Wealth of the Nation:
Scotland’s Productivity Challenge —
Technical Appendix*

Mark Mitchell† Robert Zymek‡

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Abstract

This note provides technical background information on some contents of the report Wealth of the Nation: Scotland’s Productivity Challenge (Kelly et al., 2018). The report shows that Scotland’s productivity performance is only middling in the OECD: Scotland would need to close a productivity gap of 20% to reach the top quartile of most productive OECD countries. It also illustrates that this gap can be attributed to a relatively low capital stock and low Total Factor Productivity (TFP) in equal measure. In this note, we make transparent the data and assumptions underlying these calculations.

JEL Classification codes: E01, F40, O52, R11
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†School of Economics, University of Edinburgh; mark.mitchell@ed.ac.uk.
‡School of Economics, University of Edinburgh and CESifo; robert.zymek@ed.ac.uk.
1 Introduction

This note provides technical background information on some contents of the report *Wealth of the Nation: Scotland’s Productivity Challenge*, published by the David Hume Institute on 6 September 2018 (Kelly et al., 2018). Specifically, Chapters 2 and 3 of the report informally lay out a model of the drivers of Scottish aggregate labour productivity, and place Scotland’s productivity performance in an international context. Here, we describe the underlying model formally. We then show how the model in conjunction with publicly available data on Scotland’s economy can be used to compare Scottish productivity and its components with the performance of other countries.

The workings described in this note rely heavily on the methodology for the analysis of international income differences established in the “development accounting” literature, popularised by Hall and Jones (1999), as well as some data and methods used by the latest edition of the *Penn World Table (PWT)*. Caselli (2005) offers a comprehensive overview of the development-accounting approach to international income differences. A complete description of the contents, sources and methods underlying the PWT 9.0 can be found in Feenstra et al. (2015). In addition to these articles, we use some recent results about development accounting in open economies established by Cuñat and Zymek (2018).

The purpose of the note is to make transparent the data and assumptions underlying the figures in Kelly et al. (2018) which cannot be straightforwardly compiled on the basis of readily available national statistics. We welcome comments from other interested parties on ways to improve on the data and the methods used here, or on their presentation. It is our hope that such discussions will help advance the measurement of economic activity in Scotland, and contribute to an informed debate about Scottish economics.

The remainder of this note is structured as follows. Section 2 describes the formal model underlying our analysis. Section 3 explains how to construct the data needed to operationalise the model for international productivity comparison. Section 4 briefly discussed our findings.

2 Model

2.1 Model Description

2.1.1 Preferences and Endowments

There are \( n = 1, \ldots, N \) countries — each with a representative consumer and each producing a unique, country-specific good. The representative consumer in \( n \) receives
income from capital and labour and maximises final consumption by aggregating goods from different sources with a CES aggregator:

\[ C_n = \left( \sum_{n'=1}^{N} \frac{c_{n'n}}{\theta + \theta n'} \right)^{\frac{1+\theta}{\theta}}, \quad (1) \]

where \( C_n \) is final consumption in \( n \); \( c_{n'n} \) is consumption by the representative consumer in \( n \) of the good produced by country \( n' \); and \( \theta > 0 \). The representative consumer in \( n \) maximises (1) subject to their budget constraint:

\[ \sum_{n'=1}^{N} p_{n'n} c_{n'n} \leq r_n K_n + w_n L_n, \quad (2) \]

where \( p_{n'n} \) is the price of the country-\( n' \) good in \( n \); \( K_n \) is the capital stock in \( n \); \( L_n \) are the number of hours worked by the representative consumer in \( n \) (which are taken as given); \( r_n \) is the rental rate of capital; and \( w_n \) is the hourly wage in \( n \).

### 2.1.2 Technologies

Firms in country \( n \) produce the specific country-\( n \) good using a technology described by the following production function:

\[ Q_n = \left[ \frac{A_n K_n^\alpha (h_n L_n)^{1-\alpha}}{1 - \mu} \right]^{1-\mu} \left[ \frac{1}{\mu} \left( \sum_{n'=1}^{N} \frac{x_{n'n}}{\theta + \theta n'} \right)^{\frac{1+\theta}{\theta}} \right]^{\mu}, \quad (3) \]

where \( Q_n \) is output of the country-\( n \) good; \( h_n \) is workforce skill in \( n \); \( A_n \) is total factor productivity in \( n \); \( x_{n'n} \) is use of the country-\( n' \) good as a production input in \( n \); \( \alpha \in (0,1) \) is the capital share of production; and \( \mu \in (0,1) \) is the intermediate-input share of production. Note that (3) implies that production in country \( n \) uses not only capital and labour but also some of the output of all countries as intermediate inputs.

