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Adaptive Windowing for ICI Mitigation in Vehicular Communications

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***Abstract*—The performance of orthogonal frequency division multiplexing systems in vehicular environments suffers from intercarrier interference (ICI) and the inherent non stationarity of the channel statistics. Receiver windowing constitutes an effective technique for enhancing the banded structure of the frequency-domain channel matrix, thus improving the effectiveness of a banded equalizer for ICI mitigation. However, its optimality has been verified only for stationary channels with perfectly known statistics. In non stationary channels, the second-order statistics have to be tracked and the optimal performance can be achieved at the expense of cubic complexity over the number of the subcarriers. To overcome this limitation, an adaptive windowing technique is proposed that is able to track directly an optimal receiver window in terms of average signal-to-interference noise ratio, requiring only linear complexity. Extensive simulation results verify both the ability of the proposed approach to track the time varying channel statistics and its increased robustness to channel estimation errors that are common in vehicular environments.**

I. Introduction

THE latest technological developments in vehicular communication systems, such as autonomous wireless information exchange among vehicles with the roadside infrastructure, have been employed to enhance road safety, to improve the efficiency of transportation [1] and to tackle major issues of autonomous driving [2]. In vehicular environments, especially on highways, the propagation is characterized by shadowing by other vehicles, high Doppler shifts with often sparse Doppler spectra due to a few dominant scatterers (e.g., road signs, highway overpasses) [3]. Unlike fast fading, shadowing is a non-ergodic process causing non stationarity to the channel statistics [4]. Moreover, the high Doppler shifts, introducing both time- and frequency-selectivity in the wireless channels (so called doubly selective) [5], corrupting the OFDM subchannels orthogonality and generating intercarrier interference (ICI).

Equalization of ICI has been extensively studied in the literature [6], however, most of the approached do not take into account the non stationarity of channel statistics. Despite the computational efficiency of block banded minimum-mean-square-error (MMSE) equalizers [6], achieved by band matrix approximation of the frequency-domain channel convolution matrix [7], [8], these approaches suffer from severe performance loss in high signal-to-noise ratio (SNR) regimes, [9]. To overcome this limitation, several authors suggest a time-domain pre-filtering of the input signal (windowing), in order to enforce a banded structure at the frequency domain [10],[11],[12]. This is achieved either by using a fixed predefined window (e.g. Hamming) or by estimating the one that maximizes a signal-to-interference-plus-noise ratio (SINR) criterion (e.g. maximum SINR or maximum average SINR, [11])). Among these methods, the maximum SINR exhibits the best performance, but it requires perfect knowledge of the channel impulse response (CIR) and increased computational complexity. More importantly, those methods assume channel stationarity and perfect knowledge of the channel statistics. Motivated by the aforementioned limitations, we propose a novel adaptive windowing technique, capable of tracking the variations of the channel statistics, maximizing in an adaptive manner the average SINR. This is achieved at a linear computational complexity order, over the number of the subcarriers. Extensive evaluation studies, show that the proposed technique exhibits also enhanced robustness over the channel estimation errors, which are common in vehicular environments.

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II. Problem Statement

In an OFDM transmitter (TX), the frequency-domain data stream is divided into blocks of length N and modulated by N-point inverse discrete Fourier transform (DFT). At the receiver (RX), the received blocks are demodulated by N-point DFT. Assuming time and frequency synchronization, and employing a cyclic prefix with length greater than the maximum delay spread of the channel, the input-output relation for each OFDM block can be described as

$$\mathbf{y} = \mathbf{F}\mathbf{H}_t\mathbf{F}^H\mathbf{x} + \mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{z} \tag{1}$$

