Time-dependent behaviour of bone accentuates loosening in the fixation of fractures using bone-screw systems

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Bone’s time-dependent behaviour accentuates loosening in fracture fixation using bone-screw systems

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Abstract

Objectives:
One of the typical complications reported in the treatment of fractures using devices such as locking plates or unilateral fixators is loosening. It is believed that high strains in the bone at the bone-screw interface can initiate loosening which can result in infection and further loosening. Here, we present a new theory of loosening of implants: time-dependent response of bone subjected to loads results in interfacial deformations in the bone that accumulate with cyclic loading and thus accentuate loosening.

Methods:
We employ an idealised bone-screw system in which the screw is subjected to lateral cyclic loads and trabecular bone is modelled as nonlinear viscoelastic and nonlinear viscoelastic-viscoplastic material based on recent experiments conducted by us.

Results:
The study shows that the interfacial deformation in the bone increases with cycle number and the use of nonlinear viscoelastic-viscoplastic model results in larger deformations, some of which are irrecoverable. There is an apparent trend which shows that interfacial deformations increase with increasing bone porosity.

Conclusions:
These results indicate that bone’s time-dependent behaviour will accentuate implant loosening and osteoporotic patients are at a greater risk due to loosening of screws/implants.

Keywords:
Cyclic loading; viscoelastic-viscoplastic; bone volume ratio; irrecoverable strain
Article summary:

Article focus:

- The effect of bone’s time dependent behaviour at bone-screw interface is examined
- Viscoelastic and viscoplastic models are used to simulate behaviour of an idealised bone screw system subjected to cyclic loading

Key messages:

- The simulations show that deformations at the bone-screw interface accumulate with cyclic loading
- Interfacial deformations increase with decreasing bone porosity

Strengths and limitations:

- We incorporate, for the first time, nonlinear viscoelastic and nonlinear viscoelastic-viscoplastic material properties in finite element simulations for trabecular bone
- Microstructure plays an important role but is not included in this study. We expect microstructure will have a role in time-dependent properties as well
Introduction

Many bone fracture fixation treatments, such as unilateral fixators and locking plates employ unicortical or bicortical screws that go through the bone and permit load transfer across fractured segments \(^1\). The bone-fixator construct can fail due to breakage of the device (e.g. screw or plate), which can occur due to fatigue particularly when fracture healing is delayed. Another common failure mode is screw loosening, which has been frequently reported as a complication in implant usage and some previous studies have noted that this is initiated by mechanical forces typically before any contribution from biological processes \(^2\). It has been suggested that screw loosening and failure, observed clinically, probably results from bone-screw separation events and from elevated strains \(^3\). Local bone yielding at the interface, due to high stresses/strains, can initiate loosening and result in infection and further loosening \(^4–6\).

Most orthopaedic implants have some form of load-sharing arrangement with the bone which results in cyclic stresses/strains arising at the bone implant interface due to physiological activities. Experimental investigations have shown that the separation events are a function of cycle number for a range of implants including pedicle screws \(^7,8\), dynamic hip screws \(^9\), distal tibia fracture fixations \(^10,11\) and distal radius fracture fixations \(^12\). A study on dental implants drew attention to the fact that repeated loading-unloading cycles result in alternating compression and separation of bone-screw interface \(^3\). The nature of migration suggested that it is a mechanical phenomenon or at least mechanically triggered, rather than a purely biological process \(^2\).

In computational modelling loosening at the interface has, in the past, been examined by evaluating strain level after employing time-independent elastic \(^7\) or elastoplastic \(^4,8\) constitutive models for bone. While it is recognised that bone’s response to loads is time-dependent \(^9\) and some permanent deformation arises even at low strain levels \(<3000 \mu\varepsilon\) \(^10,11\). It has also been shown that, similar to time-independent material properties such as Young’s modulus, time-dependent
properties are also related to bone volume fraction (BV/TV) \textsuperscript{12,13}. However, these time-dependent mechanical properties of bone are not generally incorporated in modelling \textsuperscript{14}.

