denoted as $\{A, B\}$. $\{A\}, \{B\}$ and $\{A, B\}$ are called sets of supporting statements (or environments) of r_1, r_2 and r_3 respectively. A and B themselves are called assumptions. If we associate $node_3$ with the supporting statements such as $\{A, B\}$ and the dependent nodes such as $\{(r_1, r_2)\}$ then $node_3$ is generally in the form of $r_3 : p \rightarrow r, \{\{A, B\}...\}, \{(r_1, r_2)...\}$ when $node_3$ has more than one justification. The collection of all possible sets of supporting environments is called the label of a node. If we use $L(r_3)$ to denote the label of $node_3$, then $\{A, B\} \in L(r_3)$. If we assume that r_1, r_2 hold without requiring any dependent relation on other nodes, then $node_1$ and $node_2$ are represented as $r_1 : p \rightarrow q, \{\{A\}\}, \{()\}$ and $r_2 : q \rightarrow r, \{\{B\}\}, \{()\}$. Therefore, we can infer a label for any node as long as its justifications are known.

The advantage of this reasoning mechanism is that the dependent and supporting relations among nodes are explicitly specified, in particular, the supporting relations among assumptions and other nodes. This is obviously useful when we want to retrieve the reasoning path. It is also helpful for belief revision. The limitation of this reasoning pattern is that we cannot infer those statements which are probably true rather than absolutely true. However, if we attach numerical degrees of belief to the elements in the supporting set of a node, we may be able to infer a statement with a degree of belief. For example, if we know A is true with probability 0.8 and B is true with probability 0.7 and A and B are probabilistically independent, then the probability of $A \wedge B$ is true is 0.56. The belief in a node is considered as the probability of its label. So for $node_3$, our belief in it is 0.56.

Definition 3: Probabilistic assumption set:[3] a set $\{A, B, ..., C\}$, denoted as $S_{A,...,C}$, is called a probabilistic assumption set for assumptions $A, B, ..., C$ if the probabilities on $A, ..., C$ are given by a probability distribution p from a piece of evidence and $\Sigma_{D \in \{A,...,C\}} p(D) = 1$. The simpliest probabilistic assumption set has two elements X and $\neg X$, denoted as $S_X = \{X, \neg X\}$. For any two assumptions in a set, it is assumed that $A_i \wedge A_j \Rightarrow \perp$ and $\vee A_j = T$ for $j = 1, ..., n$.

For two distinct probabilistic assumption sets S_A and S_B, the unified probabilistic assumption set set is defined as $S_{AB} = S_A \otimes S_B = \{(A_1, B_j) \mid A_i \in S_A, B_j \in S_B\}$ where $\otimes$ means set product and $p(A_i, B_j) = p_1(A_i) \times p_2(B_j)$. p_1 and p_2 are the probability distributions on S_A and S_B respectively.

Definition 4: Full extension of a label: assume that an environment of a

[3] Similar definition is given in [16] called auxiliary hypothesis set.

node n is $\{A_1, A_2, ..., A_l\}$ where A_i are in different probabilistic assumption sets. Because $A_1 \wedge ... \wedge A_l \equiv A_1 \wedge ... \wedge A_l \wedge (\vee B_j \mid B_j \in S_B)$, $A_1 \wedge ... \wedge A_n \to n$ and $A_1 \wedge ... \wedge A_n \wedge (\vee_j B_j \mid B_j \in S_B) \to n$ (where S_B is a probabilistic assumption set which is different from S_{A_i}), $A_1 \wedge ... \wedge A_l \wedge (\vee B_j \mid B_j \in S_B)$ is called a full extension of the environment to S_B. If there are in total m assumptions in the ATMS, then the extension $A_1 \wedge ... \wedge (\vee B_i \mid B_i \in S_B) \wedge ... \wedge (\vee C_{m-l} \mid C_{m-l} \in S_C)$ is called the full extension of the environment to all assumptions, or simply called the full extension of the environment. Similarly if every environment in a label has been fully extended to all assumptions, then we call the result the full extension of the label, denoted as $FL(n)$.

4 Implementing an ATMS Using Incidence Calculus

Abstractly if we view the set of possible worlds in incidence calculus as the set of assumptions in an ATMS, and view the calculation of the incidence sets of formulae as the calculation of labels of nodes in the ATMS, then the two reasoning patterns are similar. As incidence calculus can draw a conclusion with a numerical degree of belief on it, incidence calculus actually possesses some features of both symbolic and numerical reasoning approaches. Therefore, incidence calculus can be used both as a theoretical basis for the implementation of a probabilistic ATMS by providing both labels and degrees of belief of statements and as an automatic reasoning model to provide justifications for an ATMS.

