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Impact of Device Orientation on Error Performance of LiFi Systems

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Abstract—Most studies on optical wireless communications (OWCs) have neglected the effect of random orientation in their performance analysis due to the lack of a proper model for the random orientation. Our recent empirical-based research illustrates that the random orientation follows a Laplace distribution for a static user equipment (UE). In this paper, we analyze the device orientation and assess its importance on system performance. The reliability of an OWC channel highly depends on the availability and alignment of line-of-sight (LOS) links. In this study, the effect of receiver orientation including both polar and azimuth angles on the LOS channel gain are analyzed. The probability of establishing a LOS link is investigated and the probability density function (PDF) of signal-to-noise ratio (SNR) for a randomly-oriented device is derived. By means of the PDF of SNR, the bit-error ratio (BER) of DC-biased optical orthogonal frequency division multiplexing (DCO-OFDM) in additive white Gaussian noise (AWGN) channels is evaluated. A closed-form approximation for the BER of UE with random orientation is presented which shows a good match with Monte-Carlo simulation results. Furthermore, the impact of the UE’s random motion on the BER performance has been assessed. Finally, the effect of random orientation on the average signal-to-interference-plus-noise ratio (SINR) in a multiple access points (APs) scenario is investigated.

Index Terms—Random orientation, DCO-OFDM, bit-error ratio (BER), light-fidelity (LiFi), visible light communication (VLC).

I. INTRODUCTION

Statistical data traffic confirms that smartphones will generate more than 86% percent of the total mobile data traffic by 2021 [1]. Light-Fidelity (LiFi) as part of the future fifth generation can cope with this immense volume of data traffic [2]. LiFi is a bidirectional networked system that utilizes visible light spectrum in the downlink and infrared spectrum in the uplink [3]. LiFi offers remarkable advantages such as utilizing a very large and unregulated bandwidth, energy efficiency and enhanced security. These benefits have put LiFi in the scope of recent and future research [4]. The majority of studies on optical wireless communications assume that the device always faces vertically upwards. Although this may be for the purpose of analysis simplification or due to lack of a proper model for device orientation, in a real life scenario users hold their device in a way that feels most comfortable. Device orientation can affect the users’ throughput remarkably and it should be analyzed carefully. Even though a number of studies have considered the impact of random orientation in their analysis [5]–[13]. Device orientation can be measured by the gyroscope and accelerometer implemented in every smartphone [14]. Then, this information can be feedback to the access point (AP) by the limited-feedback schemes to enhance the system throughput [3], [15], [16].

The effect of random orientation on users’ throughput has been assessed in [5]. In order to tackle the problem of load balancing, the authors proposed a novel AP selection algorithm that considers the random orientation of user equipments (UEs). The downlink handover problem due to the random rotation of UE in LiFi networks is characterized in [6]. The handover probability and handover rate for static and mobile users are determined. The handover probability in hybrid LiFi/RF-based networks with randomly-oriented UEs is analyzed in [7]. The effect of tilting the UE on the channel capacity is studied and the lower and upper bounds of the channel capacity are derived in [8]. A theoretical expression of the bit-error ratio (BER) using on-off keying (OOK) has been derived in [9]. Then, a convex optimization problem is formulated based on the derived BER expression to minimize the BER performance by tilting the UE plane properly. A similar approach is used in [10] by finding the optimal tilting angle to improve both the signal-to-noise ratio (SNR) and spectral efficiency of M-QAM orthogonal frequency division multiplexing (OFDM) for indoor visible light communication (VLC) systems. Impacts of both UE’s orientation and position on link performance of VLC are studied in [11]. The outage probability is derived and the significance of UE orientation on inter-symbol interference is shown. The optimum polar and azimuth angles for single user multiple-input multiple-output (MIMO) OFDM is calculated in [12]. A receiver with four photodetectors (PD) is considered and the optimal angles for each PD are computed. In [13], the impact of the random orientation on the line-of-sight (LOS) channel gain for a randomly located UE is studied. The statistical distribution of the channel gain is presented for a single light-emitting diode (LED) and extended to a scenario with double LEDs. All mentioned studies assume a predefined model for the random orientation of the receiver. However, little or no evidence is presented to justify the assumed models. For the first time, experimental measurements are carried out to model the polar and azimuth angles in [17]–[19]. It is shown that the polar angle can be modeled by either the Laplace distribution (for static users) or the Gaussian distribution (for mobile users) while the azimuth angle follows a uniform distribution. Solutions to alleviate the impact of device random orientation on received SNR and throughput are proposed in [20]–[22]. In [20], the statistics of Euler rotation angles are provided based on the experimental measurements. Then, simulations of BER performance for spatial modulation using a multi-directional receiver configuration with consideration of random device orientation is evaluated in [21]. other
multiple-input multiple-output (MIMO) techniques in the presence of random orientation are studied. The authors in [22], proposed an omni-directional receiver which is not affected by device random orientation. It is shown that the omni-directional receiver reduces the SNR fluctuations and improves the user throughput remarkably. All these studies emphasize the significance of incorporating the random rotation into the analysis.

We characterize the device random orientation and investigate its effect on the users’ performance metrics such as SNR and BER in optical wireless systems. We also derive the probability density function (PDF) of SNR for randomly-orientated device. Based on the derived PDF of SNR, the BER performance of a DC biased optical OFDM (DCO-OFDM) is evaluated as a use case. A closed form approximation for BER is proposed. The impact of device orientation on BER with some interesting observations are investigated. In this study, we only consider the LOS channel gain, and the impact of higher reflections on BER performance has been investigated in our recent study [23].