The aim of this study is to examine possible loosening due to cyclic loading in an idealised bone-screw system in which trabecular bone is assigned time-dependent properties (nonlinear viscoelastic and nonlinear viscoelastic-viscoplastic). The employed properties are based on recent experiments and developed constitutive models \textsuperscript{15}. Our hypothesis is that bone-screw systems used in fracture fixation can become loose when subjected to cyclic loading due to time-dependent mechanical properties of bone and the rate of loosening is associated with BV/TV.

**Material and methods**

In our study design we considered a screw inserted in a block of bone subjected to lateral cyclic loads as shown in Fig. 1a. Figure 1a also provides the dimensions. The dimensions of the screw are similar to that used in locking plates. Taking advantage of symmetry, only half of the bone block-screw system was modelled (Fig. 1b). Exact fit between the screw and the bone was considered at the screw circumference-bone interface; while a small gap was assumed between the screw end and bone to avoid influence of any end shape effects and maintain simplicity. A frictional screw-bone interface was employed with a standard Coulomb friction coefficient of 0.3 \textsuperscript{16,17}. Screw threads were excluded. The screw was assumed to be unicortical and a 1mm thick cortex was included in the model as shown in Fig. 1a.

All external faces of the block were assumed to be fully restrained except for the face with the screw hole and symmetry boundary conditions were applied at the symmetry surface (Fig. 1a, b). Triangular cyclic forces with an amplitude of 300 N and a frequency, f=1 Hz, as shown in Fig. 1c was applied to the external end of the screw (Fig. 1a). The models were permitted 1000s of recovery time after 500 complete cycles as shown in Fig. 1c. We selected 500 cycles as we were largely interested in evaluating the trends, though this choice was also partly dictated by
the computational resources required for this highly nonlinear problem.

Figure 1

The screw and the cortical bone were modelled as linear elastic time-independent materials, with Young’s moduli of 180 GPa and 20.7 GPa, respectively and Poisson’s ratio of 0.3.

Trabecular bone was modelled using two constitutive models: a nonlinear viscoelastic (VE) model; and a nonlinear viscoelastic-viscoplastic (VEP) model. The mathematical details of these models are provided as supplementary information, which include data for each of the three BV/TV values considered.

The screw-bone system was modelled in Abaqus (v6.12 Simulia, Providence, RI, USA) using 13182 three-dimension finite elements (brick, C3D8 and tetrahedron, C3D10M). Mesh convergence studies were performed and they showed that further mesh refinement resulted in change of peak displacement of bone by less than 0.5%.

Results

Trabecular bone displacements and strains in the region around the screw were compared at three stages for different cycles: time points when the load is at its peak (300N); time points when the load is zero; and at the end of the recovery (at the end of 1000 s after cyclic loading is stopped). In Fig. 2, the rows present displacement contours at different stages and at different cycles. The columns are for different bone samples (the numerals indicate BV/TV percentage of the samples used) and for the two time-dependent material models used (VE=nonlinear viscoelastic; VEP=nonlinear viscoelastic-viscoplastic). For each case, contours along the length of the screw (Fig. 1a) and at the symmetry surface, section A-A (Fig. 1b) are shown in Fig. 2. They were exaggerated by 150 times, and the section plots are superimposed with undeformed geometry for better comparison with its
original shape. Seven representative cycles (cycle 1, 5, 10, 50, 100, 300 and 500) were selected for examining displacement variation.

Figure 2

In all cases, the peak trabecular bone displacements occurred at the interface in the middle region of the screw hole at the top, and entrance of screw hole at the bottom for all three stages. The peak displacements of trabecular bone at section A-A at selected cycles for all 6 models were extracted and are shown in Fig. 3.

Figure 3

The maximum and minimum principal strain contours are shown in Fig 4 for VEP models at 7 representative cycles.

Figure 4

Peak loading time points

At the peak loading points, the difference in displacements between the samples with different BV/TV is apparent. It is clear that the lower BV/TV samples undergo larger interfacial displacements than higher BV/TV samples. This is apparent from the displacement contours (Fig. 2a), peak displacements (Fig. 3a) and principal strain contours (Fig 4a, b).

