The Quantum Phase of Inflation

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Abstract

Inflation models can have an early phase of inflation where the evolution of the inflaton is driven by quantum fluctuations before entering the phase driven by the slope of the scalar field potential. For a Coleman-Weinberg potential this quantum phase lasts $10^{7-8}$ e-foldings. A long period of fluctuation driven growth of the inflation field can possibly take the inflaton past $\phi_*$, the value of the field where our current horizon scale crosses the horizon; alternatively, even if the field does not cross $\phi_*$, the inflaton could have high kinetic energy at the end of this phase. Therefore we study these issues in the context of different models of inflation. In scenarios where cosmological relevant scales leave during the quantum phase we obtain large curvature perturbations of $O(10)$. We also apply our results to quadratic curvaton models and to quintessence models. In curvaton models we find that inflation must last longer than required to solve the horizon problem, that the curvaton models are incompatible with small field inflation models and that there may be too large non-gaussianity. A new phase of thermal fluctuation driven inflation is proposed, in which during inflation the inflaton evolution is governed by fluctuations from a sustained thermal radiation bath rather than by a scalar field potential.

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I. INTRODUCTION

In the early stages of inflation the evolution of the inflaton field is dominated by quantum fluctuations rather than by the slope of the potential, $V'(\phi)$. In the initial stages of inflation, $V'$ is small, and $\dot{\phi}$ is not given by $-V'/(3H)$ but by the change in $\sqrt{\langle \phi^2 \rangle}$ due to quantum fluctuations leaving the horizon, freezing out, and becoming part of the large wavelength background condensate \cite{1, 2}. For a Coleman-Weinberg new inflation potential this initial phase can last for as long as $10^{7-8}$ e-foldings after which $V'(\phi)$ dominates over the fluctuations and one gets standard evolution.

In this article we discuss the quantum evolution of the inflaton field in the context of new inflation, hilltop inflation, inflection point inflation, chaotic inflation and natural inflation, and then warm (new) inflation in the weak and strong dissipative regimes. A long quantum fluctuation driven phase for the inflaton can drive it to the bottom of its potential or past $\phi_*$, the value of the inflaton field when our present horizon scale crosses the horizon. This will necessarily change our understanding of inflation. Alternatively, even if the inflaton does not cross $\phi_*$, the inflaton kinetic energy at the end of the fluctuation driven phase can be large, thereby precluding standard inflation thereafter. While the quantum phase of inflation has been known for long these possibilities have not been discussed in the literature so far. The thermal fluctuation driven phase is new and for the first time is being proposed in this paper.

We first consider new inflation with a quartic potential, $V = V_0 - \frac{1}{4} \lambda \phi^4$, which mimics the inflationary part of a Coleman-Weinberg potential when the field is far from the potential minimum \cite{1, 2}. We initially presume that our current horizon scale leaves the horizon during the standard classical rolling phase (which fixes the coupling $\lambda$ to be $\sim 10^{-14}$) and then check for the viability of this scenario after a long period of quantum evolution. We find that the field is still high up on its potential at the end of the quantum phase, and its kinetic energy is not dominant. Therefore the standard new inflation scenario can follow an initial quantum fluctuation driven phase.

We then carry out the same analysis of quadratic new inflation, inflection point inflation, chaotic inflation and natural inflation. In chaotic, inflection point and natural inflation we find that, unlike in new inflation, quantum fluctuations do not play a significant role in the evolution of the inflaton, i.e., classical evolution effectively dominates from the beginning of
We next consider scenarios where our current horizon scale crosses the horizon during the quantum phase of inflation. These are new scenarios that to our knowledge have not been considered earlier. We consider two scenarios - $\phi_* > \phi_Q > \phi_e$ and $\phi_Q > \phi_e$, where $\phi_Q$ is the value of the inflaton at the end of the quantum phase and $\phi_e$ is the value of the inflaton field at the end of inflation. We do this for new inflation and hilltop inflation models. The expressions for $\phi_*$ must be rederived for these scenarios in which the current horizon scale leaves during the quantum phase. Since $\dot{\phi}$ in is not given by $\dot{\phi} = -V'/(3H)$ but by $\sqrt{\langle \dot{\phi}^2 \rangle}$ the expression for the comoving curvature perturbation, $R_k$, too must be rederived.

For quartic new inflation models and $\phi_* > \phi_Q > \phi_e$ we find that consistency of the scenario requires the coupling $\lambda$ to be greater than $\sim 10^{-2}$. But this then implies that $R_k \approx 13$ for modes leaving during the quantum phase. Thus this scenario is ruled out. This scenario for quadratic new inflation and hilltop inflation is also ruled out due to conflict with the observed density perturbations.

For the scenario where the quantum phase lasts the entire inflationary epoch, i.e. $\phi_Q > \phi_e$, we find that it requires a very large value of $H \sim M_{Pl}$ in new inflation models. This scenario also generates too large curvature perturbations.

We then apply our results for modes leaving during the quantum phase to quadratic curvaton models [3, 4], quintessence models and to a new model of dark energy [5] where the dark energy is a condensate of quantum fluctuations generated during inflation of a very light field. If quantum fluctuations of the curvaton determine the value of the curvaton field at the end of inflation then we find that the duration of inflation must be orders of magnitude larger than the usual number of e-foldings required to solve the horizon problem. We also find that the slow roll parameter $\epsilon_H$ is determined in these models, and has a value larger than those compatible with small field inflation models such as new inflation, small field natural inflation and some hybrid inflation models with a concave downward potential. In the context of certain alternate expressions in the literature for quantum fluctuations during quadratic chaotic inflation, the non-gaussianity in the curvaton models increases to levels in conflict with observations. In the dark energy model we investigate whether the large curvature perturbations associated with the dark energy condensate during inflation are compatible with the CMB constraints on the total curvature perturbation at decoupling.

We then consider the above issues in the warm inflationary scenario in which the contin-
uous decay of the inflaton during inflation creates a thermal bath which survives throughout the inflationary phase \[6\]. In warm inflationary dynamics, \(\langle \phi^2 \rangle\) can grow initially due to fluctuations of the inflaton field in this thermal background. We study the quartic new inflation potential in the context of both weak dissipation and strong dissipation of the inflaton field. We investigate whether the inflaton reaches the minimum of its potential during the fluctuation driven phase itself. We find that in the weak dissipative regime there is no thermal fluctuation dominated phase. Therefore the potential driven phase is preceded by a quantum fluctuation driven phase, as in cold inflation. In the strong dissipative regime there is a thermal fluctuation driven phase which lasts for \(10^8\) e-foldings. This is a new phase of warm inflation not considered earlier. The further requirement that the thermal fluctuations do not take the field to the bottom of the potential requires that the scale of inflation is less than \(10^{14}\) GeV. This is two orders of magnitude less than earlier constraints \[6\]. For both dissipative regimes, after the fluctuation driven phase is over, standard warm inflation can follow, indicating the consistency of the warm inflation scenario.

Below we summarize the paper. In Sec. \[\text{II}\] we investigate the consistency of cold inflation models preceded by a quantum phase. In Secs. \[\text{III}\] and \[\text{IV}\] we consider scenarios in which the current horizon scale leaves the horizon during the quantum phase of inflation. In Sec. \[\text{V}\] we study the quantum phases of curvaton and quintessence fields. In Sec. \[\text{VI}\] we apply our ideas to warm inflation and investigate a thermal fluctuation driven phase. Sec. \[\text{VII}\] contains our conclusions.

