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Counting Triangles under Updates

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1 Problem Setting and Contributions

We consider the problem of maintaining the result of the triangle count query $Q() = \Gamma_{\text{sum}} R(A, B) \bowtie S(B, C) \bowtie T(C, A)$ under single-tuple updates to the input relations $R$, $S$, and $T$. The relations are given as key-payload maps whose keys are tuples over relation schemas, payloads are tuple multiplicities, and key lookups are (amortized) $O(1)$-time operations. A single-tuple update $\delta R(a, b) = \{ (a, b) \mapsto p \}$ to relation $R$ maps a key $(a, b)$ to a nonzero payload $p$ (positive for inserts and negative for deletes); updates to $S$ and $T$ are analogous.

The naïve maintenance approach recomputes the triangle count from scratch after each update. Computing this query using worst-case optimal join algorithms [5] takes $O(N^{1.5})$ time, where $N$ is the current size of the input database.

To incrementally maintain the triangle count under single-tuple updates, existing incremental view maintenance (IVM) approaches need linear time. For instance, under the update $\delta R(a, b)$ to $R$, the classical IVM [2] computes the delta query $\Gamma_{\text{sum}} \delta R(a, b) \bowtie S(b, C) \bowtie T(C, a)$ in $O(N)$ time because it needs to intersect two lists of possibly linearly many $C$-values that are paired with $b$ in $S$ and with $a$ in $T$. The factorized IVM [6] materializes the view $V_{ST}(B, A) = \Gamma_{B, A; \text{sum}} S(B, C) \bowtie T(C, A)$ using $O(N^2)$ space. It then computes the delta query $\Gamma_{\text{sum}} \delta R(a, b) \bowtie V_{ST}(b, a)$ in $O(1)$ time; however, updates to $S$ and $T$ still require $O(N)$ time to maintain the triangle count $Q$ and view $V_{ST}$.

This raises the question of whether the triangle count can be maintained in sublinear time. Recent work proves that no algorithm can maintain $Q$ in time $O(N^{0.5-\gamma})$ for any $\gamma > 0$, under reasonable complexity-theoretic assumptions [1]. An algorithm with sublinear maintenance time for $Q$ is not yet known.

This work introduces IVM$^\epsilon$, an IVM approach that maintains the triangle count in amortized sublinear time. IVM$^\epsilon$ partititions each input relation into two parts, heavy and light, based on the degrees of data values, the database size, and a parameter $\epsilon$. It then adapts the maintenance strategy to different heavy-light combinations of parts of the input relations to achieve worst-case sublinear maintenance. As the database evolves under updates, IVM$^\epsilon$ rebalances the partitions to account for a new database size and updated degrees of data values. While this rebalancing may take superlinear time, it remains sublinear per update.

Given a database of size $N$ and $\epsilon \in [0, 1]$, IVM$^\epsilon$ maintains the triangle count in $O(N^{\max\{1, 1-\epsilon\}})$ amortized time while using $O(N^{1+\min\{\epsilon, 1-\epsilon\}})$ space. It thus defines a continuum of approaches exhibiting a space-time tradeoff based on $\epsilon$.

\textsuperscript{*} An extended version of this work is available online [3].
We adapt the maintenance strategy to each skew-aware view to ensure sub-linear update time. While most of these views admit sublinear delta computation, few exceptions require linear-time maintenance. For these exceptions, IVM\(^{e}\) precomputes the update-independent parts of delta queries as *materialized views* and uses them to speed up the delta evaluation. Such auxiliary views also require maintenance, yet their maintenance cost is sublinear for single-tuple updates.

Figure 1 shows the materialized views used by IVM\(^{e}\) to maintain the triangle count query. The size of the view \(V_{RS}(A, C)\) is upper-bounded by the size of the result of the join of \(R_h(A, B)\) and \(S_l(B, C)\) in two distinct ways. One can iterate over all \((a, b)\) pairs in \(R_h\) and then find the \(C\)-values in \(S_l\) for each \(b\). Since \(S_l\) contains only tuples with light \(B\)-values, there are at most \(N^e\) distinct \(C\)-values for each \(B\)-value. This gives an upper bound of \(O(|R_h| \cdot N^e) = O(N^{1+e})\).