At peak loading time points, similar displacement contours were observed for models which included viscoplasticity (VEP) and those that were modelled using nonlinear viscoelasticity alone (VE). Small differences were, however, observed after the samples had experienced relatively larger number of cycles and for trabecular bone with low BV/TV (e.g. 15VEP had slightly higher displacement compared to the 15VE case). These differences between VE and VEP models were negligible at the highest BV/TV considered in the study (e.g. between 35VE and 35VEP). As expected, Fig. 3a shows that the maximum displacement experienced
by trabecular bone increases with the number of cycles, however this increase was found to be small; the largest increase for the lowest BV/TV sample with VEP model was 15%. Lower BV/TV in conjunction with viscoplasticity produced larger maximum displacements.

By examining the maximum (Fig. 4a) and minimum (Fig. 4b) principal strain contours, we observe that the strain experienced by trabecular bone increases with the number of cycles. It is worth noting that in current study the magnitude of strains in bone during loading phase were generally below 0.5% and 0.7% for tension and compression, respectively, which are the typically reported yielding strains in literature.\textsuperscript{18–20} Another apparent observation is that the trabecular bone experiences higher strain in compression than in tension (Fig. 4a and 4b). Similar to observation from displacements, strain magnitude increases with decrease in BV/TV of trabecular bone.

**Zero loading time points**

In the initial cycles, most of the deformation is recovered upon unloading (Fig. 2b), however, as the number of cycles increase, deformations accumulate with increasing number of cycles at zero load time points. This indicates development of a gap between the screw and the bone. This is also demonstrated when comparing peak displacements as shown in Fig. 3b. As expected, the VEP models have much larger peak deformation upon unloading compared to VE models, especially for lower BV/TV samples (Fig. 3b). Similar to the loading phase, the displacements of trabecular bone at zero loading time points were related to trabecular bone’s BV/TV, the deformations were found to increase with decrease in BV/TV. The largest increase for the lowest BV/TV sample with VEP model (15VEP) was more than 700% from cycle 1 to 500. It is important to note that at zero load time points, there are residual bone displacements not only when using viscoplastic models but also when employing viscoelastic models; these residual displacements cannot be obtained if time-independent elastic properties are used which require that the deformation to recover instantaneously upon unloading.
Figure 4c and 4d show the maximum and minimum principal strain upon unloading with increased cycle number for VEP models. The majority of strain is recovered upon unloading (note the difference in scale of the contour plots for peak loading time points and zero loading time points), but residual strains accumulate with cycle numbers and these residual strains are associated with bone’s BV/TV. We also found that the accumulation of principal strains, both maximum and minimum, is more rapid for bone with lower BV/TV.

Recovery

All the six models investigated in this study were allowed to recover for 1000 s under zero force condition after 500 cycles of loading. As expected, the VE models show almost complete recovery of displacement, whereas there is residual displacement with VEP models (Fig. 2c). These irrecoverable deformations were found to be related to BV/TV (Fig. 2c); irrecoverable deformations increased with decreasing BV/TV. The highest irrecoverable deformation was found for the bone with lower BV/TV (Fig. 3b), and irrecoverable deformation was found to be negligible for bone with the highest BV/TV considered (Fig. 3b). This is also apparent from principal strain contours in Fig 4e and 4f. The bone with lower BV/TV experiences the relatively higher irrecoverable minimum and maximum principal strain. This observation shows that the viscoplasticity plays an important role.

Discussion

The study finds that, in a bone-screw system subjected to cyclic loading, inclusion of time-dependent properties for trabecular bone predicts separation between the screw and the bone which increases with increasing cycle number. Incorporation of viscoplasticity results in larger deformation, some of which is irrecoverable. Interfacial deformation was found to follow a trend based on bone volume fraction (BV/TV) with porous bone experiencing larger deformations (or separation between
Numerical simulation of loosening due to mechanical forces in a bone-screw system subjected to cyclic loading has not been possible previously, as time-dependent material constitutive model of bone has not been included before. Recent multiple-load-creep-unload-recovery experimental study followed by development of time-dependent constitutive models for trabecular bone has permitted their use in this clinically relevant study. A number of previous studies, both clinical and experimental, have reported that the migration of implant or loosening of screw is a function of time or cycle numbers. Taylor and Tanner suggested that the migration of implant is a mechanical phenomenon or at least mechanically triggered, rather than a biological process. As demonstrated by previous in vitro studies, our simulations show that deformation is a function of cycle numbers for both loading and unloading phases. Inclusion of time-dependent properties implies that deformation accumulates with increasing cycle numbers, in contrast to conventional time-independent finite element analyses, in which the deformation or strain remains unchanged with increasing number of cycles.