Notations: |·| expresses the absolute value of a variable; $\tan^{-1}(y/x)$ is the four-quadrant inverse tangent. Further, $[·]^T$ stands for transpose operator. We note that throughout this paper, unless otherwise mentioned, angles are expressed in degrees. The Gaussian distribution with mean, $\mu_G$, and variance, $\sigma^2_G$, is denoted by $\mathcal{N}(\mu_G, \sigma^2_G)$.

II. SYSTEM MODEL

A. LOS Channel Gain

An open indoor office without reflective objects for optical wireless downlink transmission is considered in this study. The geometric configuration of the downlink transmission is illustrated in Fig. 1. It is assumed that an LED transmitter (or AP) is a point source that follows the Lambertian radiation pattern. Furthermore, the LED is supposed to operate within the linear dynamic range of the current-power characteristic curve to avoid the nonlinear distortion effect. The LED is fixed and oriented vertically downward.

The direct current (DC) gain of the LOS optical wireless channel between the AP and the UE is given by [24]:

$$ H = \frac{(m+1)A_{PD}}{2\pi d^2} \frac{g_t \cos^m \phi \cos \psi \cdot \text{rect} \left( \frac{\psi}{\Psi_c} \right)}{d^2}, $$

where $\text{rect}(\frac{\psi}{\Psi_c}) = 1$ for $0 \leq \psi \leq \Psi_c$ and 0 otherwise; $A_{PD}$ is the PD physical area; the Euclidean distance between the AP and the UE is denoted by $d$ with $(x_a, y_a, z_a)$ and $(x_u, y_u, z_u)$ as the position of the AP and UE in the Cartesian coordinate system, respectively; the Lambertian order is $m = \frac{1}{\log_2(\cos \Phi_{1/2})}$ where $\Phi_{1/2}$ is the transmitter semiangle at half power. The incidence angle with respect to the normal vector to the UE surface, $n_u$, and the radiance angle with respect to the normal vector to the AP surface, $n_{tx} = [0, 0, -1]$, are denoted by $\phi$ and $\psi$, respectively. These two angles can be obtained by using the analytical geometry rules as $\cos \phi = \mathbf{d} \cdot \mathbf{n}_{tx}/d$ and $\cos \psi = -\mathbf{d} \cdot \mathbf{n}_u/d$ where $\mathbf{d}$ is the distance vector from the AP to the UE and "·" is the inner product operator. The gain of the optical concentrator is given as $g_t = \frac{\varsigma^2}{\sin^2 \Psi_c}$ with $\varsigma$ being the refractive index and $\Psi_c$ is the UE field of view (FOV). After some simplifications, (1) can be written as:

$$ H = \frac{H_0 \cos \psi}{d^m+2} \cdot \text{rect} \left( \frac{\psi}{\Psi_c} \right), $$

where $H_0 = \frac{(m+1)A_{PD}g_t h^m}{2\pi}$; and $h = |z_u - z_a|$ is the vertical distance between the UE and the AP as shown in Fig. 1.

B. Rotation in the Space

A convenient way of describing the orientation is to use three separate angles showing the rotation about each axes of the rotating local coordinate system (intrinsic rotation) or the rotation about the axes of the reference coordinate system (extrinsic rotation). Current smartphones are able to report the elemental intrinsic rotation angles yaw, pitch and roll denoted as $\alpha$, $\beta$ and $\gamma$, respectively [25]. Here, $\alpha$ represents rotation about the $z$-axis, which takes a value in range of $[0, 360)$; $\beta$ denotes the rotation angle about the $x$-axis, that is, tilting the device toward or away from the user, which takes value between $-180^\circ$ and $180^\circ$; and $\gamma$ is the rotation angle about the $y$-axis, that is, tilting the device right or left, which is
between the projection of $Z_{n}$ shown in Fig. 1, $	heta$ angles. That is, spherical coordinate system using the azimuth, top this page.

Before analyzing user’s performance metrics such as average SNR and BER, let us define the critical elevation (CE), $\theta_{ce}$, which defines the elevation angle at the boundary of the field of view of the receiver. As shown in Fig. 3, the CE angle for a given position of UE, $(x_u, y_u)$, and user’s direction, $\Omega$, is the elevation angle for which $\psi = \Psi_c$. Thus, $\theta \geq \theta_{ce}$ results in $\psi \geq \Psi_c$, and the channel gain would be zero based on (1). This angle depends on both the UE position and its direction, $\Omega$ which is given as follows:

$$\theta_{ce} = \cos^{-1}\left(\frac{\cos \Psi_c}{\sqrt{\lambda_1^2 + \lambda_2^2}}\right) + \tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right),$$

where the coefficients $\lambda_1$ and $\lambda_2$ are given as:

$$\lambda_1 = \frac{r}{d} \cos\left(\Omega - \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right)\right),$$

$$\lambda_2 = \frac{h}{d},$$

where $r = \sqrt{(x_u - x_a)^2 + (y_u - y_a)^2}$ is the horizontal distance between the AP and the UE. Proof of (6) is provided in Appendix A. As can be seen from (7), the parameter $\lambda_1$ contains the direction angle, $\Omega$. The physical concept of positive $\lambda_1$ is that the UE is facing to the AP while if it is not facing to the AP, $\lambda_1$ is negative. On the other hand, since always $z_a < z_u$, we have $\lambda_2 > 0$. It should be mentioned that the acceptable range for $\theta_{ce}$ is $[0, 90]$ as the polar angle, $\theta$, given in (4) takes values between $0^\circ$ and $90^\circ$. Note that for a given location of UE, the minimum CE angle, $\theta_{th}$, is obtained for $\Omega = \pi + \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right) \equiv \Omega_{th}$ which is given as:

$$\theta_{th} = \Psi_c + \sin^{-1}\left(\frac{h}{d}\right) - \frac{\pi}{2}.$$
TABLE I: Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AP location</td>
<td>$(x_a, y_a, z_a)$</td>
<td>$(0, 0, 2)$</td>
</tr>
<tr>
<td>LED half-intensity angle</td>
<td>$\Psi_{1/2}$</td>
<td>$60^\circ$</td>
</tr>
<tr>
<td>PD responsivity</td>
<td>$R_{PD}$</td>
<td>1 A/W</td>
</tr>
<tr>
<td>Physical area of a PD</td>
<td>$A_{PD}$</td>
<td>1 cm$^2$</td>
</tr>
<tr>
<td>Refractive index</td>
<td>$n$</td>
<td>1</td>
</tr>
<tr>
<td>Downlink bandwidth</td>
<td>$B$</td>
<td>10 MHz</td>
</tr>
<tr>
<td>Number of subcarriers</td>
<td>$N_s$</td>
<td>1024</td>
</tr>
<tr>
<td>Noise power spectral density</td>
<td>$N_0$</td>
<td>$10^{-21}$ A$^2$/Hz</td>
</tr>
<tr>
<td>Conversion factor</td>
<td>$\eta$</td>
<td>3</td>
</tr>
<tr>
<td>Vertical distance of UE and AP</td>
<td>$h$</td>
<td>2 m</td>
</tr>
</tbody>
</table>

Fig. 5: The effect of changing $\Omega$ and $\theta$ on the LOS channel gain with $\Psi_e = 90^\circ$, for different positions and elevation angles $\theta = 41^\circ$ (solid lines), $\theta = \theta_{th}$ (dash lines).

Fig. 6: The effect of different FOV on having a zero LOS, $Pr\{H = 0\}$. Also $\Omega_1 = \min\{\Omega_1, \Omega_2\} = \max\{\Omega_1, \Omega_2\}$, where:

$$\Omega_1 = \cos^{-1}\left(\cos \Psi_e - \kappa_2\right) + \tan^{-1}\left(\frac{y_a - y_a}{x_u - x_a}\right)$$

and

$$\Omega_2 = -\cos^{-1}\left(\cos \Psi_e - \kappa_2\right) + \tan^{-1}\left(\frac{y_a - y_a}{x_u - x_a}\right).$$

Proof: See Appendix B

The LOS channel gain versus $\Omega$ for locations of $L_1$ and $L_5$ (see the inset of Fig. 4) with $\theta = \theta_{th}$ (dash line) and $\theta = 41^\circ \geq \theta_{th}$ (solid line) are shown in Fig. 5. Note that for $L_1$ and $L_5$, we have $\theta_{th} = 25.24^\circ$ and $\theta_{th} = 25.88^\circ$, respectively. As can be seen, if $\theta < \theta_{th}$, then, $\forall \Omega \in [0, 360)$, LOS channel gain is always non-zero (dash lines). Based on the proposition, the range of $\Omega$ for which the LOS channel gain is non-zero with $\theta = 41^\circ > \theta_{th}$ is $[0, 167.8) \cup [282.2, 360]$ for $L_1$ and $[70.1, 318]$ for $L_2$.

It can be inferred that the range of $\Omega$ for which the LOS channel is active. Let’s denote this range as $R_{\Omega, \theta}$. This range can be determined according to the following Proposition.

Proposition. For a given UE’s location, the range of $\Omega$ for which the LOS channel gain is non-zero is $[0, 2\pi]$ if $\theta$ is smaller than or equal to a threshold angle $\theta_{th} = \Psi_e + \sin^{-1}\left(\frac{h}{d}\right) - \frac{\pi}{2}$. Otherwise it is given as:

$$R_{\Omega, \theta} = \begin{cases} [0, \Omega_1) \cup (\Omega_2, 2\pi), & \text{if } \Lambda'(\Omega_1) < 0 \setminus (\Omega_1, \Omega_2), & \text{if } \Lambda'(\Omega_1) \geq 0, \end{cases}$$

where $\Lambda'(\Omega) = -\kappa_1 \sin\left(\Omega - \tan^{-1}\left(\frac{y_a - y_a}{x_u - x_a}\right)\right) + \kappa_2$ with:

$$\kappa_1 = \frac{r}{d} \sin \theta, \quad \kappa_2 = \frac{h}{d} \cos \theta.$$
can be obtained as:
\[
R_{\Omega} = R_{\Omega,\theta}|_{\theta=90^\circ}.
\] (12)

Proof of this corollary is similar to the proof of proposition 1. Noting that the worst elevation angle that leads to the smallest range of \( \Omega \) is \( \theta = 90^\circ \). The physical concept of \( R_{\Omega} \) is that when the UE faces the AP, we have \( \Omega \in R_{\Omega} \). Otherwise, if the UE faces the opposite direction of the AP, \( \Omega \not\in R_{\Omega} \). In fact, \( R_{\Omega} \) provides a stable range for which the user can change the elevation angle between 0 and 90 without experiencing the AP out of its FOV. We note that the range given in (12) is valid if \( \Psi_e \geq \cos^{-1}(\frac{\dot{\gamma}}{r}) \) (this condition can be readily seen by substituting \( \theta = 90^\circ \) in (10) and then replacing the results in (11)).

**IV. Bit-Error Ratio Performance**

In this section, we evaluate the BER performance of DCO-OFDM in LiFi networks. We initially derive the SNR statistics on each subcarrier, then based on the derived PDF of SNR, the BER performance is assessed. Note that the PDF of the SNR derived in this study is the conditional PDF given the location and direction of the UE. Therefore, having the statistics of the user location, the joint PDF of the SNR with respect to both UE orientation and location can be readily obtained.