where $\mathbf{F}$ denotes the $N \times N$ DFT matrix, $\mathbf{x}$ and $\mathbf{y}$ are the $N \times 1$ transmitted and received symbol vectors of each OFDM block, respectively, with $\mathcal{E}\{\mathbf{x}\mathbf{x}^H\} = \mathbf{I}_N$, $\mathbf{H}_t$ and $\mathbf{H}$ denote the channel convolution $N \times N$ matrices at the time and the frequency domain respectively, and $\mathbf{z}$ denotes the $N \times 1$ additive complex white Gaussian noise (AWGN) vector with $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \sigma_z^2\mathbf{I}_N)$. In doubly-selective channels, $\mathbf{H}$ is typically a non-diagonal matrix whose off-diagonal elements are due to ICI. Let us denote the K-banded approximation of the frequency-domain channel matrix as $\mathbf{H}_B = \mathbf{B} \circ \mathbf{H}$, with K non-zero elements at each row, where $\mathbf{B}$ is a $N \times N$ binary matrix with lower and upper bandwidth $K/2$ and all ones within its band, while $\circ$ denotes the element-wise product. Therefore, the banded MMSE-based soft-decision symbol vector is given by

$$\tilde{\mathbf{x}} = \mathbf{H}_B^H \left(\mathbf{H}_B \mathbf{H}_B^H + \sigma_z^2 \mathbf{I}_N\right)^{-1} \mathbf{y}. \tag{2}$$

Applying a time-domain window $\mathbf{w}$ at the RX, prior to the DFT operation of each OFDM block, we get the following output

$$\mathbf{y}_{\mathbf{w}} = \mathcal{C}(\mathbf{w})\mathbf{y} = \mathbf{F}\mathcal{D}(\mathbf{w})\mathbf{F}^H \mathbf{y}, \tag{3}$$

where $\mathcal{D}(\cdot)$, $\mathcal{C}(\cdot)$ denote the diagonal and the circulant matrix respectively, of the argument vector. In this case, the MMSE-based soft-decision output is given by

$$\tilde{\mathbf{x}}_{\mathbf{w}} = \mathbf{H}_B^H \left(\mathbf{H}_B \mathbf{H}_B^H + \sigma_z^2 \mathcal{C}(\mathbf{w})\mathcal{C}(\mathbf{w})^H\right)^{-1} \mathbf{y}_{\mathbf{w}}. \tag{4}$$

An optimal design criterion in terms of average signal-to-noise-interference ratio (SINR) is expressed as [10]:

$$\mathbf{w}^\star = \arg\max_{\mathbf{w}} \frac{\mathcal{E}\{P_s\}}{\mathcal{E}\{P_{ni}\}} \tag{5}$$

where $P_s = \|\mathbf{B} \circ (\mathcal{C}(\mathbf{w})\mathbf{H})\|_F^2$ is the signal power and $P_{ni} = \|\mathbf{B}^c \circ (\mathcal{C}(\mathbf{w})\mathbf{H})\|_F^2 + \sigma_z^2 \|\mathcal{C}(\mathbf{w})\|_F^2$ is the noise plus interference power while $\mathbf{B}^c$ is the complementary matrix of $\mathbf{B}$ and $\mathcal{E}\{\cdot\}$ denote the statistical mean.

III. Proposed Windowing Technique

As mentioned in the Introduction, a common assumption, that simplifies the ICI windowing is that the channel remains stationary. In this case, the channel statistics can be obtained once for all the OFDM block transmissions. When there are no estimation errors, the maximum average SINR criterion (5) can perform identically to the instantaneous channel state information (CSI) case [11]. However in practical scenarios, the channels are likely to be even non-stationary [4] due abrupt speed changes and shadowing, leading to erroneously estimated CSI. On the other hand, ICI mitigation based on second-order statistics (SOS) can be more robust but the SOS have to be tracked and the optimal performance can be achieved at the expense of cubic complexity. A promising approach to overcome this problem, would be to adaptively estimate the unknown channel statistics and then solve the problem (5) for each update. On this premise, we need block-averaged expressions for the statistical quantities of (5), i.e.,

$$\bar{P}_s(m) \triangleq \lambda \bar{P}_s(m-1) + P_s(m) \tag{6}$$

$$\bar{P}_{ni}(m) \triangleq \lambda \bar{P}_{ni}(m-1) + P_{ni}(m) \tag{7}$$

where m is the OFDM block index, λ is the forgetting factor while $P_s(m) = \|\mathbf{B} \circ (\mathcal{C}(\mathbf{w}(m))\mathbf{H}(m))\|_F^2$ and $P_{ni}(m) = \|\mathbf{B}^c \circ (\mathcal{C}(\mathbf{w}(m))\mathbf{H}(m))\|_F^2 + \sigma_z^2 \|\mathcal{C}(\mathbf{w})\|_F^2$. Note that when the channel is stationary, then $\lim_{m\to\infty} \bar{P}_s(m) = \mathcal{E}\{P_s\}$ and $\lim_{m\to\infty} \bar{P}_{ni}(m) = \mathcal{E}\{P_{ni}\}$.