Inclusion of time dependent properties implies that upon unloading although significant proportion of deformation is recovered, residual deformations do exist. So while the screw returns to its undeformed and unstrained configuration, the bone’s deformation recovery lags behind and this lag increases with each loading cycle. As would be expected, inclusion of viscoplasticity results in the unloaded deformations being larger due to a proportion being irrecoverable. The latter is apparent when deformations following 1000s of recovery are compared in which the deformations with the nonlinear viscoelastic model recover almost entirely.

During the loading phase the deformations in the bone are forced to follow those of the screw. Consequently, bone deformations only show a small increase with increasing number of cycles as the time lag between the bone response and the instantaneous screw response increases. At the peak loading time point in the first cycle there is no difference between deformations from VE and VEP models (Fig.
3), however, with increasing number of cycles the differences start emerging. Thus, the study shows that the inclusion of viscoplasticity not only produces larger deformations in the unloaded phase but also at the peak loading time points.

The strong positive relationship between trabecular bone’s BV/TV and its time-independent properties has been previously reported $^{19,24}$. In general, bone with higher BV/TV has a larger stiffness and yields at larger loads, though it is recognised that the bone’s micro-structure also has an important role. In other words, the denser trabecular bone has a better ability to resist the applied forces and undergoes lower deformations. This trend was consistently observed with the three samples we considered. The deformations at different stages followed a clear trend based on trabecular bone’s BV/TV; porous bone (lower BV/TV) experiences higher deformation in comparison to less porous bone. This is true for loading and unloading time points and even for recovery modelled using viscoplasticity. It has also been reported in a number of studies that the bone with low-density (e.g. due to osteoporosis) is at higher risk of implant instability; these studies include in vitro experiments $^{22,25,26}$, finite element simulations $^{27,28}$ and clinical findings $^{29}$. Basler et al$^{22}$ found that the displacement was strongly correlated to initial BV/TV ($r^2= 0.95$) which implies that it is also related to implant migration. Consistent results are observed in the current study; a low BV/TV bone not only experiences higher interfacial deformation (i.e. at higher risk to occur screw instability) in comparison to denser bone, but also the deformation increases more rapidly with increasing cycle number.

This study considers primary stability soon after the operation and before any biologically driven bone remodelling has occurred in a manner similar to several previous computational $^{4,7}$ and in vitro $^{22,23}$ studies. Primary stability relies on interlocking and frictional bone-screw contact phenomena. The cycle-dependent deformation results in the screw hole becoming enlarged and that will cause the frictional resistance at the bone-screw interface to reduce with increasing loading cycles. Donaldson et al$^4$ reported that in unilateral external fixators there exist push-in and pull-out forces that accompany axial loading. These push-in and pull-out
forces in conjunction with increasing screw hole diameter and decreasing frictional resistance can result in increased risk of loosening particularly in low BV/TV bone.

