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Counting Triangles under Updates

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1 Problem Setting and Contributions

We consider the problem of maintaining the result of the triangle count query $Q() = \Gamma_{\text{sum}} R(A,B) \bowtie S(B,C) \bowtie T(C,A)$ under single-tuple updates to the input relations $R$, $S$, and $T$. The relations are given as key-payload maps whose keys are tuples over relation schemas, payloads are tuple multiplicities, and key lookups are (amortized) $O(1)$-time operations. A single-tuple update $\delta R(a,b) = \{(a,b) \mapsto p\}$ to relation $R$ maps a key $(a,b)$ to a nonzero payload $p$ (positive for inserts and negative for deletes); updates to $S$ and $T$ are analogous.

The naïve maintenance approach recomputes the triangle count from scratch after each update. Computing this query using worst-case optimal join algorithms \cite{6} takes $O(N^{1.5})$ time, where $N$ is the current size of the input database.

To \textit{incrementally} maintain the triangle count under single-tuple updates, existing incremental view maintenance (IVM) approaches need linear time. For instance, under the update $\delta R$ to $R$, the classical IVM \cite{2} computes the delta query $\Gamma_{\text{sum}} \delta R(a,b) \bowtie S(b,C) \bowtie T(C,a)$ in $O(N)$ time because it needs to intersect two lists of possibly linearly many $C$-values that are paired with $b$ in $S$ and with $a$ in $T$. The factorized IVM \cite{6} materializes the view $V_{ST}(B,A) = \Gamma_{B,A,\text{sum}} S(B,C) \bowtie T(C,A)$ using $O(N^2)$ space. It then computes the delta query $\Gamma_{\text{sum}} \delta R(a,b) \bowtie V_{ST}(b,a)$ in $O(1)$ time; however, updates to $S$ and $T$ still require $O(N)$ time to maintain the triangle count $Q$ and view $V_{ST}$.

This raises the question of whether the triangle count can be maintained in sublinear time. Recent work proves that no algorithm can maintain $Q$ in time $O(N^{0.5-\gamma})$ for any $\gamma > 0$, under reasonable complexity-theoretic assumptions \cite{1}. An algorithm with sublinear maintenance time for $Q$ is not yet known.

This work introduces IVM$^\epsilon$, an IVM approach that maintains the triangle count in \textit{amortized sublinear} time. IVM$^\epsilon$ partitions each input relation into two parts, heavy and light, based on the degrees of data values, the database size, and a parameter $\epsilon$. It then adapts the maintenance strategy to different heavy-light combinations of parts of the input relations to achieve worst-case sublinear maintenance. As the database evolves under updates, IVM$^\epsilon$ rebalances the partitions to account for a new database size and updated degrees of data values. While this rebalancing may take superlinear time, it remains sublinear per update.

Given a database of size $N$ and $\epsilon \in [0,1]$, IVM$^\epsilon$ maintains the triangle count in $O(N^{\max\{1,1-\epsilon\}})$ amortized time while using $O(N^{1+\min\{1,1-\epsilon\}})$ space. It thus defines a continuum of approaches exhibiting a space-time tradeoff based on $\epsilon$.

* An extended version of this work is available online \cite{3}.
S

a,b
count query. The size of the view $V$ tion, few exceptions require linear-time maintenance. For these exceptions, IVM

parts:

Q

values in $R$
b,c

Alternatively, one can iterate over all $(b, c)$ pairs in $S_l$ and then find the $A$-values in $R_h$ for each $b$. Since $R_h$ contains only tuples with heavy $A$-values, there are at most $N' = N^{1-\epsilon}$ distinct $A$-values. This gives an upper bound of $O(|S_l| \cdot N^{1-\epsilon}) = O(N^{2-\epsilon})$. The overall space complexity is the minimum of the bounds. The space analysis for $V_{ST}$ and $V_{TR}$ is analogous.

We explain our adaptive strategy on a single-tuple update $\delta R_*(a, b)$ to relation $R$. This update can affect either the heavy or light part of $R$, hence the *
symbol; we assume that checking whether a is heavy or not in R is a constant-
time operation. Updates to the other two relations are handled similarly.

Figure 2 shows the deltas of the views affected by the update \( \delta R_a(a, b) \) to
the heavy or light part of R. The symbol * stands for h or l. The delta \( \delta V_{ST} \) is empty
since \( V_{ST} \) does not refer to R. The evaluation order of deltas is from left to right.

