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Comparison of MCMC approaches with an application to volcano earthquake processes

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Abstract: In this paper we consider statistical modelling of volcanic earthquake data. In particular, we investigate the use of Bayesian analysis with Markov Chain Monte Carlo (MCMC) to estimate the parameters of point process models, and make inferences on the models, applied to data collected from the Tungurahua volcano in Ecuador.

Keywords: Bayesian modelling; Eruption forecasting; Point processes.

1 Introduction

This paper aims to use statistical modelling to describe the occurrence of volcanic earthquakes. The main approach taken is that of using Bayesian analysis with Markov Chain Monte Carlo (MCMC) to fit point process models to the available data, collected from the Tungurahua volcano in Ecuador.

2 Dataset and modelling

This dataset was recorded in July 2013, and consists of a series of event times which were picked from a stretch of seismic data to identify the individual earthquakes. The dataset was examined in a study by Bell et al. (2018).

The events started at 6:00 on 13 July, and the eruption occurred at 11:46 on 14 July. The event rate grew increasingly up until eruption. Plots of the data show that the event rate grows at an increasing rate up to the...
eruption, with the inter-spike interval (ISI) duration changing from over 10 minutes to below 30 seconds. The ISIs are “quasi-periodic”, being more regular than would be seen if the events followed a Poisson process, and thus not independent (Bell et al., 2018).

Applying a material failure approach to describe the physical processes leading a volcanic system to an eruption, the accelerating rate of earthquakes is described by a power law relationship (Bell et al., 2018):

\[ \lambda(t) = k(t_f - t)^{-p}, \]

where \( k \) is a constant (related to the amplitude of the signal), \( t_f \) is the time of eruption, and \( p = \frac{1}{\alpha - 1} \) is a parameter describing the non-linearity of acceleration. At time \( t_f \), the rate becomes instantaneously infinite, representing the eruption (Bell et al., 2018). In the model, \( \lambda(t) \) is the intensity used in the inhomogeneous gamma (IG; parameter \( \alpha \)) point process.

Details of the MCMC implementation in PyMC3 were investigated, including the sampling method used and the initialisation process. Attributes of the MCMC chain, such as convergence, were examined. Posterior checks were performed using simulated data, to sense check whether the model appears appropriate. The fit of the model was assessed further using statistical methods.

The MCMC approaches considered included:

- No-U-Turn sampler (Hoffman and Gelman, 2014);
- Metropolis;
- Slice sampling (Neal, 2003).

Alternative models were also investigated, and their fits compared to that of the given inhomogeneous gamma model: inhomogeneous Poisson (IP), inhomogeneous inverse Gaussian (IIG) and inhomogeneous Weibull (IW) models.

3 Results

Figure 1 gives comparison of the MCMC trace plots for the methods for the IG model. Figures 2 and 3 show posterior plots using MCMC sampled values for the IG model. A Kolmogorov-Smirnov goodness of fit approach (Barbieri et al., 2001; Ogata, 1988) gives an effective method of comparison of various possible models.
2.6 NUTS further analysis

2.6.1 Convergence

To test convergence, 10 chains were run in parallel, starting at randomly selected initial points. It can be seen from the trace plot in Figure 2.6 that all of the chains appear to have converged to the same distribution. Moreover, the parameter means of each of the chains (shown in Table 2.1) did not vary by more than 0.1 for $\alpha$, $t_f$, $p$, and by no more than 1 for $k$, from the values presented in Table 2.2, which also indicates convergence.