### 2.1.3 Market structure

All goods and factor markets are assumed to be perfectly competitive. Capital and labour cannot move across borders.

Final consumption cannot be traded, but all country-specific goods can be traded between countries subject to an “iceberg” trade cost: if one unit of good is shipped from \( n' \) to \( n \), only a fraction \( 1/\tau_{n'n} < 1 \) arrives in \( n \). We impose the normalisation \( \tau_{nn} = 1 \) for all \( n \). Therefore, denoting by \( p_{n'} \) the price of the country-\( n' \) good in its home market,

\[ p_{n'n} = \tau_{n'n} p_{n'}. \quad (4) \]
Market clearing requires

\[ Q_n = \sum_{n'=1}^{N} \tau_{nn'} (c_{nn'} + x_{nn'}) , \]  

(5)
i.e. the output of country-\( n \) goods needs to equal the amount of country-\( n \) goods used in final consumption and production by all countries (inclusive of the portion of goods lost in transit).

### 2.2 Model Implications

#### 2.2.1 Expenditure-Side Real GDP

Given the assumptions made in Section 2.1, it is straightforward to show that the “ideal” price index of the representative consumer in country \( n \) is

\[ P_n \equiv \left( \sum_{n'=1}^{N} p_{n'n}^{\theta} \right)^{-1/\theta}. \]  

(6)
The real GDP of country \( n \) at consumer prices can then be written as

\[ Y_n^e \equiv \frac{r_n K_n + w_n L_n}{P_n} = \left( \frac{p_n}{P_n} \right)^{1-\mu} A_n K_n^\alpha (h_n L_n)^{1-\alpha}, \]  

(7)
where the second equality follows from the model’s equilibrium conditions. \( Y_n^e \) is described as “expenditure-side real GDP” in PWT since version 8.0 (Feenstra et al., 2015). It corresponds to the traditional measure of “real income” used in international comparisons: income evaluated at local consumer prices. Differences in expenditure-side real GDP across countries reflect differences in the consumption possibilities afforded by a countries’ GDPs — and are thus closely related to differences in welfare.

#### 2.2.2 Output-Side Real GDP

Since edition 8.0, PWT also features a second real-GDP concept: “output-side” real GDP. It is defined as the revenue function of a country, evaluated at constant prices.

In the model of Section 2.1, the revenue function of country \( n \) can be defined as

\[ R_n (p_n, P_n; K_n, H_n) \equiv \]  

\[ \max_{\{x_{n'n}\}_{n'}} \left\{ \frac{A_n K_n^\alpha (h_n L_n)^{1-\alpha}}{1 - \mu} \right\}^{1-\mu} \left[ \frac{1}{\mu} \left( \sum_{n'=1}^{N} x_{n'n}^{\theta} \right)^{\frac{1+\theta}{\theta}} \right]^{\mu} - \sum_{n'=1}^{N} p_{n'n} x_{n'n}. \]  

(8)
Evaluating (8) at constant prices, we obtain

\[ Y_n^\alpha \equiv R_n (p, P; K_n, H_n) = \pi A_n K_n^\alpha (h_n L_n)^{1-\alpha}, \tag{9} \]

where \( \pi \equiv (p/P_n)^{1/(1-\mu)} \) is a constant. Therefore, differences in output-side real GDP across countries reflect purely reflect differences in countries productive capacity – capital endowments, hours worked, labour productivity and TFP. Output-side GDP does not take into account to what extent prices allow countries’ productive capacity to be turned into consumption possibilities.\(^1\)

### 2.2.3 Productivity

Defining aggregate productivity as expenditure-side real GDP per hour worked (i.e. the real consumption possibilities a country generates per work hour), we obtain

\[ y_e^n \equiv \frac{Y_e^n}{L_n} = k_n^\alpha \times h_n^{1-\alpha} \times \left( \frac{p_n}{P_n} \right)^{\frac{1}{1-\mu}} \times A_n, \tag{10} \]

where \( k_n \equiv K_n/L_n \) is the capital stock per (wo)man hour.