\section{Cold Inflation}

We first present the relevant equations for studying the quantum evolution phase in cold inflation models. There are three scenarios that we consider - i) the quantum phase ends before our current horizon scale crosses the horizon at time \(t_*\), ii) the quantum phase ends after \(t_*\) but before the end of inflation at \(t_e\), and iii) the quantum phase lasts for the entire duration of inflation. In the first scenario our focus is on determining whether the inflaton ends the quantum phase with conditions suitable for a subsequent period of classical inflation (thereby confirming the validity of existing models of inflation which allow a quantum phase). For the other two cases, we investigate (in Secs. \[\text{III}\] and \[\text{IV}\]) the feasibility of models where the current horizon scale leaves during the quantum phase.
\( \phi_Q < \phi_* \)

During the quantum evolution phase in cold inflation, field fluctuations about an initial value \( \phi_0 \) grow as

\[
\langle \delta \phi^2 \rangle = \frac{1}{(2\pi)^3} \int_H^{aH} d^3k |\phi_k|^2 = \left( \frac{H}{2\pi} \right)^2 \times N(t),
\]

where \( k \) is the comoving momentum, \( N(t) = H(t - t_0) \) is the number of e-foldings since the beginning of inflation at \( t_0 \), and we have only integrated over modes outside the horizon which can act as part of the homogeneous background field. For small field inflation models we ignore any initial \( \phi_0 \). \(^1\) (We have also assumed an effectively massless field, \( m_{\text{eff}} \ll H \).)

Therefore

\[
\phi = \phi_0 \pm \left( \frac{H}{2\pi} \right) \sqrt{H(t - t_0)}
\]

(we take \( \phi_0 > 0 \)), and \( \dot{\phi} \) during the quantum phase is then given by eq. (8.3.12) of Ref. 1 as

\[
\dot{\phi}_q = \frac{H^2}{4\pi \sqrt{H(t - t_0)}}.
\]

The period of quantum evolution lasts while

\[
|\dot{\phi}_q| \gg |\dot{\phi}_V| = \left| -\frac{V'}{3H} \right|,
\]

that is, for values of \( \phi \) satisfying

\[
\frac{3H^4}{8\pi^2} \gg |\phi - \phi_0| |V'(\phi)|
\]

As we shall see, this condition is satisfied only for small field inflation models such as New inflation.

To confirm that the universe is potential energy dominated during and at the end of the fluctuation driven epoch we compare \( \langle \dot{\phi}^2 \rangle \) with \( V(\phi) \). Now

\[
\langle \dot{\phi}^2 \rangle = \frac{1}{(2\pi)^3} \int_H^{aH} d^3k |\phi_k|^2
\]

Ignoring the gradient \((k^2/a^2)\) term in the equation of motion for \( \phi_k \),

\[
\ddot{\phi}_k + 3H \dot{\phi}_k + V''(\phi) \phi_k = 0
\]

\(^1\) For standard classical inflation one argues that due to quantum fluctuations the initial value of the inflaton should not be less than \( H/(2\pi) \). This argument is not relevant when one is studying quantum fluctuation driven evolution. Nevertheless for small field models, presuming \( \phi_0 > H/(2\pi) \), we impose \( \phi_* > H/(2\pi) \). \( \phi_0 \) actually depends on the inflaton dynamics as the universe approaches the inflationary epoch.
and,
\[ \dot{\phi}_k \approx -\frac{V''(\phi)}{3H} \phi_k. \]  
(8)

Then
\[ \langle \dot{\phi}^2 \rangle = \left[ \frac{V''(\phi)}{3H} \right]^2 \frac{1}{(2\pi)^3} \int_H^{aH} d^3k |\phi_k|^2 \]  
(9)
\[ = \left[ \frac{V''(\phi)}{3H} \right]^2 \langle \delta\phi^2 \rangle \]  
(10)

We now apply the above results to specific models of inflation.

A. Quartic new inflation

For a Coleman-Weinberg potential, modelled as \( V = V_0 - \frac{\lambda}{4} \phi^4 \) for small \( \phi \), the quantum phase occurs for \( \phi < \phi_Q \), where, using eq. (4) and eqs. (2) and (3),

\[ 2\phi_Q = H \left( \frac{60}{\lambda} \right)^{1/4} \]  
(11)

for \( \phi_0 \approx 0 \). Using eq. (2), the number of e-foldings till \( t_Q \) is \( N_Q = H(t_Q - t_0) = 8/\sqrt{\lambda} \). For the scenario with \( \phi_Q < \phi_* , \lambda \sim 10^{-14} \) for GUT-scale inflation and one can see that \( N_Q \approx 10^8 \), i.e., \( 10^8 \) e-foldings occur before one gets to classical evolution in this new inflation model. \( \phi_Q \approx 10^3H \). For \( \phi_Q < \phi_* \) the number of e-foldings of inflation after the inflaton field crosses \( \phi_* \) is given by eq. (8.62) of Ref. [8]) as

\[ N(\phi_* \rightarrow \phi_e) = \frac{8\pi}{M_{Pl}^2} \int_{\phi_*}^{\phi_e} \frac{V}{V'} d\phi \]  
(12)
\[ = \frac{3H^2}{2\lambda} \left( \frac{1}{\phi^2_*} - \frac{1}{\phi^2_e} \right) \]  
(13)

and taking \( \phi_e \approx (V_0/\lambda)^{1/4} \) we get \( \phi_* \approx 10^6H \), which is larger than \( \phi_Q \) as presumed. (Typically one uses the slow roll condition \( |V''| \ll 9H^2 \) to obtain \( \phi_e = (3/\lambda)^{1/2}H \) [8]. But then \( V(\phi_e) < 0 \). We instead take \( \phi_e \approx (V_0/\lambda)^{1/4} \).)

\( V(\phi_Q) = V_0 - H^4/\pi^4 \), and with \( V_0 \approx 0.1H^2M_{Pl}^2 \), \( V(\phi_Q) \approx V_0 \). Using eq. (11), \( \langle \dot{\phi}^2 \rangle = \lambda^2 \phi^6/H^2 \) and for \( \phi \leq \phi_Q \), \( \langle \dot{\phi}^2 \rangle \ll V_0 \), i.e., conditions for inflation remain valid during the quantum phase and at the end of the quantum phase.

\[^2\text{Our value of } \phi_Q \text{ differs slightly from that obtained in Ref. [1], possibly because of some factors in eq. (8.3.12) of Ref. [1].}\]
The quantum evolution is statistical and thus \( \phi = (H/2\pi) \sqrt{H(t - t_0)} \) reflects the magnitude of the displacement due to quantum fluctuations averaged over many Hubble volumes. Even for the case where \( |\dot{\phi}_q| \gg |\dot{\phi}_V| \) on average, there will be some regions where the classical dynamics dominates the quantum. However the number of such regions will be relatively small.

**B. Quadratic new inflation**

We now consider a quadratic model of new inflation. For quadratic new inflation with \( V = V_0 - m^2 \phi^2/2 \) the slow roll conditions \( |V''| \ll 9H^2 \) and \( |V'M_\text{Pl}|/V | \ll (48\pi)^{1/2} \) give \( m^2 \ll 9H^2 \) and \( \phi \ll (H^2/m^2)M_\text{Pl} \). We take \( \phi_c = V_0^{1/2}/m \), as in Ref. [9]. From eqs. (2), (3) and (4), \( \phi_Q = 0.2(H/m)H \). Then from eq. (2), \( N_Q = 1.6H^2/m^2 \).

For quadratic new inflation the WMAP bounds on \( 1 - n_s \) imply GUT-scale inflation [9]. Then for \( H = 10^{-6}M_\text{Pl}, N = 60 \) and using eq. (12)

\[
\phi_* = \phi_c e^{-20m^2/H^2}
\]

The condition that \( \phi_Q < \phi_* \) then gives \( m/H < 0.8 \). Setting \( R_k = H^2/(2\pi\dot{\phi})_* = 5 \times 10^{-5} \) and using \( \dot{\phi} = -V'/3H \), one gets \( m/H = 0.03 \) or 0.35. But \( 1 - n_s = 2m^2/(3H^2) \) and WMAP observations imply that \( 0.025 < 1 - n_s < 0.049 \) (at 68% C.L.)[10], thereby allowing only \( m/H = 0.03 \). Then \( \phi_Q = 7H \) and \( N_Q = 2000 \). Using eq. (10) we can also note that \( V(\phi_Q) \approx V_0 \gg \langle \dot{\phi}^2 \rangle \) during the quantum phase, and so the universe is potential energy dominated during and at the end of the fluctuation driven phase. Thus the scenario with \( \phi_Q < \phi_* \) is consistent.