Alternatively, one can iterate over all \((b, c)\) pairs in \(S_l\) and then find the \(A\)-values in \(R_h\) for each \(b\). Since \(R_h\) contains only tuples with heavy \(A\)-values, there are at most \(N^{1-e}\) distinct \(A\)-values. This gives an upper bound of \(O(|S_l| \cdot N^{1-e}) = O(N^{2-e})\). The overall space complexity is the minimum of the bounds. The space analysis for \(V_{ST}\) and \(V_{TR}\) is analogous.

We explain our adaptive strategy on a single-tuple update \(\delta R_*(a, b)\) to relation \(R\). This update can affect either the heavy or light part of \(R\), hence the

<table>
<thead>
<tr>
<th>Materialized View Definition</th>
<th>Space Complexity</th>
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</thead>
<tbody>
<tr>
<td>(Q() = \bigcup_{u,v,w \in {h,l}} \Gamma_{\text{sum}} R_u(A, B) \bowtie S_v(B, C) \bowtie T_w(C, A))</td>
<td>(O(1))</td>
</tr>
<tr>
<td>(V_{RS}(A, C) = \Gamma_{A,C;\text{sum}} R_h(A, B) \bowtie S_l(B, C))</td>
<td>(O(N^{1+\min{e,1-e}}))</td>
</tr>
<tr>
<td>(V_{ST}(B, A) = \Gamma_{B,A;\text{sum}} S_h(B, C) \bowtie T_l(C, A))</td>
<td>(O(N^{1+\min{e,1-e}}))</td>
</tr>
<tr>
<td>(V_{TR}(C, B) = \Gamma_{C,B;\text{sum}} T_h(C, A) \bowtie R_l(A, B))</td>
<td>(O(N^{1+\min{e,1-e}}))</td>
</tr>
</tbody>
</table>

Fig. 1. The materialized views used by IVM\(^{e}\) for a database of size \(N\) and \(e \in [0,1]\).
Fig. 2. Computing the deltas of the views from Figure 1 for an update $\delta R_s(a,b)$ to the heavy or light part of $R$. The symbol $*$ stands for $h$ or $l$. The delta $\delta V_{ST}$ is empty since $V_{ST}$ does not refer to $R$. The evaluation order of deltas is from left to right.

<table>
<thead>
<tr>
<th>Delta Evaluation Strategy</th>
<th>Time Complexity</th>
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</thead>
<tbody>
<tr>
<td>$\delta Q_{+hh}(\cdot) = \delta R_s(a,b) \cdot \sum T_h(C,a) \cdot S_h(b,C)$</td>
<td>$O(N^{1-\epsilon})$</td>
</tr>
<tr>
<td>$\delta Q_{+hl}(\cdot) = \delta R_s(a,b) \cdot V_{ST}(b,a)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\delta Q_{+lh}(\cdot) = \delta R_s(a,b) \cdot \sum T_h(C,a) \cdot S_l(b,C)$ or $\delta Q_{+lh}(\cdot) = \delta R_s(a,b) \cdot \sum S_l(b,C) \cdot T_h(C,a)$</td>
<td>$O(N^{\min{\epsilon,1-\epsilon}})$</td>
</tr>
<tr>
<td>$\delta Q_{+ll}(\cdot) = \delta R_s(a,b) \cdot \sum S_l(b,C) \cdot T_l(C,a)$</td>
<td>$O(N^\epsilon)$</td>
</tr>
<tr>
<td>$\delta Q(\cdot) = \delta Q_{+hh}(\cdot) + \delta Q_{+hl}(\cdot) + \delta Q_{+lh}(\cdot) + \delta Q_{+ll}(\cdot)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\delta V_{RS}(a,C) = \delta R_h(a,b) \cdot S_l(b,C)$</td>
<td>$O(N^\epsilon)$</td>
</tr>
<tr>
<td>$\delta V_{TR}(C,b) = \delta R_l(a,b) \cdot T_h(C,a)$</td>
<td>$O(N^{1-\epsilon})$</td>
</tr>
</tbody>
</table>

symbol; we assume that checking whether $a$ is heavy or not in $R$ is a constant-time operation. Updates to the other two relations are handled similarly.

Figure 2 shows the deltas of the views affected by the update $\delta R_s(a,b)$ and their time complexity when evaluated from left to right. In all but one case, the complexity is determined by the number of $C$-values that need to be iterated over. Computing the deltas involves multiplying the payloads of matching tuples and, if $C$ is not in the target view schema, summing them over $C$-values.

