Approach to form long-range ion-pair molecules in an ultracold Rb gas

Adam Kirrander,1,2,* Seth Rittenhouse,3,4 Marco Ascoli,5 Edward E. Eyler,4 Phillip L. Gould,4 and H. R. Sadeghpour2
1School of Chemistry, University of Edinburgh, West Mains Road, Edinburgh EH9 3JL, United Kingdom
2ITAMP, Harvard-Smithsonian Center for Astrophysics, Cambridge, Massachusetts 02138, USA
3Department of Physics and Astronomy, Western Washington University, Bellingham, Washington 98225, USA
4Physics Department, University of Connecticut, 2152 Hillside Rd., Storrs Connecticut 06269-3046, USA

*adam.kirrander@ed.ac.uk

We propose a realizable and efficient approach for exciting long-range ion-pair molecules, known as heavy Rydberg states, in an ultracold $^85\text{Rb}$ gas via Feshbach resonances. Heavy Rydberg states, the molecular analogs of electronic Rydberg states, with the electron replaced by an atomic anion, offer opportunities for novel physics, including the formation of ultracold anions and strongly coupled plasmas. We map the positions, lifetimes, and Franck-Condon factors of heavy Rydberg resonances across a wide range of excitation energies, and we calculate the rates of formation for the most promising transitions through long-lived intermediate interferometric resonances.

DOI: 10.1103/PhysRevA.87.031402

We calculate the positions and lifetimes of HRS resonances in Rb$_2$ by solving the nonadiabatic close-coupled equations in Rb$_2$ by solving the nonadiabatic close-coupled equations for nuclear motion on the \textit{ab initio} potential-energy curves for states 1 – 7 $\Sigma^+$ from Park \textit{et al.} [14], shown in Fig. 1. The ion-pair potential included in Fig. 1 is given in a.u. by

\[
V_{\text{ion}}(R) = D_{\text{Rb}\text{-Rb}} - \frac{1}{R} - \frac{a_{\text{Rb}^+} + a_{\text{Rb}^-}}{2R^2},
\]

where the polarizabilities are $a_{\text{Rb}^+} = 9.11$ a.u. [15] and $a_{\text{Rb}^-} = 526.0$ a.u. [16], with the experimental ion-pair dissociation energy $D_{\text{Rb}^+\text{-Rb}^-} = 29771.59$ cm$^{-1}$ [17,18]. Only the $1\Sigma^+_g$ and $1\Sigma^+_u$ symmetries support ion-pair states, corresponding to $0^+_g$ and $0^+_u$ states when accounting for spin-orbit coupling. In these calculations, we account for the coupling of the ion-pair potential and the covalent Rydberg potential curves near $R_c = 38$ a.u. and 52 a.u. by constructing the respective matrix elements. Our calculations focus on the five highest-energy \textit{ab initio} potential-energy curves (PECs) shown in Fig. 1 and the ion-pair potential given by Eq. (1). The two lowest $1\Sigma^+_g$ PECs are separated from the rest by a large energy and therefore have insignificant influence on the dynamics. Chibisov \textit{et al.} report on asymptotic exchange interactions and couplings [19], but since radial nonadiabatic coupling matrix elements have been unavailable until very recently [20], we fit the nonadiabatic couplings at each avoided crossing using
FIG. 1. (Color online) Ab initio potential-energy curves (PECs) for $\text{Rb}_2$ from Park et al. [14]. The first seven adiabatic PECs and the ion-pair potential [see Eq. (1)] are shown for the symmetry $\Sigma_g^+$, and the lowest PEC for $\Sigma_u^+$, and $\Sigma_u^+$ PECs, is shown, as well as the one-photon excitation to states below the $5p + 5p$ threshold near the avoided crossings with the ion-pair potential.

a two-state approximation [21]. Our couplings are in general accord with those reported in Ref. [20]. The resulting set of diabatic states and couplings reproduce the adiabatic $\text{ab initio}$ potentials accurately. In effect, this approach closely resembles the Landau-Zener approximation [22]. In total, seven sets of two-state avoided crossings are included. The results of these calculations remain reasonably stable with respect to variations in the strength of the couplings by up to $\pm 30\%$, in particular in terms of the positions of resonances, as can be seen in Fig. 2. Note, however, that the calculated lifetimes are more sensitive to changes in the nonadiabatic couplings, indicative of the interferometric nature of the resonances (see Ref. [23] and below).