We first analyze the access patterns of the skew-aware delta views: (1) For
\( \delta Q_{shh}(\ ) = \delta R_a(a, b) \cdot \sum_C T_h(C, a) \cdot S_h(b, C) \)
\( \delta Q_{shi}(\ ) = \delta R_a(a, b) \cdot V_{ST}(b, a) \)
\( \delta Q_{shi}(\ ) = \delta R_a(a, b) \cdot \sum_C T_h(C, a) \cdot S_h(b, C) \) or
\( \delta R_a(a, b) \cdot \sum_C S_h(b, C) \cdot T_h(C, a) \)
\( \delta Q(\ ) = \delta Q_{shh}(\ ) + \delta Q_{shi}(\ ) + \delta Q_{shi}(\ ) \)
\( \delta V_{RS}(a, C) = \delta R_h(a, b) \cdot S_l(b, C) \)
\( \delta V_{TR}(C, b) = \delta R_l(a, b) \cdot T_h(C, a) \)

<table>
<thead>
<tr>
<th>Delta Evaluation Strategy</th>
<th>Time Complexity</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta Q_{shh}(\ ) = \delta R_a(a, b) \cdot \sum_C T_h(C, a) \cdot S_h(b, C) )</td>
<td>( O(N^{1-\epsilon}) )</td>
</tr>
<tr>
<td>( \delta Q_{shi}(\ ) = \delta R_a(a, b) \cdot V_{ST}(b, a) )</td>
<td>( O(1) )</td>
</tr>
<tr>
<td>( \delta Q_{shi}(\ ) = \delta R_a(a, b) \cdot \sum_C T_h(C, a) \cdot S_h(b, C) ) or</td>
<td>( O(N^{\min{\epsilon,1-\epsilon}}) )</td>
</tr>
<tr>
<td>( \delta R_a(a, b) \cdot \sum_C S_h(b, C) \cdot T_h(C, a) )</td>
<td>( O(N^\epsilon) )</td>
</tr>
<tr>
<td>( \delta Q(\ ) = \delta Q_{shh}(\ ) + \delta Q_{shi}(\ ) + \delta Q_{shi}(\ ) )</td>
<td>( O(1) )</td>
</tr>
<tr>
<td>( \delta V_{RS}(a, C) = \delta R_h(a, b) \cdot S_l(b, C) )</td>
<td>( O(N^\epsilon) )</td>
</tr>
<tr>
<td>( \delta V_{TR}(C, b) = \delta R_l(a, b) \cdot T_h(C, a) )</td>
<td>( O(N^{1-\epsilon}) )</td>
</tr>
</tbody>
</table>

Fig. 2. Computing the deltas of the views from Figure 1 for an update \( \delta R_a(a, b) \) to
the heavy or light part of R. The symbol * stands for h or l. The delta \( \delta V_{ST} \) is empty
since \( V_{ST} \) does not refer to R. The evaluation order of deltas is from left to right.

An insert \((a, b)\) into R may promote a from light to heavy in R or may
increase the heavy-light threshold such that some A-values change from heavy
to light. Without rebalancing the partitions, our assumptions on the number of
B-values paired with a or the number of heavy A-values may become invalid.

IVM* loosens the partition threshold to amortize the cost of rebalancing
over multiple updates. Instead of the actual database size \( N \), the threshold now
depends on a variable $M$ for which the invariant $\left\lfloor \frac{1}{4}M \right\rfloor \leq N < M$ always holds. If the database size violates one of the limits, we perform major rebalancing where we double or halve $M$ to satisfy the invariant again, repartition the input relations using the new threshold $M'$, and recompute the auxiliary views. The time complexity of this operation is $O(M^{1+\min(\epsilon,1-\epsilon)})$, which is amortized over at least $\left\lceil \frac{1}{4}M \right\rceil$ updates between two major rebalancing steps.

IVM$^\epsilon$ also enforces the following two invariants: The number of tuples with the same value of the partitioning attribute is less than $\frac{3}{2}M^\epsilon$ in each light part and at least $\frac{1}{2}M^\epsilon$ in each heavy part. If any of the two invariants is violated, we perform minor rebalancing where we move at most $\left\lceil \frac{3}{2}M^\epsilon \right\rceil$ tuples from one part to another and update the affected views. The time complexity of this operation is $O(M^{\epsilon+\max(\epsilon,1-\epsilon)})$, which is amortized over at least $\left\lceil \frac{1}{2}M^\epsilon \right\rceil$ updates between two minor rebalancing steps for the same value of the partitioning attribute.

In conclusion, both rebalancing steps together take $O(M^{\max(\epsilon,1-\epsilon)})$ amortized time. Since each single-tuple update can be realized in time $O(M^{\max(\epsilon,1-\epsilon)})$ and $M = O(N)$, IVM$^\epsilon$ needs $O(N^{\max(\epsilon,1-\epsilon)})$ overall amortized time. The extended version of this work presents a detailed complexity analysis of IVM$^\epsilon$ [3].

3 Beyond the Triangle Query

IVM$^\epsilon$ can be applied to any query but may not always yield asymptotic improvements over existing approaches. It can achieve sublinear maintenance for the counting variants of acyclic queries, e.g., 3-path and 4-path, and cyclic queries, e.g., Loomis-Whitney and 4-cycle. Different semirings can be used to specify operations on the payloads [6]; we used here $(\mathbb{Z}, +, \cdot, 0, 1)$ to express counting. An early prototype implementation of IVM$^\epsilon$ on top of DBToaster [4] shows several factors performance improvement over classical and factorized IVM.

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References