Table 2.1: Mean values of parameters for each of 10 randomly started chains

<table>
<thead>
<tr>
<th>Chain</th>
<th>$\alpha$</th>
<th>$k$</th>
<th>$t_f$</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.476</td>
<td>340.827</td>
<td>1.422</td>
<td>1.508</td>
</tr>
<tr>
<td>2</td>
<td>2.474</td>
<td>339.679</td>
<td>1.420</td>
<td>1.501</td>
</tr>
<tr>
<td>3</td>
<td>2.475</td>
<td>340.564</td>
<td>1.421</td>
<td>1.501</td>
</tr>
<tr>
<td>4</td>
<td>2.476</td>
<td>338.639</td>
<td>1.418</td>
<td>1.495</td>
</tr>
<tr>
<td>5</td>
<td>2.476</td>
<td>341.698</td>
<td>1.422</td>
<td>1.506</td>
</tr>
<tr>
<td>6</td>
<td>2.477</td>
<td>339.795</td>
<td>1.420</td>
<td>1.500</td>
</tr>
<tr>
<td>7</td>
<td>2.475</td>
<td>340.430</td>
<td>1.421</td>
<td>1.503</td>
</tr>
<tr>
<td>8</td>
<td>2.474</td>
<td>340.166</td>
<td>1.420</td>
<td>1.500</td>
</tr>
<tr>
<td>9</td>
<td>2.475</td>
<td>339.651</td>
<td>1.420</td>
<td>1.500</td>
</tr>
<tr>
<td>10</td>
<td>2.475</td>
<td>341.572</td>
<td>1.423</td>
<td>1.509</td>
</tr>
</tbody>
</table>

Gelman-Rubin statistic

The Gelman-Rubin statistic is very close to 1 for all the parameters, indicating that there is no clear evidence against convergence (Gelman and Rubin, 1992).

$\hat{R}(\alpha) = 1.00001$, $\hat{R}(k) = 1.00026$, $\hat{R}(p) = 1.00021$, $\hat{R}(t_f) = 1.00022$
4 Modelling volcano earthquake processes

be explained by these parameters being correlated with each other, as can be noted from Figure 2.9 which shows scatter plots of the parameter values from the samples drawn from the posterior (generated from the code in Appendix A.1 provided by Dr. A. Bell (2017)). Lower (higher) values of $k$ tend to occur with lower (higher) values of $p$ and $t_f$.

For $\alpha$, there is no such visible autocorrelation with the other parameters, as the scatter plots of $k$, $t_f$, and $p$ with $\alpha$ shows a lack of discernible trends. The NUTS sampler works effectively in such situations where some of the parameters are correlated, however this significantly slows down other samplers like the Metropolis and Slice sampler, in line with the earlier findings.

2.6.3 Thinning

Thinning is a process by which every $k$-th iteration of the chain is kept, and the rest are discarded. This has the effect of reducing autocorrelation within the chain, and reduces Monte Carlo error. However, the discarded values still carry some information about the posterior distributions. Thus, given that there are no practical restrictions on memory storage in this case, thinning is not considered necessary.

2.6.4 Results

The posterior distributions, with 95% highest posterior density intervals (HPDIs), are shown below in Figure 2.10, with values given in Table 2.2. The resulting HPDIs are reasonably narrow, and the distributions have regular unimodal shapes.

![Figure 2.10: Posteriors (excluding burn-in), 20k iterations NUTS](image)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>95% HPDI</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>2.476</td>
<td>0.117</td>
<td>[2.252, 2.710]</td>
</tr>
<tr>
<td>$k$</td>
<td>339.893</td>
<td>33.207</td>
<td>[285.102, 408.778]</td>
</tr>
<tr>
<td>$t_f$</td>
<td>1.420</td>
<td>0.060</td>
<td>[1.310, 1.539]</td>
</tr>
<tr>
<td>$p$</td>
<td>1.501</td>
<td>0.175</td>
<td>[1.175, 1.852]</td>
</tr>
</tbody>
</table>

An IG model was found to produce satisfactory results. It was demonstrated that the MCMC chain appears to converge to the correct stationary distribution, providing reasonable posterior estimates. From review of simulated data, and Q-Q and K-S plots, it was found that the IG model fits the July 2013 data very well. A small number of outliers (around 5% of the data) was noted, and found to correspond to spikes with long preceding ISIs. Some lack of fit was also found in the middle quantiles of the K-S plot, however this only slightly breached the 95% error bounds.

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**References**


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