It has been suggested that bone ingrowth/ongrowth occurs if the micromotions are less than 40-50 µm$^{30}$. If physiological loads give rise to bone-implant relative micro-movements of the order of 100 - 200 µm then they inhibit bone in-growth, resulting in the formation of a fibrous tissue layer around the prosthesis$^{31}$, and eventual loosening of the implant$^{32}$. Some previous experiments conclude that the threshold value of micromotion for osseointegration is between 30 and 150 µm$^{33}$. Although our study considers an idealised system, with a screw inserted in a block of bone, it is tempting to examine quantitative values of the interfacial motions; they are smaller than the threshold value of micromotion required in the formation of a fibrous tissue layer. However, it is important to note that in the idealised system used in this study, the offset at which the load is applied is perhaps similar to that in a locking plate; devices with larger offset (e.g. unilateral fixators) will result in much larger forces at the bone-screw interface. Moreover it has been previously shown that deformations increase nonlinearly as the external plate/frame bends$^{7}$, a phenomenon not included in this study. We only applied 500 cycles (as nonlinear simulation takes considerable computational resources), however the trends show the separation continues to increase. While the maximum and minimum principal strains are generally lower than the typically reported values of yield they are not too far from yield values after 500 cycles of load application, particularly for the lowest BV/TV sample considered. It is also important to note that while minimum principal strain occurs primarily in the direction radial to the screw that maximum principal strain is in the hoop direction. Steiner et al$^{34}$ reported that bone damage may occur due to the screw insertion process itself, and the most bone damage occurs within a 300µm radial distance of the screws. In this study, we assumed that the screw inserted into bone without any damage. Consequently, this study is valid with respect to trends rather than actual quantitative values.

This study has a few limitations. We use single frequency of 1Hz. It is reasonable to expect that the quantitative results will vary with the choice of frequency$^{35}$.
however, the trends in the frequency range that are not too dissimilar to the one selected here are likely to be maintained. We found a clear trend with variation in BV/TV, but we only considered three samples, a statistical analysis that considers the influence of different BV/TV was not possible. So, while we expect the trend to be generally followed we deliberately did not develop a relationship with respect to BV/TV. Previous studies on time-independent mechanical properties of bone have shown that in addition to BV/TV, microstructure plays an important role. We expect microstructure will have a role in time-dependent properties as well. As the focus of this study was to examine the radial separation at the bone-screw interface and not slippage due to pull-out forces screw threads were not included. Screw threads are likely to reduce loosening due to slippage when pull-out or push-in forces are applied. One the other hand, they generate stress/strain concentrations at the interface which will may also accentuate loosening. We only applied 500 cycles; larger number of cycles will increase the predicted separation. Lastly, in this study, time-independent elastic material properties were employed for cortical bone and screw; the time-dependent effect on screw is negligible, but cortical bone too will have time-dependent behaviour. Inclusion of time-dependent behaviour for cortical bone is likely to accentuate screw loosening even further.

Current study is more advanced compared to conventional finite element analyses on bone-screw interface mechanics in which only time-independent material properties that are either elastic or inelastic are assigned to bone. However, it does not consider a full bone fixator construct or the influence of healing. Importance of time-dependent effects at bone-screw interface was discussed at least a couple of decades ago, but these have not been previously included in models due to lack of experimental data and absence of time dependent constitutive models for bone. The developed nonlinear viscoelastic-viscoplastic constitutive models permit simulation of time and cycle dependent response to be included in the analysis of bone screw systems.

It can be concluded that implant loosening due to physiological activities can be predicted computationally by employing models that incorporate time-dependent
behaviour of bone. Osteoporotic patients with lower bone-volume ratio are at a greater risk of implant loosening. It is known that secondary healing can be promoted by application of cyclic loads that cause interfragmentary motion between fractured segments. Clinically this implies that the need for application of cyclic loads has to be balanced by the risk of loosening before healing occurs.

Acknowledgement

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Reference


Figure Legend

Figure 1. Geometry of the bone-screw system showing symmetry surface with location of load application (a); section A-A (b); load application - each model was subjected to 500 cycles of triangular load of 300 N amplitude followed by 1000 s of recovery (c)

Figure 2. Displacement (um) contours at symmetry surface and section A-A for 7 representative cycles at the time points when load at its peak (300N) (a); at the time points when load is zero (b) and recovery after 1000 s (c). They are exaggerated by 150 times, and the section plots are superimposed with undeformed geometry for better comparison with its original shape

Figure 3. Peak displacement experiences of trabecular bone at section A-A for both VE and VEP models at the time points when load is at its peak (300N) (a); at the time points when load is zero and after 1000 s recovery (b)

Figure 4. Maximum principal strain (a, c, e) and minimum principal strain (b, d, f) contours at symmetry surface and section A-A for 7 representative cycles at the time points when load at its peak (300N) (a, b); at the time points when load is zero (c, d) and recovery after 1000 s (e, f). Strain is expressed as a percentage (%)
Figure 1

a) [Diagram showing dimensions and labels for cortical bone, trabecular bone, and screw.]

b) [Section A-A showing dimensions and a screw symbol labeled \( \varphi 6.5 \).]