**C. Other models of inflation**

We have also investigated the quantum phase of inflation for inflection point inflation, chaotic inflation and natural inflation. A quantum phase during inflection point inflation about the saddle point \( \bar{\phi}_0 \) is mentioned in Refs. [11, 12] (though the criterion for quantum evolution is a bit different than ours) and it is assumed that the field is out of the range for quantum evolution for the cosmologically relevant phase of inflation. We did not consider this scenario further. The existence of the saddle point requires a certain fine-tuned relation
between parameters in the lagrangian. Refs. [13, 14] consider deviations from this relation. In Ref. [14] the deviation is parametrised by a variable $\beta$. For weak scale SUSY with $m \sim 100 - 1000 \text{ GeV}$, one has $\bar{\phi}_0 \sim 10^{14-15} \text{ GeV}$ and $V_0 \sim 10^{32-34} \text{ GeV}^4$, and CMB observations require $\beta \sim O(10^{-10})$ [14]. (Also see Refs. [12, 13].) For such values the quantum phase condition in eq. (5) with $\phi$ replaced by $\bar{\phi}_0$ is valid only for $|\phi - \bar{\phi}_0| < \sim 10^{-6} \text{ GeV}$ whereas $H \sim 10^{-2-3} \text{ GeV}$. Therefore one can ignore the quantum phase for these models as the quantum phase range is much smaller than $H$. Ref. [14] also considers scenarios without finetuning of $\beta$ but again $|\phi - \bar{\phi}_0| \ll H$.

For quadratic and quartic chaotic inflation there is no quantum phase of inflation for $\phi < \phi_{QG}$ where $\phi_{QG}$ is the value of the $\phi$ for which quantum gravity effects become important ($V(\phi_{QG}) = M^4_{\text{Pl}}$). Natural inflation with a potential of the form $V = \Lambda^4[1+\cos(\phi/f)]$ [15, 16] where $\phi$ lies between 0 and $2\pi f$, and we take $\phi < \pi f$, also does not have a quantum phase of inflation (unless the field value is extremely small, $\phi/f < 10^{-6}$).

D. The quantum condition

Another approach to identifying the quantum phase is to compare the evolution of the inflaton due to quantum fluctuations and classical slow roll at an instant in time, over a time interval $\Delta t = H^{-1}$ [1, 17] i.e., to check if

$$\delta \phi_q = \frac{H}{2\pi} > \delta \phi_V = \dot{\phi}_V H^{-1}$$

$$= \frac{V'(\phi)}{3H^2}$$

This approach checks if instantaneously the quantum jump overrides the classical evolution, while the condition in eq. (2) is a measure of whether quantum evolution dominates over classical evolution averaged over longer time durations. One notices that the condition in eq. (15) is similar to that for eternal inflation $\mathcal{K} \equiv \delta \phi_q/[0.61\dot{\phi}_V H^{-1}] \gtrsim 1$ [18].

Comparing the above approach and that in eqs. (2) and (3), we see that the quantum evolution of the field in eq. (2) goes as $\dot{\phi}_q(t) \sim \sqrt{N(t)} = \sqrt{Ht}$, which means $\dot{\phi}_q \sim 1/\sqrt{N} \sim 1/\sqrt{t}$. In other words $\dot{\phi}_q$ decreases over time. Thus if one is comparing this motion with the classical motion and if the latter has approximately constant velocity, as expected in a slow-roll regime, then it implies that at late time eventually the classical motion always dominates. Since the quantum kicks on the scalar field are a random process, it might seem
that there should be no dependence on the past history of the quantum evolution that should enter in comparing whether quantum or classical motion dominates at a given time. In fact at any given instant the RMS quantum kick in eq. (15) is of order $H$, and so the RMS velocity at any instant from quantum kicks is of order $H^2$. So from this point of view, one might think the relevant quantities to compare is whether this RMS velocity from quantum evolution, which is independent of past history, is the correct quantity to compare against the classical velocity, as in eq. (15). However the history of the evolution should enter the comparison. Even though at any given instant, the velocity from quantum kicks is some approximately constant value, the direction is random. As such, with increasing time steps, there is an ever increasing number if possible paths that the $\phi$ field could have followed. If one asks what is the net motion from the quantum kicks after some interval of time, on average it increases only as $\sqrt{t}$. So a time derivative over an increasing time interval actually is decreasing as $1/\sqrt{t}$. On the other hand, the classical evolution of the field, even if slow, is steadily always moving in the same direction. As such, the longer one waits, the greater the chance that the $\phi$ field has arrived at increasing field values due to classical rather than quantum evolution.

III. $\phi_e > \phi_Q > \phi_*$

We now consider the scenario where the quantum fluctuations take the inflaton beyond $\phi_*$ and so our current horizon scale leaves the horizon when inflaton evolution is dominated by quantum fluctuations. In this section we consider the scenario in which the fluctuations do not take the inflaton past $\phi_e$. Since in the previous section we found that the quantum phase is important only for small field inflation models here we investigate only new inflation and hilltop inflation models.

To obtain $\phi_*$ we break the evolution from $\phi_*$ to $\phi_Q$, and from $\phi_Q$ to $\phi_e$. Then

$$\mathcal{N}(\phi_* \to \phi_e) = \int_{t_*}^{t_e} H dt$$
\[
\begin{align*}
\int_{\phi_*}^{\phi_e} H \frac{d\phi}{\dot{\phi}} &= 0 \quad \text{(17)} \\
\int_{\phi_*}^{\phi_Q} H \frac{d\phi}{\dot{\phi}} + \int_{\phi_Q}^{\phi_e} H \frac{d\phi}{\dot{\phi}} &= 0 \quad \text{(18)} \\
\frac{8\pi^2}{H^2} \int_{\phi_*}^{\phi_q} \phi \, d\phi + \frac{8\pi}{M_{\text{Pl}}^2} \int_{\phi_Q}^{\phi_e} \frac{V(\phi)}{-V'(\phi)} \, d\phi &= 0 \quad \text{(19)} \\
\frac{4\pi^2}{H^2} (\phi^2 - \dot{\phi}^2) + \frac{8\pi}{M_{\text{Pl}}^2} \int_{\phi_Q}^{\phi_e} \frac{V(\phi)}{-V'(\phi)} \, d\phi &= 0 \quad \text{(20)}
\end{align*}
\]

where we have used eqs. (2) and (3) for \( \dot{\phi} \) for \( \phi < \phi_Q \).

The comoving curvature perturbation is given by

\[
\mathcal{R} = \psi_{\text{com}} = \psi + H \delta \tau \quad \text{(21)}
\]

where \( \psi \) is the curvature potential, \( \delta \tau = \delta \phi / \dot{\phi} \) is the time-displacement to go from a generic slicing with generic \( \delta \phi \) to the comoving slicing with \( \delta \phi_{\text{com}} = 0 \). Evaluating the rhs in the flat gauge in which \( \psi = 0 \) gives

\[
\mathcal{R}_k = H \frac{\delta \phi_k}{\dot{\phi}} \quad \text{(22)}
\]

We first evaluate the rhs at horizon crossing. In the flat gauge, \( \delta \phi_k \) is given by \( H/(2\pi) \). \( \dot{\phi} \) represents the motion of the background condensate of long wavelength modes. Therefore \( \dot{\phi} \) is given by eq. (10), Then

\[
\mathcal{R}_k = \frac{3H^3}{2\pi |V''(\phi_*)|\dot{\phi}_*} \quad \text{(23)}
\]

where \( \phi_* \) is given by the value at horizon exit obtained from eq. (20). (The sign of \( \mathcal{R}_k \) is suppressed as the relevant quantity is the 2-point function \( \sim |\mathcal{R}_k|^2 \).)

While considering times after horizon exit the background should only include those superhorizon modes that are of longer wavelength than the mode being investigated. Therefore after horizon exit both \( \delta \phi_k \) and \( \dot{\phi} \) in eq. (22) evolve as in the standard classical roll picture and one gets constancy of \( \mathcal{R}_k \) outside the horizon as usual. (Quantum evolution of \( \phi \) will continue as more and more modes cross the horizon and add to the condensate of fluctuations. But the condensate value of \( \phi_* \) relevant in eq. (23) will not include modes that leave the horizon after the mode being investigated.)