**A. SNR Statistics**

The received electrical SNR on \( k \)-th subcarrier of a LiFi system can be acquired as:
\[
S = \frac{R_{PD}^2 H^2 P_{opt}^2}{(K-2)\eta^2 \sigma_k^2},
\] (13)
where the PD responsivity is denoted by \( R_{PD} \); \( H \) is the LOS channel gain given in (1); \( P_{opt} \) is the transmitted optical power; \( K \) is the total number of subcarriers with \( K/2 - 1 \) subcarriers bearing information. Furthermore, \( \eta \) is the conversion factor [26]. The condition \( \eta = 3 \) can guarantee that less than 1% of the signal is clipped so that the clipping noise is negligible [3, 27]. In (13), \( \sigma_k = N_0 B/K \) is the noise power on \( k \)-th subcarrier where \( N_0 \) stands for the noise spectral density and \( B \) represents the modulation bandwidth. Based on the experimental measurement of the device orientation, it is shown in [17] that the LOS channel gain, \( H \), follows a clipped Laplace distribution as:
\[
f_H(h) = \exp\left( \frac{-h - \mu_H}{b_H} \right) - \exp\left( \frac{-h_{\min} - \mu_H}{b_H} \right), \quad h_{\min} \leq h \leq \mu_H,
\]
\[
2 - \exp\left( \frac{-h - \mu_H}{b_H} \right) - \exp\left( \frac{-h_{\min} - \mu_H}{b_H} \right), \quad h_{\min} \leq \mu_H \leq h,
\]
\[
\exp\left( \frac{-h_{\min} - \mu_H}{b_H} \right) - \exp\left( \frac{-h - \mu_H}{b_H} \right), \quad \mu_H \leq h_{\min} \leq h.
\]
where \( h_{\min} = \sqrt{b_H^2/2} \). The parameters \( \mu_H \) and \( \sigma_H \) are the mean and standard deviation of the elevation angle, which are obtained based on the experimental measurements. For static users, they are reported as \( \mu_H = 41^\circ \) and \( \sigma_H = 7.68^\circ \). Proof of (15) is provided in Appendix C. Furthermore, for the detailed proof of (14), we refer to Eq. (56) and (57) of [17]. The mean and scale factor of channel gain, \( \mu_H \) and \( b_H \) respectively, are:
\[
\mu_H = \frac{H_0}{d_{m+2}} \left( \lambda_1 \sin \theta_H + \lambda_2 \cos \theta_H \right), \quad b_H = \frac{H_0}{d_{m+2}} b_0 \sqrt{\lambda_1^2 + \lambda_2^2},
\]

where \( H_0 \) is given below (2). The factors, \( \lambda_1 \) and \( \lambda_2 \), are given in [7]. The support range of \( f_H(h) \) is \( h_{\min} \leq h \leq h_{\max} \) where \( h_{\min} \) and \( h_{\max} \) are given as:
\[
h_{\min} = \begin{cases} H_0 \cos \Psi_e, & \cos \psi < \cos \Psi_e \\ \frac{H_0}{d_{m+2}} \min\{\lambda_1, \lambda_2\}, & \text{o.w} \end{cases}, \quad (18)
\]
\[
h_{\max} = \begin{cases} \frac{H_0}{d_{m+2}} \lambda_2, & \lambda_1 < 0 \\ \frac{H_0}{d_{m+2}} \sqrt{\lambda_1^2 + \lambda_2^2}, & \lambda_1 \geq 0 \end{cases}. \quad (19)
\]

The cumulative distribution function (CDF) of LOS channel gain can be also obtained by calculating the integral of (14), which is given as:
\[
F_H(h) = c_H + \exp\left( \frac{-h - \mu_H}{b_H} \right) - \exp\left( \frac{-h_{\min} - \mu_H}{b_H} \right), \quad h_{\min} \leq h \leq \mu_H,
\]
\[
2 - \exp\left( \frac{-h - \mu_H}{b_H} \right) - \exp\left( \frac{-h_{\min} - \mu_H}{b_H} \right), \quad h_{\min} \leq \mu_H \leq h,
\]
\[
\exp\left( \frac{-h_{\min} - \mu_H}{b_H} \right) - \exp\left( \frac{-h - \mu_H}{b_H} \right), \quad \mu_H \leq h_{\min} \leq h.
\]
where \( \mu_H = \sqrt{b_H^2/2} \) and with the support range of \( s \in (s_{\min}, s_{\max}) \), where \( s_{\min} = \sqrt{5} b_0 h_{\min}^2 \) and \( s_{\max} = \sqrt{5} b_0 h_{\min}^2 \), with \( h_{\min} \) and \( h_{\max} \) given in (18) and (19), respectively.

By calculating the integral, \( F_S(s) = \int_{s_{\min}}^{s} f_S(s) ds \), the CDF of SNR on \( k \)-th subcarrier can be obtained. The CDF of SNR can be also acquired by substituting \( h = \frac{s}{\sqrt{S_0}} \) in (20), i.e., \( F_S(s) = F_H\left( \frac{s}{\sqrt{S_0}} \right) \).

Fig. 7 shows the PDF and CDF of the received SNR obtained from analytical results compared with the
Monte-Carlo simulation results. The UE is located at position $L_1$, the transmitted optical power is $3.2$ W and UE’s FOV is $90^\circ$. The results are provided for two directions: $\Omega = 45^\circ$ and $\Omega = 225^\circ$. Other simulation parameters are given in Table II. As it can be seen, the analytical models for both PDF and CDF of the received SNR match the simulation results. The factor $c_H$ for $\Omega = 45^\circ$ is 0. This factor for $\Omega = 225^\circ$ is 0.975 for simulation results and 0.979 for analytical model. These results confirm the accuracy of the analytical model.

### B. BER Performance

In this subsection, we aim to evaluate the effect of UE orientation on the BER performance of a LiFi-enabled device as one use case. BER is one of the common metrics to evaluate the point-to-point communication performance. Assuming the M-QAM DCO-OFDM modulation, the average BER per subcarrier of the communication link can be obtained as \[ P_e = \int_{s_{\text{min}}}^{s_{\text{max}}} P_c(s) f_S(s) \, ds, \tag{22} \]
where $P_c$ determines the BER of M-QAM DCO-OFDM in additive white Gaussian noise (AWGN) channels, which can be obtained approximately as \[ P_c(s) \approx \frac{4}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{3s}{M-1}}\right), \tag{23} \] where $Q(\cdot)$ is the Q-function. Substituting (21) and (23) into (22) and calculating the integral from $s_{\text{min}}$ to $s_{\text{max}}$, we get the average BER of the $M$-QAM DCO-OFDM in AWGN channels with randomly-oriented UEs. After calculating the integral and some simplifications, the approximated average BER is given as:

\[
\hat{P}_e \approx \begin{cases} 
-\Delta_0 + \frac{1}{2}c_H c_M, & \mu_H \leq h_{\text{min}} \\
\hat{P}_e(s_{\text{min}}^2) + \frac{1}{2}c_H c_M, & h_{\text{min}} < \mu_H \leq h_{\text{max}} 
\end{cases} , \tag{24}
\]

where
\[
\Delta_0 = \frac{2}{\log_2 M} \left(1 - \frac{1}{\sqrt{M}}\right) \exp \left(\frac{\mu_H - h_{\text{min}}}{b_H}\right) \left(2 - \exp \left(-\frac{h_{\text{max}} - \mu_H}{b_H}\right)\right),
\tag{25}
\]

The proof is provided in Appendix D.

Note that if the UE is tilted optimally towards the AP, the BER is minimum. For any arbitrary location and direction of UE, the optimum tilt (OT) angle is defined as the angle that provides maximum channel gain \[ \bar{c}_H \]. This angle is $\theta_{\text{ot}} = \tan^{-1}\left(\frac{\lambda_H}{\lambda_2}\right)$ and the average BER for this tilt angle is $\hat{P}_e \approx \hat{P}_e(S_{\text{ot}}^2)$ (since $c_H = 0$).

Figure 8 illustrates the BER performance of 4-QAM DCO-OFDM for three scenarios: i) a vertically upward UE, ii)
a UE with a fixed polar angle and without random orientation
iii) a realistic scenario in which the polar angle follows a
Laplace distribution that considers the random orientation, i.e.,
\( \theta \sim \mathcal{L}(\mu_\theta, b_\theta) \). Here, we assume \( \mu_\theta = 41^\circ \) and \( b_\theta = 5.43^\circ \) as reported in [17] based on the experimental measurements.

Other simulation parameters are given in Table I. The results
are provided for the UE’s location of \( L_1 = (3, 3) \) and with
\( \Psi_e = 60^\circ \). For this location, \( \theta_{\text{rot}} \approx 65^\circ \). Some interesting
observations can be seen from the results shown in this figure.

As can be seen, for \( \Omega = 45^\circ \), the vertically upward UE falls
behind the other two scenarios. Because for \( \theta > 0 \), the UE
will be tilted towards the AP (see the results shown in Fig. [4]).
Also, the gap between the exact and approximate BER is
small which confirms the accuracy of the BER approximation.

One interesting observation is that after \( P_{\text{opt}} > 2 \) W
and \( P_{\text{opt}} > 2.5 \) W, the BER does not decrease and is saturated
for \( \theta = 41^\circ \) and \( \theta = \theta_{\text{rot}} \), respectively. This is due to
the constant term in (24), i.e., \( \frac{1}{2} \mathcal{H}_{\text{CM}} \), will be dominant compared
to the power-dependent term, i.e., \( P_e(s) \). In other words,
due to the random orientation, there are cases that LOS link is
out of the UE’s FOV and data is lost. These results highlight
the significance of considering the random orientation in the
performance assessment. The BER performance of second and
third scenarios can still be better if \( \theta = \theta_{\text{rot}} \approx 65^\circ \). For \( \theta = \theta_{\text{rot}} \)
the maximum LOS channel gain is achieved and under this
condition the BER is minimum. This fact underlines that the
device orientation is not always destructive. Furthermore, with
\( \theta = \theta_{\text{rot}} \) the UE’s random orientation has the minimum effect
on the BER. We note that for a given location and \( \Omega \), the \( P_e \)
given in (24) is always bounded to the BER of \( P_e(s) \) obtained
for \( \theta = \theta_{\text{rot}} \) as it provides the maximum LOS channel gain.

The BER results of \( \theta = \theta_{\text{rot}} \) are just provided for a comparison
purpose however, the users tends to keep their smartphone
with \( \theta = 41^\circ \) (when doing sitting activities) according to the
experimental measurements [17].

C. UE’s Random Motion

In this subsection, we will include the effect of UE’s random
motion even though the user is static in addition to the random
orientation on the BER performance. Note that here, the
random UE’s motion encompass small movements in \( x \), \( y \)
and \( z \) directions, which are modeled as Gaussian distributions.

Hence, the UE’s location at each realization is given as:

\[
(x_u, y_u, z_u) = (x_0, y_0, z_0) + (\Delta x, \Delta y, \Delta z),
\]

(26)

where \( \Delta x \sim \mathcal{N}(\mu_{\Delta x}, \sigma_{\Delta x}^2) \), \( \Delta y \sim \mathcal{N}(\mu_{\Delta y}, \sigma_{\Delta y}^2) \) and \( \Delta z \sim \mathcal{N}(\mu_{\Delta z}, \sigma_{\Delta z}^2) \).

The location \((x_0, y_0, z_0)\) denotes the mean
point that the UE fluctuates around. It is noted that typically
the variation of the UE’s height (along \( z \) axis) is less than the
variation along \( x \) and \( y \) axes.

Fig. [9] shows the effect of random motion along with random
orientation on the BER performance. In these simulations, we
assume that \( \sigma_x = \sigma_y = 5\sigma_z = \sigma \). The results are presented
for three values of \( \sigma \), which are 0.05 m, 0.1 m and 0.15 m.

Note that for \( \sigma = 0.15 \) m, the deviation of the UE’s location
from the mean point, \((x_0, y_0, z_0)\), can be in the range of
\(-3\sigma = -45 \text{ cm} \) to \( 3\sigma = 45 \text{ cm} \). This corresponds to high

![Fig. 9: The effect of random orientation with/without random motion on BER performance of a UE located at the arbitrary position of L1.](image_url)

UE’s motion which is very low probable for normal human
activities. Here, the modulation order is considered to be \( M = 4 \).
The simulations are carried out for a UE located at \( L_1 = (3, 3) \)
different \( \Omega \) and \( \mu_\theta \). The UE’s FOV is assumed to be
90° for these simulations. As it can be seen, with \( \sigma \in \{0.05, 0.1\} \), the gap between the results when random motion
is included, is indeed negligible. For the case of \( \Omega = 135^\circ \)
and \( \mu_\theta = 41^\circ \) and with \( \sigma = 0.15 \) m, the gap is still small.
For \( \Omega = 45^\circ \) and \( \mu_\theta = 41^\circ \) (or \( \mu_\theta = 65^\circ \)) with \( \sigma = 0.15 \) m, the
gap grows in high transmitted power.

D. Multiple APs Scenario

To investigate the effect of multiple APs on the error
performance of a randomly-oriented UE, we consider two
APs located at \((-2, 0)\) and \((2, 0)\) as shown in Fig. [10].

The signal-to-interference- plus-noise ratio (SINR) can be obtained
as:

\[
\Upsilon = \frac{R_{\text{PD}}^2 H_0^2 \rho_{\text{opt}}}{(K-2)\eta^2 (\sigma_k^2 + \Upsilon)}.
\]

(27)

where \( H_0 \) is the LOS channel gain between the desired AP and
the PD; \( \Upsilon \) is the interfering power from other APs on the \( k \)
subcarrier. Other parameters are defined below (13). Here, with
the consideration of two APs, the interference from the other
AP on the \( k \)th subcarrier is \( I = R_{\text{PD}}^2 H_{\text{in}}^2 \rho_{\text{opt}}/(\eta^2) \),
where \( H_{\text{in}} \) is the channel gain between the interfering AP
and the UE. Note that the desired AP is selected based on
the received signal intensity metric. Fig. [11] shows the

![Fig. 10: Geometry of two APs with interference consideration. APs are located at \((-2, 0)\) and \((2, 0)\) on the ceiling.](image_url)
average SINR versus different horizontal distances between the UE and first AP (as depicted in Fig. 10). The average is taken over different random orientations following a Laplace distribution based on the experimental measurements, i.e., $\theta \sim \mathcal{L}(41^\circ, 5.43^\circ)$. Note that mobility is not considered in these results and at each location the user is assumed to be sitting. The simulation parameters are given in Table I and the UE’s FOV is assumed to be $90^\circ$. The transmitted optical power per AP is supposed to be 1 W as multiple APs require lower transmit power to cover the room in comparison to the single AP case. The PDF of SINR for $r_h = 1$ with $\Omega = 0$ and $r_h = 4.5$ with $\Omega = 90^\circ$ are presented. For the former the average SINR is about 82 while for the latter, it is about 16. Note also that the PDF of SINR shows similar Laplacian distributions as in the SNR case.

V. CONCLUSIONS AND FUTURE WORKS

We analyzed the device orientation and assessed its importance on system performance. The PDF of SNR for randomly-orientated device is derived, and based on the derived PDF, the BER performance of DCO-OFDM in AWGN channel with randomly-orientated UEs is evaluated. An approximation for the average BER of randomly-oriented UEs is calculated that closely matches the exact one. The role of CE angle that guarantees having LOS link in the UE’s FOV is investigated. Furthermore, the significant impact of being optimally tilted towards the AP on the BER performance is shown. We also studied the effect of the UE’s random motion on the BER performance. We note that even though we considered DCO-OFDM, the methodology can be readily extended to other modulation schemes, which can be the focus of future studies. Furthermore, other performance metrics such as throughput and user’s quality of service can also be assessed. Also, the device orientation impact can be evaluated in a cellular network with consideration of non-line-of-sight links.

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APPENDIX

A. Proof of (6)

Recalling that $\cos \psi = -d \cdot n_u/d$, replacing for $d = [x_u - x_a, y_u - y_a, z_u - z_a]^T$ and $n_u = [\sin \theta \cos \omega, \sin \theta \sin \omega, \cos \theta]^T$ and also noting that $\omega = \Omega + \pi$, we have:

$$\cos \psi = \frac{(x_u - x_a) \sin \theta \cos \Omega + (y_u - y_a) \sin \theta \sin \Omega - (z_u - z_a) \cos \theta}{\sqrt{(x_u - x_a)^2 + (y_u - y_a)^2 + (z_u - z_a)^2}}$$

$$= \frac{\sqrt{(x_u - x_a)^2 + (y_u - y_a)^2 + (z_u - z_a)^2}}{\sqrt{x_u - x_a} + (y_u - y_a) + (z_u - z_a) \cos \psi} \cos \theta, \quad (28)$$

For a given location of UE and a fixed angle of $\Omega$, by using the simple triangular rules, $\cos \psi$ can be represented as:

$$\cos \psi = \lambda_1 \sin \theta + \lambda_2 \cos \theta = \sqrt{\lambda_1^2 + \lambda_2^2} \cos \left(\theta - \tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right)\right), \quad (29)$$

where $\lambda_1$ and $\lambda_2$ are given as:

$$\lambda_1 = \frac{r}{d} \cos \left(\Omega - \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right)\right), \quad (30)$$

$$\lambda_2 = \frac{h}{d} \cos \theta. \quad (31)$$

According to the definition of critical elevation angle, if $\theta = \theta_{ce}$, then, $\cos \psi = \cos \Psi_c$. Therefore, (29) results in:

$$\theta_{ce} = \cos^{-1}\left(\frac{\cos \Psi_c}{\sqrt{\lambda_1^2 + \lambda_2^2}}\right) + \tan^{-1}\left(\frac{\lambda_1}{\lambda_2}\right). \quad (32)$$

This completes the proof of the derivation of CE angle.

B. Proof of Proposition

For a given location of UE and a fixed elevation angle, one other representation of $\cos \psi$ given in (28) would be as a function of $\Omega$:

$$\cos \psi = k_1 \cos \left(\Omega - \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right)\right) + k_2 \cos \lambda(\Omega), \quad (32)$$

where the coefficients $k_1$ and $k_2$ are given as:

$$k_1 = \frac{r}{d} \sin \theta, \quad k_2 = \frac{h}{d} \cos \theta. \quad (33)$$

Note that since $\theta \in [0, 90]$, we have $k_1 \geq 0$ and $k_2 \geq 0$. As mentioned for $\theta = \theta_{ce}$, we have $\cos \psi = \cos \Psi_c$. Then, solving $\Lambda(\Omega) - \cos \Psi_c = 0$ for $\Omega$, the roots are $\Omega_{1,2} = \min\{\Omega_1, \Omega_2\}$ and $\Omega_{1,2} = \max\{\Omega_1, \Omega_2\}$, where $\Omega_1$ and $\Omega_2$ are given as follow:

$$\Omega_1 = \cos^{-1}\left(\frac{\cos \Psi_c - k_2}{k_1}\right) + \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right), \quad (34)$$

$$\Omega_2 = -\cos^{-1}\left(\frac{\cos \Psi_c - k_2}{k_1}\right) + \tan^{-1}\left(\frac{y_u - y_a}{x_u - x_a}\right). \quad (35)$$
For the special case of $\Psi_c = 90^\circ$, (34) is simplified as:

$$
\Omega_1 = \cos^{-1} \left( - \frac{h \cot \theta}{r} \right) + \tan^{-1} \left( \frac{y_a - y_b}{x_a - x_b} \right),
$$

$$
\Omega_2 = - \cos^{-1} \left( - \frac{h \cot \theta}{r} \right) + \tan^{-1} \left( \frac{y_a - y_b}{x_a - x_b} \right).
$$

Using the sinuous function properties if $\Lambda(\Omega) \leq 0$ for $\Omega \in [\Omega_{r1}, \Omega_{r2}]$, then the derivative of $\Lambda(\Omega)$ at $\Omega = \Omega_{r1}$ is negative, i.e., $\frac{d\Lambda(\Omega)}{d\Omega}|_{\Omega=\Omega_{r1}} < 0$. For simplicity of notation, let’s denote $\Lambda'(\Omega) = \frac{d\Lambda(\Omega)}{d\Omega}|_{\Omega=\Omega_{r1}}$. Using (32), we have $\Lambda'(\Omega) = - \kappa_1 \sin \left( \Omega - \tan^{-1} \left( \frac{y_a - y_b}{x_a - x_b} \right) \right) + \kappa_2$. Therefore, the range of $R_{\Omega}$ that guarantees $\Lambda'(\Omega) > 0$ would be $[0, \Omega_{r1}] \cup [\Omega_{r2}, 2\pi]$. Similarly, if $\Lambda(\Omega) \geq 0$ for $\Omega \in (\Omega_{r1}, \Omega_{r2})$, then the derivative of $\Lambda(\Omega)$ at $\Omega = \Omega_{r1}$ is positive, i.e., $\frac{d\Lambda(\Omega)}{d\Omega}|_{\Omega=\Omega_{r1}} > 0$. Consequently, in this case the range of $R_{\Omega}$ that ensures $\Lambda'(\Omega) > 0$ would be $[\Omega_{r1}, \Omega_{r2}]$. This completes the proof of Proposition.

### C. Proof of (15)

Using (29), the CDF of $\cos \Psi$ can be obtained as:

$$
F_{\cos \Psi}(\tau) = \Pr \{ \cos \Psi \leq \tau \} = \Pr \left\{ \sqrt{\lambda_1^2 + \lambda_2^2} \cos (\theta - \tan^{-1} \left( \frac{\lambda_1}{\lambda_2} \right)) \leq \tau \right\}
$$

$$
= 1 - F_\theta \left( \cos^{-1} \left( \frac{\tau}{\sqrt{\lambda_1^2 + \lambda_2^2}} \right) + \tan^{-1} \left( \frac{\lambda_1}{\lambda_2} \right) \right).
$$

where $F_\theta(\theta)$ is the CDF of the elevation angle, $\theta$. Under the assumption of Laplacian model for the elevation angle, $F_\theta(\theta)$ is given as [17]:

$$
F_\theta(\theta) = \begin{cases} 
\frac{1}{2} \exp \left( \frac{-\theta - \mu_\theta}{\beta_\theta} \right), & \theta < \mu_\theta \\
1 - \frac{1}{2} \exp \left( - \frac{-\theta - \mu_\theta}{\beta_\theta} \right), & \theta \geq \mu_\theta.
\end{cases}
$$

Note that with reported values for $\mu_\theta$ and $\beta_\theta$ from [17], we have $\left(G(\frac{\pi}{2}) - G(0)\right) \approx 1$. Therefore,

$$
F_\theta(\theta) \approx \begin{cases} 
\frac{1}{2} \exp \left( \frac{-\theta - \mu_\theta}{\beta_\theta} \right), & \theta < \mu_\theta \\
1 - \frac{1}{2} \exp \left( - \frac{-\theta - \mu_\theta}{\beta_\theta} \right), & \theta \geq \mu_\theta.
\end{cases}
$$