Unit: mm

Force (N)

0 1 2 500 1500

Triangular cyclic loading Recovery

c) [Graph showing force over time with triangular cyclic loading and recovery phases.]
Figure 2
Figure 3
Figure 4
Supplementary information

The VE model was based on Schapery’s nonlinear viscoelastic constitutive law \(^1\),
given by Manda et al. \(^2\)

\[
\varepsilon_{ve}(t) = g_0 D_g \sigma + \int_0^t \Delta D(\psi^t - \psi^\tau) \frac{d(g_2 \sigma)}{d\tau} d\tau \tag{1}
\]

\[
\psi^t = \int_0^t \frac{d\tau'}{\alpha_T(\tau') \alpha_e(\tau')}
\tag{2}
\]

where \(D_g\) is instantaneous compliance, \(g_0, g_1, g_2\) and \(\alpha_\sigma\) are stress-dependent nonlinear viscoelastic parameters expressed as second order polynomial equations \(^2\), \(\sigma\) is applied stress and \(\psi^t\) is reduced time. The effects of temperature (\(\alpha_T\)) and other environment variables (\(\alpha_e\)) are not considered; consequently these two parameters are unity. The transient compliance, \(\Delta D\), in equation (1) is represented by Prony series as

\[
\Delta D(\psi^t) = \sum_1^n D_n[1 - \exp(-\lambda_n \psi^t)]
\tag{3}
\]

where \(D_n\) is \(n\)th coefficient of the Prony series associated with the reciprocal of \(n\)th retardation time, \(\lambda_n\).

Manda et al. \(^2\) used this model to evaluate parameters for 19 trabecular bone samples; three samples, with BV/TV of 15\%, 25\% and 35\%, were selected from this study. The corresponding parameters and their definitions are provided in Table 1.
Table 1: The nonlinear viscoelastic parameters along with linear Prony coefficients and irrecoverable strains at multiple stress levels for three different BV/TV $^1$

<table>
<thead>
<tr>
<th>BV/TV</th>
<th>Linear coefficients at Cycle I</th>
<th>Cycle no.</th>
<th>$\varepsilon_{\text{static}}$ (%)</th>
<th>$\sigma^N$</th>
<th>Nonlinear VE parameters</th>
<th>$\varepsilon_{\text{irrec}}$ (%)</th>
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<tr>
<td>15%</td>
<td>$[D_0]$ = $6.40 \times 10^{-3}$</td>
<td>I</td>
<td>0.2</td>
<td>0.36</td>
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<td>1.00</td>
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<td>0.91</td>
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<td></td>
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<td>1.17</td>
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<td>13.01</td>
<td>0.70</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td>$\lambda_3$ = $7.05 \times 10^{-1}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

BV/TV is the bone volume fraction, $D_n$ is the instantaneous compliance in 1/MPa, $D_n$ (n = 1, 2, 3) are transient compliance coefficients in 1/MPa, $\lambda_n$ (n = 1, 2, 3) are reciprocal of nth retardation time in Prony series in s$^{-1}$, $\varepsilon_{\text{static}}$ is the applied static strain in each loading cycle, $\sigma^N$ is the stress corresponding to plateau stress in the Nth loading cycle in MPa. Parameters $g_0, g_1, g_2, \gamma$ are stress-dependent nonlinear viscoelastic (VE) parameters and $\varepsilon_{\text{irrec}}$ is the irrecoverable strain exist at the end of each loading cycle.