Equation (10) shows that differences in aggregate productivity across countries can be attributed to four drivers: differences in capital-stocks per hour worked \( (k_n) \); differences in workforce skill \( (h_n) \); differences in the output-expenditure price ratio (“terms of trade”, \( p_n/P_n \)); and differences in TFP \( (A_n) \).\(^2\)

Data on \( y_e^n, k_n, h_n, p_n/P_n \) and \( A_n \) across countries can be obtained from PWT 9.0. However, the United Kingdom is only reported as a single political entity in this database. In the following, we describe how we construct data on these variables for Scotland.

\(^1\)See Cuñat and Zymek (2018) for more details on the comparison between expenditure-side and output-side real GDP in standard open-economy models.

\(^2\)Note that, if there are prohibitive barriers to international trade,

\[
\frac{p_n}{P_n} = \frac{p_n}{\sum_{n'=1}^N \left( \tau_{n'n} p_{n'} \right)^{-\theta}} \xrightarrow{\tau_{n'n} \to \infty} 1, \forall n' \neq n.
\]

Traditionally, development accounting has been performed under the assumption that countries can be represented as closed economies. As the above shows, there are only three drivers of productivity in equation (10) if we assume that countries are closed. (Caselli, 2005) However, recent research shows that accounting for variation in the “terms of trade” across countries is important for understanding measured international productivity differences. (Feenstra et al. 2015; Cuñat and Zymek, 2018)
3 Data

3.1 Productivity

Productivity as defined above — that is, expenditure-side real GDP per hour, \( y_n^e \) — can be calculated easily for all OECD countries, including the UK but not Scotland, using the data in PWT 9.0: \( y_n^e = \frac{cgdpe}{h} \). Note that the variable \( cgdpe \) is reported in current purchasing-power adjusted dollars in PWT.

For Scotland, we take GDP statistics in current GBP from the Scottish government website. We convert them into constant 2011 dollars by dividing by the UK GDP price level at 2011 base from the PWT National Accounts supplement (\( vgdp/qgdp \)) and multiplying with the 2011 dollar-Sterling exchange rate from PWT (\( 1/xr \)). We then turn them into current purchasing-power adjusted dollars by multiplying with the PPP-per-2011-national-price ratio for the UK from PWT (\( cgdpe/rgdpe \)).

This allows us the present comparable productivity statistics for OECD countries, including the UK and Scotland, for the year 2014 — the latest year for which data is available from PWT 9.0. These statistics are shown in Figure 1 of Kelly et al. (2018), for convenience reprinted here as Figure A1. In the figure, all productivity numbers

\[ \text{Figure A1: OECD Productivity, 2014} \]

Note: Productivity is measured as expenditure-side real GDP per hour. Sources: PWT 9.0, Scottish Government, and own calculations.

An assumption implicit in these calculations is that price indices calculated for the UK as a whole are a reasonably close approximation for Scottish price indices. This assumption is would be violated if Scottish consumers purchase very different products from the UK average, or if the price of the same products were very different in Scotland from the rest of the UK. We believe any departures of Scottish price indices from their UK-wide counterparts arising from such differences are likely to be sufficiently minor as not to bias our productivity numbers significantly.

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are reported with Denmark as the base country (=100). We chose Denmark as the base because it is exactly at the border to the top quartile of the OECD productivity distribution, and its population size is similar to Scotland’s.

The figure shows that Scotland sits around the productivity median in the OECD. It is more productive than many of the OECD’s poorer member countries, but slightly less productive than the UK as a whole. Moreover, Scotland’s productivity is 20% lower than Denmark’s − implying a sizeable gap between Scottish productivity and the productivities found in the top quartile of OECD economies. Scotland is also 4% less productive than Finland and 13% less productive than Austria, both of which are relatively small OECD countries by population size.

3.2 Capital Stocks

Capital stocks per hour worked for OECD economies are readily available from PWT 9.0: \( k_n = ck/h \).

For Scotland, we take the annual time series of aggregate gross fixed capital formation in current GBP from the Scottish government website. We convert it into constant 2011 dollars by dividing by the UK investment price level at 2011 base from the PWT National Accounts supplement (\( vgfcf/qgfcf \)) and multiplying with the 2011 dollar-Sterling exchange rate from PWT (\( 1/xr \)). This gives us a time series of real aggregate investment in Scotland (in constant 2011 USD).

We now compute the Scottish capital stock using the perpetual inventory method, following Caselli (2005)\(^4\)

\[ K_{nt} = I_{nt} + (1 - \delta_{nt}) K_{nt-1}^c, \]  

where \( K_{nt}^c \) is the capital stock of a country \( n \) in year \( t \) in constant dollars; \( I_{nt} \) is aggregate real investment; and \( \delta_{nt} \in (0, 1) \) is the depreciation rate of capital. Given an appropriate calibration of \( \delta_{nt} \), the size of the capital stock in some initial year 0, and a time series of real aggregate investment for \( t = 1, ..., T \), the formula in (11) can be used to calculate the capital stock in \( n \) for all \( t = 1, ..., T \).