Finally, by recalling the definition of the CE angle given in (6), $F_{\cos \Psi_c}(\cos \Psi_c)$ can be approximately obtained as:

$$
F_{\cos \Psi_c}(\cos \Psi_c) \approx \begin{cases} 
1 - \frac{1}{2} \exp \left( \frac{-\theta - \mu_{\theta_c}}{\beta_{\theta_c}} \right), & \theta_c < \mu_{\theta_c} \\
\frac{1}{2} \exp \left( - \frac{-\theta - \mu_{\theta_c}}{\beta_{\theta_c}} \right), & \theta_c \geq \mu_{\theta_c}.
\end{cases}
$$

This completes the proof of (15).

### D. Proof of (24)

Substituting (21) and (23) into (22), we have:

$$
P_c = c_0 \int_{s_{\min}}^{s_{\max}} Q \left( \sqrt{\frac{3s}{M-1}} - \frac{1}{\sqrt{s}} \exp \left( - \left| \sqrt{s} - \sqrt{S_0 h_{\mu H}} \right| \right) \right) ds
$$

$$
+ c_{\text{HM}} \int_{s_{\min}}^{s_{\max}} Q \left( \sqrt{\frac{3s}{M-1}} \right) \delta(s) ds
$$

with $c_0$ and $c_{\text{HM}}$ given as:

$$
c_0 = \frac{2b_1 \sqrt{S_0}}{\sqrt{2 - \exp \left( - h_{\max} \mu_{\mu H} b_1 \right)}} ,
$$

$$
c_{\text{HM}} = \frac{4}{\log_2 M} \left( 1 - \frac{1}{\sqrt{M}} \right).
$$

Note that if $s_{\min} = 0$, the second integral in (40) is $c_{\text{HM}} Q(0) = \frac{\pi c_0 a_0}{2}$, and referring to the definition of $c_H$, it is zero for $s_{\min} > 0$. Thus, the second integral can be expressed as $\frac{\pi c_0 a_0}{2}$ and we need to simplify the first integral. For simplicity of notation, let define $c_1 = \sqrt{\frac{S_0}{M-1}}$, $c_2 = \sqrt{S_0} h_{\mu H}$ and $c_3 = \sqrt{S_0} b_1$. Furthermore, let $x = \sqrt{s}$, thus, the first integral in (40) can be rewritten as (42) given at the top of the next page. The right side of (42) is based on the behavior of PDF of SNR. It can be either single exponential (if $c_2 \geq \sqrt{s_{\min}}$) or double exponential (if $\sqrt{s_{\min}} < c_2 \leq \sqrt{s_{\max}}$), for example, see results shown in Fig. 7. Noting that

$$
\int Q(c_1 x) e^{-x^2} dx = \frac{c_0 e^{-c_1^2} Q(c_1)}{2} + \frac{c_0 e^{-c_1^2} Q(c_1 x - \frac{1}{2c_1 c_3})}{2} \left( 1 - 2Q \left( c_1 x - \frac{1}{2c_1 c_3} \right) \right)
$$

also for given values of $c_1$, $c_2$ and $c_3$, we have $Q(c_1 c_2) \approx Q(c_1 \sqrt{s_{\max}})$ and also since $h_{\mu H} >> b_1$, then, $e^{\frac{c_3}{c_2}} \approx c_0$. Hence, $P_c$ can be approximated by (43) presented at the top of the next page. By substituting for the values of $c_0$, $c_1$, $c_2$, $c_3$ and noting that $\sqrt{s_{\min}} = \sqrt{S_0} h_{\min}$ and $\sqrt{s_{\max}} = \sqrt{S_0} h_{\max}$ (43) can be rewritten as:

$$
P_c \approx \begin{cases} 
\frac{1}{2} \exp \left( \frac{-\theta - \mu_{\theta_c}}{\beta_{\theta_c}} \right), & \theta_c < \mu_{\theta_c} \\
1 - \frac{1}{2} \exp \left( - \frac{-\theta - \mu_{\theta_c}}{\beta_{\theta_c}} \right), & \theta_c \geq \mu_{\theta_c}.
\end{cases}
$$

This completes the proof of (24).

### REFERENCES


\[
\int_{\sqrt{s_{\min}}}^{\sqrt{s_{\max}}} Q(c_1 x) e^{-\frac{x-c_2}{\tau_3}} dx = \begin{cases} 
\int_{\sqrt{s_{\min}}}^{\sqrt{s_{\max}}} Q(c_1 x) e^{-\frac{x-c_2}{\tau_3}} dx, & c_2 \leq \sqrt{s_{\min}} \\
\int_{\sqrt{s_{\min}}}^{\sqrt{s_{\max}}} Q(c_1 x) e^{-\frac{x-c_2}{\tau_3}} dx + \int_{c_2}^{\sqrt{s_{\max}}} Q(c_1 x) e^{-\frac{x-c_2}{\tau_3}} dx, & \sqrt{s_{\min}} < c_2 \leq \sqrt{s_{\max}}
\end{cases}
\] (42)

\[
\begin{array}{ll}
P_e \approx & -c_0 c_3 + \frac{c_1 c_3 M}{2}, \\
& 2c_0 c_3 Q(c_1 c_2) \left( 2 - e^{-\frac{c_2 - \sqrt{s_{\max}}}{\tau_3}} \right) + \frac{c_1 c_3 M}{2}, \quad \sqrt{s_{\min}} < c_2 \leq \sqrt{s_{\max}}.
\end{array}
\] (44)