Our aim is to generalize the optimal average SINR design criterion for the case of non-stationary environments. Designing an adaptive window based on (6)-(7) will be robust and able to track the rapidly changing conditions which usually occur in vehicular communications.

Proposition 1. *An optimally designed adaptive window for ICI mitigation based on the maximum SINR for the m-th OFDM block is given by $\mathbf{w}^\star(m) = \mathbf{\Lambda}(m)^{-\frac{1}{2}} \mathbf{v}^\star(m)$, where*

$$\mathbf{v}^\star(m) = arg\max_{\mathbf{v}} \frac{\mathbf{v}^H \left(\mathbf{\Lambda}^{-\frac{1}{2}}(m)\mathbf{R}(m)\mathbf{\Lambda}^{-\frac{1}{2}}(m)\right)\mathbf{v}}{\mathbf{v}^H\mathbf{v}} \tag{8}$$

with $\mathbf{\Lambda}(m) = \lambda\mathbf{\Lambda}(m-1) + \mathcal{D}\big(\mathbf{F}^H\mathbf{H}(m)\mathbf{H}^H(m)\mathbf{F}\big) + \sigma_z^2\mathbf{I}_N,$

$$\textit{and } \mathbf{R}(m) = \lambda\mathbf{R}(m-1) + \sum_{k=1}^{N} \left[\mathbf{\Xi}(m,k)\mathbf{\Xi}^H(m,k)\right]^* \tag{9}$$

where $\mathbf{\Xi}(m,k) \triangleq \mathcal{D}(\mathbf{F}_{|k})\mathbf{F}\mathbf{H}(m)\mathcal{D}(\mathbf{B}_{|k})$ and $\mathbf{F}_{|k}$ denotes the k-th row of $\mathbf{F}$.

Proof. c.f. Appendix. □

Hence, the design of the adaptive window would require the computation of the maximum eigenvector of the matrix $\mathbf{Q}(m) \triangleq \mathbf{\Lambda}^{-\frac{1}{2}}(m)\mathbf{R}(m)\mathbf{\Lambda}^{-\frac{1}{2}}(m)$ at each OFDM block which is expressed via (8). Direct methods would require high complexity and memory requirements (e.g., $\mathcal{O}(N^3)$). On the other hand, several iterative schemes exist for evaluating efficiently the principal eigenvectors of large matrices (e.g., $\mathcal{O}(N^2)$). Two standard methods are Lanczos-type and those that are based on the power method. Although the Lanczos methods require less iterations for evaluating the subspace of a symmetric matrix given a random initialization, two attractive properties of the power iteration method are, its robustness and the fact that using a starting subspace close to the subspace of interest can lead to a very fast solution. Those motivated us to propose a scheme that builds on the power method ending up with the following update scheme,

$$\mathbf{v}(m) = \mathbf{Q}(m)\mathbf{v}(m-1) \tag{10}$$

where $\mathbf{v}(m-1)$ is the dominant eigenvector of the $(m-1)$-th OFDM block. Once we obtain the current block update of the dominant eigenvector $\mathbf{v}(m)$, the optimal window can be computed by

$$\mathbf{w}(m) = \mathbf{\Lambda}^{-\frac{1}{2}}(m)\mathbf{v}(m)/\|\mathbf{v}(m)\|_2. \tag{11}$$