Now we will show how to manage assumptions in the ATMS in the way we manage sets of possible worlds in incidence calculus. Here we look at an example (from [16]).

Example 1 Assume that there are the following nodes in an ATMS:

assumed nodes:	$n_1 :< b \to a, \{\{V\}\}, \{(V)\} >$	$n_2 :< c \to a, \{\{W\}\}, \{(W)\} >$
	$n_3 :< d \to b, \{\{X\}\}, \{(X)\} >$	$n_4 :< d \to c, \{\{Y\}\}, \{(Y)\} >$
	$n_5 :< e \to d, \{\{Z\}\}, \{(Z)\} >$	
premise node:	$n_6 :< e, \{\{\}\}, \{()\} >$	
derived nodes:	$n_7 :< d \to a, \{\{X, V\}, \{Y, W\}\}, \{(n_1, n_3), (n_2, n_4)\} >$	
	$n_8 :< e \to a, \{\{Z, X, V\}, \{Z, Y, W\}\}, \{(n_7, n_5)\} >$	
	$n_9 :< a, \{\{Z, X, V\}, \{Z, Y, W\}\}, \{(n_6, n_8)\} >$	
assumption nodes:	$< X, \{\{X\}\}, \{(X)\} >$,	$< V, \{\{V\}\}, \{(V)\} >$, ...

The label of node a is $Bel(a) = Pr((Z \wedge X \wedge V) \vee (Z \wedge Y \wedge W))$. Given that probabilities on different assumptions are $p_1(V) = .7; p_2(W) = .8; p_3(X) = .6; p_4(Y) = .75; p_5(Z) = .8$, and they are probabilistically independent, the belief in a is $Bel(a) = 0.6144$ which is calculated based on $FL(a)$. A different calculation procedure can also be found in [16] which produces the same result.

Now let us see how his problem can be solved in incidence calculus theories. Suppose that we have the following six incidence calculus theories

$$< S_V, \varrho_1, P, \{b \to a, T\}, i_1(b \to a) = \{V\}, i_1(T) = S_V >$$

$< S_W, \varrho_2, P, \{c \rightarrow a, T\}, i_2(c \rightarrow a) = \{W\}, i_2(T) = S_W >$
$< S_X, \varrho_3, P, \{d \rightarrow b, T\}, i_3(d \rightarrow b) = \{X\}, i_3(T) = S_X >$
$< S_Y, \varrho_4, P, \{d \rightarrow c, T\}, i_4(d \rightarrow c) = \{Y\}, i_4(T) = S_Y >$
$< S_Z, \varrho_5, P, \{e \rightarrow d, T\}, i_5(e \rightarrow d) = \{Z\}, i_5(T) = S_Z >$
$< S_E, \varrho_6(E) = 1, P, \{e\}, i_6(e) = S_E >$

where $S_V = \{V, \neg V\}$, ..., $S_Z = \{Z, \neg Z\}$, and $S_E = \{E, \neg E\}$ are probabilistic assumption sets. If we assume that sets of $S_X, ..., S_Z, S_E$ are probabilistically independent, the combination of the first five theories produces an incidence calculus theory $< S_7, \varrho_7, P, \mathcal{A}_7, i_7 >$ in which the joint set is $S_7 = S_Z \otimes S_X \otimes S_V \otimes S_Y \otimes S_W$. Combining this theory with the sixth incidence calculus theory[4] we obtain $i(e \wedge \phi_1) = S_E ZXV S_Y S_W$, $i(e \wedge \phi_2) = S_E ZYW S_X S_V$, $i(e \wedge \phi_1 \wedge \phi_2) = S_E ZXVYW$, if we let $e \rightarrow d \wedge d \rightarrow b \wedge b \rightarrow a = \phi_1$ and $e \rightarrow d \wedge d \rightarrow c \wedge c \rightarrow a = \phi_2$. Because $e \wedge \phi_1 \rightarrow a$, $e \wedge \phi_2 \rightarrow a$ and $e \wedge \phi_1 \wedge \phi_2 \rightarrow a$, the following equation $i_*(a) = S_E ZXV S_Y S_W \cup S_E S_X S_V ZYW$ holds. So $p_*(a) = wp(i_*(a)) = 0.6144$. Similarly we can also obtain $i_*(d \rightarrow a)$, $i_*(e \rightarrow a)$ as:

$$i_*(d \rightarrow a) = S_E S_Z XV S_Y S_W \cup S_E S_Z YW S_X S_V$$
$$i_*(e \rightarrow a) = S_E ZXV S_Y S_W \cup S_E ZYW S_X S_V$$

Therefore the following equations $i_*(d \rightarrow a) \equiv FL(d \rightarrow a)$, $i_*(e \rightarrow a) \equiv FL(e \rightarrow a)$ and $i_*(a) \equiv FL(a)$ hold. Here the symbol $\equiv$ is read as "equivalent to". An incidence set of a formula (or its lower bound) is equivalent to the full extension of the label of a node means that for any element in the incidence set there is one and only one conjunction part in $FL(*)$.