We solve the close-coupled equations for nuclear motion in the standard diabatic representation,

$$\Psi''(R) = W(R)\Psi(R), \quad (2)$$

with $\Psi$ being an $N \times N$ matrix, each column a linearly independent solution, and $\Psi''$ indicating the second derivative with respect to $R$. The matrix $W$ consists of

$$W(R) = \frac{2M}{\hbar^2} V(R) - k^2, \quad (3)$$

where the $N \times N$ matrix $V$ contains the diagonal potentials and the off-diagonal coupling elements. The diagonal matrix $k$ contains the asymptotic channel wave vectors $k_i = (2M/\hbar^2)\epsilon_i$ where $M$ is the reduced mass of $^{85}\text{Rb}_2$ and $\epsilon_i$ is a diagonal matrix such that $\epsilon_i = E - E_i$, where $E$ is the total energy and $E_i$ is the threshold energy in each channel $i$.

Equation (2) is solved using the log-derivative method [24], which propagates the log-derivative matrix $Y(R) = \Psi'(R)\Psi^{-1}(R)$ instead of propagating the wave function $\Psi$ directly. The matrix is propagated out to the matching radius $R_f$, where it is used to calculate the wave function in the form $\Psi = F - Gk$, where $F$ and $G$ are diagonal $N \times N$ matrices containing energy-normalized Milne functions [25]. These coincide with analytic Coulomb and Riccati-Bessel functions when the polarization term in Eq. (1) vanishes. The density of states is calculated from the energy derivative of the cumulative eigenphase of the scattering matrix. The lifetimes $\tau$ are obtained from the width at half maximum, $\Gamma$, for each resonance.

Figure 3 shows the lifetimes of the resonances in the energy range 23 815 to 25 601 cm$^{-1}$, beginning below the $5s + 5p$ limit and continuing up above the $5p + 5p$ limit. They are shown in terms of their lifetimes, since this is a crucial aspect when choosing a suitable target state for excitation. Each calculated resonance can be assigned a principal quantum number $n_{pp}$ and a quantum defect $\mu$ by reference to the Rydberg formula (see, e.g., Ref. [26]). The overall progression of states in the calculated range corresponds to that typical of low-lying heavy Rydberg states, with a strong energy dependence of the quantum defects and many interloper states [26,27]. The slow transition to “pure” HRS behavior in terms of quantum defects and density of states, compared with electronic Rydberg states, can be attributed to the large reduced mass and the extensive range of internuclear distances where the potentials deviate significantly from the pure Coulomb potential. The resonances display the same interferometric modulations of the lifetimes that were previously observed in LiF [23]. The large reduced mass and the large internuclear separation make the heavy Rydberg states robust with regard to changes in the total angular momentum $J$, with only very small shifts in resonance positions due to changes in the rotational angular momentum. We therefore anticipate little thermal broadening and rotational dependence of the formation rates.

Our excitation scheme relies on a vertical transition from a Feshbach resonance in the lowest $1\Sigma_g^+$ and $3\Sigma_u^+$ electronic states directly into heavy Rydberg resonances in the excited
1$\Sigma^+_g$ manifold, which we take as representative of the 0$^2$g$^-$ and 0$^2$g$^+$ symmetries that support the heavy Rydberg states. The wave function for the Feshbach resonance is well approximated by the wave function for the last bound state to the ionic HRS, with the relevant covalent transition moment proportional to the atomic $d_{5s-6p}$ transition and a single-photon transition from the initial Feshbach resonance to the final state is possible. The single-photon Rabi frequency $\Omega_1$ for this process is

$$\Omega_1 = |E_d| f_{hrs,i} \sin \xi,$$

assuming that the covalent-covalent dipole transition moment is much larger than the covalent-ionic moment, $D_{cov,i} \gg D_{ion,i}$, and that the Franck-Condon overlap $f_{hrs,i}$ is dominated by the wave function close to the classical turning point where its amplitude is the largest.