Note that in the proposed algorithm, only one step of the power method, given by (10)-(11), is executed per each OFDM block, reducing further the computational complexity. However, the overall technique still requires quadratic complexity over the number of the subcarriers, since, according to (9), N matrix-matrix products with $\mathcal{O}(N)$ complexity are required for each OFDM block. To overcome this problem and achieve linear complexity, we remove the summation term in (9) and we properly select index k as follows:

$$\mathbf{Q}(m) \approx \lambda\mathbf{Q}(m-1) + \mathbf{\Lambda}^{-\frac{1}{2}}(m)\big[\mathcal{J}\big]^*\mathbf{\Lambda}^{-\frac{1}{2}}(m) \tag{12}$$

with $\mathcal{J} \triangleq \mathbf{\Xi}(m,k)\mathbf{\Xi}^H(m,k)$, $k = \text{mod}(m-1,N)+1$, where $\text{mod}(\cdot,N)$ denotes the modulo-N operation, and thus, $k \in [1,N]$. To justify this approximation note that $\text{rank}\big(\mathbf{R}(m)\big) = \text{rank}\big(\mathcal{J}\big) = K$ which implies that all matrices span the same subspace and share common eigenvectors. Moreover, since K is usually very small, one iteration of the power method using (10) provides a good approximation to the dominant

Algorithm 1 Adaptive Windowing for the m-th OFDM block

Input: $\mathbf{H}_t(m), \mathbf{v}(m-1), \mathbf{\Lambda}(m-1)$

Output: $\mathbf{w}(m)$

1: $\mathbf{\Lambda}(m) = \lambda\mathbf{\Lambda}(m-1) + \mathcal{D}(\mathbf{H}_t(m)\mathbf{H}_t^H(m)) + \sigma_z^2\mathbf{I}_N$

2: $k = \mod(m-1, N) + 1$

3: $\mathbf{\Xi}(m,k) = \mathcal{D}(\mathbf{F}_{|k})\mathbf{H}_t(m)\mathbf{F}^H\mathcal{D}(\mathbf{B}_{|k})$

4: Obtain $\mathbf{w}(m)$ via (11) and (13)

eigenvector. Replacing with the approximation (12) into (10) and observing that $\mathbf{v}(m-1) \approx \mathbf{Q}^2(m-1)\mathbf{v}(m-2)$, the adaptive window can be expressed as:

$$\mathbf{v}(m) \approx \lambda\mathbf{v}(m-1) + \mathbf{\Lambda}^{-\frac{1}{2}}(m)\mathbf{\Xi}(m,k)\tilde{\mathbf{v}}(m-1) \tag{13}$$

where $\tilde{\mathbf{v}}(m-1) \triangleq \mathbf{\Xi}^H(m,k)\mathbf{\Lambda}^{-\frac{1}{2}}(m)\mathbf{v}(m-1)$. Through simulation results (c.f. Section IV), it is verified that approximation (12) holds for $\lambda < 1$.

The proposed adaptive windowing technique is summarized in Algorithm 1. For initialization we set $\mathbf{v}(m-1) = \mathbf{0}$ and $\mathbf{\Lambda}(m-1) = \mathbf{0}$. The lines 1-3 are for the update of the sample correlation matrices which cost $\mathcal{O}(NL)$ since matrix $\mathbf{H}_t$ is a sparse matrix with L non-zero elements at each row. Line 4 is for the update of the dominant eigenvector, i.e. the windowing filter, which also has $\mathcal{O}(NK)$ complexity, since matrix $\mathbf{B}_{|k}$ is a sparse matrix with K non-zero elements at each row. The overall complexity order of the Algorithm 1 is $\mathcal{O}(N(L+K))$ since in our case $L, K \ll N$.

Remark (blind estimation): The proposed adaptive windowing technique requires the estimated CSI, as it is the case with the classical method based maximum SINR criterion. However in our case, the channel matrix expression $\mathbf{H}(m)$ is the channel convolution matrix in the frequency domain, and potentially, it can be estimated *blindly*, e.g. [13], thus avoiding the costly operation of channel estimation.

IV. Performance Evaluation and Discussion

To evaluate the performance of the proposed technique we adopt the IEEE 802.11p standard for vehicular communications [14]. We consider an OFDM system with 64 subcarriers where the TX sends uncoded quadrature phase-shift keying (QPSK) symbols through a double-selective channel. In the RX we implement a banded MMSE equalization to mitigate the ICI effect with band size $K = 3$, given by (4). For comparison, we have considered: 1) the Hamming window, 2) the optimal SINR window based on the instantaneous CSI [11], 3) the optimal mean SINR window based on the statistical properties of the channel [11] and 4) the optimal mean-square-error (MSE) case where no channel truncation. The channel power delay profile (PDP) corresponds to the PDP of the ITU Vehicular channel type A. The subcarrier frequency spacing was set to $F = 0.15625$ MHz and the carrier frequency to $f_c = 5.85$ GHz. The maximum relative speed between the TX and the RX was set to 140kmh, i.e., $f_d = \frac{f_c}{F \cdot c}v = 0.0049$ where c is the speed of light and v is vehicles relative speed. The results have been averaged over 100 Monte-Carlo realizations and the MSE curves have been weighted over a window of $T_w = 30$ OFDM samples.

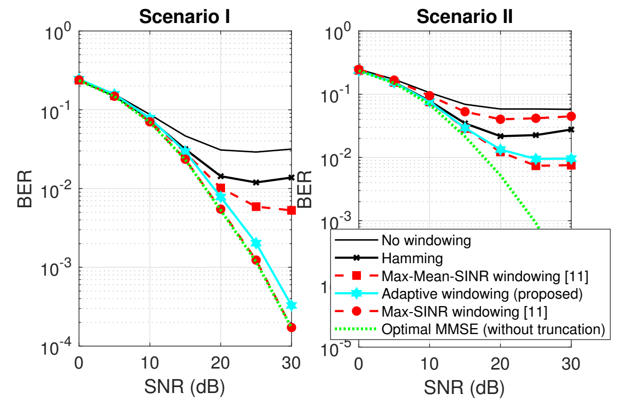

Fig. 1. Performance evaluation of the proposed technique with $N = 64$, $f_d = 0.0049$ (140kmh) and $K = 3$.

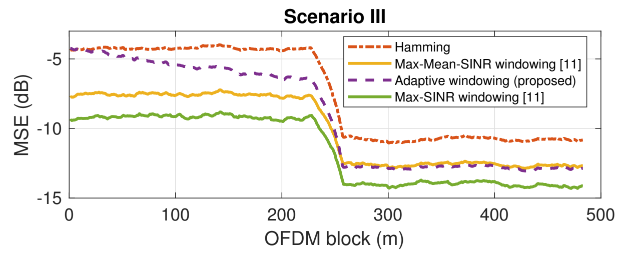

Fig. 2. MSE tracking curve where $f_d = 0.0049$ (140kmh) changes to $f_d = 0.0024$ (70kmh) and shadowing at 256-th OFDM block, at SNR=30dB.

Setup: We consider three scenarios, namely: a) Scenario I, perfect CSI and SOS at the RX. In scenario II, we consider several realistic imperfections, modeling both the channel estimation errors and outdated channel knowledge. We assume that the non-coherent environment introduces $\pm$3m/s fluctuation of the relative speed and hence the Doppler spread changes at each OFDM block. Also, due to the fast varying environment, the estimated channel differs from the true one at the m-th OFDM block, due to the delay of the feedback path and the outdated estimation, i.e.,

$$\mathbf{H}^{\text{true}}(m) = \alpha\mathbf{H}(m-1) + \sqrt{1-|\alpha|^2}\mathbf{N}(m) \tag{14}$$

where $\alpha = J_0(2\pi f_d)$ with J_0 denote the zero-order Bessel function of the first kind and each entry of $\mathbf{N}(m)$ is independently random drawn from $\mathcal{CN}(0,1)$. In scenario III, we consider that a sudden change of the vehicle speed (from 140kmh to 70kmh) as well as loss of the line-of-sight component of the channel results into variation of the SOS.

Discussion: In Fig. 1 we show the bit-error-rate (BER) versus the signal-to-noise-ratio (SNR). The performance of the proposed technique is almost optimal for Scenario I and II with $\lambda = 0$ and $\lambda = 0.8$ respectively. Fig. 2 evaluates the

$$\text{MSE} \triangleq \sum_{m=1}^{T_w} \frac{\|\mathbf{x}(m) - \tilde{\mathbf{x}}_{\mathbf{w}}(m)\|_2^2}{\|\mathbf{x}(m)\|_2^2} \tag{15}$$

under Scenario III . The proposed technique is able to converge and track to the optimal case, i.e., Max-Mean-SINR curve, after the abrupt change. In Fig. 3 the convergence of the proposed technique is verifed under Scenarios I and II. For each scenario we consider two vehicle speeds, 70kmh ($f_d = 0.0024$) and 140kmh ($f_d = 0.0049$). For Scenario I, the adaptive technique converges almost immediately at 1dB higher than the max-SINR case, which is caused by the

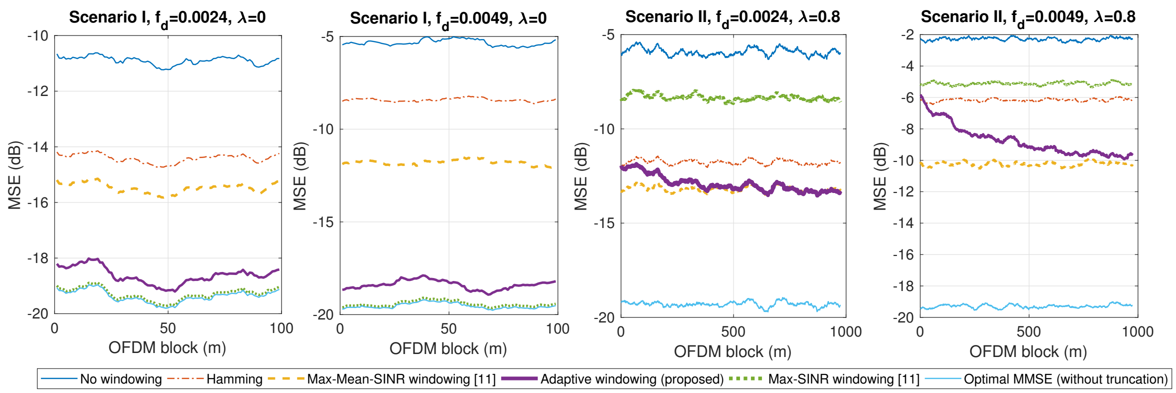

Fig. 3. MSE convergence curves of the proposed adaptive technique with $N = 64$, SNR = 30dB and $K = 3$.

one-iteration approximation of the dominant eigenvector using (10). For Scenario II, the adaptive technique converges to the max mean-SINR optimum after a number of OFDM blocks, depending on the vehicle speed. Note that the initialization of the adaptive window has been set to the Hamming window while the forgetting parameter was set to $\lambda = 0.8$.

In conclusion, through extensive simulation results we have verified the ability of the proposed approach to track the time varying channel statistics which are common in vehicular communication scenarios.

Appendix

Let us consider the m-th OFDM block index, then based on the properties of the Hadamard product we have that

$$P_s = \sum_{n=1}^{N} \|\mathbf{E}_{nn}\mathcal{C}(\mathbf{w})\mathbf{H}\mathcal{D}(\mathbf{B}_n)\|_F^2 = \mathbf{w}^H \left[\sum_{n=1}^{N} \mathbf{R}_n\right]^* \mathbf{w} \quad (16)$$

where $(\cdot)^*$ denotes the conjugate and $\mathbf{R}_n = \mathcal{D}(\mathbf{F}_n)\mathbf{F}^H\mathbf{H}\mathcal{D}(\mathbf{B}_n)\mathbf{H}^H\mathbf{F}\mathcal{D}(\mathbf{F}_n)$. The first equality in (16) holds since matrix $\mathbf{E}_{nn}$ is orthonormal with only one unit value at the n-th row and n-th column and zeros elsewhere [15, p.110], while the second holds because $\|\mathbf{E}_{nn}\mathcal{C}(\mathbf{w})\mathbf{H}\mathcal{D}(\mathbf{B}_n)\|_F^2 = \mathrm{tr}(\mathbf{J}_n(\mathbf{w})\mathbf{R}_n\mathbf{J}_n(\mathbf{w})^H) = \mathbf{w}^H\mathbf{R}_n\mathbf{w}$ where $\mathbf{J}_n(\mathbf{w})$ is a matrix with zero rows except for the n-th row which is equal to $\mathbf{w}$ with $\mathbf{E}_{nn}\mathcal{C}(\mathbf{w}) = \mathbf{E}_{nn}\mathbf{F}\mathcal{D}(\mathbf{w})\mathbf{F}^H = \mathbf{J}_n(\mathbf{w})\mathcal{D}(\mathbf{F}_n)\mathbf{F}^H$. Considering the denominator term we have that