Theorem 1 *Given an ATMS, there exists a set of incidence calculus theories such that the reasoning result of the ATMS is equivalent to the result obtained from the combination of these theories. For any node d_i in an ATMS, $L(d_i) \setminus L(\perp)$ is equivalent to the incidence set of formulae d_i in incidence calculus.*

The proof is given in [17].

Example 2 Following the story in Example 1, suppose we are told later that f is also observed and there is a rule $f \rightarrow \neg c$ with degree .8 in the knowledge base. That is, three more nodes in the ATMS are used as shown below.

assumed node: $< f \rightarrow \neg c, \{\{U\}\}, \{(U)\} >$
premise node $< f, \{\{\}\}, \{()\} >$
assumption node $< U, \{\{U\}\}, \{(U)\} >$
and assumption sets $S_U = \{U, \neg U\}$, $S_F = \{F, \neg F\}$.
Here S_F is created to support premise node f.

In the ATMS, we can infer that one environment of node c is $\{E, Z, Y\}$ and one environment of node $\neg c$ is $\{F, U\}$. So the *nogood* environment is $\{E, X, Y, F, U\}$. The belief in node a needs to be recomputed in order to redistribute the weight of conflict on the other nodes. The revised belief in a is 0.366 given in [16].

Similar to Example 1, in incidence calculus two more incidence calculus theories are constructed from the assumed node and the premise node. Combining

[4] The combination sequence does not affect the final result. Here in order to show the result explicitly, we take these two steps.

these two theories with the final one we obtained in Example 1, we have $W_0 = \{UZY\}^5$, $i_*(a) = \{ZXV \cup ZYW\} \setminus W_0$. Therefore $wp(\{UZY\}) = 0.48$ which is the weight of conflict and $p'_*(a) = wp(\{ZXV \cup ZYW\}) \setminus \{UZY\}) = 0.366$ which is our belief in a. Both of these results are the same as those given in [16], but the calculation of belief in node a and the weight of conflict are based on incidence calculus theory.

5 Conclusions

Existing papers discuss the unification of an ATMS with numerical uncertain reasoning mechanisms [5, 6, 8, 11, 14, 15, 16, 19, 20]. The closest work to ours is described in [16]. In their paper the relations between the ATMS and the Dempster-Shafer theory of evidence is discussed. They claimed that the relation between the two theories is that the ATMS can be used to represent DS inference networks. More precisely, their result is that a set of belief functions can be equivalently translated into a corresponding ATMS system. In such systems the reasoning procedure is carried out as a normal ATMS together with performing the appropriate calculations of uncertainty values. However a formal proof of equivalence between the two theories is missing. We claim that incidence calculus, though closely related to DS theory [2, 3], also has strong similarities to the ATMS. These have allowed us to produce a proof of the equivalence between the two forms of inference.

The discussion in this paper tells us that incidence calculus itself is a unification of both symbolic and numerical approaches. It can therefore be regarded as a bridge between the two reasoning patterns. This result also gives theoretical support for research on the unification of the ATMS with numerical approaches. In incidence calculus structure, both symbolic supporting relations among statements and numerical calculation of degrees of belief in different statements are explicitly described. For a specific problem, incidence calculus can either be used as a support based symbolic reasoning system or be applied to deal with numerical uncertainties. This feature cannot be provided by pure symbolic or numerical approaches independently.

Another advantage of using incidence calculus to make inferences is that it doesn't require the problem solver to provide justifications. The whole reasoning procedure is performed automatically. The inference result can be used to produce the ATMS related justifications. The calculation of degrees of beliefs in nodes are based on the hypothesis that each assumption is in one auxiliary set and all these sets are probabilistically independent. Further work will consider the more general situation, that is, several assumptions are in one set as individual elements and there is a probability distribution on it.

[5] In order to state the problem clearly, we use UZY instead of $UZYS_XS_WS_VS_ES_F$.

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