We set \( \delta_{nt} \) to match the UK capital depreciation rate reported in PWT 9.0 (\texttt{delta})\(^5\). The Scottish annual time series of aggregate gross fixed capital formation only starts in 1998, which makes this our “year zero.” As is standard in applications of the perpetual inventory method, we impose that

\[ K_{n0}^c = \frac{I_{n0}}{\delta_n + \gamma_n}, \]  

\(^4\)This approach has a considerable pedigree in the international-comparisons literature. See Caselli (2005) for a discussion of the underlying assumptions, and their limitations.

\(^5\)Again, we are assuming here that the UK’s average capital depreciation rate is not too different from Scotland’s.
where $\delta_n$ is the average depreciation rate in country $n$; and $\gamma_n$ is the average growth rate of real investment. Equation (12) represents the relationship between capital stock and investment which we would expect to prevail in steady state, given (11). For the UK, and by our assumption for Scotland, the average depreciation rate was .04 in the 1998-2014 period. The average growth rate of real investment for Scotland was .02. This gives us enough information to compute an annual time series of constant-dollar capital stocks for Scotland from 1998 to 2014. For consistency with PWT, we convert it into current purchasing-power adjusted dollars by multiplying with the capital-PPP-per-2011-national-price ratio for the UK from PWT ($ck/rkna$).

![Figure A2: Scottish Capital Stock, 1998-2014](image)

Note: Capital stock/GDP is $K_n/Y_n^e$. $K_n, Y_n^e$ for Scotland are calculated as described in the text. Sources: PWT 9.0, Scottish Government, and own calculations.

Figure A2 plots the capital/GDP ratio for Scotland which arises from our computations. The figure shows that this ratio has increased from a value of around 2 in 1998 to a value of around 3.5 by 2014. There is also some sense that the rise in capital/GDP ratio has levelled off since 2010. The numbers we obtain pass a basic plausibility test. Feenstra et al. (2015, Online Appendix) show for a sample of 143 countries in 2005 that the median value of capital/GDP is 2.7, corresponding to the dashed line in the figure. They find that the large majority of capital/GDP ratios are in the range 2-4. The capital/GDP ratio for Scotland which we have calculated is thus in the same ballpark as the corresponding ratios of many other countries.

### 3.3 Workforce Skill

For OECD economies, PWT 9.0 provides the standard index of workforce skill (“human capital”) in international comparisons: $h_n = hc$. 
This index is calculated for each country based on the average number of years of formal schooling in the adult population. Letting $s_n$ denote the average years of schooling in country $n$,

$$
\ln h_n = \begin{cases} 
.134 \times s_n & \text{if } s_n \leq 4 \\
.101 \times (s_n - 4) + .134 \times 4 & \text{if } 4 < s_n \leq 8 \\
.068 \times (s_n - 8) + .101 \times 4 + .134 \times 4 & \text{if } 8 < s_n 
\end{cases}
$$

(13)

The form of equation (13) reflects evidence about the economic returns to schooling from Mincer wage regressions (see Hall and Jones, 1999; Caselli, 2005 for details). To construct the corresponding value of the index for Scotland, we require information on the average years of schooling in the Scottish adult population.

PWT relies on schooling data from Barro and Lee (2013) and Cohen and Leker (2014). These datasets contain information on the average years of schooling in the adult population for the UK as a whole, but not for Scotland. The UK years of schooling are reported in column 1 of Table A1 for the years 2000-2014.

<table>
<thead>
<tr>
<th>Year</th>
<th>UK</th>
<th>Scotland</th>
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</thead>
<tbody>
<tr>
<td>2000</td>
<td>12.6</td>
<td>12.9</td>
</tr>
<tr>
<td>2001</td>
<td>12.7</td>
<td>13.0</td>
</tr>
<tr>
<td>2002</td>
<td>12.8</td>
<td>13.1</td>
</tr>
<tr>
<td>2003</td>
<td>12.9</td>
<td>13.1</td>
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<tr>
<td>2004</td>
<td>13.0</td>
<td>13.2</td>
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<tr>
<td>2007</td>
<td>13.2</td>
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<td>2008</td>
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<td>2010</td>
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<tr>
<td>2011</td>
<td>13.5</td>
<td>13.6</td>
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<td>2013</td>
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</tr>
<tr>
<td>2014</td>
<td>13.6</td>
<td>13.8</td>
</tr>
</tbody>
</table>

Table A1: Average Years of Schooling in the Adult Population, 2000-2014

Sources: PWT 9.0, EUROSTAT, and own calculations (see text).

We were unable to find data on the average years of schooling in the Scottish adult population. However, EUROSTAT reports data on the percentage of the UK and
Scottish populations with “less than primary, primary and lower secondary education (levels 0-2)”; with “upper secondary and post-secondary non-tertiary education (levels 3-4)”; and with “tertiary education (levels 5-8)”. Assuming that, in a UK context, the average person in the first category has had 10 years of formal schooling; the average person in the second had 13 years of schooling; and the average person in the third had 16 years of schooling, we arrive at the numbers reported in columns 2 and 3 of Table A1.

As the table shows the average number of years of schooling in the UK adult population based on EUROSTAT data and our assumption is virtually identical to the average number reported in the Labor supplement of PWT 9.0 for the 2000-2014 period. This suggests that our assumption about the average years of schooling in the different education categories is reasonable. Since Scotland has a slightly higher population share with education levels 3-4 and 5-8, we find that Scotland’s average years of schooling are slightly higher than those for the UK population as a whole. Correspondingly, after applying the formula in (13), we obtain a slightly higher index of workforce skill for Scotland than PWT 9.0 reports for the UK as a whole.

### 3.4 Terms of Trade

From equations (7) and (9), it is clear that the “terms of trade” term in the productivity accounting equation (10) can be thought of equivalently as the ratio of expenditure-side and output-side real GDP:

\[
\frac{Y^e_n}{Y^o_n} = \frac{1}{\pi} \left( \frac{p_n}{P_n} \right)^{\frac{1}{1-\pi}}.
\] (14)

That ratio is available in PWT 9.0 for all OECD countries: \( \frac{\text{cgdpe}}{\text{cgdpo}} \). Variations of this ratio across countries reflects variations in the ratio of the expenditure-side and output-side PPP price indices used in PWT. The expenditure-side PPP is the traditional measure of differences in purchasing power of a given nominal income across countries. The output-side PPP deflates nominal incomes in a manner designed to better reflect the productive capacity of countries. PWT constructs it from the expenditure-side PPP by subtracting countries’ weighted import prices and adding their weighted export prices (see Feenstra et al., 2015).

Constructing an output-side PPP for Scotland would be a difficult and data-intensive exercise. Therefore, we avail ourselves of a short cut. The equilibrium conditions of the model described in Section 2.1 imply that

\[
m_{nn} = \left( \frac{p_n}{P_n} \right)^{-\theta},
\] (15)

\(^{6}\)“Levels” here correspond to the International Standard Classification of Education (ISCED).
where $m_{nn}$ is the share of country-$n$ expenditure on its own goods. Combining (14) and (15), we obtain
\[
\ln \frac{Y_{ne}}{Y_{no}} = - \ln \pi - \frac{1}{\theta (1 - \mu)} \ln m_{nn}.
\]
(16)

For 43 countries, including the UK, the share of spending on non-imported output for 2014 can be obtained from the World Input Output Database (2016 release; see Timmer at al., 2015). We plot their expenditure-side/output-side real GDP ratio from PWT 9.0 against this share in Figure A3.

As can be seen from the figure, there is a negative relationship between the two variables – just as equation (16) would suggest. Moreover, this relationship is very close for most countries: the $R^2$ of the line of best fit (shown as a red dashed line in the figure) is .50. It has a constant equal to -.06, and a coefficient equal to -.39. Assuming that departures from this line largely reflect measurement error, we can infer that $\pi = 1.06$ and $\theta (1 - \mu) = 2.56$ would represent a reasonable calibration of our model. Combining this with the 2014 share of spending on non-imported output for Scotland – which can be calculated from the Scottish government’s input-output table –, we obtain a “terms of trade” term for the Scottish economy.

Figure A3: Expenditure/Output Real GDP and Own Spending Share, 2014
Sources: PWT 9.0, WIOD (2016 release), and own calculations.

There are two possible objections to our short-cut approach to calculating Scotland’s terms of trade. The first is that equations (14) and (15) are derived from a model of international trade based on very specific, and restrictive assumptions. However, it is by now well-known that isomorphic expressions can be derived from a large class of standard quantitative trade models, based on a
3.5 Capital Share

In line with equation (10), we impose a single capital share in production ($\alpha$) for all countries. This capital share is based on the output-side-GDP-weighted ($cgdpo$) capital share for OECD countries reported in PWT 9.0 ($1 - 1abh$). The value we obtain is $\alpha = .41$.