We point out an interesting fact that both \( \delta \phi_k \) and \( \dot{\phi} \) in \( \mathcal{R}_k \) are quantum variables in the quantum phase of inflation. Therefore in the quantum phase we expect that there will be an increase in the cosmic variance because of the stochastic nature of the evolution of \( \phi \).
A. Quartic new inflation

For quartic new inflation, eq. (20) implies

\[ \phi_*^2 = \phi_Q^2 - \left[ \frac{\mathcal{N}H^2}{4\pi^2} - \frac{3H^2}{2(60\lambda)^{1/2}} + \frac{3}{8\pi^2\lambda^{3/2}} \frac{H^4}{V_0^{1/2}} \right]. \]  

(24)

The third term in the square brackets is larger than the second only for \( H > 2M_{\text{Pl}} \). Note that limits such as \( V_0^{1/4} < 10^{16} \text{ GeV} \) and \( H < 10^{13} \text{ GeV} \) are derived for classical inflation and not valid in the quantum phase. Nevertheless, \( H > 3M_{\text{Pl}} \) implies \( V(\phi) > M_{\text{Pl}}^4 \) which would put us in the realm of quantum gravity. We take \( H < M_{\text{Pl}} \) and ignore the third term in the square brackets above.

To check for consistency of this scenario, the quantity in square brackets should be positive. We also impose \( \phi_* > H/(2\pi) \). These conditions give a lower bound and an upper bound on \( \lambda \) respectively, namely,

\[ \frac{60}{\mathcal{N}^2} < \lambda < \frac{240}{(\mathcal{N} + 1)^2}. \]  

(25)

(Imposing \( \phi_Q < \phi_e \) only gives a weak bound \( H < 2M_{\text{Pl}} \).) For GUT-scale inflation with \( \mathcal{N} \approx 60 \) this implies

\[ 0.02 < \lambda < 0.06. \]  

(26)

The cosmologically relevant scales leave inflation over 8 e-foldings. For all these scales to leave during the quantum phase for GUT-scale inflation the condition is \( 0.02 < \lambda \). For inflation models with scales varying from the electroweak scale to the Planck scale, \( \mathcal{N} \) ranges from 30-65 and we get a lower bound of \( (1 - 7) \times 10^{-2} \) for \( \lambda \) for all these models. Clearly these values of \( \lambda \) are interesting but one has to also check for constraints from the the power spectrum.

From eq. (23)

\[ \mathcal{R}_k = \frac{1}{2\pi} \frac{H^3}{\lambda \phi_*^3}. \]  

(27)

Using the expression for \( \phi_* \) in eq. (24) in eq. (27) we get

\[ \mathcal{R}_k = \frac{4\pi^2}{\lambda \left[ 2 \left( \frac{60}{\lambda} \right)^{1/2} - \mathcal{N} \right]}^{1/2}. \]  

(28)
For GUT-scale inflation with $N = 60$ we take $\lambda = 4 \times 10^{-2}$ consistent with eq. (26) and get

$$R_k \approx 13.$$  \hspace{1cm} (29)

Such a large value of the curvature perturbation is ruled out by observations. Therefore this scenario, despite its mild constraint on the inflaton coupling, namely, $\lambda \sim 10^{-2}$, is not feasible.

It may be surprising that though in the quantum phase $\dot{\phi}$ is larger than the classical value, one gets such a large value of $R_k$. However note that while $\dot{\phi}_q \gg \dot{\phi}_V, \sqrt{\langle \dot{\phi}^2 \rangle}$ which enters the expression for $R_k$ is $3 \dot{\phi}_V$. Now the expression for $R_k$ for classical evolution, using eq. (22) and $\dot{\phi}_V = -V'/(3H)$, is

$$R_k = \frac{1}{2\pi} \frac{3H^3}{\lambda \phi_e^2}.$$  \hspace{1cm} (30)

Comparing this with eq. (27) one sees that the expressions are similar. Yet, though $\lambda$ in eq. (27) is large compared to $\lambda$ in eq. (30), $R_{k,q} \gg R_{k,cl}$ because it happens that $\phi_e$ is much smaller than in the standard classical evolution scenario. As shown earlier, the value of $\phi_e$ for the classical case for $\lambda = 10^{-14}$ and $N = 60$ is $\phi_{e,cl} = 10^6 H$. However, for the value of $\lambda = 4 \times 10^{-2}$ and $N = 60$ one gets, from eq. (24), $\phi_{e,q} = 0.7H$.

To understand why $\phi_e$ is much smaller for the quantum evolution scenario note that i) $\phi_e \sim 1/\sqrt{\lambda}$ is much smaller in the quantum scenario than for the classical scenario as $\lambda$ is large, and ii) a larger $\dot{\phi}_q$ in eq. (18) requires a larger displacement in $\phi$ to obtain the required number of e-foldings. Both these effects push the inflaton to a much smaller value of $\phi$ at the time of horizon exit for a given value of $N$.

**B. Quadratic new inflation**

For quadratic new inflation eq. (20) implies

$$\phi_e^2 = \phi_Q^2 - \left[ \frac{NH^2}{4\pi^2} - \frac{3H^4}{4\pi^2 m^2 \ln \left( \frac{\phi_e}{\phi_Q} \right)} \right].$$  \hspace{1cm} (31)

Using expressions for $\phi_e$ and $\phi_Q$ from Sec. II B we get

$$\phi_e^2 = \phi_Q^2 - \left[ \frac{NH^2}{4\pi^2} - \frac{3H^4}{4\pi^2 m^2 \ln \left( \frac{1.7 M_{Pl}}{H} \right)} \right].$$  \hspace{1cm} (32)

The constraint that $\phi_Q > \phi_e$ implies

$$\frac{m^2}{H^2} > \frac{3}{N} \ln \left( \frac{1.7 M_{Pl}}{H} \right).$$  \hspace{1cm} (33)
while the requirement that $\phi_s > H/(2\pi)$ implies
\[
\frac{1.6}{N+1} + \frac{3}{N+1} \ln\left(\frac{1.7M_{Pl}}{H}\right) > \frac{m^2}{H^2}.
\] (34)

Consistency of the above constraints implies
\[
H > 1.7M_{Pl}e^{-\frac{N+1}{2}}
\] (35)

For GUT-scale inflation with $N = 60$ this requires $H > 10^6 \text{GeV}$.

To obtain the curvature perturbation we use eq. (23) with $\phi_s$ given by eq. (31).
\[
\mathcal{R}_k = \frac{3}{2\pi} \frac{H^3}{m^2 \phi_s}.
\] (36)

Now for $\phi_Q > \phi_s$ the curvature perturbation satisfies
\[
\mathcal{R}_k > \frac{3}{2\pi} \frac{H^3}{m^2 \phi_Q} = \frac{2H}{m}
\] (37)

For this to be consistent with observations would require $m/H > 10^5$. But such a large value of $m/H$ is in conflict with the upper bound in eq. (34). Therefore this scenario is not feasible. (For electroweak scale inflation the lower bound on $H$ is too large ($3 \times 10^{12} \text{GeV}$) and so we do not consider it.)