The study by Manda et al. $^2$ found that trabecular bone response to mechanical loads comprises both recoverable irrecoverable deformations. Our VEP model incorporated these, from the data available from the above study, by employing a viscoplastic constitutive model along with the nonlinear viscoelastic model. In this, the total strain rate is given by

$$\dot{\varepsilon}_{\text{total}} = \dot{\varepsilon}_{ve} + \dot{\varepsilon}_{vp} \quad (4)$$

where $\dot{\varepsilon}_{ve}$ and $\dot{\varepsilon}_{vp}$ are viscoelastic and viscoplastic strain rates, respectively. The viscoplastic strain rate based on Perzyna model $^3$, is given by

$$\dot{\varepsilon}_{vp} = \eta \left( \frac{\varepsilon}{\sigma} \right)^N \frac{\partial G}{\partial \sigma} \quad (5)$$

where $\eta$ is the viscoplastic parameter, $G$ is viscoplastic potential function. The
Macaulay brackets, $<.>$, indicate that the viscoplastic strain rate is non-zero only when $\left( \frac{F}{\sigma_y} \right)^N > 0$. The terms $\sigma_y^0$ and $N$ are material parameters. We based the yield function $F$ on extended Drucker-Prager criterion

$$F = \tau - \alpha p - \kappa(\varepsilon_{vp})$$

where $\tau$ is deviator shear stress; $p = -\frac{1}{3} tr(\sigma)$ is the equivalent pressure stress; $\alpha = \tan \theta$ is a pressure-sensitivity parameter related to friction angle $\theta$; $\kappa(\varepsilon_{vp})$ is viscoplastic hardening function, which is a function of effective viscoplastic strain, given by

$$\kappa(\varepsilon_{vp}) = \kappa_0 + \kappa_1 [1 - \exp(-\kappa_2 \varepsilon_{vp})]$$

where $\kappa_0$ is the initial yield stress, $\kappa_1$ is the saturated stress for a fully-hardened material, $\kappa_2$ is the transition rate between $\kappa_0$ and $\kappa_0 + \kappa_1$. The viscoplastic potential function is given by

$$G = \tau - \beta p$$

where $\beta = \tan \theta'$ is a parameter related to the dilation angle $\theta'$. In this study, the friction angle $\theta$ and dilation angle $\theta'$ were assumed to be 46° and 0°, respectively, based on previous investigation on bone. Consequently, $\alpha$ and $\beta$ were equal to 1.035 and 0, respectively.

The evaluated viscoplastic parameters for three BV/TV bone samples considered in this study are given in Table 2. These parameters were used as input to the user defined material (UMAT) for trabecular bone samples, which was implemented in Abaqus 6.12 (Simulia, Providence, RI, USA).
Table 2. The values of the viscoplastic parameters for three different BV/TV

<table>
<thead>
<tr>
<th>BV/TV</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$N$</th>
<th>$\eta$ (s$^{-1}$)</th>
<th>$\kappa_0$ (MPa)</th>
<th>$\kappa_1$ (MPa)</th>
<th>$\kappa_2$</th>
<th>$\sigma_0^2$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>1.035</td>
<td>0</td>
<td>3</td>
<td>$4.34 \times 10^{-2}$</td>
<td>$2.42 \times 10^{-3}$</td>
<td>2.11</td>
<td>$4.62 \times 10^{2}$</td>
<td>3.38</td>
</tr>
<tr>
<td>25%</td>
<td>1.035</td>
<td>0</td>
<td>3</td>
<td>$4.91 \times 10^{-3}$</td>
<td>$1.35 \times 10^{-10}$</td>
<td>4.11</td>
<td>$3.50 \times 10^{2}$</td>
<td>5.75</td>
</tr>
<tr>
<td>35%</td>
<td>1.035</td>
<td>0</td>
<td>3</td>
<td>$5.35 \times 10^{-2}$</td>
<td>$4.03 \times 10^{-1}$</td>
<td>10.40</td>
<td>$5.00 \times 10^{2}$</td>
<td>26.70</td>
</tr>
</tbody>
</table>

BV/TV is the bone volume fraction, $\alpha$ is a pressure-sensitivity parameters related to friction angle, $\beta$ is a parameter related to dilation angle, $\sigma_0^2$ and $N$ are material constant, $\eta$ is a viscosity parameter, $\kappa_0$ is the initial yield stress, $\kappa_1$ is the saturated stress for the fully-hardened material, $\kappa_2$ is the transition rate between $\kappa_0$ and $\kappa_0 + \kappa_1$. 
References: