An embedded segmental $k$-means model for unsupervised segmentation and clustering of speech

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Abstract
Unsupervised segmentation and clustering of unlabelled speech are core problems in zero-resource speech processing. Most competitive approaches lie at methodological extremes: some follow a Bayesian approach, defining probabilistic models with convergence guarantees, while others opt for more efficient heuristic techniques. Here we introduce an approximation to a segmental Bayesian model that falls in between, with a clear objective function but using hard clustering and segmentation rather than full Bayesian inference. Like its Bayesian counterpart, this embedded segmental $k$-means model (ES-KMeans) represents arbitrary-length word segments as fixed-dimensional acoustic word embeddings. On English and Xitsonga data, ES-KMeans outperforms a leading heuristic method in word segmentation, giving similar scores to the Bayesian model while being five times faster with fewer hyperparameters. However, there is a trade-off in cluster purity, with the Bayesian model’s purer clusters yielding about 10% better unsupervised word error rates.

Index Terms: zero-resource speech processing, word discovery, word segmentation, unsupervised learning, acoustic embeddings

1. Introduction
The growing area of zero-resource speech processing aims to develop unsupervised methods that can learn directly from raw speech audio in settings where transcriptions, lexicons and language modelling texts are not available. Such methods are crucial for providing speech technology in languages where transcribed data are hard or impossible to collect, e.g., unwritten or endangered languages [1]. In addition, such methods may shed light on how human infants acquire language [2, 3].

Several zero-resource tasks have been studied, including acoustic unit discovery [4–6], unsupervised representation learning [7–9], query-by-example search [10, 11] and topic modelling [12, 13]. Early work mainly focused on unsupervised term discovery, where the aim is to automatically find repeated word- or phrase-like patterns in a collection of speech [14–16]. While useful, the discovered patterns are typically isolated segments spread out over the data, leaving much speech as background. This has prompted several studies on full-coverage approaches, where the entire speech input is segmented and clustered into word-like units [17–20].

Two such full-coverage approaches have recently been applied to the data of the Zero Resource Speech Challenge 2015 (ZRSC), giving a useful basis for comparison [21]. The first is the Bayesian embedded segmental Gaussian mixture model (BES-GMM) [22]: a probabilistic model that represents potential word segments as fixed-dimensional acoustic word embeddings, and then builds a whole-word acoustic model in this space while jointly doing segmentation. The second is the recurring syllable-unit segmenter (SylSeg) [23], which is a cognitively motivated, fast, heuristic method that applies unsupervised syllable segmentation and clustering and then predicts recurring syllable sequences as words. These two models are representative of two methodological extremes often used in zero-resource systems: either a Bayesian approach is used, defining probabilistic models with convergence guarantees [6, 19, 22], or heuristic techniques are used in pipeline approaches [18, 23].

Here we introduce an approximation to BES-GMM that falls in between these two extremes. The embedded segmental $k$-means (ES-KMeans) algorithm uses hard clustering and segmentation, rather than full Bayesian inference. Nevertheless, it has a clear objective function, in contrast to heuristic methods such as SylSeg. Compared to BES-GMM, it has the advantage of fewer hyperparameters and a simpler optimization algorithm since probabilistic sampling is not necessary; ES-KMeans is therefore more efficient, while still having a principled objective.

Hard approximations have been used since the start of probabilistic modelling in supervised speech recognition [24–26], and also in more recent work to improve the efficiency of an unsupervised Bayesian model [27]. We are therefore following in a long tradition of using hard approximation. However, all of these studies applied it in frame-by-frame modelling approaches, while our approach operates on embedded representations of whole speech segments. There is a growing focus on such acoustic word embedding methods [11, 28–32], since they make it possible to easily and efficiently compare variable-duration speech segments in a fixed-dimensional space.

We analyze how this approximation affects speed and accuracy relative to the original BES-GMM and the SylSeg method. On English and Xitsonga data, we show that ES-KMeans outperforms SylSeg in word segmentation and gives similar scores to BES-GMM, while being five times faster. However, the cluster purity of ES-KMeans falls behind that of the other two models. We show that the higher purity for BES-GMM results from a tendency towards smaller clusters which, unlike in ES-KMeans, can also be varied using hyperparameters.

2. The embedded segmental $k$-means model
Starting from standard $k$-means, we describe the embedded segmental $k$-means (ES-KMeans) objective function and algorithm.

2.1. From $k$-means to ES-KMeans objective function
Given a speech utterance consisting of acoustic frames $y_{1:M} = y_1, y_2, \ldots, y_M$ (e.g., MFCCs), our aim is to break the sequence up into word-like segments, and to cluster these into hypothesized word types.

If we knew the segmentation (i.e., where word boundaries occur), the data would consist of several segments of different durations, as shown at the bottom of Figure 1. To cluster these,
we need a method to compare variable-length vector sequences. One option would be to use an alignment-based distance measure, such as dynamic time warping. Here we instead follow an acoustic word embedding approach [11, 28], an embedding function \( f_e \) is used to map a variable length speech segment to a single embedding vector \( x \in \mathbb{R}^D \) in a fixed-dimensional space, i.e., segment \( y_{t_1:t_2} \) is mapped to a vector \( x_i = f_e(y_{t_1:t_2}) \), illustrated as coloured horizontal vectors. The idea is that speech segments that are acoustically similar should lie close together in \( \mathbb{R}^D \), allowing segments to be efficiently compared directly in the embedding space without requiring alignment.

Embedding all segments in the data set would give a set of vectors \( \mathcal{X} = \{x_i\}_{i=1}^N \), which could be clustered into \( K \) hypothesized word classes using k-means, as shown at the top of Figure 1.

Standard k-means aims to minimize the sum of squared euclidean distances to each cluster mean: 
\[
\min_{\mu_1, \ldots, \mu_K} \sum_{c=1}^K \sum_{x \in \mathcal{X}_c} \|x - \mu_c\|^2 
\]
where \( \{\mu_1, \ldots, \mu_K\} \) are the cluster means, \( \mathcal{X}_c \) are all vectors assigned to cluster \( c \), and element \( z_i \) in \( x \) indicates which cluster \( x_i \) belongs to. The standard algorithm alternates between reassigning vectors to the closest cluster means, and then updating the means.

Standard k-means would be appropriate if the segmentation was known, but this is not the case in this zero-resource setting. Rather, the embeddings \( \mathcal{X} \) can change depending on the current segmentation. For a data set of \( S \) utterances, we denote the segmentations as \( \mathcal{Q} = \{q_i\}_{i=1}^S \), where \( q_i \) indicates the boundaries for utterance \( i \). \( \mathcal{X}(\mathcal{Q}) \) is used to denote the embeddings under the current segmentation. Our aim now is to jointly optimize the cluster assignments \( z \) and the segmentation \( \mathcal{Q} \). Under what objective should these be optimized?

One option would be to extend the standard k-means objective and optimize 
\[
\min_{\mathcal{Q}, z} \sum_{c=1}^K \sum_{x \in \mathcal{X}_c \cap \mathcal{X}(\mathcal{Q})} \|x - \mu_c\|^2
\]
where \( \mathcal{X}_c \cap \mathcal{X}(\mathcal{Q}) \) are embeddings assigned to cluster \( c \) under segmentation \( \mathcal{Q} \). But this is problematic: imagine the extreme of inserting no boundaries over an utterance, resulting in a single embedding and only a single term in the summation; any other segmentation would result in more terms in the summation, likely giving an overall worse score—even if all embeddings are close to cluster means. Instead of assigning a score per segment, we deal with this by assigning a score per frame. This score is given by the score achieved by the segment to which that frame belongs, implying that segment scores are weighted by duration:
\[
\min_{\mathcal{Q}, z} \sum_{c=1}^K \sum_{x \in \mathcal{X}_c \cap \mathcal{X}(\mathcal{Q})} \text{len}(x) \|x - \mu_c\|^2
\]

where \( \text{len}(x) \) is the number of frames in the sequence on which embedding \( x \) is calculated.

The overall ES-KMeans algorithm initializes word boundaries randomly, and then optimizes (1) by alternating between optimizing segmentation \( \mathcal{Q} \) while keeping cluster assignments \( z \) and means \( \{\mu_1, \ldots, \mu_K\} \) fixed (top to bottom in Figure 1), and then optimizing the cluster assignments and means while keeping the segmentation fixed (bottom to top in the figure).

2.2. Segmentation

Under a fixed clustering \( \mathcal{Q} \), the objective (1) becomes
\[
\min_{z} \sum_{x \in \mathcal{X}(\mathcal{Q})} \text{len}(x) \|x - \mu^*_z\|^2 = \min_{z} \sum_{x \in \mathcal{X}(\mathcal{Q})} d(x)
\]
where \( \mu^*_z \) is the mean of the cluster to which \( x \) is currently assigned (according to \( z \)), and \( d(x) \triangleq \text{len}(x) \|x - \mu^*_z\|^2 \) is the “score” of embedding \( x \) (lower \( d \) is better).

Equation (2) can be optimized separately for each utterance, so we want to find the segmentation \( q_i \) for each utterance that gives the minimum of the sum of the scores of the embeddings under that segmentation. This is exactly the problem addressed by the shortest-path algorithm (Viterbi) which uses dynamic programming to solve this problem efficiently [33, §2.17].

Let \( q_i \) be the number of frames in the hypothesized segment (word) that ends at frame \( t \): if \( q_i = j \), then \( y_{t-j+1:t} \) is a word.\footnote{For an utterance with frames \( y_{1:M} \), a sequence of \( q \)'s ending with \( q_M \) specifies a unique segmentation; boldface \( q \) is this sequence of \( q \)’s.}

We define forward variables \( \gamma[t] \) as the optimal score up to boundary position \( t \):
\[
\gamma[t] = \min_{j=1}^{t-1} \{d(f_e(y_{t-j+1:j})) + \gamma[t-j] \}
\]

Starting with \( \gamma[0] = 0 \), we calculate (3) for \( 1 \leq t \leq M - 1 \). We keep track of the optimal choice (arg min) for each \( \gamma[t] \), and the overall optimal segmentation is then given by starting from the final position \( t = M \) and moving backwards, repeatedly choosing the optimal boundary.

2.3. Cluster assignments and mean updates

For a fixed segmentation \( \mathcal{Q} \), the objective (1) becomes
\[
\min_{z} \sum_{c=1}^K \sum_{x \in \mathcal{X}_c \cap \mathcal{X}(\mathcal{Q})} \text{len}(x) \|x - \mu_c\|^2
\]

When the means \( \{\mu_1, \ldots, \mu_K\} \) are fixed, the optimal reassignments (5) follow standard k-means, and are guaranteed to improve (1) since the distance between the embedding and its assigned cluster mean never increases:
\[
z_i = \arg \min_{c} \{\text{len}(x_i) \|x_i - \mu_c\|^2 \} = \arg \min_{c} \|x_i - \mu_c\|^2
\]

Finally, we fix the assignments \( z \) and update the means:
\[
\mu_c = \frac{1}{\sum_{x \in \mathcal{X}_c} \text{len}(x)} \sum_{x \in \mathcal{X}_c} \text{len}(x) x \approx \frac{1}{N_c} \sum_{x \in \mathcal{X}_c} x
\]
the approximation, which is exact if all segments have the same duration, to again match standard k-means, with \( N \), the number of embeddings currently assigned to cluster \( c \). The complete ES-KMeans algorithm is given below. Since the segmentation, clustering and mean updates each improve (1), the algorithm will converge to a local optimum.

Algorithm 1 The embedded segmental k-means algorithm.

1: Initialize segmentation \( Q \) randomly.
2: Initialize cluster assignments \( \pi \) randomly.
3: repeat
4: for \( i = \text{randperm}(1 \text{ to } S) \) do
5: Select utterance \( i \)
6: t \leftarrow M_i
7: while \( t \geq 1 \) do
8: \( q_t \leftarrow \arg \min_{j=1}^{M_i} \{ d(f_t, y_{j-1:t-1}) + \gamma[t-j] \} \)
9: \( t \leftarrow t - q_t \)
10: end while
11: Assign new embeddings \( X(q_t) \) to clusters using (5).
12: Update means using (6).
13: end for
14: until convergence

2.4. The Bayesian embedded segmental GMM

In previous work [20, 22], we proposed a very similar model, but instead of k-means, we used a Bayesian GMM as whole-word clustering component (top of Figure 1). This Bayesian embedded segmental GMM (BES-GMM) served as inspiration for ES-KMeans; we briefly discuss their relationship here.

A Bayesian GMM treats its mixture weights \( \pi \) and component means \( \mu_k \) as random variables rather than point estimates, as is done in a regular GMM. We use conjugate priors: a Dirichlet prior over \( \pi \) and a spherical-covariance Gaussian prior over \( \mu_k \). All components share the same fixed spherical covariance matrix \( \Sigma \). The model is then formally defined as:

\[
\pi \sim \text{Dir}(\alpha/K) \quad (7) \quad \mu_k \sim \mathcal{N}(\mu_0, \sigma_0 \Sigma) \quad (9) \\
z_i \sim \pi \quad (8) \quad \kappa \sim \mathcal{N}(\mu_\kappa, \sigma_\kappa \Sigma) \quad (10)
\]

Under this model, component assignments and a segmentation can be inferred jointly using a collapsed Gibbs sampler [34]. Full details are given in [20], but the Gibbs sampler looks very similar to Algorithm 1: the Bayesian GMM gives likelihood terms ("scores") in order to find an optimal segmentation, while the segmentation hypothesizes the boundaries for the word segments which are then clustered using the GMM. However, for BES-GMM, component assignments and segmentation are sampled probabilistically, instead of making hard decisions.

The link between the two models emerges asymptotically. It can be shown that standard k-means results from a GMM as the variances approach zero [35, 36]. In a similar way it can be shown that the Gibbs sampling equations for segmentation and component assignments for BES-GMM (as given in [20]) approach (3) and (5), respectively, in the limit \( \sigma^2 \to 0 \), when all other hyperparameters are fixed.

Without giving a full complexity analysis, we note that because ES-KMeans only considers the closest cluster, it is more efficient than BES-GMM, where all components are considered when assigning embeddings to clusters and during segmentation (since embedding "scores" are obtained by marginalizing over all components). ES-KMeans can also be trivially parallelized, since both segmentation and cluster assignment can be performed in parallel for each utterance. This parallelized algorithm is still guaranteed to converge, though possibly to a different local optimum than Algorithm 1 since updates are in a different order. Parallelizing BES-GMM is also possible, but the guarantee of converging to the true posterior distribution is lost [27]. We do not consider parallelization in this work, but rather keep the two algorithms as close as possible (using the same update order) to allow for direct comparison.

2.5. Heuristic recurring syllable-unit word segmentation

We also compare to the ZRSC submission of Räsänen et al. [23]. Their system, which we refer to as SylSeg, relies on a novel cognitively motivated unsupervised method that predicts boundaries for syllable-like units, and then clusters these units on a per-speaker basis. Using a bottom-up greedy mapping, recurring syllable cluster sequences are then predicted as words.

SylSeg has the benefit of being much simpler in terms of computational complexity and implementation than ES-KMeans or BES-GMM. But, in contrast to the heuristic methodology followed in SylSeg, both ES-KMeans and BES-GMM have clear overall objective functions that they optimize, the one using hard clustering, the other using Bayesian inference.

3. Experiments

3.1. Experimental setup and evaluation

Evaluation is performed on the two ZRSC data sets: an English corpus of around 5 hours of speech from 12 speakers, and a Xitsonga corpus of around 2.5 hours from 24 speakers [37]. We also use a separate English set of around 6 hours for development.

As in [20, 22, 38], we use several metrics to evaluate against ground truth forced alignments. By mapping every discovered word token to the ground truth token with which it overlaps most and then mapping every cluster to its most common word, average cluster purity and unsupervised word error rate (WER) can be calculated. By instead mapping every token to the true phoneme sequence with which it overlaps most, the normalized edit distance (NED) between all segments in the same cluster can be calculated; lower NED is better, with scores from 0 to 1. Word boundary F-score evaluates segmentation performance by comparing proposed and true word boundaries; similarly, word token F-score measures the accuracy of proposed word token intervals. Word type F-score compares the set of unique phoneme mappings (obtained as for NED) to the set in the true lexicon. See [38] for full details.

Our implementation of ES-KMeans follows as closely as possible that of BES-GMM in [22]. Both use uniform downsampling as embedding function \( f: \) a segment is represented by keeping 10 equally spaced MFCCs and flattening these [39]. Both models use unsupervised syllable pre-segmentation [23] to limit allowed word boundaries. \( K \) is set to 20% of the number of first-pass segmented syllables. Word candidates are limited to span at most 6 syllables, and at least 200 ms. For BES-GMM we use simulated annealing, an all-zero vector for \( \mu_0, \sigma_0^2 = \sigma^2 / \kappa_0, \kappa_0 = 0.05, \alpha = 1, \sigma^2 = 0.001 \). See [22] for full details.

3.2. Results and analysis

Table 1(a) shows the performance of the three models on the English and Xitsonga corpora. Some of the SylSeg scores are unknown since these were not part of the ZRSC evaluation [23]. Compared to BES-GMM, ES-KMeans achieves worse purity, WER and NED, but similar boundary, token and type F-scores. This comes with a 5× improvement in runtime. ES-KMeans

\footnote{We allow more than one cluster to be mapped to the same word.}
Table 1: (a) Performance of models on the two test corpora. Lower NED is better. Runtimes for SylSeg* are rough estimates, obtained from personal communication with the authors [23]. (b) English development set performance of BES-GMM as the variance is varied.

<table>
<thead>
<tr>
<th></th>
<th>English (%)</th>
<th>Xitsonga (%)</th>
</tr>
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<tr>
<td></td>
<td>SylSeg</td>
<td>ES-KMeans</td>
</tr>
<tr>
<td>Cluster purity</td>
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</tr>
<tr>
<td>WER</td>
<td>-</td>
<td>73.2</td>
</tr>
<tr>
<td>NED</td>
<td>71.1</td>
<td>71.6</td>
</tr>
<tr>
<td>Boundary $F$</td>
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<td>62.2</td>
</tr>
<tr>
<td>Token $F$</td>
<td>12.4</td>
<td>18.1</td>
</tr>
<tr>
<td>Type $F$</td>
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<td>18.9</td>
</tr>
<tr>
<td>Runtime (s)</td>
<td>100*</td>
<td>193</td>
</tr>
</tbody>
</table>

Figure 2: The 5 biggest clusters for ES-KMeans and BES-GMM. Circle radii are according to cluster size; shading indicates purity. The cluster-to-word mapping is also shown.

Figure 3: Spectrograms for random tokens from the cluster of ES-KMeans mapped to “be” in Figure 2. The portion of each true word which is covered by the segment is shown in bold.

When reading the results in Table 1(b) from bottom to top, note that the results of BES-GMM do not seem to converge to ES-KMeans, even though $\sigma^2$ is tending towards 0 (where the two models should be equivalent). This is because, based on the recommendation in [40], we set the variance of the prior on the component means of BES-GMM as $\sigma^2 = \sigma^2 / \kappa_0$ (see end of §3.1), so the prior variance on the component means is tied to the fixed data variance. Under these conditions, the asymptotic equivalence of BES-GMM and ES-KMeans no longer holds. Murphy [40] explains that this coupling is a sensible way to incorporate prior knowledge of the typical spread of data, and here we indeed show how this helps our Bayesian model; this principled way of including priors is not possible in ES-KMeans or SylSeg. When setting $\sigma^2 = 1$ (rather than tying it), the results of BES-GMM match ES-KMeans when $\sigma^2 = 0.00001$.

4. Conclusion

We introduced the embedded segmental $k$-means model (ES-KMeans), a method that falls in between the fully Bayesian embedded segmental GMM (BES-GMM) and the cognitively motivated heuristic SylSeg method. Its word segmentation performance is on par with BES-GMM and superior to SylSeg, but cluster purity is worse than both other methods. In terms of efficiency, it is 5 times faster than BES-GMM, but half as fast as SylSeg. Despite using hard clustering and segmentation, ES-KMeans still has a clear objective function and it is guaranteed to converge (to a local optimum), in contrast to SylSeg. It also has far fewer hyperparameters than BES-GMM, although we show that this is what gives the latter the upper hand. How to balance these trade-offs between speed, performance and having a clear objective will ultimately depend on the downstream task on which the model is applied.

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5. References


