Consistent robustness analysis (CRA) identifies biologically relevant properties of regulatory network models

Citation for published version:

Digital Object Identifier (DOI):
10.1371/journal.pone.0015589

Link:
Link to publication record in Edinburgh Research Explorer

Document Version:
Publisher's PDF, also known as Version of record

Published In:
PLoS One

Publisher Rights Statement:
This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

General rights
Copyright for the publications made accessible via the Edinburgh Research Explorer is retained by the author(s) and / or other copyright owners and it is a condition of accessing these publications that users recognise and abide by the legal requirements associated with these rights.

Take down policy
The University of Edinburgh has made every reasonable effort to ensure that Edinburgh Research Explorer content complies with UK legislation. If you believe that the public display of this file breaches copyright please contact openaccess@ed.ac.uk providing details, and we will remove access to the work immediately and investigate your claim.

Download date: 22. Dec. 2018
Consistent Robustness Analysis (CRA) Identifies Biologically Relevant Properties of Regulatory Network Models

Treenut Saithong¹,⁴, Kevin J. Painter²,³, Andrew J. Millar¹,³*¹

¹ Department of Biological Sciences, Institute of Molecular Plant Sciences, University of Edinburgh, Edinburgh, United Kingdom, ² Department of Mathematics and Maxwell Institute for Mathematical Sciences, School of Mathematical and Computer Sciences, Heriot-Watt University, Edinburgh, United Kingdom, ³ Centre for Systems Biology at Edinburgh, University of Edinburgh, Edinburgh, United Kingdom, ⁴ School of Bioresources and Technology, King Mongkut’s University of Technology Thonburi, Bangkok, Thailand

Abstract

Background: A number of studies have previously demonstrated that “goodness of fit” is insufficient in reliably classifying the credibility of a biological model. Robustness and/or sensitivity analysis is commonly employed as a secondary method for evaluating the suitability of a particular model. The results of such analyses invariably depend on the particular parameter set tested, yet many parameter values for biological models are uncertain.

Results: Here, we propose a novel robustness analysis that aims to determine the “common robustness” of the model with multiple, biologically plausible parameter sets, rather than the local robustness for a particular parameter set. Our method is applied to two published models of the Arabidopsis circadian clock (the one-loop [1] and two-loop [2] models). The results reinforce current findings suggesting the greater reliability of the two-loop model and pinpoint the crucial role of TOC1 in the circadian network.

Conclusions: Consistent Robustness Analysis can indicate both the relative plausibility of different models and also the critical components and processes controlling each model.

Introduction

Mathematical modelling has established itself as a complementary means to study the complexity of biological systems. Through its capacity to integrate extensive data from diverse sources [3-5], modelling has contributed greatly to our understanding of the mechanisms governing organismal behaviour [1,2,6–10], as exemplified by the JWS online (http://jjj.biochem.sun.ac.za/) [11] and BioModels (http://www.ebi.ac.uk/biomodels-main/) [12] databases.

The fitting of models to data necessitates the determination of parameters describing processes of the biological system [13–15]. However, parameters obtained through experimental measurement are condition-dependent, while the measuring process itself is costly with respect to technique, expense, and time. Optimisation provides an alternative and increasingly popular method to estimate the model parameters [16]. Implementing the optimisation requires an appropriate measure to compare the experimental data with simulated results and the first test of a model’s suitability lies in its capacity to “fit” the biological data. However, a considerable drawback in using optimisation to estimate parameters for complex models is that multiple parameter sets may “fit” the data equally [1,17].

An analysis of the robustness of the system is the logical next step to address the uncertainties arising from considering only “goodness of fit”. While the notion of model robustness is interpreted broadly in the literature, the robustness of a biological system is mainly defined as a property of a biological function [15,18]. Measurement of the robustness of a biological system therefore relates to the determination of the effect of certain perturbations on the biological function. In this context, the biological function is inferred by “the behaviour of a dynamical system”—such as a gene expression waveform or the period of a sustained oscillation. These behaviours could be among the targets used in the optimisation process. Hence, the reference to model robustness here is specifically defined as the persistence of the data equally [1,17].

A number of studies have previously demonstrated that “goodness of fit” is insufficient in reliably classifying the credibility of a biological model. Robustness and/or sensitivity analysis is commonly employed as a secondary method for evaluating the suitability of a particular model. The results of such analyses invariably depend on the particular parameter set tested, yet many parameter values for biological models are uncertain.

Results: Here, we propose a novel robustness analysis that aims to determine the “common robustness” of the model with multiple, biologically plausible parameter sets, rather than the local robustness for a particular parameter set. Our method is applied to two published models of the Arabidopsis circadian clock (the one-loop [1] and two-loop [2] models). The results reinforce current findings suggesting the greater reliability of the two-loop model and pinpoint the crucial role of TOC1 in the circadian network.

Conclusions: Consistent Robustness Analysis can indicate both the relative plausibility of different models and also the critical components and processes controlling each model.

Abstract

Background: A number of studies have previously demonstrated that “goodness of fit” is insufficient in reliably classifying the credibility of a biological model. Robustness and/or sensitivity analysis is commonly employed as a secondary method for evaluating the suitability of a particular model. The results of such analyses invariably depend on the particular parameter set tested, yet many parameter values for biological models are uncertain.

Results: Here, we propose a novel robustness analysis that aims to determine the “common robustness” of the model with multiple, biologically plausible parameter sets, rather than the local robustness for a particular parameter set. Our method is applied to two published models of the Arabidopsis circadian clock (the one-loop [1] and two-loop [2] models). The results reinforce current findings suggesting the greater reliability of the two-loop model and pinpoint the crucial role of TOC1 in the circadian network.

Conclusions: Consistent Robustness Analysis can indicate both the relative plausibility of different models and also the critical components and processes controlling each model.


Copyright: © 2010 Saithong et al. This is an open-access article distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original author and source are credited.

Funding: The Centre for Systems Biology at Edinburgh is a Centre for Integrative Systems Biology supported by BB/SRC and EPSRC through award BB/D019621. TS was supported by the postgraduate studentship from Higher Educational Strategic Scholarships for Frontier Research from the Royal Thai Government. The funders had no role in study design, data collection and analysis, decision to publish, or preparation of the manuscript.

Competing Interests: The authors have declared that no competing interests exist.

* E-mail: andrew.millar@ed.ac.uk
model or pinpointing the weaknesses of a failed model. Bifurcation analysis applied to two published models for the Xenopus cell cycle oscillator [20,21] indicated that the later model is more robust, thus cementing its position as the more realistic model than based on biological evidence alone. In a similar manner, Zeilinger et al (2006) [17] demonstrated that three distinct models for the Arabidopsis circadian clock could be distinguished through robustness analysis.

Robustness/sensitivity analysis can also be used to pinpoint the specific factors or processes affecting a system, indicating how the system maintains functionality in spite of internal or environmental perturbations [22,23]. Furthermore, robustness analysis reveals insight into the importance of model parameters on the model behaviours [24]. A variety of techniques have been developed to determine the robustness of a system, for example bifurcation analysis [25–27], control analysis (CA) [28–31] and Infinitesimal Response Curve (IRC) [32]. To summarise such analyses and compare across the systems, Kitano (2007) [33] proposed a method to quantify the robustness through a single factor. The above methods reveal different insights into the robustness of a system, for example bifurcation analysis [25–27], control analysis (CA) [28–31] and Infinitesimal Response Curve (IRC) [32]. To summarise such analyses and compare across the systems, Kitano (2007) [33] proposed a method to quantify the robustness through a single factor. The above methods reveal different insights into the robustness of distinct system properties, for example bifurcation analysis can determine the exact space of the parameters giving desired system performance (e.g. periodic solution for oscillator) [25–27], while CA and IRC can quantify the dynamic changes of the system in applied differentiated perturbations [9,34–36]. Although CA and IRC provide precise analytical measurements, these methods evaluate the robustness around a fixed point in parameter space and the subsequent results are therefore potentially biased to a specific parameter set. The inherent impact of parameters to model robustness is hard to separate [13–15] and it becomes exaggerated in mechanistic modelling, where the focus is on correct interactions rather than the used parameters.

The circadian clock is a fundamental biological process of organisms ranging from unicellular (e.g. Synechococcus cyanobacterium) to multi-cellular [37–39]. Its network is believed to be composed of a negative feedback loop structure which generates a robust 24h-period oscillation. While the molecular mechanism of the circadian clock has been extensively studied in the cyanobacterium [37,38,40], fungi [Neurospora crassa] [37,38,41,42], insects (Drosophila melanogaster) [37,38,43] and mouse [37,39], for plants (Arabidopsis thaliana) the network has recently been established [38,44–46]. A series of Arabidopsis circadian clock models were constructed following the proposal of its molecular network. Locke et al (2005) [1] created an initial “one-loop” model based on the hypothesis of Alabadi et al (2001) [44], which proposed a negative feedback loop of three genes (Figure S1a); two redundant gene encoding MYB transcription factors, LATE ELONGATED HYPOCOTYL (LHY) and CIRCADIAN CLOCK ASSOCIATED 1 (CCA1), and a gene encoding the pseudo-response regulator protein, TIMING OF CAB EXPRESSION 1 (TOCI). A system of seven ordinary differential equations (ODEs) containing 25 parameters was proposed to describe the regulation of the circadian clock for the one-loop model. While the simulated results of this model fitted experimental data from the wild-type (see Figure S1a), it failed to match mutant data, for example short period oscillations observed in the hygr1 hygr2 double mutant plant [1,2]. To match these data, Locke et al (2005b) [2] derived a second model (the two-loop model) through addition of hypothetical genes ‘X’ and ‘Y’. The hypothetical gene ‘X’ was added to extend the time-delay in the model and incorporate an indirect activation of LHY/CCA1 by TOCI [whose mechanism is unclear] [44,47]. An additional loop connects with TOCI in the original loop in an interlocking fashion as illustrated in Figure S1b. The extensions resulted in a system of 13 ODEs and 58 parameters. Simulations of the two-loop model match additional experimental data, including the hygr1 hygr2 double mutant.

Parameter optimisation to fit such data can reveal multiple parameter sets spanning large tracts of parameter space. Until the parameters are measured experimentally, it is desirable to determine the sensitivity/robustness of a model circuit independent of the chosen parameter set and here we propose a strategy that determines this intrinsic robustness of a model. The method is applied to the one- and two-loop models for the Arabidopsis circadian clock, where we take advantage of the previously globally-optimised parameter sets produced by Locke et al (unpublished data) as an initial input for the method. We demonstrate that robustness corroborates the perceived greater credibility of the two-loop model, which is more robust, as well as matching more data than the one-loop model. Our analysis leads to biological inference on the core processes governing this network.

Results

Analysis of Arabidopsis circadian clock models

The proposed method, Consistent Robustness Analysis (CRA), was applied to analyse two published models of the Arabidopsis circadian clock. The circadian clock in Arabidopsis is appropriate for a number of reasons. Firstly, circadian clocks are believed to be highly robust in comparison to other cellular processes (for example, calcium or glycolytic oscillations) [48]. Secondly, previous studies have demonstrated that the one-loop model failed to capture a critical behaviour of the Arabidopsis circadian clock that was replicated in the two-loop model. A critical test of the procedure introduced here is to determine whether it can extend understanding beyond the better fit of the two-loop model. We describe the analysis of the two-loop model in detail, summarise the main results from a similar analysis of the one loop model and interpret the results biologically.

1. Robustness analysis of the two-loop Arabidopsis circadian clock model

(1) Reference parameter set selection. The input to our analysis exploits earlier work by Locke et al (unpublished data), in which 50 low cost-of-fit parameter sets were generated following global optimisation to the semi-quantitative cost function (see [1] for details). One of these, set 0, was described previously [2]. A set of reference parameter sets was selected as described in Methods (see also Figure 1a). Hierarchical Clustering (HCL) and Principal Component Analysis (PCA) were employed to measure the distances between the parameter sets. Figure S2a plots the HCL results for the full 50 parameter sets and the asterisks mark those selected for the second phase of the analysis. We note that, among the seven selected parameter sets, four (sets 9, 12, 13, and 14) are distant from set 0, while the others (sets 27 and 39) are located close to set 0. Re-optimisation of each of the selected parameter sets to the fully-quantitative cost function (fitting to data) was performed through a simulated annealing algorithm (5000 steps) to yield the locally optimised parameter sets, termed L0, L9, L12, L13, L14, L27 and L39, to be used in the later sensitivity analysis. The simulations given from L0 match the data very well (Figure S1b) compared to those from the rest whose simulated oscillations showed low amplitude (L12 and L13; Figure S3), abnormal shape (L12 and L27; Figure S3), and short period in continuous darkness (all of parameter sets; Figure S3). The unequal fit quality of the selected parameter sets indicated that the re-optimisation to quantitative cost function is required to refine the initial results obtained from exhaustive search across the semi-quantitative
criteria of Locke et al (unpublished data). To illustrate the span of the selected parameter sets used in the analysis, the re-optimised parameter values are plotted in Figure S2b.

(2) One-dimensional sensitivity analysis. One-dimensional analysis was performed through six-fold (plus and minus) perturbations of each parameter in each of the reference sets. Figure 2a displays the sensitivity for the two-loop model for $L_0$: results for each of the other parameter sets can be found in Figure S4. Sensitivity/robustness is determined first by calculating the change to the cost-of-fit due to each parameter perturbation and then normalising within the parameter set according to Equation (2). The sensitivity coefficients are determined from Equations (3-4) and summarised in Figure 2b (scales from highest in white to lowest in black). The normalisation is required because the unperturbed cost-of-fit and the maximum perturbed cost-of-fit are different for each parameter set. This means that absolute values for sensitivity coefficients can only be compared within a column (i.e. across the parameters within a particular parameter set). Nevertheless, there are similar trends with respect to the sensitivity of a particular parameter across the different sets. Distinct sets of sensitive parameters are calculated for each parameter set (according to the classification criterion in Equation (5) with $m = 1$) and listed in Table 1. Between five and thirteen sensitive parameters were determined for each set, resulting in a pool of 27/58 parameters being identified as sensitive at least once (Table 1). The discrepancy in which parameters are identified as sensitive for the various parameter sets highlights the fact that model sensitivity depends on the parameter set as well as the network circuit. On the other hand, certain parameters were repeatedly classified as sensitive across the diverse parameter sets. The frequency with which the parameters are identified as sensitive is tabulated in Figure 3 (see also Figure S5) and, stipulating that a particular parameter must be identified as sensitive in at least 50% of sets, we determine the eight “consistently-sensitive” parameters listed in the final column of Table 1. These eight consistently sensitive parameters are $P_2$ ($n_1$: max. light-dependent $LHY$ transcription), $P_{13}$ ($n_2$: max. $TOC1$ transcription rate), $P_{15}$ ($n_3$: constant of $LHY$ inhibiting $TOC1$ transcription), $P_{16}$ ($g_3$: constant for $TOC1$ transcription), $P_{40}$ ($n_6$: constant for $Y$ transcription), $P_{42}$ ($m_{12}$: max. degradation rate of $Y$ mRNA), $P_{52}$ ($g_6$: constant for $Y$ transcription), and $P_{54}$ ($b$: Hill coefficient for $TOC1$ transcription) (see also in Data S1). Since the classification of a sensitive parameters is subject to the strictness of the classification criterion ($m$ in Equation (5); see also Figures S5-S6), we varied $m$ to test the appropriateness of the employed value. For $m = 0.5$ (Figure S6a) or $m = 2$ (Figure S6b), we obtained respectively too many and too few sensitive parameters, suggesting that the applied criterion of $m = 1$ is relatively more sensible.

Figure 1. Consistent robustness analysis scheme. (a) Schematic demonstrating the proposed consistent robustness analysis method which aims to acquire the universal robustness property of a model producing similar results in wide regions of reasonable parameter space, illustrated in (b). doi:10.1371/journal.pone.0015589.g001
The sensitivity analysis pinpoints the influence of specific molecular processes, entities or parts of the genetic network: 7/8 of the consistently sensitive parameters describe transcription processes, 4/8 relate to the TOC1 gene while 7/8 relate to evening-phase genes, TOC1 and Y. Ranking the sensitivity coefficients, the most sensitive parameters correspond to TOC1 transcription.

Further insight arises through the distribution of the parameter sensitivities. The distribution curves for sensitivity coefficient ($S_{size}$) are plotted for the individual reference sets and presented in Figure 4. Comparing with Figure S2a, parameter sets located close to each other in parameter space show similar parameter sensitivity distributions, e.g. L0, L27, and L39 show a comparable pattern of the distribution curve (left-skewed with a small divided peak) while others demonstrate a seemingly random shape. The distribution of parameter sensitivity probably therefore reflects the relative position of the parameter set in parameter space. In the reverse direction, the distribution of parameter sensitivity for a new parameter set might be predicted from knowing its relative location within the parameter space.

(3) Two-dimensional sensitivity analysis. Greater insight into the robustness of the parameter space is obtained through two-dimensional sensitivity analysis. The long numerical time required to perturb across two dimensions in parameter space prevents an exhaustive analysis: the focus here is therefore on the most sensitive parameters as revealed in Step 2 (Figure 1). Insensitive parameters are expected to give rise to flat and smooth distributions (for example, see Figure S5). The

Figure 2. Sensitivity of the two-loop model of Arabidopsis circadian clock. The (a) sensitivity with respect to the parameters in the two-loop Arabidopsis circadian clock model using L0 (the reoptimised parameter set from set 0). The heatmap plots the sensitivity (white = sensitive, black = robust) of the model at all parameters (rows) and perturbations (columns). Similar plots for other reference parameter set shown in Figure 5a for sensitivity. (b) The sensitivity coefficients ($S_{size}$ - left panel and $S_{Schoppy}$ – right panel) of the two-loop model for all reference parameter sets are plotted as a heatmap in which high sensitivity is shown in white, scaling to low sensitivities in black. The sensitivity coefficients of a parameter (row) in each reference parameter set (column) were independently determined from the cost function normalised within the reference set. Note that $S_{size}$ and $S_{Schoppy}$ (Figure S5) are broadly consistent, indicating that either method is reasonable. doi:10.1371/journal.pone.0015589.g002
characteristics of the parameter surface can be inferred through limited investigations in “meaningful areas” of the most sensitive parameter space. A pair of highly sensitive parameters within a set were chosen and perturbed pair-wise. Examples of the parameter space surface of set L0 are plotted in Figures 5a–5e: the surface in highly sensitive regions is coarse, with a deep hole corresponding to where the optimal solution (red star) lies. Similar results for the other six reference sets are illustrated in Figure S7.

The parameter surface is an atlas of model sensitivity on the parameter coordinates, its nature demonstrating the range of model behaviours tested by the cost function at any given parameter set. Besides the main information, the efficiency of the optimisation procedure is also illustrated in these 3D maps, where the reference parameter sets were always located at the lowest point of the surfaces.

2. Robustness analysis of the one-loop Arabidopsis circadian clock model

A similar analysis was performed for the one-loop model and we state the main results for brevity. Again, the initiating globally-optimised parameter sets are provided by Locked et al (unpublished data). In contrast to the two-loop model, the robustness analysis demonstrated that the model is extremely sensitive to a specific minor group of parameters, which are generally conserved across all reference parameter sets. Overall, 7/25 sensitive parameters were identified, all of which relate to molecular processes of TOC1 (transcription, translation, transportation and degradation). With the same consistency cut off (50%), three consistently sensitive parameters were defined as followed: P13 (n2: max. TOC1 transcription rate), P14 (g2: Constant of activation by TOC1), and P15 (m4: constant Max. rate of TOC1 mRNA degradation). The results indicate that TOC1 transcription is the crucial process within the one-loop model. The sensitivity to TOC1 in both the one-loop and two-loop models highlights its importance at the heart of the Arabidopsis circadian clock network.

3. Robustness analysis and Model plausibility

The two Arabidopsis clock models both express similar patterns with respect to sensitivity of the specific molecular components/processes, yet the degree of their sensitivity diverges. The robustness of the two models was compared through the DOR according to Equation (7). Figures 6a and 6b compare robustness between the one-loop and the two-loop models across all parameters at the largest perturbations. Robustness of the most sensitive parameters in each model (suggested by Figure 2b and marked by an arrow in Figure 6b), is graphed across its full perturbation range in Figure 6c. The robustness difference between the one-loop and two-loop models (determined through the most sensitive parameter pointed by arrows in Figure 6c) demonstrates that the two-loop model is far more robust than the one-loop model for all parameters and across the perturbation range. Robustness can be considered as an essential property for most biological systems (particularly circadian clocks) and our analysis indicates the two-loop model is much more plausible as a model for the Arabidopsis circadian clock, reinforcing similar suggestions based on biological evidence [45,47,49–51]. Furthermore, it indicates that the analytical process developed here gives a reasonable measure for determining the robustness of the system, rather than its robustness at a particular point in parameter space.

Discussion

Simple robustness analyses have limited relevance in systems biology. The measured robustness of a model to local parameter changes can vary according to the starting parameter set, exemplified by the distinct sets of sensitive parameters (SP) identified for each reference parameter set (Table 1), and in most biological systems only a minority of parameter values have been fixed by experimental measurements. “Global” analysis methods
avoid this limitation by testing many starting parameter sets, to derive broader conclusions about the circuit of the model rather than the particular dynamics of one parameter set. These are often the most relevant to guide experimental work, because molecular and genetic studies commonly manipulate the model circuit rather than modulating parameter values.

The Consistent Robustness Analysis (CRA) developed here aims to identify a set of consistently sensitive parameters, for a range of biologically-reasonable parameter sets, which we term reference parameter sets. The method is more strategic than previous robustness analysis [52], as it focuses on parameter sets that best allow the model to match a full set of training data, avoiding time-consuming sensitivity analysis of parameter sets that cannot describe the biology of interest. It is still computationally costly, because multiple parameter sets that match the data must first be identified [1,2]. Parameter sets that represented different dynamics (different parts of parameter space) were then manually selected, though this could in principle be automated.

The CRA approach has identified a subset of parameters for the Arabidopsis clock models that prove to be consistently sensitive for

---

**Figure 3. Percent consistency of sensitive parameters in the two-loop Arabidopsis circadian clock model.** The percent consistency of the sensitive parameters (using $m = 1$) among the reference parameter sets was plotted according to (a) the genes and (b) molecular processes (TL = translation and T = Transportation). The consistently sensitive parameters, marked by black bars, were classified based on a 50 percent consistency cut-off.

doi:10.1371/journal.pone.0015589.g003

**Figure 4. Distribution of sensitivity coefficients of parameters in two-loop Arabidopsis circadian clock model.** The histograms demonstrate the distribution of the sensitivity coefficient ($S_{size}$) within each reference parameter set of the two-loop model. This shows the frequency distribution of parameters of the model displaying similar magnitudes of sensitivity.

doi:10.1371/journal.pone.0015589.g004
Figure 5. Two-dimensional sensitivity analysis of the two-loop Arabidopsis circadian clock model. The parameter surface obtained from a 2D sensitivity analysis of the two-loop model according to the (a–e) highly and (f) lowly sensitive parameters in set $L_0$: $P_{8} (r_2)$-TOC1 protein transportation to cytosol, $P_{13} (n_2)$-max. TOC1 transcription rate, $P_{19} (p_2)$-rate constant of TOC1 mRNA translation, $P_{23} (m_6)$-max. rate of light independent cytoplasmic TOC1 degradation, $P_{48} (k_1)$-Y protein in cytosol degradation, and $P_{54} (b)$-Hill coefficient of activation by protein Y. The red star illustrates the position of the reference parameter set which is always coincident with the minimum cost on the parameter surface. X and Y axis represent the perturbation of sensitive parameters while Z axis is the cost function corresponding to the parameter perturbation.

doi:10.1371/journal.pone.0015589.g005

Figure 6. Comparison of model robustness. The robustness of the one-loop and two-loop Arabidopsis circadian clock models is compared through their best-fitting parameter set ($L_0$ for two-loop and $L_{26}$ for one-loop). The degree of robustness ($DOR$) of all parameters in both models at (a) 1/6 times perturbation, and (b) 6 times perturbation is plotted against the perturbed costs. The $DOR$ of the most sensitive parameter in both models, as pointed out by the arrow in (b), was selected to plot across its full perturbation range in (c).

doi:10.1371/journal.pone.0015589.g006
multiple parameter sets. For the two-loop model, only eight consistently sensitive parameter (CSP), mostly involved in TOC1 transcription, were identified from an overall pool of 27 locally sensitive parameters (SP), suggesting wide variation between the sets of sensitive parameters (or genes/molecular processes) classified from each reference parameter set (Table 1). These “consistently sensitive parameters” (see Table 1) suggested two features: (1) the importance of TOC1 transcriptional regulation in both models, as the parameters involved were always more than half of the whole set of sensitive parameters, and (2) the importance of the evening feedback loop involving TOC1 and Y in the two-loop model, compared to the loop involving LHY/CCA1 and TOC1, as the majority of consistently sensitive parameters related to TOC1 or Y function, compared to only few of them (<15%) relating to LHY/CCA1 or X. These traces are consistent with the results of an independent study of the two-loop model, which also inferred the dominance of TOC1 transcription in controlling the model behaviours and properties [16].

In our relatively simple models, these results can be understood relatively easily, as follows:

1. Multiple experimental results support the importance of TOC1 for circadian clock function. Manipulating TOC1, by loss-of-function mutants and transgenic over-expression or constitutive expression, severely alters circadian period and phase [43,46,48,49] or may lead to arrhythmia [53]. Reflecting this importance, TOC1 RNA and proteins are the components that interlock the feedback loops of the two-loop model. The range of available data may be biased, however, because TOC1 was the earliest clock mutant described in Arabidopsis [54].

2. The relative importance of the evening loop in the two-loop model may be related to rhythm generation or to the input of light signals that regulate clock components. The two-loop model was constructed to account for the short-period rhythms of lhy, cca1 double null mutant plants [2]. Accordingly, the evening feedback loop between TOC1 and Y was required to sustain short-period rhythms in the model in a simulated lhy, cca1 double null mutant: the model is relatively robust to the abolition of LHY/CCA1 function. No such constraint was placed upon the simulated Y null mutant, which becomes arrhythmic [2]. In the later, three-loop model [8] the Y null mutant remains rhythmic. Robustness analysis of the three-loop model might be expected to show greater robustness to parameter changes in the evening feedback loop, in contrast to the sensitivity of this loop in the two-loop model.

3. Many of the data sets used in our analysis reflect regulation under constant light or in light:dark cycles, where the lights-on and lights-off signals at dawn and dusk both participate in entraining the Arabidopsis clock [55,56]. In the two-loop model, these signals are mediated by the light-activated transcription of LHY/CCA1 and of Y, respectively. The importance of the TOC1-Y loop in our results is consistent with simulations of the two-loop model under different photoperiods, where entrainment by the Y-mediated lights-off signal dominated the LHY/CCA1-mediated lights-on signal [8].

Finally, parameters related to transcription were extremely influential in both models. While the impact of transcription on the circadian rhythms in plants is unclear, an experimental study for the mammalian circadian clock has been undertaken by Dibner et al, demonstrating that reduction in global transcriptional rates resulted in resilient expression of core clock genes, for instance short rhythmic period and low amplitude [57]. Post-translational regulation is represented much less in the models than in current data on the clock mechanisms of several organisms [58–60]. The data available to construct these models, in contrast, strongly emphasised transcriptional regulation. Our results highlight the locations in the model where this emphasis should be revisited and confirmed experimentally: in the processes relating to the consistently sensitive TOC1 transcriptional parameters, for example, whereas there is less evidence from our analysis that modelling of LHY/CCA1 transcription needs to be revisited.

The plausibility of models can be impartially distinguished through comparing model-specific robustness using the parameter-independent robustness analysis (CRA) proposed in this work. While we acknowledge that robustness has been variously defined in the literature, the employment of the DOR definition here is a convenient and simple mathematical measure to quantify changes in model behaviours and compare differences between models. We further note the plausibility of this particular definition is strengthened by a number of recent publications using a similar measurement [16,52]. While neither DOR, as defined in Equation 7, nor CRA can exclude the effect of redundancy (as described in [61]) from the robustness, this factor is still usable as a means to contrast robustness in diverse models: the redundancy effect is trivial in a small genetic network model and can be avoided in larger models by confining the degree of perturbation to a relatively small range with respect to the null mutation.

The comparison of the DOR both at the most sensitive parameter (Figure 6c) and across a full range of parameters within a parameter set (Figures 6a and 6b) suggests the greater plausibility of the two-loop model of Arabidopsis circadian clock, correlating with the previous assertion that the one-loop model contains a number of weak points. Circadian clock systems, in particular, require a degree of sensitivity to external environmental signals, e.g., light, for entrainment purposes, but should be highly robust to the internal (parameters) variations, as found in the more plausible two-loop model. The accuracy in determining model robustness here is expected to increase with the number of analysed reference sets, however in practice this is confined by the solutions of the optimisation.

As the CRA method has provided reasonable results for these relatively simple models, it is likely to provide greater advantages in analysing the larger models of more complex biological regulators, including plant clock models that include additional components known from the literature [43].

Conclusions

Recently, robustness has been proposed a validating property of biological models: a reliable model should be highly robust. The analytical approach to characterising the real robustness of a model is therefore of the utmost importance. Herein, we created a new robustness analysis method called “consistent robustness analysis” which intends to evaluate model robustness independently of operating parameters. This novel method allows us new comprehension into the given model: (1) the sensitive parameters of the model at a given parameter set, (2) the “consistently sensitive parameters” specific to the model, (3) the distribution of parameter sensitivity within the model, and (4) the parameter surface. In addition, we initiated a benchmark factor, (DOR or DOS), to evaluate the plausibility of various models (of differing complexity) by comparing the normalised magnitude of the model robustness. The success of this new method was demonstrated through the study of two Arabidopsis circadian clock models (one-loop and two-loop) with its results conferring both physically and biologically reasonable outcomes. The consistently sensitive parameters successfully pinpointed the TOC1 transcription as the sensitive component and the molecular processes controlling the model behaviours, whereas DOR indicated the much greater plausibility of the two-loop model compared with the one-loop model, supporting many biological findings.
Methods

Modelling through fitting to the data

The optimisation process identifies parameter sets that minimise
an appropriate cost function: a set of criteria or desired properties
that a “good model” should satisfy. The cost function typically
compares or quantifies the mismatch between the behaviours
of the model and the real system, for example experimental data sets
and/or qualitative criteria from observed biological behaviour [1].
In the analysis performed in Results section, the cost function
compares simulated results with experimental data from various
sources/conditions (see also Table S3 in Data S2) [44,50,62,63]
using a least-square formula [64]. A low cost-of-fit is thus expected
to give a good representation of the system. However, for the large
parameter spaces typical of complex models, it is unlikely that a
unique minimising parameter set exists and similar fitting results
may be obtained from widely spaced parts of the parameter space.
Furthermore, experimental data is collected under various
conditions in different laboratories, thus altering specific
parameters. Consequently, the extent to which robustness of a model can
validly be determined from a single parameter set is uncertain.

Consistent Robustness Analysis (CRA)

We propose a new analysis to address some of the limitations
highlighted above. The aim is to understand system robustness by
performing sensitivity analyses using multiple parameter sets that
yield reasonable model behaviour, as judged by the full cost
function. Figure 1a illustrates our algorithm, consisting of three
phases: (1) selection of the reference parameter sets, (2)
one-dimensional sensitivity analysis – determination of sensitive
parameters and (3) two-dimensional sensitivity analysis – investigation
of parameter surface.

(1) Selection of the reference parameter sets. The first
phase ensures model sensitivity is tested across wide regions of
parameter space rather than at a specific point. Initially, global
optimisation was performed to obtain a number of parameter sets
yielding a reasonable fit to the data while covering a broad region
of the parameter space [1,2]. From this larger set, a subset of
reference parameter sets was chosen according to three criteria:
low cost-of-fit, biologically sensible parameter values and a
significant distance between the reference parameter sets. The
distance was evaluated using standard techniques (e.g. clustering
methods) and the reference parameter sets were chosen at distant
locations to ensure broad coverage of parameter space. Finally,
following the selection of the parameter sets from the global
optimisation, local optimisation is performed on each selected set
to obtain the (locally) best-fitting reference parameter sets (see also
Figure 1b). The range of parameter space covered is displayed as
the span of parameter values (Figure S2b).

(2) One-dimensional sensitivity analysis. In the second
phase, for each of the locally-optimised reference parameter sets a
one-dimensional sensitivity analysis was performed through
stepwise alteration of each parameter across a 36-fold range of
values, centred on its value in the reference parameter set. The
sensitivity of the model to a particular parameter was measured
through the cost-of-fit (cost function).

In the following we denote by $k = 1 \ldots N_k$ the reference
parameter sets, $j = 1 \ldots N_j$ to denote the parameters within each
set and $i = -N_i, \ldots, +N_i$ to denote the perturbation where $-$ and $+$
respectively represent negative and positive perturbations. Thus,
$C_{i,j,k}(x_e, x_{m_{i,j,k}})$ is the least-square cost function (Equation 1)
calculated at the $i$th perturbation to the $j$th parameter in the $k$th
reference parameter set, where $x_e$ represents an experimental data
set to be compared with its counterpart $x_{m_{i,j,k}}$ calculated through
simulation of the model. The cost function is normalised within
each reference parameter set with respect to its maximum
computed across all parameters and perturbations, to allow
meaningful comparisons among parameters despite difference in
the cost-of-fit of each reference parameter set:

$$C_{i,j,k}(x_e, x_{m_{i,j,k}}) = \left( \frac{x_{m_{i,j,k}} - x_e}{x_e} \right)^2$$

(1)

$$NC_{i,j,k} = \frac{C_{i,j,k}(x_e, x_{m_{i,j,k}})}{\max_{i=-N_i, N_i,j=1,N_j} C_{i,j,k}(x_e, x_{m_{i,j,k}})}$$

(2)

For each parameter $j$ in each reference parameter set we
determine two “sensitivity coefficients$: $S_{\text{size}}^j$ representing the magnitude and $S_{\text{choppy}}^j$ inferring the smoothness/variation of the
calculated sensitivity.

$$S_{\text{size}}^j = \frac{\sum_{i=-N_i}^{N_i} |NC_{i,j,k} - \max_{i=-N_i, N_i,j=1,N_j} (NC_{i,j,k})|}{2N_i}$$

(3)

$$S_{\text{choppy}}^j = \frac{\sum_{i=-N_i}^{N_i-1} |NC_{i+1,j,k} - NC_{i,j,k}|}{2N_i}$$

(4)

The above sensitivity coefficients are used to determine
“sensitive parameters” through their means and standard
deviations within each reference parameter set. For a $k$th parameter
set, the $j$th parameter is subsequently defined as sensitive if

$$S_{\text{size}}^j \geq \text{Mean}_{j=1,N_p} S_{\text{size}}^j + m \times SD_{j=1,N_p} S_{\text{size}}^j \quad \& \quad S_{\text{choppy}}^j \geq \text{Mean}_{j=1,N_p} S_{\text{choppy}}^j + m \times SD_{j=1,N_p} S_{\text{choppy}}^j$$

(5)

where the parameter $m$ indicates the strictness within which
sensitivity is defined.

To determine the consistently sensitive parameters, we calculate
the frequency for which a particular parameter is classified as
sensitive across $N_r$ reference parameter sets. We denote by $N_j$ the
number of parameter sets for which the $j$th parameter is classified as
sensitive according to Equation (5) and define $PC_j$ as the
percentage consistency for each parameter according to

$$PC_j = \frac{N_j}{N_r} \times 100$$

(6)

(3) Two-dimensional sensitivity analysis. The final phase
is a two-dimensional sensitivity analysis: two of the most sensitive
parameters determined by the previous analysis were chosen and
perturbed simultaneously using a similar procedure of parameter
perturbation and sensitivity measurement. Through variation of
two parameters, we can obtain greater understanding of the
surface structure of the sensitivity space via 3D plots of the cost-of-
fit.

Consistent Robustness Analysis (CRA)
Model Robustness Comparison

Direct and unbiased comparison of the robustness between models presents a number of challenges: models display varying complexity with respect to both topology and the number of parameters. For this study, the sensitivity between the models is compared through the degree of robustness (DOR). For each model, we compute DOR for whichever parameter j is the most sensitive within the best-fit-simulation parameter set k. DOR is defined as the inversion of the degree of sensitivity (DOS), defined as follows:

\[ \text{DOS}_{i,j,k} = \frac{C_{i,j,k}(x_{r}x_{m,j,k}) - C_{i=0,j,k}(x_{r}x_{m0,j,k})}{C_{i=0,j,k}(x_{r}x_{m0,j,k})} \]  

where \( i = 0 \) locates the zero perturbation point (at which parameter values are identical to the reference parameter set) and \( J \) denotes the most sensitive parameter according to Equation (5) of the parameter set \( k \).

Supporting Information

Data S1 Model equations and parameters. The systems of ODE equations describing one-loop and two-loop Arabidopsis circadian clock models and their corresponding best fit parameter sets.

Data S2 List of experimental data for modelling Arabidopsis circadian clock. The summary of experimental data used for matching the model simulations.

Figure S1 Simulations showing fit to data for the one-loop and two-loop Arabidopsis circadian clock model using the best-fit parameter sets.

References


Acknowledgments

We thank Dr. Ozgur Biringer-Akman and Dr. Kieron D. Edwards for useful discussions, and James CW Locke for providing unpublished good parameter sets of one-loop and two-loop Arabidopsis circadian clock models.

Author Contributions

Conceived and designed the experiments: TS. Performed the experiments: TS. Analyzed the data: TS KJP AJM. Wrote the paper: TS KJP AJM.