We first analyze the access patterns of the skew-aware delta views: (1) For $\delta Q_{+hh}$, we iterate over at most $N^{1-\epsilon}$ $C$-values in $T_h$ for the given $a$ and then look up in $S_h$ for each $(b,c)$; (2) For $\delta Q_{+hl}$, we look up in the materialized view $V_{ST}$ for the given $(a,b)$; (3) For $\delta Q_{+lh}$, we either iterate over at most $N^{1-\epsilon}$ $C$-values in $T_h$ for the given $a$ and look up in $S_l$ for each $(b,c)$, or we iterate over at most $N^\epsilon$ $C$-values in $S_l$ for the given $b$ and look up in $T_h$ for each $(c,a)$; (4) For $\delta Q_{+ll}$, we iterate over at most $N^\epsilon$ $C$-values in $S_l$ for the given $b$ and then look up in $T_l$ for each $(c,a)$. Then, summing these partial deltas and updating $Q$ take constant time. The views $V_{RS}$ and $V_{TR}$, which facilitate updates to $T$ and respectively to $S$, are maintained for updates to distinct parts of $R$. Computing $\delta V_{RS}$ and updating $V_{RS}$ requires iterating over at most $N^\epsilon$ $C$-values in $S_l$ for the given $b$; similarly, computing $\delta V_{TR}$ and updating $V_{TR}$ involves at most $N^{1-\epsilon}$ heavy $C$-values in $T_h$. The final step of IVM updates the (heavy or light) part of $R$ that corresponds to $\delta R_s$ in (amortized) $O(1)$ time. Overall, IVM maintains the views from Figure 1 under single-tuple updates to any of the input relations in $O(N^\max\{\epsilon,1-\epsilon\})$ time using $O(N^{1+\min\{\epsilon,1-\epsilon\}})$ space.

An insert $(a,b)$ into $R$ may promote $a$ from light to heavy in $R$ or may increase the heavy-light threshold such that some $A$-values change from heavy to light. Without rebalancing the partitions, our assumptions on the number of $B$-values paired with $a$ or the number of heavy $A$-values may become invalid.

IVM loosens the partition threshold to amortize the cost of rebalancing over multiple updates. Instead of the actual database size $N$, the threshold now
depends on a variable $M$ for which the invariant $|\frac{1}{4}M| \leq N < M$ always holds. If the database size violates one of the limits, we perform major rebalancing where we double or halve $M$ to satisfy the invariant again, repartition the input relations using the new threshold $M'$, and recompute the auxiliary views. The time complexity of this operation is $O(M^{1+\min\{\epsilon, 1-\epsilon\}})$, which is amortized over at least $\lceil \frac{1}{4}M \rceil$ updates between two major rebalancing steps.

IVM$^\epsilon$ also enforces the following two invariants: The number of tuples with the same value of the partitioning attribute is less than $\frac{3}{2}M'$ in each light part and at least $\frac{1}{2}M'$ in each heavy part. If any of the two invariants is violated, we perform minor rebalancing where we move at most $\lceil \frac{3}{2}M' \rceil$ tuples from one part to another and update the affected views. The time complexity of this operation is $O(M^{\epsilon+\max\{\epsilon, 1-\epsilon\}})$, which is amortized over at least $\lceil \frac{1}{2}M' \rceil$ updates between two minor rebalancing steps for the same value of the partitioning attribute.

In conclusion, both rebalancing steps together take $O(M^{\max\{\epsilon, 1-\epsilon\}})$ amortized time. Since each single-tuple update can be realized in time $O(M^{\max\{\epsilon, 1-\epsilon\}})$ and $M = O(N)$, IVM$^\epsilon$ needs $O(N^{\max\{\epsilon, 1-\epsilon\}})$ overall amortized time. The extended version of this work presents a detailed complexity analysis of IVM$^\epsilon$ [3].

3 Beyond the Triangle Query

IVM$^\epsilon$ can be applied to any query but may not always yield asymptotic improvements over existing approaches. It can achieve sublinear maintenance for the counting variants of acyclic queries, e.g., 3-path and 4-path, and cyclic queries, e.g., Loomis-Whitney and 4-cycle. Different semirings can be used to specify operations on the payloads [6]; we used here $(\mathbb{Z}, +, \times, 0, 1)$ to express counting. An early prototype implementation of IVM$^\epsilon$ on top of DBToaster [4] shows several factors performance improvement over classical and factorized IVM.

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References