For resonances just below the 5s + 6p dissociation limit, near the avoided crossing around $R_c = 38$ a.u., the relevant covalent transition moment is proportional to the atomic $d_{5s-6p}$ transition and a single-photon transition from the initial Feshbach resonance to the final state is possible. The single-photon Rabi frequency $\Omega_1$ for this process is

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Rabi frequency $\Omega_2$ is

$$\Omega_2 = \frac{|Ed_{5\rightarrow5p}|^2}{\Delta} |f_{\text{res,1}} \sin \xi|,$$

(8)

where $\Delta$ is the detuning from the intermediate state. If we assume that the both photons come from the same laser and that the detuning is half the final-state energy with respect to the $5p + 5p$ threshold, giving a detuning of approximately $\Delta \approx 40 \text{ cm}^{-1}$, we have $d_{5\rightarrow5p} = \frac{3\mu_0}{4\pi} A_{5\rightarrow5p}/2 = 2.99 \text{ a.u.}$, using $A_{5\rightarrow5p} = 3.81 \times 10^7 \text{ s}^{-1}$. The mixing angles $\xi$ and the Franck-Condon overlap $f_{\text{res,1}}$ are taken from Table I. Again, using modest intensities, $10^3 \text{ W/cm}^2$, excellent Rabi frequencies $\Omega_2 \sim 2\pi \times 10^3 \text{ kHz}$ are obtained. Since the intermediate state is far detuned, the laser intensity can be further increased without significant ionization, up to $10^7\text{–}10^9 \text{ W/cm}^2$.

Using Fermi's golden rule, the rate $\mathcal{R}$ is obtained as

$$\mathcal{R} = 2\pi \Omega^2 \tau,$$

(9)

where $\tau$ is the lifetime and the resonances are treated as having Lorentzian shapes. If we illuminate a sample of trapped $^{85}\text{Rb}$ molecules with $10 \text{ ns}$ pulses of $100 \text{ mW}$ peak power focused to a diameter of $100 \mu\text{m}$, the HRS excitation probability per shot will be on the order of $10^{-3}\text{–}10^{-7}$ per molecule, using the rates from Table I. We note here an important advantage of starting with bound molecules instead of trying to produce HRS by photoassociation from the continuum: the existence of a Franck-Condon window in $R$ which can be varied by choice of the initial molecular state.

In conclusion, we propose a scheme for exciting heavy Rydberg states in an ultracold $^{85}\text{Rb}$ gas. It relies on direct excitation from Feshbach resonances or weakly bound molecules to heavy Rydberg states (HRS) just below the avoided crossings between covalent potential-energy curves and the long-range ion-pair potential. Purely ionic HRS can form by, for instance, stepwise excitation using a chirped laser pulse and can be detected by field ionization. These states have long lifetimes, and the nonadiabatic mixing at the avoided crossings provides favorable dipole transition moments. In the future, we anticipate being able to improve the predictions by including $ab\text{ initio}$ couplings and to account for spin-orbit interactions.

This work was supported by the National Science Foundation through a grant for the Institute for Theoretical Atomic, Molecular, and Optical Physics at Harvard University and Smithsonian Astrophysical Observatory and by a National Science Foundation grant to the University of Connecticut.

We are grateful to Z. Pavlović for the calculations of the wave functions for the Feshbach resonances. The authors acknowledge helpful discussions with S. Yelin and E. Kuznetsova, as well as G. Pichler (H.R.S.), K. Lawley and T. Ridley (A.K.), and B. L. Johnson (S.R.).