C. Hilltop inflation

For the potential
\[
V = V_0 - \lambda \frac{\phi^p}{M_p^{p-4}} \quad p > 2,
\] (38)

$\mathcal{N}$ is obtained from eq. (20) as
\[
\mathcal{N} = \frac{4\pi^2}{H^2} (\phi_Q^2 - \phi_s^2) + \frac{8\pi}{M_{Pl}^2} \frac{V_0 M_p^{p-4}}{\lambda (p-2)} \left( \frac{1}{\phi_Q^{p-2}} - \frac{1}{\phi_e^{p-2}} \right)
\] (39)

and so
\[
\phi_s^2 = \phi_Q^2 - \left[ \frac{\mathcal{N} H^2}{4\pi^2} - \frac{H^2}{4\pi^2} \frac{8\pi}{M_{Pl}^2} \frac{V_0 M_p^{p-4}}{\lambda (p-2)} \left( \frac{1}{\phi_Q^{p-2}} - \frac{1}{\phi_e^{p-2}} \right) \right].
\] (40)

Using $H = [(8\pi/3)V_0/M_{Pl}^2]^{1/2}$
\[
\frac{1}{\phi_Q^{p-2}} - \frac{1}{\phi_e^{p-2}} = \frac{1}{\left[ \frac{H^2}{M_{Pl}^2} \frac{V_0 M_p^{p-4}}{\lambda} \right]^{p-2}} - \frac{1}{\left[ \frac{V_0 M_p^{p-4}}{\lambda} \right]^{p-2}}
\] (41)
Considering $H^2 \ll M_{\text{Pl}}^2$, we ignore $1/\phi_0^{p-2}$. For $p > 4$ and $H \ll M$ (the potential in eq. (38) is an effective potential for $\phi < M$, and we only consider $\phi > H/(2\pi)$) one gets from eq. (23) (using Mathematica)

$$R_k = 0.5 \frac{1}{\lambda^p} \left( \frac{M}{H} \right)^{p-4}$$

which is greater than 1.

IV. $\phi_Q > \phi_e$

We now consider the scenario where evolution in the inflationary phase is entirely dominated by quantum fluctuations, i.e., $\phi_Q > \phi_e$. Then

$$N(\phi_0 \to \phi_e) = \frac{4\pi^2}{H^2} (\phi_e^2 - \phi_*^2), \quad (43)$$

and so

$$\phi_* = \left[ \phi_e^2 - \frac{N}{4\pi^2} H^2 \right]^{\frac{1}{2}}. \quad (44)$$

A. Quartic new inflation

For quartic new inflation, as discussed earlier, $\phi_e \approx (V_0/\lambda)^{1/4}$. The condition $\phi_Q > \phi_e$ implies that

$$H > 1.8 M_{\text{Pl}}. \quad (45)$$

(This is close to the limit $H < 3M_{\text{Pl}}$ for classical gravity to be valid.) From eq. (44),

$$\phi_e^2 > N H^2/(4\pi^2). \quad (46)$$

(Imposing $\phi_* > H/(2\pi)$ gives a similar bound with $N \to N + 1$.) For Planck scale inflation we take $N = 65$. This then implies

$$H < \frac{0.02 \pi^2}{\sqrt{\lambda}} M_{\text{Pl}}. \quad (46)$$

Combining the above two bounds one finds that for inflation to be in the quantum phase during its entire duration requires a large Hubble parameter during inflation and only a reasonable upper bound on $\lambda$, i.e.,

$$H > 1.8 M_{\text{Pl}} \quad \text{and} \quad \lambda < 1 \times 10^{-2}. \quad (47)$$

The comoving curvature perturbation is given by eq. (27) with $\phi_*$ as in eq. (44). Once again we obtain a large value of $R_k (> 17)$ which rules out this scenario.
B. Quadratic new inflation

The condition that \( \phi_Q > \phi_e \) implies

\[
H > 1.7 M_{\text{Pl}} \quad (48)
\]

and the condition that \( \phi_e^2 > N H^2 / (4\pi) \) in eq. (44) implies that

\[
\frac{3}{2N} M_{\text{Pl}}^2 > m^2. \quad (49)
\]

For Planck scale inflation \( N = 65 \), and \( 0.15 M_{\text{Pl}} > m \). The curvature perturbation in eq. (36) satisfies

\[
R_k > \frac{3}{2\pi} \frac{H^3}{m^2 \phi_e}. \quad (50)
\]

Using eqs. (48) and (49) we get \( R_k > 26 \) which is in conflict with observations.

V. OTHER SCALAR FIELDS

A. Curvaton

We now consider scenarios where a field other than the inflaton evolves due to its quantum fluctuations during inflation. In the curvaton scenario, quantum fluctuations of a field \( \sigma \) with a flat potential is responsible for the density perturbations in the Universe \[3, 4\]. (Also see Refs.\[20–23\].) The curvature perturbation in these scenarios is given by

\[
\zeta = \frac{4 \rho_r \zeta_r + 3 \rho_\sigma \zeta_\sigma}{4 \rho_r + 3 \rho_\sigma} \quad (51)
\]

\[
\approx r \zeta_\sigma \quad (52)
\]

where the subscript \( r \) refers to radiation, and the variable \( r \) is the ratio of the curvaton energy density \( \rho_\sigma \) to the total energy density \( \rho \) just before the curvaton decays. One presumes that \( \zeta_r \) is negligible and it is ignored.

The curvature spectrum due to the curvaton field is obtained from the expression for \( \zeta_\sigma \) at the time when the curvaton starts to oscillate after inflation when \( H \) falls to \( m_\sigma \) at \( t_{osc} \).

\[
\zeta_{\sigma,k} = \frac{2}{3} \left. \frac{\delta \sigma_k}{\sigma} \right|_{osc} \quad (53)
\]
Since $\delta\sigma_k$ is constant outside the horizon, we take $\delta\sigma_k(t_{osc})$ to be $H/(2\pi)$. We take $\sigma(t_{osc})$ to be the value $\sigma_e$ at the end of inflation at $t_e$. $\sigma_e = \sigma_{cl}(t_e) \pm \delta\sigma_q(t_e)$. If $\sigma_e$ is determined by quantum fluctuations then

$$\zeta_\sigma = \frac{2}{3} \frac{\delta\sigma_k}{\delta\sigma(t_e)}. \quad (54)$$

For a curvaton mass $m_\sigma$, $\delta\sigma(t_e)$ is either $[H/(2\pi)] N_e^{3/2}$ or $3H^4/(8\pi^2 m_\sigma^2)$, depending on whether the condensate of curvaton fluctuations does not or does attain the asymptotic value by $t_e$. ($N_e$ is the total number of e-foldings of inflation.) If it does not, then

$$\zeta_\sigma = \frac{2}{3} \frac{1}{N_e^2}. \quad (55)$$

Now $\zeta_\sigma = \zeta/r$ where $\zeta = 5 \times 10^{-5}$, and $10^{-2} < r < 1$. (The lower limit on $r$ comes from the relation between $r$ and the non-gaussianity parameter $f_{NL}$, namely $f_{NL} = 5/(4r)$ [4], and the upper limit on $f_{NL}$ of $O(100)$ [24].) This then implies

$$2 \times 10^4 < N_e < 2 \times 10^8. \quad (56)$$

Therefore curvaton models where quantum fluctuations determine $\sigma_e$ and where $\delta\sigma$ has not reached the asymptotic value by $t_e$ require more e-foldings of inflation than standard models of inflation. For quartic new inflation models the upper limit on $N_e$ is of the same order as the duration of the quantum phase.

The condition that the curvaton has not reached its asymptotic limit (for a quadratic curvaton potential) is [25]

$$\frac{2m_\sigma^2}{3H^2} N_e \ll 1. \quad (57)$$

Combining this with the lower bound on $N_e$ above gives

$$\frac{m_\sigma^2}{H^2} \ll 8 \times 10^{-5}. \quad (58)$$

Now the scalar power spectrum spectral index in the curvaton scenario is [4]

$$n_s = 1 - 2\epsilon_H + 2\eta_{\sigma\sigma}. \quad (59)$$

where $\epsilon_H = -\dot{H}/H^2$ and $\eta_{\sigma\sigma} = M_{Pl}^2/(8\pi) \partial^2 V/\partial\sigma^2$. For a quadratic curvaton potential, $\eta_{\sigma\sigma} = m_\sigma^2/(3H^2)$. Then for $n_s = 0.963$ [24]

$$\frac{m_\sigma^2}{3H^2} \approx \epsilon_H - 0.02. \quad (60)$$
Therefore the above upper bound on $m_2^2/H^2$ further implies that in quadratic curvaton models where quantum fluctuations determine $\sigma_e$ and $\delta \sigma$ has not reached the asymptotic value by $t_e, \epsilon_H = 0.02$. \footnote{This was also pointed out to us by K. Dimopoulos.} Small field inflation models such as new inflation, small field natural inflation and some hybrid inflation models with a concave downward potential would not satisfy this criterion \cite{26, 27}.

If $N_e \gg (3H^2)/(2m^2)$ one uses the asymptotic value for the curvaton fluctuations in Eq. (64). Then using the observed value of $\zeta$ and the bounds on $r$ one finds

$$9 \times 10^{-5} < \frac{m_\sigma}{H} < 9 \times 10^{-3}.$$ \hspace{1cm} (61)

This is in conflict with the bound $m_\sigma/H \lesssim 10^{-4}$ obtained in Sec. IIIB of Ref. \cite{28} from CMB constraints. (We believe there is an error in the derivation of the bound in Ref. \cite{28}.) Once again, Eq. (60) implies that $\epsilon_H = 0.02$ which is incompatible with small field inflation models. The lower bound on $H/m_\sigma$ from Eq. (61) and the lower bound on $N_e$ above also implies that inflation must last much longer than $10^4$ e-foldings.

If $\sigma_e$ is determined by $\sigma_{cl}(t_e)$ rather than by quantum fluctuations then $\delta \sigma_q(t_e) \ll \sigma_e$ and

$$\zeta_{\sigma} \ll \frac{2}{3} \frac{\delta \sigma_k}{\delta \sigma(t_e)}.$$ \hspace{1cm} (62)

and one can only conclude that $N_e < 2 \times 10^8$ and $m_\sigma/H > 9 \times 10^{-5}$ respectively in the scenarios where the curvaton fluctuations do not and do attain their asymptotic value.

As an aside, we comment on certain existing results in the literature which may be relevant for the curvaton scenario. The asymptotic value of the quantum fluctuations for the curvaton

$$\langle \sigma^2 \rangle = \frac{3H^4}{8\pi^2m_\sigma^2}.$$ \hspace{1cm} (63)

may be used in eq. (53) as in Refs. \cite{25}. However it may be shown (via eq. (4.11) of Ref. \cite{29}, eq. (13) of Ref. \cite{30} and eq. (14) of Ref. \cite{31}) that on including the time variation of $H$ during quadratic chaotic inflation the asymptotic value of the fluctuations during inflation of any light scalar field goes as

$$\langle \sigma^2 \rangle = \frac{3H^4}{16\pi^2m^2},$$ \hspace{1cm} (64)

where $m$ is the inflaton mass. If one uses eq. (64) then one gets

$$\zeta_{\sigma} = \left(\frac{2}{3}\right)^{\frac{3}{2}} \frac{m}{H} \approx 1.6 \times 10^{-2}.$$ \hspace{1cm} (65)
which would need \( r \approx 3 \times 10^{-3} \) to agree with the observed density perturbations. Such a small value of \( r \) will imply \( f_{NL} \approx 400 \) which is in conflict with bounds on \( f_{NL} \) [24]. In models with non-quadratic terms in the curvaton potential \( f_{NL} \) can be small even when \( r \) is small [32, 33] because the coefficient of the term proportional to \( 1/r \) in \( f_{NL} \) in the presence of non-quadratic terms can be very small. These models are then not ruled out (though Ref. [35] indicates that very small values of \( r \) are not favoured).

### B. Quintessence

In quintessence models the dark energy is associated with a slowly rolling background field \( Q \). Since the potential must be flat so that potential energy dominates the kinetic energy (and the pressure is negative) we verify whether quantum evolution can dominate over the slow classical evolution. Consider the potential

\[
V = V_0 \left( \frac{M_{Pl}}{Q} \right)^p
\]

(66)

The condition in eq. (4) implies that quantum evolution dominates for

\[
\frac{H^2}{4\pi\sqrt{N}} > \frac{p}{8\pi} \left( \frac{M_{Pl}}{Q} \right) H M_{Pl}
\]

(67)

(We have presumed that the de Sitter result for the fluctuations is valid in our current epoch of acceleration.) For \( Q \sim M_{Pl} \) [36] and \( H \ll M_{Pl} \) the above inequality is not satisfied. Note that any perturbations generated in the current accelerating phase leave the horizon and can not be detected.

In the model of quintessential inflation [37] the quintessence field plays the role of the inflaton at early times. The potential is as in eq. (66) with \( p = 4 \). The value of \( Q \) today is approximately its value at the beginning of the radiation dominated era and is about \( 8M_{Pl} \). Once again, for \( H \ll M_{Pl} \) quantum evolution does not dominate during the current accelerated phase.

In Ref. [5] the dark energy today is associated with a frozen condensate of fluctuations of a field \( \varphi \) generated during inflation. The field, which has a quadratic potential, is almost massless during inflation and evolves due to quantum fluctuations, similar to the evolution of the inflaton in the quantum phase that has been studied above. As the Universe evolves after inflation the superhorizon fluctuations remain frozen as a condensate at a value \( \varphi_c = \)
and dominate the energy density of the Universe today ($H_I$ is the Hubble parameter during inflation). The condensate behaves like a slowly rolling quintessence field today with the equation of state of dark energy. Inflation in this model occurs at a low scale of $5\,\text{TeV}$. The curvature perturbation generated during inflation due to quintessence field fluctuations is given by eq. (23) which implies

\[ R_k = \frac{3H^2 \delta \varphi}{m^2 \varphi} \]

\[ = \sqrt{6}H_I/m \gg 1. \]  

CMB fluctuations are influenced by the curvature perturbation at decoupling. The contribution of the dark energy to the curvature perturbation at decoupling will be $R_k(t_{\text{dec}}) * f$ where

\[ f = \frac{(\rho + p)\varphi}{(\rho + p)}|_{\text{dec}} \]

\[ = \frac{\varphi^2}{\rho_{\text{dm}}}|_{\text{dec}} \]

\[ = \frac{m^2 H_{\text{dec}}^4}{9\pi H_{\text{dec}}^4 M_{\text{Pl}}^2} \]

and we have used $\dot{\varphi} = -m^2 \varphi/(3H)$ and approximated $\varphi$ by the value $\varphi_c$ above (as also in Ref. [5]). To obtain the value of the curvature perturbation at decoupling one assumes that $\delta \varphi/\varphi$ remains constant during slow roll because both $\delta \varphi$ and $\varphi$ have the same equation of motion (as in curvaton models). Once the field starts oscillating when $H \sim m$, then $R_k \sim \delta \rho/\rho|_\varphi \sim \delta \varphi/\varphi \sim \text{constant}$. Therefore $R_k(t_{\text{dec}}) = \sqrt{6}(H_I/m)(m/H_I)^2$. Then

\[ fR_k(t_{\text{dec}}) = \frac{\sqrt{6}m^3 H_{\text{I}}^3}{9\pi H_{\text{dec}}^4 M_{\text{Pl}}^2} \]

\[ \ll 5 \times 10^{-5}, \]

where $m \lesssim H_0$, the Hubble parameter today, as indicated in this model. Thus the curvature perturbation at decoupling due to the quintessence condensate will not give rise to a large net curvature perturbation. Therefore the curvature perturbation due to the quintessence field condensate in this model is not in conflict with CMB observations.

VI. WARM INFLATION

In warm inflation [6], dissipative effects are important during inflation so that radiation production occurs concurrently with inflationary expansion. The basic equation for describ-
ing the evolution of an inflaton field that dissipates energy is of a Langev
in form \[38, 39\]
\[\ddot{\phi} + [3H + \Upsilon]\dot{\phi} - \frac{1}{a^2(t)}\nabla^2 \phi - \frac{\partial V}{\partial \phi} = \zeta. \tag{75}\]

In this equation, $\Upsilon \dot{\phi}$ is a dissipative term and $\zeta$ is a fluctuating force. Both are effective
terms, arising due to the interaction of the inflaton with other fields. In general these
two terms are related through a fluctuation-dissipation theorem, which would depend on
the statistical state of the system and the microscopic dynamics. Although the statistical
state can be quite general, all studies so far have focused on the thermal state and we will
restrict our consideration here to that also. Thus the evolution of the inflaton field has to
be calculated in a thermal background.

The above dynamics need not be restricted just to the period when there is a potential
driven inflation period. The above dynamics could also occur previous to such a period. This
point leads to a new type of inflation phase in which while inflation occurs, the inflaton rather
than being governed by the potential $V$ is instead governed by the thermal background which
produces large fluctuations in the inflaton field. This is similar to the quantum fluctuation
driven inflation discussed in the previous sections, except now the fluctuations are thermal
rather than quantum.

To examine the initial period during this thermal fluctuation dominated inflation phase,
we first need to evaluate $\phi_T$ which is the value of $\phi$ when evolution due to the fluctuations
is no longer dominant. Following the same procedure as done for the cold inflation case,
we need to evaluate the equivalent of eq. (8.3.12) of Ref.\[1\] and eq. (3.11) of Ref. \[2\] to
obtain $\dot{\phi}$ due to fluctuations and due to the potential and ascertain till when the former
dominates. Fluctuations in warm inflation are obtained from a Langevin equation derived
using a real-time formalism of thermal field theory \[39, 40\].

In treating warm inflation, one caveat is important. In general the inflaton dynamics is a
non-equilibrium problem. Whether the case of cold or warm inflation, certain assumptions
are already being made about this dynamics when one writes down the evolution equation. In
cold inflation, the basic assumption is the inflaton is evolving at effectively zero temperature
and interactions with other fields are negligible. In the case of warm inflation, one is assuming
there is a thermal state and the inflaton interaction with other fields is significant. In this
case there will be a point in time, $t_d$, when these conditions are realized. Previous to that
time, in principle one would need to calculate the full quantum field dynamics and determine
the statistical state and its evolution. This is beyond the scope of this paper. Here what we will assume is either dissipation effects are important and evolve as eq. (75) or else they are not important and to a good approximation the evolution is the same as the cold inflation case. If \( t_d \) is sufficiently early, then the entire fluctuation dominated era can be calculated based on the evolution eq. (75). However if \( t_d \) occurs fairly close to the onset of the slow-roll period, then this can allow for possibilities which combine both quantum and thermal fluctuation regimes before potential driven inflation.

There are two dissipative regimes that must be considered depending on whether in eq. (75) \( \Upsilon \leq 3H \), which is called the weak dissipative regime, or \( \Upsilon > 3H \), which is called the strong dissipative regime. For all these cases, the Langevin equation for the modes of the inflaton field can be solved, as done in Ref. [41]. Our interest here is in the long wavelength modes, for which Ref. [41] finds the solution coincides with the homogeneous solution, as if the effect of the noise force had no effect. Thus calculation of \( \langle \delta \phi^2 \rangle \) in both these cases is found to have the same general form as for cold inflation and, following eqs. (7.3.10-7.3.12) of Ref.[1],

\[
\langle \delta \phi^2 \rangle = \frac{1}{(2\pi)^3} \int_H^{aH} d^3k |\phi_k|^2 = \int_H^{aH} \frac{dk}{k} P_\phi(k) ,
\]

where only the inflaton power spectrum \( P_\phi(k) \equiv k^3|\phi_k|^2/(2\pi^2) \) is different for the various cases. The limits of the integral admit only those modes that have left the horizon during inflation. Below we consider a quartic new inflation potential.

**A. Weak dissipation**

In the scenario in which \( \Upsilon < 3H \) the slow roll of the inflaton is because of the Hubble damping term in the equation of motion, so in particular the inflaton mass \( m < 3H \) must hold in order that a slow-roll regime is eventually achieved. The analysis for this case is very similar to the cold inflation case, except the expression for the inflaton fluctuation is different. In this regime the inflaton fluctuation at freeze-out gives \( P_\phi(k) = (3\pi/4)^{1/2}HT \) \cite{38,39,41} and

\[
\langle \delta \phi^2 \rangle = \left( \frac{3\pi}{4} \right)^{1/2} HT \times N(t) . \tag{77}
\]

(This may be compared with eq. (1), \( \langle \delta \phi^2 \rangle = (H/2\pi)^2 \times N(t) \). Then, for \( \phi_0 \approx 0 \),

\[
\phi = \left( \frac{3\pi}{4} \right)^{1/2} (HT)^{1/2} \sqrt{H(t-t_0)} , \tag{78}
\]
and the evolution rate of the inflaton in the thermal fluctuation driven phase is

\[ \dot{\phi}_{\text{th}} = \frac{1}{2} \left( \frac{3\pi}{4} \right)^{\frac{1}{4}} H^{\frac{3}{2}} T^{\frac{1}{2}} \frac{1}{\sqrt{H(t-t_0)}}. \]  

(79)

Thermal fluctuations will dominate the evolution of \( \phi \) as long as

\[ \frac{1}{2} \left( \frac{3\pi}{4} \right)^{\frac{1}{4}} H^{\frac{3}{2}} T^{\frac{1}{2}} \frac{1}{\sqrt{H(t-t_0)}} \gg -\frac{V'}{3H} = \frac{\lambda \phi^3}{3H}. \]  

(80)

Defining \( \phi_T \) as the largest value for which thermal fluctuations dominate, \( \phi_T \approx H^{\frac{3}{4}} T^{\frac{1}{4}} / \lambda^{\frac{1}{4}} \).

The temperature during the thermal phase must be determined. Any residual radiation from initial conditions will rapidly redshift away during any inflation epoch, and so in order for a thermal fluctuation dominated phase of inflation to exist, there must be a source of radiation production. Although in general this is a problem of nonequilibrium quantum field theory, following our statements at the start of this section, we will assume the radiation is produced by the background component of the scalar field, \( \phi \), which is controlled by eq. (75) without the noise force, for which the radiation produced is

\[ \rho_r = \frac{\Upsilon \phi^2}{4H}. \]  

(81)

In the weak dissipative regime, \( \dot{\phi} = -V'/(3H) \), and for the quartic new inflation potential of Sec. II this gives

\[ \rho_r = \frac{\lambda^2 \Upsilon \phi^6}{36H^3}. \]  

(82)

Since the radiation energy density increases with \( \phi \), an estimate of the largest radiation energy density produced will be for \( \phi \sim \phi_T \). Equating the expression eq. (82) with \( \rho_r \approx g_s T^4 \), the temperature can be expressed in terms of the other quantities, and we find \( T \sim 0.2\lambda^{1/5} \Upsilon^{2/5} H^{3/5} / g_s^{2/5} \ll H \). The latter inequality follows since \( \lambda \ll 1 \) and in the weak dissipative regime \( \Upsilon < H \). Thus we conclude that in the weak dissipative case for the new inflation quartic potential, there is never a thermal fluctuation driven regime during inflation. The dynamics before potential driven inflation will be the same as for the quantum fluctuation dominated phase in cold inflation.

### B. Strong dissipation

In the strong dissipation case \( P_\phi(k) = \sqrt{\pi/4} (\Upsilon H)^{\frac{3}{4}} T \) (see eq. (34) of Ref. [41]). Results will now be calculated during the fluctuation era for the quartic new inflation model.
Using the above expression for $P_\phi(k)$ in eq. (76), $\langle \delta \phi^2 \rangle$ in the fluctuation dominated regime can be obtained as

$$
\langle \delta \phi^2 \rangle = \left( \frac{\pi}{4} \right)^{\frac{1}{4}} (\Upsilon H)^{1/2} T \times N(t).
$$

(83)

Then, for $\phi \approx 0$,

$$
\phi = \left( \frac{\pi}{4} \right)^{\frac{1}{4}} (\Upsilon H)^{\frac{1}{2}} T^{\frac{1}{2}} \sqrt{H(t-t_0)},
$$

(84)

and so

$$
\dot{\phi}_{\text{th}} \approx (\Upsilon H)^{\frac{1}{2}} T^{\frac{1}{2}} H \frac{1}{2\sqrt{H(t-t_0)}}.
$$

(85)

The thermal fluctuations will dominate the $\phi$ evolution from the potential as long as

$$
(\Upsilon H)^{\frac{3}{8}} T^{\frac{1}{4}} H^{\frac{1}{2}} \sqrt{H(t-t_0)} \gg \lambda \phi^3.
$$

(86)

If fluctuations dominate evolution till $\phi_T$ then eq. (86) implies that

$$
\phi_T = (\Upsilon H)^{3/8} T^{1/4} / \lambda^{1/4}.
$$

(87)

The temperature during the thermal fluctuation dominated phase must now be determined using eq. (81), where in the strong dissipative regime $\dot{\phi} = -V'/T$. To estimate the maximum the radiation energy density will be during the thermal fluctuation inflation regime, eq. (87) is used from which we find,

$$
T \sim \lambda^{1/5} g^{2/5}_* (\Upsilon H)^{1/2}.
$$

(88)

In order for this regime to be thermal dominated requires $T > H$, which implies the condition

$$
\Upsilon \gtrsim g^{4/5}_* H^{2/5} H.
$$

(89)

In this regime, from eqs. (84) and (87)

$$
N_T = \frac{(\Upsilon H)^{\frac{1}{4}}}{(\lambda T)^{\frac{1}{4}}},
$$

(90)

The curvature power spectrum for strong dissipation is given by

$$
P^\frac{3}{2}_R = \frac{H \Upsilon}{|V'|} P^\frac{1}{2}_\phi
$$

$$
= 2\pi^{\frac{1}{4}} (\lambda T)^{\frac{1}{4}} N^\frac{3}{2}_k,
$$

(91)

23
with \( N_k \approx H \Upsilon / (2\lambda \phi_k^2) \). (The expression for \( N_k \) is that for the weak dissipative regime with \( 3H \) replaced by \( \Upsilon \), as can be surmised from eq. \[17\].) Setting \( N_k = 60 \) and \( P_R^{\frac{3}{2}} = 10^{-5} \) gives \( N_T = (H \Upsilon)^{\frac{3}{4}} / (T^\lambda)^{\frac{3}{4}} = 10^8 \). One can verify that \( V(\phi_T) \approx V_0 - (\Upsilon / H)^{\frac{3}{2}} TH^3 / 4 \approx V_0 - \lambda^{1/5} (\Upsilon H)^2 / (4g_{*}^{2/5}) \). In order that \( V(\phi_T) \approx V_0 \) and so the fluctuations do not take the inflaton to the bottom of the potential, it requires the condition \( 10V_0 / (\lambda^{7/20} M_{Pl})^4 \ll 1 \), which holds for \( V_0^{1/4} \sim 10^{14} \text{GeV} \). This is two orders of magnitude less than earlier constraints based on CMBR density fluctuation measurements \[7\].

In this regime, we also must confirm that the kinetic energy of the field fluctuations does not dominate the potential during the fluctuation driven epoch. The kinetic energy of the field fluctuations is obtained from

\[
\langle \dot{\phi}^2 \rangle = \int_{k_F}^{\infty} \frac{dk}{k} |\delta \phi(k)|^2,
\]

where \( \delta \phi(k) = (k^3 / 2\pi^2)^{\frac{1}{2}} \phi_k \) and \( k_F = (\Upsilon H)^{1/2} \) is the freeze-out scale in the strong dissipative regime \[39\]. Using eq. (38) of Ref. \[41\] for \( \delta \phi(k) \),

\[
\dot{\delta \phi}(k) = -\left( \frac{\pi}{4} \right)^{\frac{1}{2}} \frac{k^2}{\Upsilon a^2} (\Upsilon H)^{\frac{3}{4}} T^{\frac{1}{4}},
\]

we get from eq. (93),

\[
\langle \dot{\phi}^2 \rangle \approx \Upsilon^{1/2} H^{5/2} T,
\]

which is much smaller than \( V_0 = 0.1H^2 M_{Pl}^2 \) since \( T \) and \( \Upsilon \) are much smaller than \( M_{Pl} \). Also from eq. (95) it follows that \( \rho_r \) is much less than \( V_0 \).

This result has demonstrated a new thermal fluctuation inflation phase. There can be other variations to the above scenario. In the strong dissipative regime slow-roll motion only requires the condition on the inflaton mass \( m < \Upsilon \), thus in general the inflaton mass can be much bigger than the Hubble parameter. This feature, combined with the presence of a radiation component and dissipative dynamics can lead to other possible dynamics previous to the potential driven inflation phase. One example is if the dissipation dynamics is not initially active, previous to time \( t_d \), the inflaton field now is massive and evolves like the cold inflation case, in that only a \( 3H \dot{\phi} \) term damps its evolution. In such a case for a massive inflaton field the estimate for \( \langle \delta \phi^2 \rangle \) differs from that in eq. (11) and rather it is eq. (7.3.13) of Ref. [1].
VII. CONCLUSION

In this article we have investigated the evolution of the inflaton due to quantum fluctuations and studied its possible consequences. If the field travels far on its potential due to quantum fluctuations or acquires a large kinetic energy then standard inflation where the field rolls due to the slope of its potential will not subsequently occur. This will dramatically alter the inflationary scenario and significantly affect the density spectrum. For a given potential a priori one can not predict the impact of quantum fluctuations on the evolution of the inflaton. For example, for GUT-scale inflation the quantum phase lasts for $10^8$ e-foldings for a Coleman-Weinberg potential (approximated by a quartic potential). In contrast, for quadratic new inflation the quantum phase lasts for 2000 e-foldings. For chaotic inflation, inflection point inflation and for natural inflation, the quantum phase is negligible and classical rolling is important from the beginning of inflation. One might have expected quantum fluctuations to be relevant for all small field inflation models but it is not so for small field natural inflation.

For new inflation models, if one assumes that our current horizon scale left the horizon during classical slow roll, then the earlier quantum phase ends with the inflaton far from the minimum of its potential and with sub-dominant kinetic energy. This allows for the standard classical rolling inflationary phase to follow. If cosmologically relevant scales leave the horizon during the quantum phase, which is subsequently followed by a classical phase, then for quartic new inflation this requires that the coupling $\lambda$ is greater than $10^{-2}$. We have derived the expression for the curvature perturbation which is valid for the quantum phase. We get a large value for the curvature perturbation for modes that leave during the quantum phase. This then rules out this scenario. We also consider a scenario where the quantum phase lasts for the entire inflationary epoch which is also ruled out because of the large curvature perturbations. Our conclusions are similar for quadratic new inflation.

We have also studied curvaton and quintessence models where quantum evolution can be relevant. For curvaton models, if the curvaton evolves during inflation due to quantum fluctuations which determine the curvaton field value at the end of inflation, we find that the number of e-foldings of inflation must be orders of magnitude more than the usual minimum value required to solve the horizon problem. We also find that the slow roll parameter $\epsilon_H$ is determined and has a value that is incompatible with small field inflation models. We further
point out that newer results for asymptotic values of scalar fields during inflation can lead
to large non-gaussianity in conflict with observations. Quantum fluctuations in quintessence
models do not lead to any inconsistencies with observations though in the condensate dark
energy model the curvature perturbation associated with the condensate field is large during
inflation.

We have studied quartic new inflation in the context of warm inflation (weak dissipation
and strong dissipation regimes). We find that as in cold inflation about $10^8$ e-foldings of
inflation occur before inflaton evolution is driven by the slope of the potential. However
in the weak dissipative regime, the fluctuation driven phase is due to quantum fluctuations
while in the strong dissipative regime it is due to thermal fluctuations. Quantum fluctuations
of the inflaton in a thermal background are larger than in vacuum, and the condition that the
inflaton is not driven to the minimum of its potential by fluctuations in the strong dissipative
regime requires that the scale of inflation must be less than $10^{14}$ GeV. In both dissipative
regimes the kinetic energy of the inflaton at the end of the fluctuation driven phase is much
less than the potential energy, thereby allowing for the standard warm inflation scenario
with a slowly rolling inflaton field driven by the potential to commence after the fluctuation
driven phase is over. This confirms the robustness of the warm inflation scenario.

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