We impose a single capital share in production for expositional simplicity: it allows us to perform the productivity decomposition in Section 4 in a fairly straightforward manner. Country-specific capital shares would complicate the decomposition, and may require a richer and more intricate description of countries’ production technologies than the one we have adopted here. Nevertheless, we should note that imposing a common capital share across countries is not necessarily realistic. PWT data indicates that even among OECD countries the capital share may vary. Whether such observed variation reflects genuine differences in the average production technologies countries employ, or measurement difficulties, remains an open question. We cannot satisfactorily resolve this issue here, but hope that future research on productivity will return to it.

3.6 Total Factor Productivity

Section 3.1-3.5 describe in detail how to compute aggregate productivity for all OECD economies and Scotland, as well as the contribution of capital stocks, workforce skill and the terms of trade. Based on this data, TFP can now be calculated as the residual component of productivity, which accounts for all observed variation that cannot be attributed to the other three drivers. This is the standard definition of TFP in international income and productivity comparisons: it captures all differences between countries which contribute to differences in productivity but cannot be quantified directly.

4 Findings

Figure 4 in Kelly et al. (2018) uses the data assembled in Section 3 to decompose Scotland’s productivity performance into four drivers of productivity introduced in range of different microfoundations. (Arkolakis et al., 2012) The second possible objection is that the model equations we use are expressed in terms of CES ideal price indices – while the PPPs from the PWT, which we are trying to predict, are not. These PPPs are calculated using the Fisher-Geary-Khamis method, which only approximates ideal price indices. Yet Cuñat and Zymek (2018) show that the approximation is very close in the present context.

In particular, our assumption of Cobb-Douglas technologies imposes a unitary substitution elasticity between capital and labour. One reason for differences in the capital share across countries may be that the elasticity of substitution is, in fact, greater than one.

The PWT capital share equals one minus the measured labour share. See Elsby et al. (2013) for a discussion of the inherent difficulty in comparing labour shares across space and time.
Section 2 – capital stocks, workforce skill, terms of trade, and TFP. For convenience, the figure is reprinted as Figure A4 below. As in Figure A1, each country’s standing with respect to a particular variable is expressed relative to the base country Denmark (=100).

The first notable feature of Figure A4 is that, for the most part, OECD economies differ little in their workforce skill (top left panel) and their terms of trade (bottom left panel). On both indicators, Scotland’s performance is relatively favourable, but since across-OECD differences are small, this has little relevance for Scotland’s overall productivity performance in an OECD context.

![Workforce Skill](image1)

![Capital Stock](image2)

![Terms of Trade](image3)

![Total Factor Productivity](image4)

**Figure A4: Drivers of OECD Productivity, 2014**

Note: Each driver corresponds to one term in equation (10). By definition, the product of terms corresponds to the productivity levels shown in Figure A1. Sources: For sources and underlying calculations, see Section 3.

The second notable feature is that Scotland sits in the bottom half of OECD countries in terms of its capital stock (top right panel) and TFP (bottom right panel). These two drivers alone account for the full productivity gap between Scotland and Denmark, with the capital stock and TFP each responsible for roughly half of it.

As discussed in Kelly et al. (2018), both business and government investment has been consistently lower in Scotland than in the rest of the OECD – which helps explain Scotland’s relatively low capital stock per work hour. By definition, the factors behind Scotland’s low TFP are less tangible. Kelly et al. (2018) point towards the business environment, the size and quality of firms, management quality and population demographics as possible contributors to low TFP.

One goal of the development accounting literature has been to reduce the reliance on indirectly observed TFP in explaining income and productivity differences. It has
attempted to do so by introducing more complex models of the drivers of productivity, and by improving the measurement of these drivers. (Caselli, 2005) Here, we have laid out a relatively simply and conventional model of the drivers of productivity, and we have attempted to give a complete account of our best attempt to quantify the role of these drivers for Scotland. Yet it is possible that the model we have used is too simple, and likely that our data compilation leaves room for improvement — and that this explains the large part of Scotland’s productivity under-performance relative to other OECD countries which our exercise attributes to TFP. We hope that a lively discussion of the sources and methods we have employed, and future work in this area, will improve our ability to identify the causes of low productivity — in Scotland, and beyond.
References