$$P_{si}(m) = \|\mathcal{C}(\mathbf{w})\mathbf{H}\|_F^2 - \|\mathbf{B} \circ (\mathcal{C}(\mathbf{w})\mathbf{H})\|_F^2 + \sigma_z^2\|\mathcal{C}(\mathbf{w})\|_F^2 \quad (17)$$

$$= \mathbf{w}^H\left(\sigma_z^2\mathbf{I}_N + \mathcal{D}(\mathbf{F}^H\mathbf{H}\mathbf{H}^H\mathbf{F}) - \mathbf{R}\right)\mathbf{w} \quad (18)$$

where the last equality holds by using (16) in (17) and with $\|\mathcal{C}(\mathbf{w})\|_F^2 = \mathbf{w}^H\mathbf{w}$ and $\|\mathcal{C}(\mathbf{w})\mathbf{H}\|_F^2 = \mathbf{w}^H\mathcal{D}(\mathbf{F}^H\mathbf{H}\mathbf{H}^H\mathbf{F})\mathbf{w}$.

Based on the previous we conclude that (5) can be expressed as a generalized eigenvector estimation problem, i.e., $\mathbf{R}\mathbf{w}^\star = \eta_m(\mathbf{\Lambda}-\mathbf{R})\mathbf{w}^\star$ with $\mathbf{\Lambda} \triangleq \sigma_z^2\mathbf{I}_N + \mathcal{D}(\mathbf{F}^H\mathbf{H}\mathbf{H}^H\mathbf{F})$. Let $\mathbf{v} \triangleq \mathbf{\Lambda}^{1/2}\mathbf{w}$, then we can formulate the equivalent standard eigenvector estimation problem, namely $\mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{R}\mathbf{\Lambda}^{-\frac{1}{2}}\mathbf{v}^\star = \kappa_m\mathbf{v}^\star$. Note that $\kappa_m = \frac{\eta_m}{1+\eta_m}$, where η_m is the maximum eigenvalue and since the matrices $\mathbf{R}$ and $\mathbf{\Lambda} - \mathbf{R}$ are positive semi-definite (they can be written as Gram matrices), Since the function $f(\eta) = \frac{\eta}{1+\eta}$ is strictly increasing, the eigenvector of the κ_m-th eigenvalue corresponds to the eigenvector of η_m. Taking the statistical mean of the involved matrices we have $\mathcal{E}\{\mathbf{R}\}\mathbf{w}^\star = \eta_m\mathcal{E}\{(\mathbf{\Lambda} - \mathbf{R})\}\mathbf{w}^\star \Rightarrow \mathcal{E}\{\mathbf{\Lambda}\}^{-\frac{1}{2}}\mathcal{E}\{\mathbf{R}\}\mathcal{E}\{\mathbf{\Lambda}\}^{-\frac{1}{2}}\mathcal{E}\{\mathbf{\Lambda}\}^{1/2}\mathbf{w}^\star = \kappa_m\mathcal{E}\{\mathbf{\Lambda}\}^{1/2}\mathbf{w}^\star \Rightarrow \mathcal{E}\{\mathbf{\Lambda}\}^{-\frac{1}{2}}\mathcal{E}\{\mathbf{R}\}\mathcal{E}\{\mathbf{\Lambda}\}^{-\frac{1}{2}}\mathbf{v}^\star = \kappa_m\mathbf{v}^\star$ which results into the Proposition 1 by replacing the expectation with the sample-based average.

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