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Counting Triangles under Updates

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1 Problem Setting and Contributions

We consider the problem of maintaining the result of the triangle count query

\[ Q() = \Gamma_{\text{sum}} R(A,B) \bowtie S(B,C) \bowtie T(C,A) \]

under single-tuple updates to the input relations \( R, S, \) and \( T \). The relations are given as key-payload maps whose keys are tuples over relation schemas, payloads are tuple multiplicities, and key lookups are (amortized) \( O(1) \)-time operations. A single-tuple update \( \delta R(a,b) = \{ (a,b) \mapsto p \} \) to relation \( R \) maps a key \( (a,b) \) to a nonzero payload \( p \) (positive for inserts and negative for deletes); updates to \( S \) and \( T \) are analogous.

The naïve maintenance approach recomputes the triangle count from scratch after each update. Computing this query using worst-case optimal join algorithms [5] takes \( O(N^{1.5}) \) time, where \( N \) is the current size of the input database.

To incrementally maintain the triangle count under single-tuple updates, existing incremental view maintenance (IVM) approaches need linear time. For instance, under the update \( \delta R(a,b) \), the classical IVM [2] computes the delta query \( \Gamma_{\text{sum}} \delta R(a,b) \bowtie S(b,C) \bowtie T(C,a) \) in \( O(N) \) time because it needs to intersect two lists of possibly linearly many \( C \)-values that are paired with \( b \) in \( S \) and with \( a \) in \( T \). The factorized IVM [6] materializes the view \( V_{ST}(B,A) = \Gamma_{B,A;\text{sum}} S(B,C) \bowtie T(C,A) \) using \( O(N^2) \) space. It then computes the delta query \( \Gamma_{\text{sum}} \delta R(a,b) \bowtie V_{ST}(b,a) \) in \( O(1) \) time; however, updates to \( S \) and \( T \) still require \( O(N) \) time to maintain the triangle count \( Q \) and view \( V_{ST} \).

This raises the question of whether the triangle count can be maintained in sublinear time. Recent work proves that no algorithm can maintain \( Q \) in time \( O(N^{0.5-\gamma}) \) for any \( \gamma > 0 \), under reasonable complexity-theoretic assumptions [1]. An algorithm with sublinear maintenance time for \( Q \) is not yet known.

This work introduces IVM\(^\epsilon\), an IVM approach that maintains the triangle count in amortized sublinear time. IVM\(^\epsilon\) partitions each input relation into two parts, heavy and light, based on the degrees of data values, the database size, and a parameter \( \epsilon \). It then adapts the maintenance strategy to different heavy-light combinations of parts of the input relations to achieve worst-case sublinear maintenance. As the database evolves under updates, IVM\(^\epsilon\) rebalances the partitions to account for a new database size and updated degrees of data values. While this rebalancing may take superlinear time, it remains sublinear per update.

Given a database of size \( N \) and \( \epsilon \in [0, 1] \), IVM\(^\epsilon\) maintains the triangle count in \( O(N^{\max\{\epsilon,1-\epsilon\}}) \) amortized time while using \( O(N^{1+\min\{\epsilon,1-\epsilon\}}) \) space. It thus defines a continuum of approaches exhibiting a space-time tradeoff based on \( \epsilon \).

* An extended version of this work is available online [3].
S, a, b over all (a, b) pairs in R ≲ \( R_h(A, B) \) and then find the C-values in \( S_l \) for each b. Since \( S_l \) contains only tuples with light B-values, there are at most \( N^\epsilon \) distinct C-values for each B-value. This gives an upper bound of \( O(|R_h| \cdot N^\epsilon) = O(N^{1+\epsilon}) \).

Alternatively, one can iterate over all (b, c) pairs in \( S_l \) and then find the A-values in \( R_h \) for each b. Since \( R_h \) contains only tuples with heavy A-values, there are at most \( \frac{N}{\epsilon} = N^{1-\epsilon} \) distinct A-values. This gives an upper bound of \( O(|S_l| \cdot N^{1-\epsilon}) = O(N^{2-\epsilon}) \). The overall space complexity is the minimum of the bounds. The space analysis for \( V_{ST} \) and \( V_{TR} \) is analogous.

We explain our adaptive strategy on a single-tuple update \( \delta R_*(a, b) \) to relation \( R \). This update can affect either the heavy or light part of \( R \), hence the *
symbol; we assume that checking whether a is heavy or not in R is a constant-time operation. Updates to the other two relations are handled similarly.

Figure 2 shows the deltas of the views from Figure 1 for an update $\delta R_a(a, b)$ and their time complexity when evaluated from left to right. In all but one case, the complexity is determined by the number of $C$-values that need to be iterated over. Computing the deltas involves multiplying the payloads of matching tuples and, if $C$ is not in the target view schema, summing them over $C$-values.

We first analyze the access patterns of the skew-aware delta views: (1) For $\delta Q_{shh}$, we iterate over at most $N^{1-\epsilon}$ $C$-values in $T_h$ for the given $a$ and then look up in $S_h$ for each $(b, c)$; (2) For $\delta Q_{shl}$, we look up in the materialized view $V_{ST}$ for the given $(a, b)$; (3) For $\delta Q_{slh}$, we either iterate over at most $N^{1-\epsilon}$ $C$-values in $T_h$ for the given $a$ and look up in $S_l$ for each $(b, c)$, or we iterate over at most $N^\epsilon$ $C$-values in $S_l$ for the given $b$ and look up in $T_h$ for each $(c, a)$; (4) For $\delta Q_{sll}$, we iterate over at most $N^\epsilon$ $C$-values in $S_l$ for the given $b$ and then look up in $T_l$ for each $(c, a)$. Then, summing these partial deltas and updating $Q$ take constant time. The views $V_{RS}$ and $V_{TR}$, which facilitate updates to $T$ and respectively to $S$, are maintained for updates to distinct parts of $R$. Computing $\delta V_{RS}$ and updating $V_{RS}$ requires iterating over at most $N^\epsilon$ $C$-values in $S_l$ for the given $b$; similarly, computing $\delta V_{TR}$ and updating $V_{TR}$ involves at most $N^{1-\epsilon}$ heavy $C$-values in $T_h$. The final step of IVM$^*$ updates the (heavy or light) part of $R$ that corresponds to $\delta R_a$ in (amortized) $O(1)$ time. Overall, IVM$^*$ maintains the views from Figure 1 under single-tuple updates to any of the input relations in $O(N^{\max\{\epsilon,1-\epsilon\}})$ time using $O(N^{1+\min\{\epsilon,1-\epsilon\}})$ space.

An insert $(a, b)$ into $R$ may promote $a$ from light to heavy in $R$ or may increase the heavy-light threshold such that some $A$-values change from heavy to light. Without rebalancing the partitions, our assumptions on the number of $B$-values paired with $a$ or the number of heavy $A$-values may become invalid.

IVM$^*$ loosens the partition threshold to amortize the cost of rebalancing over multiple updates. Instead of the actual database size $N$, the threshold now

<table>
<thead>
<tr>
<th>Delta Evaluation Strategy</th>
<th>Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta Q_{shh}(\epsilon) = \delta R_a(a, b) \cdot \sum C T_h(C, a) \cdot S_h(b, C)$</td>
<td>$O(N^{1-\epsilon})$</td>
</tr>
<tr>
<td>$\delta Q_{shl}(\epsilon) = \delta R_a(a, b) \cdot V_{ST}(b, a)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\delta Q_{slh}(\epsilon) = \delta R_a(a, b) \cdot \sum C T_h(C, a) \cdot S_l(b, C)$</td>
<td>$O(N^{\min{\epsilon,1-\epsilon}})$</td>
</tr>
<tr>
<td>$\delta Q_{shl}(\epsilon) = \delta R_a(a, b) \cdot \sum C S_l(b, C) \cdot T_h(C, a)$</td>
<td>$O(N^\epsilon)$</td>
</tr>
<tr>
<td>$\delta Q(\epsilon) = \delta Q_{shh}(\epsilon) + \delta Q_{shl}(\epsilon) + \delta Q_{slh}(\epsilon) + \delta Q_{sll}(\epsilon)$</td>
<td>$O(1)$</td>
</tr>
<tr>
<td>$\delta V_{RS}(a, C) = \delta R_h(a, b) \cdot S_l(b, C)$</td>
<td>$O(N^\epsilon)$</td>
</tr>
<tr>
<td>$\delta V_{TR}(C, b) = \delta R_l(a, b) \cdot T_h(C, a)$</td>
<td>$O(N^{1-\epsilon})$</td>
</tr>
</tbody>
</table>

Fig. 2. Computing the deltas of the views from Figure 1 for an update $\delta R_a(a, b)$ to the heavy or light part of $R$. The symbol $*$ stands for $h$ or $l$. The delta $\delta V_{ST}$ is empty since $V_{ST}$ does not refer to $R$. The evaluation order of deltas is from left to right.
depends on a variable $M$ for which the invariant $\lfloor \frac{1}{4} M \rfloor \leq N < M$ always holds. If the database size violates one of the limits, we perform major rebalancing where we double or halve $M$ to satisfy the invariant again, repartition the input relations using the new threshold $M'$, and recompute the auxiliary views. The time complexity of this operation is $O(M^{1+\min(\epsilon,1-\epsilon)})$, which is amortized over at least $\lceil \frac{1}{4} M \rceil$ updates between two major rebalancing steps.

$\text{IVM}^\epsilon$ also enforces the following two invariants: The number of tuples with the same value of the partitioning attribute is less than $\frac{3}{2}M^\epsilon$ in each light part and at least $\frac{1}{2}M^\epsilon$ in each heavy part. If any of the two invariants is violated, we perform minor rebalancing where we move at most $\lceil \frac{3}{2}M^\epsilon \rceil$ tuples from one part to another and update the affected views. The time complexity of this operation is $O(M^{\epsilon+\max(\epsilon,1-\epsilon)})$, which is amortized over at least $\lceil \frac{1}{2}M^\epsilon \rceil$ updates between two minor rebalancing steps for the same value of the partitioning attribute.

In conclusion, both rebalancing steps together take $O(M^{\max(\epsilon,1-\epsilon)})$ amortized time. Since each single-tuple update can be realized in time $O(M^{\max(\epsilon,1-\epsilon)})$ and $M = O(N)$, $\text{IVM}^\epsilon$ needs $O(N^{\max(\epsilon,1-\epsilon)})$ overall amortized time. The extended version of this work presents a detailed complexity analysis of $\text{IVM}^\epsilon$ [3].

3 Beyond the Triangle Query

$\text{IVM}^\epsilon$ can be applied to any query but may not always yield asymptotic improvements over existing approaches. It can achieve sublinear maintenance for the counting variants of acyclic queries, e.g., 3-path and 4-path, and cyclic queries, e.g., Loomis-Whitney and 4-cycle. Different semirings can be used to specify operations on the payloads [6]; we used here $(\mathbb{Z}, +, *, 0, 1)$ to express counting. An early prototype implementation of $\text{IVM}^\epsilon$ on top of DBToaster [4] shows several factors performance improvement over classical and factorized $\text{IVM}$.

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